Image Signal Processing

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Lecture 21

Space Invariance

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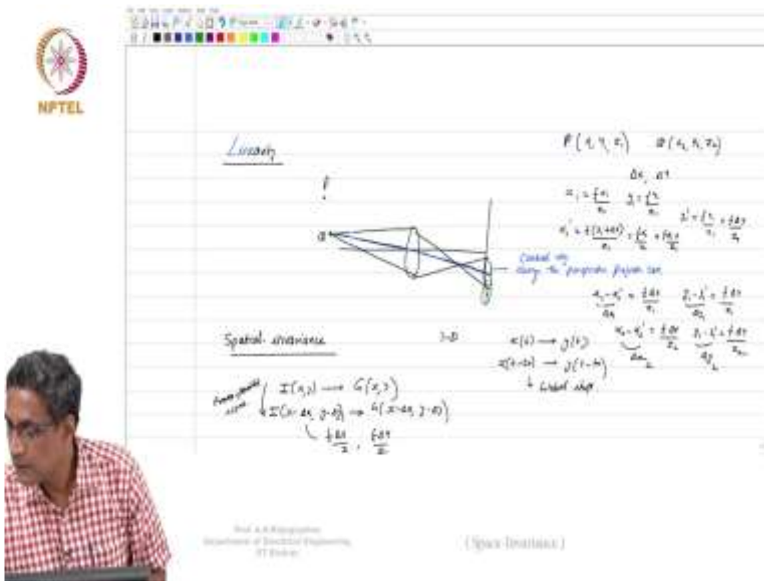

So we are saying that you know this one is so regarding linearity and space in variance, so the linearity part I hope is fairly clear. So the linearity, linearity right we saw okay last time itself that I mean you have this lens and let you have the image plane here and we said that we could have not just one point light source we could have multiple point light source and all of these now ofcourse now will typically map into some kind of a blur circle.

And wherever let us say they all probably converge, let for this point they converge here create a blur circle then the other one will have its own blur circle and then we showed that there are multiple number of them they will all add up together, so on the image plane let just that right all these blur circles will come all together just say superposition holds and then you know what intensity you will get there.

The second notion is more tricky, the second notion is about space invariance, special invariance or space invariance right as it is called, this one requires some moment of thought. Now by spacial invariance if it is of 1D case okay we say that if X of t is given to a system and if it produces Y of t, whether it is linear or not we do not care but if I shift t by t naught then for any arbitrary t naught Y should also shift by t naught, right, t naught can be positive, negative whatever it does not matter.

And this could be a system that is linear or not we do not care this shift invariance does not have anything to do with linearity they are all independent notions. So similarly what we want to ask is if I had an image, I mean that know that if I were shift it by some amount delta x delta y and here you should note that these are all global shift, this is a global shift every mn so that t naught is applied everywhere correct we do not keep changing t naught.

So the entire signal is shifted to the right or to the left exactly by t naught, so similarly we want to look at an image, assume that right I of xy goes in if I take a continuous case assume that I of xy goes in and it produces G of xy and our interpretation I think last time itself I told you that I of xy the interpretation is that this is up in hole focus image that you would have seen had this lens not been there.

And in the place where the aperture if you replace it with a lens then whatever is the image you see that becomes you G, that is the output of this imaging system. Then if you do I of let say whatever x minus x naught or delta x let say and then y minus delta y all on the image plane again delta is delta by delta y is on the image plane so it is like if we can globally shift the image to right, left, top, bottom, then G right we want to check whether G also shifts by x minus delta x y minus delta y.

If this does not happens we know that there we can declare that this is not shift invariant or this is not spacially invariance and we would have to like a system that is actually spacially invariant. Yeah?

Student: (())(03:42)

Professor: The centre of the say that again, does it means that

Student: (())(03:51)

Professor: Correct, exactly. So it basically means that if these lens itself were to bring that scene into focus then I and G will be exactly the same, G will be equal to y you know at very point to point they will match and that intensity because there is no longer blur circle there is only a point, so whatever you say with respect to pin hole the same intensity you will now see at the central ray, there is no circle around it.

