Image Signal Processing
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Lecture No. 23
2D Convolution - Part 2

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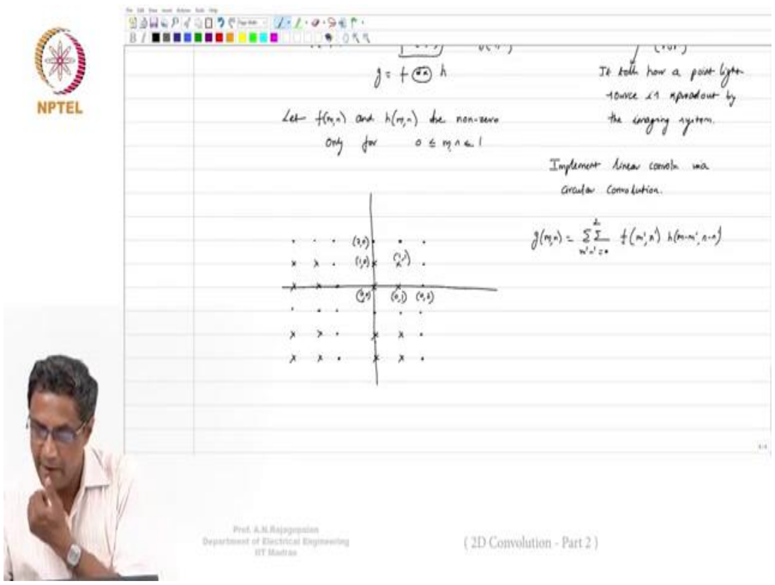

So, let us look at the case of a 2D system, it is we said that we have a linear shift invariant system, by the way did I say that this HMN is called actually a point spread function? I have to say that so the so the impulse response by the HMN this has, this called point spread function. So, what that actually means is that, it actually indicates how a point light source will get spread out, if it is if it is not in focus, it is called a point spread function.

And there are and there are various models for it, for at least for the optical blur case there are models we will look at a few of them. So, this called a PSF in short, so you have a point spread function of some 2D system, lens based and let us say that you input (what do am I using?) FMG whatever, so let us say this is FMN, this is HMN and outcomes GMN, GSF convolution.

I will indicate this by 2 stars just to indicate that it is actually 2D convolution FH. So, so point spread function because it tells how a point light source spread by the lens spread out by the imaging system. In this case ofcourse a lens based imaging system. So, it is called point spread function. Now let us kind of go to this figure now.

Again will take a very simple example just to just to just to make things easy to write here, so what we will do is we will assume that HS, so these both exists from, so let us say FMN and HMN are non-zero, let HMN be non-zero only for let us say M comma N less than or equal to 1, rest of the places on the grid they are 0. So, what that means is that on the grid we can draw, so on the grid on the grid so, on the grid when we draw, so so let us say that we have same thing will hold for both.

I will just draw one grid, so suppose I indicate by a cross that means the value exists there, so let us say this is 0 (cross), 0 comma 0 and let us say this is let us use rho comma column. So this will be 0 comma 1 you can use any any notation. I am just using some notation here. This will be like 1 comma 0 then and this will be 1 comma 1. So, we are saying that HMN and FMN they both exists like this and everywhere else they are 0.

Just to illustrate what we want to tell. You can then later on you can write your own code you can take whatever size you want, it will just blow up even to write this it will require big matrix and so on. And and then we will again go go and implement linear convolution by a circular convolution implement linear, for the same reason that I said yesterday that linear convolution were implemented by a circular convolution gives you the circular kind of structure for the for the system matrix which has certain interesting properties.

First of all it is normal therefore, it is unitarily unitarily diagonalizable and so on. At least there is a structure that you can see, implement linear convolution via circular convolution. So, then which means that which means that we need a 0 paired and cyclically repeat now but this repetition should be on the grid. Earlier we did it on, 1D yesterday that was a simple example we will just repeat now this is also simple but we have to be a little more careful now.

Again the length will be so what we have so it will be like 0 to 2 now, so, of the output. So, we will go like so I will put a 0 here, I will put a 0 so dot means it is a 0 and then 0, 0, 0 so here is 0 comma 2 and this will be like 2 comma 0 and this will be like 2 comma 1 and so on. And then and then this should now cyclically repeat on the grid, so it will be like saying that I will have this, this, this and then this, this, this.

