
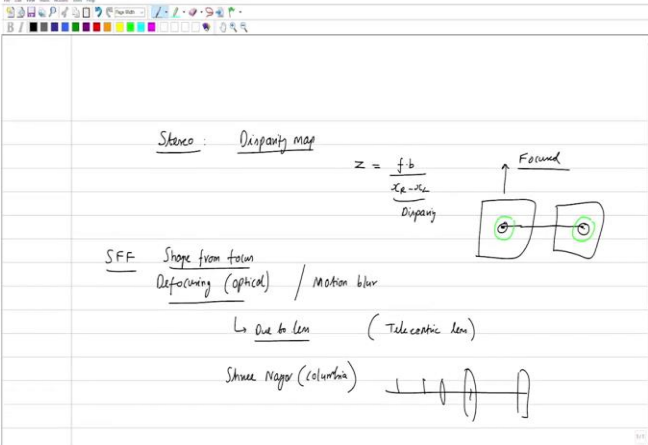


Image Signal Processing
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Lecture 29
SFF Principle

(Refer Slide Time: 00:16)

Stereo: Disparity map

$$z = \frac{fb}{x_e - x_c}$$

Disparity

Focussed

SFF: Slope from focus

Defocusing (optical) / Motion blur

↳ **Due to lens (Telecentric lens)**

Shree Nagar (columnar)

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(SFF Principle)

Then see then the advantage is that you automatically see is the following. So, it looks like looks like if I had a stack and if no pixel moves. Let me go to the good another slide and move down.

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SFF: Shape from focus / Defocusing (optical) / Motion blur

↳ Due to lens (Telecentric lens)

Shree Nagar (columnar)

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So, what this means is right if you if you capture. Ok. A number of frames like that right which is what is called as stack. Assume that you capture 30-40 frames like that. So, it is like saying that I have a z state, I cannot keep on right moving it up and I don't know I capture a bunch of frames.

Now, the nice thing you saw is that because of a telecentric set up if I look at a point here, the same point exists exactly the same place in my next frame. The same point exists exactly the same place in the last frame. Nowhere has it moved the degree of blurring around it of course changing right. Because of the fact that I am kind of moving the stage. So therefore, at some in some frames, this point is coming in focus may be here and then afterwards it is getting blurred may be or may be some other point is coming in focus. In the first frame but then it becomes blurred later ok we do not know.

Something like that is going on but what this also shows us is that that here is a cue for depth because of some means right. If now again get us go back to this kind of fictitious focus operator which we think we have with us. If you had that operator and you know and let us say if we could apply it here around this pixel or on that pixel typically it is around ok in and around that pixel. All let us say I just apply it on the pixel and then if I compare let us say that this focus measure operator says you know outputs is a value say is that ok.

This is the degree of sharpness now by a by just by looking at that it does not it is not even clear how what do I tell about that? What is that absolute number? What does it mean? But now if I go to the next frame, I ask the same operator on the same point I ask what is the degree of sharpness? Now, it says let us say earlier it said 40 now it says 30. Then I know that right it is getting blurred because my sharpness is kind of reducing.


And then and then may be right sometimes later it probably increase I do not understand somewhere in between right. At be in some place it says that it is 80. So, relatively I know ok because the underlying image is the same you know nothing is moved. There is no there is no image motion, all that is happening is that my say level of blur is changing and therefore relative to relative to the other frames you can talk about some frame where this point is coming into focus and the same holds good for every point.

It is so the nice thing about this is that as long as you can ensure that every point in the 3D object comes into focus in one frame or in some frame or the other you are fine. Then that by itself can that is why it is quite safe from focus it does not say J from D focus. Shape from focus because you have to look for look for the points look for the frame in which a point is appearing in focus. And each point can come in focus in whatever frame it wants.

Right depending upon the nature of the underlying object. Okay so that is why it is called shape from focus and why I wanted you wanted to talk about this because now that you understand let us say if I showed you one of these images you know exactly what is going on and how the lens is introducing that blur but now we are going more going one step forward and utilizing whatever we have whatever we have learnt.