Okay now the point is right so delta x delta y so what is it mean to translate an image now, correct we have to understand now because you know in 1D it is very easy we just write x of t minus t naught and ask for y of whether the output is y of t minus t naught, now as far as imaging system is concerned we have to first of all ask what does it means to shift this input image now.

The I of xy what does it mean to shift it globally, and then we can see whether the output is shifted. So here is where sometimes people tend to interpret things in a sort of a different way and end up concluding that the lens is shift invariant okay, actually it is not. If you can provide to the lens and image that is globally shifted the lens will give out an output that is globally shifted, the fact that you cannot produce this that input cannot be used to blame the lens okay, I will tell you what I actually mean by that.

Now imagine that let say now if you go back, now what are the key things right that we want again borrow? I told you we do a pin hole because there are many things that we borrow from there, in order to understand what is happening here, so one of the things we saw was the central ray in a lens okay the central ray as this boy just telling just now.

So the central ray right we know that this ray obeys this central ray okay last time I think I wrote the central ray obeys the perspectives of projection model the perspective of projection equation model whatever ray you want to call it, which means the ray out here you can apply your similarity of triangles and then you can show that the shift right if you compute I mean all that we did with respective to the focal length all that.

Now the image co-ordinate how it would be related to the word co-ordinate all of that does obeys by this central ray and what is also important to realize is that image is getting formed I mean central by the by this centre through many blur circles getting formed around the central ray. So the central ray is very-very is a kind of unique ray in that sense it has all these relations that it borrows from actually a pin hole model.

And then the blur circle is in fact formed around it, so the point is when you when you say that we want to shift I of xy, so I of xy is really a pin hole equivalent now and we are trying to say that I want to get I of x minus delta x y minus delta y and imagine that you have a point let say P in the scene, let me go back to black, so if have a point let say there is some point P here and let say the other point is some Q.

Now if P has let say co-ordinate x1 y1 z1 and Q has co-ordinates x2 y2 z2, now shifting the mean you can either talk about shifting this scene or you can talk about shifting the camera okay both are the same, it is simply a relative motion. Now if you think about just to make matters easy imagine that this camera I actually move by an amount delta x delta y okay.

I am kind of moving the camera by I mean so in the world co-ordinate I am kind of moving the camera by a amount delta x and delta y or equivalently think of the scene moving by an amount delta x delta y. Now what will happen is this co-ordinate, we know that the image co-ordinate for P will be some small x1. Before you move it will be x1 is equal to fx1 by z1, y1 will be fy1 by z1 and after you move by an amount delta x and delta y.

So you will have a new one which is x1 dash which will be f of x1 plus delta x by okay now the point is right if okay so since here z1 has not changed that is still z1 so you get fx1 by z1 plus f delta x by what is that delta x by z1, similarly you can show that y1 prime will be fy1 by z1 plus f delta y by z1.

Now if you look at the shift so it is if you look at x1 minus x1 prime the amount of shift that is given by f delta x by z1 and y1 minus y1 prime is also f delta y by z1, for Q you can say similarly show that if for Q using 2, so you can say x2 minus x2 prime will be f the point is right this guy will go like fx2 by z2, fy2 by z2 then x2 plus delta x y2 plus delta y if you just carry out this operation you will end up with f delta x by z2 and y1 minus y1 prime will be equal to f delta y by z2.

So then what it actually means is that we cannot take arbitrary 3D scene okay and start and sort of assume that now if I capture the next image which will be a pin hole image let us say that is my next I and the one that is sitting here. It is not true that all of them have moved by the same amount of delta what is this small delta x, so this is your small delta x this is your small delta y but then here it is like delta x2 deltay2, here it is delta x1 delta y1 because they both are not at the same depth with respect to the camera.

So which again brings us back to the fronto-parallel kind of a plane notion, so only if you have a front- parallel plane in front of a camera fronto-parallel in the sense at this plane is parallel to the sensor plane then whether you move the camera or whether you move the plane by delta x delta y that is the only time when all the points move by an amount which is exactly the same.