Again I have I have to repeat it cyclically either this way or that way either way does not matter. Let me repeat it like this. Then either I can go this way or I can repeat this way should not matter. Only thing is the spacing is not very uniform that is alright. And then, it sort of it will just go on and on. So, this is like cyclically repeating the pattern this is valid for both F and H.

And we are now looking at a convolution of the form G of M comma N. So, we want to write similar to the case that we did for 1D where we saw some structure in our system matrix that relates input and the output. Similarly, we want to see there is a nice structure here, so GMN will be double sum, let us say M prime N prime going from 0 to 2. And then let me take it as F here because F we can write on the say right hand side.

F of M prime, N prime then H of M minus M prime N minus N prime and let us write this down, so what is it?

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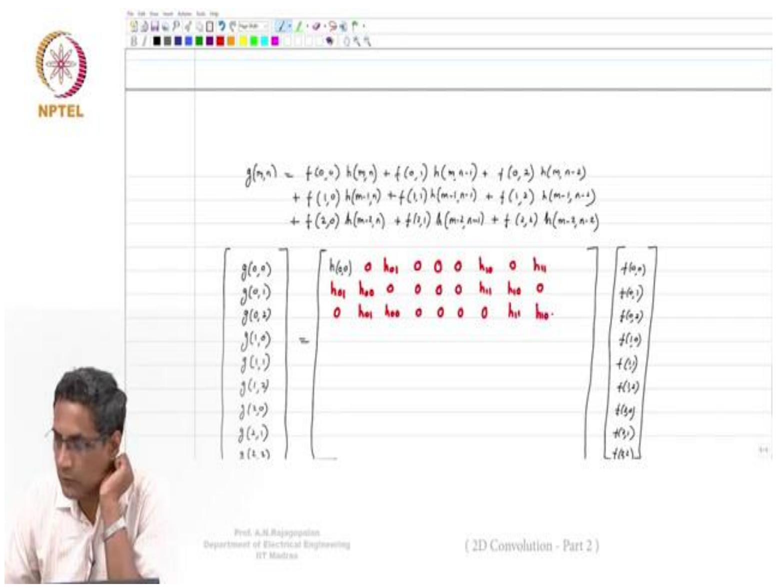

We are like GF M, N is equal to so what will be like F of 0 comma 0? H of M comma N plus 0 comma 1 H of M comma N minus 1 H of M comma N minus 2 plus, so 0 comma 2 is 1comma 0 H of M minus 1 comma N plus f of 1 comma 1 H of M minus 1comma N minus 1 plus f of 1 comma 2 H of M minus 1 comma N minus 2 then lastly f of 2 comma 0 h of M minus 2 n plus 2 comma 1 m minus 2 comma N minus 1 plus f of 2 comma 2 h of M minus 2 N minus 2 comma so there will be.

Now now, the idea is to be able to write this in this form now, so this called lexicographical ordering, what that means is even if you have you have G which is now actually matrix, so you take the first row stack it up as a column, take the next row stack it below column following that and then you going to create a create a create a 1D sort of vector out of 2D 2D 2D array. So, that is called lexicographical ordering.

So in this case what it will mean is we will write this G, will lexicographically order it such that we have G00, G01, G02, G10, G11, G12, G20, G21, G22. You will, you will understand the importance of all of this later I mean why this why we write in this form and why it is convenient to write it a lexicographical order lexicographical model because if you write it this way then then then the way that you can go from 1D to a sort of 2D transform at all becomes very easy, it makes it very convenient to actually write equations by simply borrowing from your from your 1D 1D ideas.

Now, I am going to write this as some big matrix, not so big in 9 entries we have to write here, so let us say you go up to this and then here also will lexicographically order in the same way ofcourse whatever you can also take the first column, put that as a first column, take the second column put it below but whatever you do on the left do the same thing on the right.

So, here f of so one is like row lexicographical ordering another is column lexicographical ordering. 0 1, f02, f1 and this all means that it is 0 padded 0 comma 2 is 2. F of, what is this? 1 comma 1 f of 1 comma 2 then f of 2 comma 0 2 comma 1 f of 2 comma 2. We want to able to fill all these 9 rows now. Will fill 3 rows and then after that we will just assume that you can you can actually fill it up because we may not, we do not want to spend too much time. But just to give everybody here an idea about how this will work now.