In order to solve something like an inverse problem. Now, we are asking for a 3D scene given this bunch of frames. Any doubt at this point before I go further. Ok now I will just draw this draw this set up okay and these are all real images okay captured in the lab. I do not think that right I just synthesized them or something. These are real. All right the way it looks like is the following ok. I am just going to draw this small little diagrams.

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SFF relay

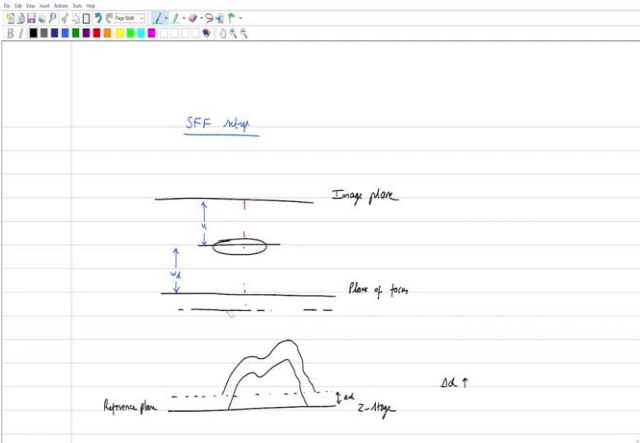


Image plane

Plane of form

Reference plane


d

z

$\Delta d \uparrow$

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SFF relay

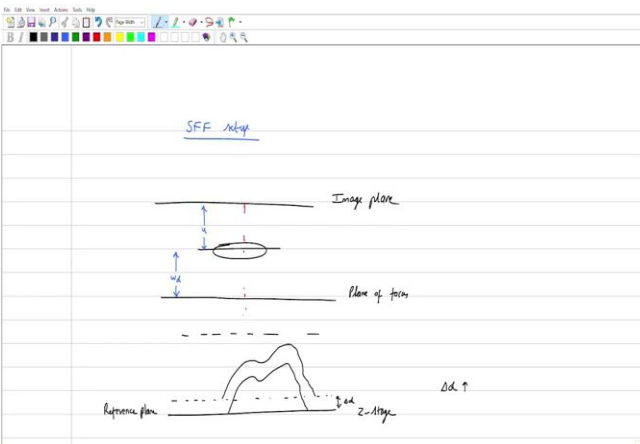


Image plane

Plane of form

Reference plane


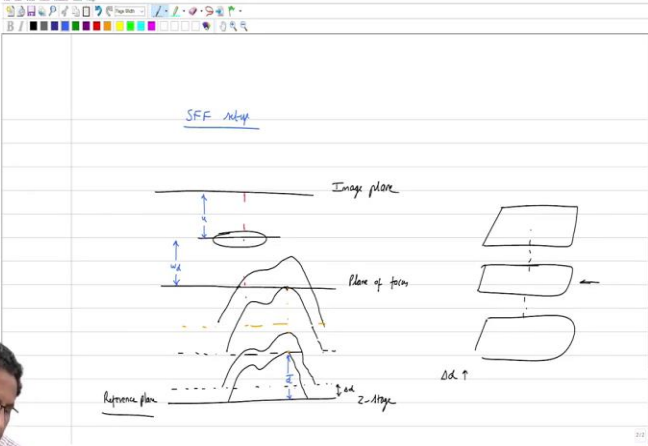
d

z

$\Delta d \uparrow$


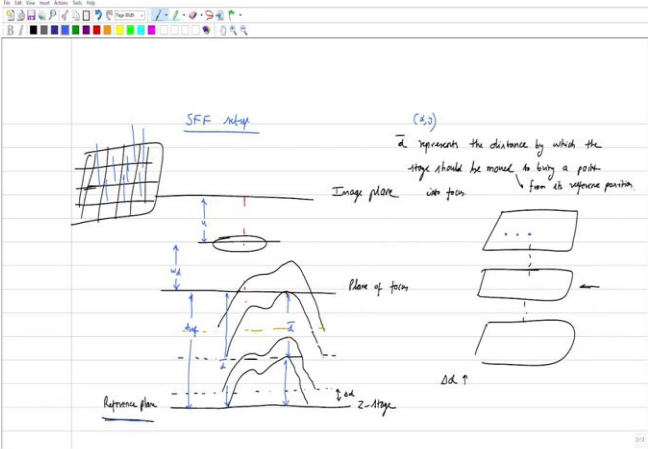
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So, the SFF setup it looks somewhat like this. You have you have your image plane here. And no of course you could also drawn it from left to right. But I am drawing it top bottom because typically it is like a Z stage and then you know move along the optical axis of. I am drawing it you know top down.

So, this is the image plane let us say. And then somewhere here is your lens. Ok so let us call this as some u should I draw it in some blue or something. So, this is u so the image plane to lens distance is u and let us say that that you have the plane of focus somewhere here. The plane of focus means that means that for that focal length of this lens whatever it is right some F . So, where this lens law getting satisfied.

So, your ωD writes ωD is calculated from the lens center so this is your ΩD which is a working distance at which we have actually talked about that means any point that is kept here will appear in focus correct. That what we mean when we say it is a working distance plane of focus means a point that that if you if you keep an object here exactly anywhere on this line it will come in focus because it is exactly the same lens law.

And then down below we have actually a 3D object let us say right here is our 3D object some shape ok some shape I have and whose shape I want to actually find out I want to find out what kind of 3D surface am I making us looking at and this 3D surface right what you what you do ok let us let us call this as z stage. This is called as z stage and then this is called reference plane ok. So, here is where we start with it.

So, we start from here ok. So we start from here and then right what we do is so for example it so with respect to this this plane ok reference plane we will take one image ok with this set up then what we do is we actually we actually move this stage up by a by an amount let us call that ΔD . Up by up by ΔD now ΔD is something that we will talk about later.