That means all of them will move by an amount which is given by f delta x by z because then all of them are at the same depth z from the lens. In no other situation will you be able to create an image that you can write as I of x plus delta or whatever x minus delta x and y minus delta y because there is no constant delta x delta y that is applicable all over the image.

So first of all even to get think about when this will happen we have to be careful, it does not mean that we can do it any time right no whenever we wish, no. It can only be done under certain conditions only this will happen okay but that is not but then the point is that what we have to understand is to say that x of t minus t naught exists similarly we have to think about when will I of x minus x naught exist and when it exist is my output G of x minus x naught.

So we have to give globally shifted input now all that we are talking about here is when can we even do that? So we can d that in a kind of fronto parallel situation so that is the case when we can actually talk about at least even going from here to here forget about what the lens is doing, I am not even brought in the lens yet, just saying if you wanted to create a pinhole shifted equivalent only at a certain situation I will get x minus x naught y minus y naught not always, correct?

So even to do this I assume a fronto parallel-scene, on a fronto-parallel scene this you can do you can move by capital delta x capital delta y and I leave it to you to think what will happen if I move the plane along z, I said this way right delta x delta y, what if I moved it along like this along z right you think about it what will happen?

So this is the only situation when you have a fronto-parallel scene and the camera moves like that in it I own plane by an amount capital X capital delta or the plane moves like capital delta X delta Y. Then all the points move by a amount with a amount small delta x small delta y given by f delta x by z and so really right so for that situation this will turn out to be everyone we will move f delta x by z and f delta y by z, delta x delta y is the actual motion in the gate world co-ordinate.

And on the image plane it will translate to f delta x by z and f delta y by z irrespective of which wherever you are on the plane all of them will get shifted by that amount. Now we want to examine what will happen to G now, so if you actually think about it so originally okay what we had was let me showed it with some green color so if you had this Q point okay it went here so we had the central ray that was actually hitting here. And then we had a kind of blur circle there.

Now when this when you are doing this delta x delta y whatever capital delta X capital delta Y shift imagine that right because of this blur circle shifts somewhere down, if you would have think of the pin hole equivalent right so what will happen because of this point instead of appearing here it would have appeared somewhere else that shift being equal to f delta x by z.

Now what will happen the moment you introduce the lens you will if things are not in focus this is a fronto parallel plane if you wish you can bring it into focus well let say it is not in focus then what will happen you will get actually a blur circle around a central ray again, because I mean right all that does happened is your blur circle has shifted from wherever it was it is gone to another place.

Another place but not arbitrarily it has shifted exactly by the same amount because the central ray has got shifted by that amount f delta x by z f delta y by z because that is the one that will follow a still a pin hole law which is the perspective this one the projection law and then all that happens is the blur and then you also know the radius is no going to change because your z has not change.

So it is identically the same blur circle coming and relocating itself somewhere nearby depending upon how much you move. So in that sense G of x y it has now moved on to G of x minus x naught and then what is not x minus x naught, x minus delta x y minus delta y, this is okay I mean do you have because this is something I want you to understand.

Because you know what latter when you read anything more advanced you should be able to map back to the simple situations and that is the reason why I feel it is important to at least know, this is fairly clear as to what we mean by this invariance notion now, because what happens is right people sometimes write that a lens is shift variant. Because what they are looking at is if I give a general 3D scene then I know I will get a space variant blur and therefore where is this notion of space invariance and so on.

What we have to remember is just as in 1D what do we say x of t minus t naught that means we are able to generate x of t minus t naught when you can generate x of t minus t naught am I getting y of t minus t naught is what we are asking. Similarly here you should look at first of all situation that will give us an I of x minus x naught, y minus y naught without even having that where is the if you cannot even give a shifted a globally shifted input what is the point in even asking whether my output is globally shifted?

So first of all the input should come shifted globally which ofcourse you know because it is a length that possess certain condition and when you can do that. But if you offer a globally shifted input to this lens it will give you a globally shifted output that is all we care as far as space as far as shift invariance is concerned, give me show it a globally shifted input I do not care how you are going to show it, if you can show that it will give you a output that is also shifted, yes?

Student: Does that mean a D scene can never yield global shift?