So now if you take G00 that means M is 0, N is 0, so f00 multiplies h00, f0 so this will be h0 comma minus 1, so 0 comma 0 minus is this so it is 0, so let us go here and let us I will enter it in this itself. So, look what is the next one 0 comma minus 2, now can somebody tell what is 0 comma minus 2? So, 0 comma minus 2 is this one, is this one which is the same as 0 comma 1.

So let us go here, fill this as h 01, I am going to write it like that. Then here f1 0, so what is multiplying f and 0 h minus 1, what is this? H minus 1 0, and what is the what is the next one? So minus 1, minus 1, so if you kind of quickly tell me what is minus 1, minus that is also 0 and then what about the next one is minus 1 comma minus 2?

So minus 1 minus 1 is here, minus 2 is here, minus 2 will be here. So, that is also 0. Then let us come to f20. So, here it is h minus 2 comma 0 so h minus 2 let me go down here h minus 2 comma 0 is this one, this is same as this is this is the same as 1 comma 0, 1 comma 0. Then 2, so we are here, so this is 2 comma so minus 2 comma minus 1, so minus 2 comma minus 1 0, minus 2 minus 1 will be here, will land here so that is a 0 and then finally, not finally, finally minus 2 comma minus 2 so minus 2 comma minus 2 look here and then you go there and that will be 1 comma1. So 1 comma 1 h of 1 comma 1.

And now I wanted it will be G01, what should I write here? First entry, somebody tell me or you want to see that grid again? 0 comma? 0 comma 1 then?

Student: (())(13:32)

Professor: h 0 comma 0 then? 0, then? What do I write here? No, is it what h01? Are you sure? What comes here? This is so you are looking at 0 comma 1 and you are looking at 1 comma 0 so it is 0 comma 1 so it will be minus 1 comma 1. So you look at what is h minus 1 comma 1. So, h minus 1 is this and 1 minus is 0, minus 1 comma 1 is 0. So, be careful. Next, what next? 1 comma 1 so you have to write for 1 comma 1 so we have finished 4 entries 1, 2, 3, 4 gone.

1 cooma 1 so 1 comma 1 is sitting here so that is you are looking at 0 so minus 1 comma 0. So, what is minus 1 comma 0? So minus 1 comma 0 is this guy, so 0. Then, next? May be you just make a guess that may be that also ought to be 0 do make guess. 2 comma so this is what? Minus 1 comma minus 1, so what is that? 0. Then let us go to 2 comma 0 so this will be minus 2 comma1. So minus 2 comma 1 is this 1 that is 1 comma 1 what is it, am I correct? Minus 2 comma 0 , 1 comma 0 no-no minus 2 comma 1 it is not minus 2, minus 2 comma 1.

So minus 2 comma 1 will mean we are here that is 1 comma 1. You guys are following, right? 1 comma 1 then 2 comma 1, so which is here so where are we? So minus 2 comma 0 so minus 2 comma 0. So minus 2 comma 0 is this guy which is 1 cooma 0 10 then 2 comma 2 so that will be where are we here 0 comma 1 so it will be minus 2 comma minus 1. So, minus instead of minus 1 is 0. Third one, G0 to 0 then next 0 comma 2, so look at 0 comma 2 so 0 comma 2 is 0 comma 1 now. It is 0 1 why you have to think so much, this one is a simplest.

0 comma 1 and then 0 comma 0 now 0 comma 0 third one then come to the next one. This is what when I look at 0 comma 2 so it is minus 1 comma 2 so minus 1 comma 2 will be minus 1 is here and then 2 is 0, correct, so 0 then minus 1 comma minus no minus 1 comma 1 (what happened?) 0 comma 2 where are we have we doing for 1 comma 1 now, we are doing for 1 comma 1 we have finished these 4 we are doing for 1 comma 1.

So, here and 0 comma 2 so it is minus 1 comma 1. So minus 1 comma 1 is 0, so minus so mins minus 1 is this one, 1 is this is one so that is 0, then let us go to the next one. 1 comma 2 it is like minus 1 comma 0, 0. Then 2 0 so we are looking at minus 2 comma 2 where are we? So, last one you have 2 comma 2 so this should be minus 2 comma 0 so minus 2 comma 0 go the correct way, so minus 2 is like two rows down and minus 2 comma 0 is this guy this is 1 0. And now I am going I am going to fill up the rest.