Just assume that it is it is a greater translator along the along this z axis ok. So, here is a optical axis of the cameras is this ok. If I show like that you do not see right. So ok so here is your here is your optical axis like so you are moving it along that right. Now, assume that assume that I actually move the stage up right there. So ok from here to here is really a really a translation of ΔD . Now, how to choose this ΔD and already we will talk about later.

Let us say that I just shifted up by ΔD therefore what will happen so this guy will now does to come up. Right and then ok I am kind of roughly drawing it ok so then you are there ok. And then and then you again move it another by another ΔD another ΔD . I am going to keep doing let us say that let us say that you are somewhere here okay. You we are traverse several ΔD s. And some ok no not there let us see I wanted to okay now let us say that let us say that read we have somewhere here ok.

And then from here. All right I mean I assume ok what I what I want to show is maybe I should go little lower. Ok. So, let us say I am here and suppose from here I draw I want to show something like that. It is not exact but then I just want you to understand that right there will come a point right when this guy which is which is the peak of this surface.

It will actually hit that plane. Ok. Will hit or hit your plane of focus which means that which means that all through read if you would if you are watching this point you all through I mean I know in the beginning it would have been blurred because it is further off from your plane of focus you get brought it up a little closer. It would have become a little more focused and then and then you keep doing it. Maybe it was it came much more into focus but right now at this point it is perfectly in focus correct.

And then of course and then then you go on right I mean you do not stop there then maybe then may be at some other point right. You draw again and then and then this object kind of goes like that right and so on. So, the idea is to be able to sweep through the whole object right take a bunch of frames so that so that you can actually sweep through the whole object.

So, the idea is this so for example, at some frame ok so because you move ΔD ΔD ΔD ΔD , let us say some you move by what are m number of frames. Let us say at the m th frame right so as far as the point here is concerned right this came into focus m th point and after that it became blurred because of the fact that you will now see out of the focal plane.