Professor: Exactly, so in that case it would not be a global shift, that is our point so even getting this I suppose you call the next one the I dash there only a certain conditions when you can actually get a this one global shift. What will happen if you shift it along if you shift the image plane what do you think will happen?

Let say it is a fronto parallel plane actually that holds even if you have a 3D scene by the way that law actually it can be mapped as a homography I will leave it to you, it is actually an homography, those two images you know you will get another image right, think of a do not even look at the lens, look at a pinhole case. I have an I now I kind of move the image plane along z I will get some I dash.

The first question that you might want to ask is can I relate I and I dash through some means through homography? The answer is yes and it is irrespective to whether the scene is 3D or not, you do not need a fronto parallel plane by the way. But I am saying but that is not the I dash we want that will not give you delta x delta y, is this okay?

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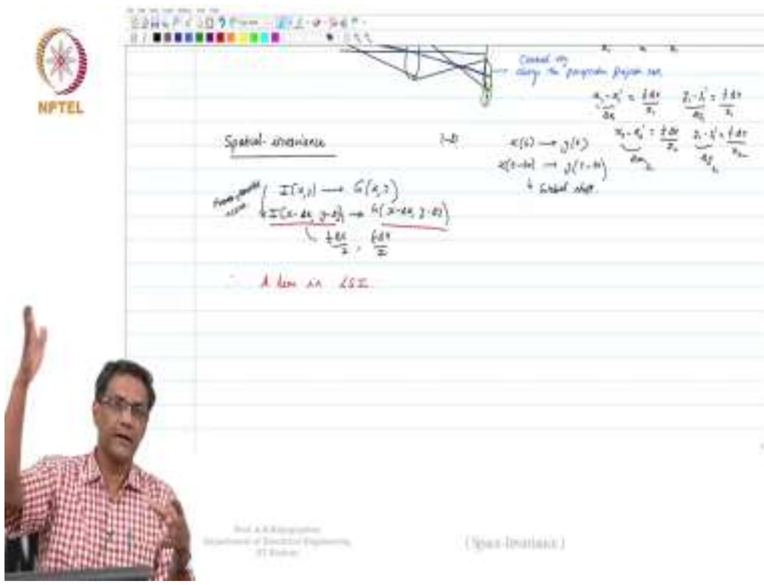

Then so therefore right so therefore overall if you all say agree upon this that you know if you give a shifted input then it will of course give a shifted output the output exactly being shifted by the same delta x delta y know alpha times delta x and alpha times delta y naught on a image plane moves exactly by the same amount which that means that and therefore a lens is actually LSI that means its linear as well as shift invariant.

Now what you have to kind of say realize that if you had a 3D scene then ofcourse it will introduce the blur that will spacially varying that does not render lens as shift invariant simply means that this scene is producing different amounts of blur let say in different part of the scene but if you showed fronto parallel scene then what you would get is a spacially invariant blur because all the blur circle will be at the same radius and therefore you will get just one uniform blur.

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So let me just show you an example of that here, okay so look at this, this image is an example of this I just picked up from the web okay, picked it up so I able to see it do I zoom it, so this is the case of space invariant blur, you know that you can see it is not very sharp but then the sharpness does not or the degree of blur does not change as you walk through the image, wherever you go it is a same blur.

So this is like a plane wall this is like a wall and therefore you can imagine that either it will be totally focused or it will be totally blur but the blur is going to be uniform in the sense that its invariant it is a same wherever you go.