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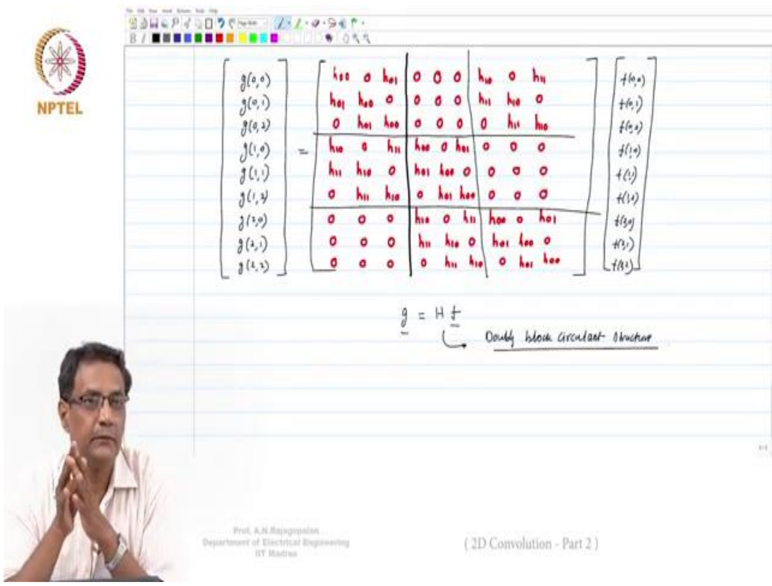

And I leave it to you to leave it to you to know just is it verify, so now like what we have is G lexicographically ordered F lexicographically ordered and now this is again in the form in the form G is equal to hx. So, it is of the form G is equal to hx and hf. Now what kind of structure do you see here? So, if I see here h11 is repeating, no it is not repeating but something is going on, correct-correct, very good.

So, what you have to do is you have to look at it like this, now if you do a partitioning and if you do this partitioning that is why I said that if I directly wrote it you will figure it out. Now see if you were to partition it like that what do you observe now? You observe that each of these sub-matrices is by itself a circulant, see h00, 0 h01 then h01 h00 then 0 h01 h00, so it is 000 h100 h11 h11 so each one of these is actually circulant.

But then it does not stop there each of these matrices themselves repeat circulantly. See this guy appears here, this guy appears to the right, this guy appears to its right then again this one comes back here, this one comes back there and this one comes back there. So, when you have 2D system and then when you write down this kind of matrix vector relationship you realize that you get a matrix that is actually now if you look at it is it circulant node? It is not.

Correct I mean directly if you see this row is repeating, it is not. Whether then it has some other structure and such a structure is called, do you know what it is called? It is called doubly block circulant, So, as far as h is concerned it is what is called a doubly block circulant because it is it is circulant block wise and then doubly because each block itself is circulant, So, that is why it has its name doubly block circulant.

The block circulant we mean that all the all these all these matrices just just come come in a circulant way but each one of them being circulant within itself makes a kind of doubly block circulant. And all these has implications I mean nice things about these things is either way we get used to just look at the summation and then walk away, say that anywhere way there will also give us the same result but the other way to look at it is to actually look at look at look and see this structure that that actually lies there and which and and you know as it as it should happen then it turns out turns out that when you have something like this to diagonalize this guy just as you needed 1D DFT DFT matrix to that we have not talked about yet but somebody here was asking yesterday.

So, similar to that just as DFT matrix has his ability 1D DFT matrix has ability to diagonalize the circular matrix it turns out that a 2D DFT can will actually diagonalize a doubly block circular matrix, so just natural and so naturally happens that when you go from 1D to 2D it automatically whatever was vehicle there that was originally 1D DFT becomes now now you have to kind of go to a 2D DFT and that will help you actually diagonalize a matrix of this kind.

And those are things that we see later but at this point of period we just want to know that there is a nice way to relate input and output just as in 1D LTI systems you do there you see some structure and all. Similarly, you can extend that notion to the case when you have a 2D system and within a 2D system also structure exists and then such things are possible.