So, it would so it will be like gradually coming into focus then it hits the plane it is perfectly in focus then again it becomes blurred. Now, imagine some other point here right so for example, this guy right it does not come into the focus in the same frame right as this because this came into focus in a different frame. This would have come into focus in some other frame right after you lifted it some more.

So, the whole idea is that is that in that stack then so if you now imagine this stack right which is like with the frame that you captured for every one of these ΔD s. So, imagine the lower moses when you are at this is a reference plane then you say up ΔD may be may be this one and then there are several more a lot more that you have captured and then you have a whole stack know.

So, now what we are saying is as long as every point in this 3D object. At least at least appears in appears in appears and focus now in some frame or the other ok. Then you know the cue for that because then see then the point is this right if you know how do you how do you really think about think about estimating depth. Let me say that let me say that for this point and I am going

to indicate something right. I am going to say that this this this quantity I am going to call this as D bar.

Similar to disparity, that we had the right. I am here I am going to call a quantity D bar. So, what is D bar? Omega wait a minute no D bar is which way. D bar the other way. Not this be a little careful ok when I say that now ok. Now, d bar is from here to here. Ok so, at the time when you start right. When you started here it is a reference plane ok which is here which is here where we start.

So, from there how much should you move right so here what this means is d bar is the quantity that that basically represents but how much this state should be moved right in order in order for this point to appear in focus. Let me write that down. D bar D bar represents because the rest of quantity you know what they mean. Ok so D bar represents D bar represents the distance by which the stage should be moved to bring a point into focus into focus should we moved from it is from it is reference position reference position. So, it means right there is there is there is not so there is D bar for every point correct. D bar D bar is not really a really noble value.

So, for this point this is the D bar that is it for some other point right. So, for example for this guy right which is lying here know so that is where we have started know. It is a reference plane right. I think this got wiped out. Okay. Here is our is a reference plane so for this point right so D bar is going to be all the way from here to here.

And if for example if I knew from where I was starting see D bar itself will give you give you the surface profile. If you simply plot D bar you know that something is more something is less therefore you get up get a profile. So on the exact height then you should know this know lets me call this as D ref D ref let say something right which I know. Because I am going to know from where I start.

So, for no D ref if I if I simply do D ref minus D bar and if you are interested in this as the height. You want to call this as the height then all that you have to lose for every point is simply you simply have to subtract D bar from actually D ref to get the height of the object. It does not matter you look at it look at you compute height or you simply compute D bar both are both are equivalent to the D bar itself will give you give you how the surfaces undulating. So, the whole


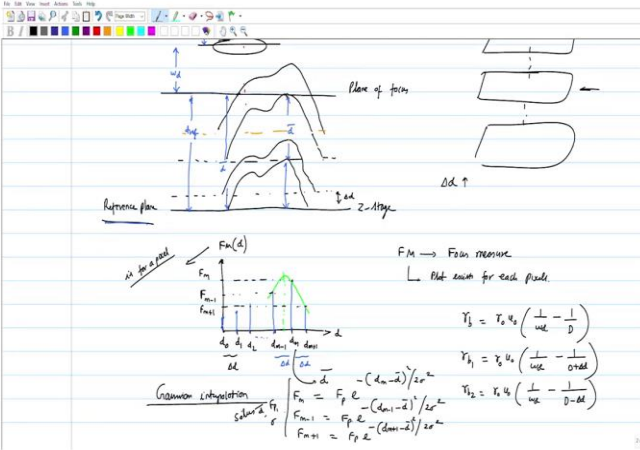
so the whole thing now boils down to so what this means is that if by some means if you can compute \bar{D} at every at every you know at every this one ok location.

You can compute \bar{D} at you know every this one location. So, it is like saying that that you know you have a \bar{D} for this is your \bar{D} for this. So, \bar{D} is like a function of X, Y right. It will change all over the image and therefore the output right of this whole thing should be something like another grid which will be of the same special resolution as their image ok. It should have the same special resolution as a image. And on this all that you would do is plot the plot here \bar{D} .