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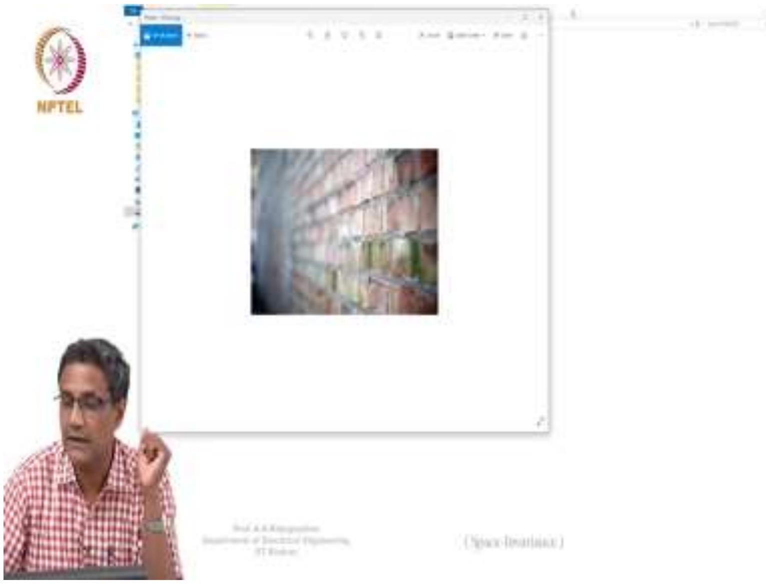

Whereas if you had scene like this it is a okay if you had something like that okay wall which is actually inclined there is also a wall by the way it is like a library you know and you can see it is like a library rack and you see that the blur is varying and this all optical look at this no motion here, it is all optical kind of a defocusing.

And you can see that the blur is changing as you right as the camera focused now as the camera is focused only with respect to objects that are near and then those objects that are far away will get blurred. So this is an example of how a space variant blur looks like and you should be able to map it back to your back to the way that you could actually visualize how this image might have got and formed.

So you have a 3D scene and how would different blur circles and all must have come together to give you this kind of an image so that is the image formation part and we will again get back to that latter okay, why this formation why this can help us understand many other things also, it is not only to understand image formation in pin hole and image formation in lens, it is also something that we can exploit later for other things. In fact you know what? When you see this it get tells you something, what does it tells you?

Student: Right side of the (())(20:34) is closer to the viewer

Professor: Closer to the viewer or closer to the camera let us say and what is it? Someone I thought something I heard from here

Student: (())(20:47)

Professor: Exactly right she is correct, what it is saying is that the scene that you are actually looking at has a 3D variation because the blur is changing, otherwise you would not have fond a change in blur, either it would have been all focused or it would have been all blurred the same way you would not have got a variation in the degree of blur.

Looks like in the front is less and then somewhere else its more which means that the lens is actually looking at a 3D scene, so it is like acting as a Q for depth so if you are smart you should be able to exploit that to may be tell that this scene is actually 3D and then even go further and sort of may be make inferences about which object is where, right how far away and so on.

We will come back to that later what is called depth from focus and so on. But at this point of time okay it is enough to just you know what walk through these examples.

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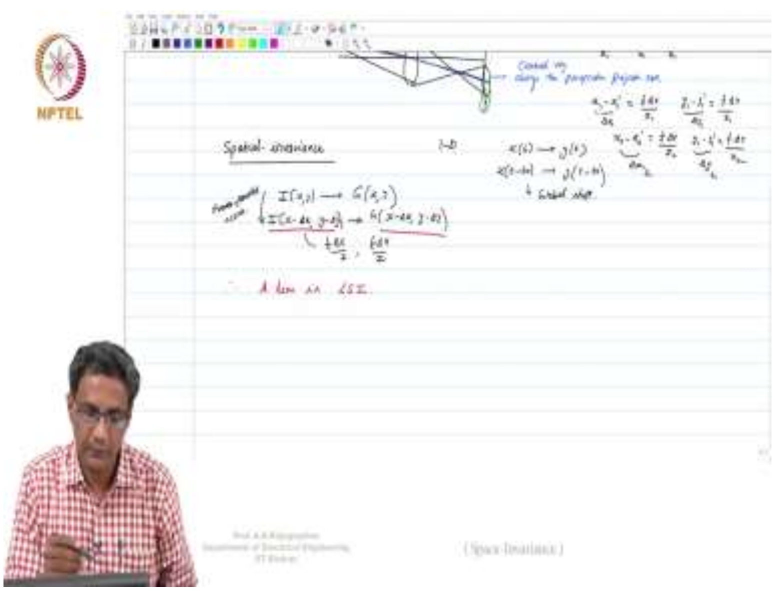

Alright now (what happened) okay, so lens is LSI