So, it will mean that right some will be high some will be low whatever right. Some may be very high whatever it is right. So, all this all this spikes right that they are plotting so these spikes are real \bar{D} and they but they in turn tell you what is the surface now. This is ok. So, which means that which means that right if let us say that somebody had actually \bar{D} blurred is emitted. Blurring means removing the blur it said something had said that you know what this is all this is all blurred and first of all you cannot use I mean use your use parallax idea. Because nothing is moving right

So, in this case even if you had blur you will get nothing. Right do not do not go for actually \bar{D} blurring utilize whatever the lens system is telling you in order to be able to recover some other information. So, I think so there so that is why I said now right is the V new sense or a see right new sense. So, this is like this is where blur makes new sense. So, now the point is now we will ask how do we how do we actually do this now.

(Refer Slide Time: 15:37)

The slide content includes the following elements:

- Diagrams:**
 - A diagram showing a 'Plane of focus' and a 'Reference plane' with a 'z-Offset' between them.
 - A diagram showing a 'Plot for each pixel' with a focus measure curve $F_M(d)$ and a peak at d_0 .
 - A diagram showing a 'Focus measure' plot with a peak at d_0 and a value F_{max} .
- Text:**
 - $F_M \rightarrow$ Focus measure
 - Plot for each pixel
- Formulas:**
 - $$Y_1 = r_s u_s \left(\frac{1}{u_s} - \frac{1}{D} \right)$$
 - $$Y_2 = r_s u_s \left(\frac{1}{u_s} - \frac{1}{D-d} \right)$$
 - $$Y_3 = r_s u_s \left(\frac{1}{u_s} - \frac{1}{D-d} \right)$$
- Equations:**
 - $$F_n = F_p e^{-\frac{(d_n - d_0)^2}{2\sigma^2}}$$
 - $$F_{n+1} = F_p e^{-\frac{(d_{n+1} - d_0)^2}{2\sigma^2}}$$
 - $$F_{n+1} = F_p e^{-\frac{(d_{n+1} - d_0)^2}{2\sigma^2}}$$
- Text:**
 - Continuous interpolation
 - Subpixel accuracy



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What will be that operator what should we do in order to be able to get a sense for sense for this sense for the you know degree of sharpness and all that. So, all that banks upon something called I know a plot a plot that is called a focus ok let me write this as FM, FM of D and this let us say it is D and FM stands for Focus Measure focus measure. So, it is some operator ok which which we will also you will also see.

So, this operator is something that when you when you ok to keep it at a pixel it will tell you give you some sense for sharpness. So, now this plot right. You will this plot exists for each pixel for each pixel. So, it is not like a plot for the entire know it is not a plot for the entire image every pixel.

You will have this plot and what this plot will tell you is what does simply means this let us say that it at D naught which is where you are starting which is which is a reference. At D naught if you apply and again it this this plot exists for each pixel. So, this plot is for a pixel. Or is it you know it is it is being evaluated for a pixel all I want to say at a pixel what is right so this plot is being evaluated at a pixel using may be right some small sort of a neighborhood may be or it could simply that to see pixel itself.

So, at a pixel you are looking at inner frame all right so as you can imagine that you have all these frames so you are going know see I so you are standing somewhere here in one frame. And

at a pixel and here and you are asking applying this focus operator and asking what is the value right and suppose let us edit tells you that the value.

You know so the sharpness is somewhat like this then we asking the see next frame up r. Go up and look for at the same location look for the sharpness again and suppose in this case it turns out that let us say it turns out that D_1 which is the next frame that means these two are separated by actually a ΔD .

Because here this ΔD apart and we and we going to keep ΔD uniform because we do not want to make them make the math complicated. As it is very simple technique so that is it is strength. So, D_1 is this so let us say let us say D_1 shows up there and then again let us say let us say D_2 . It is something else some something else and then and then we can go on. Like this and let us say at some place let me call that $DM - 1$. Let me say that met me say that I know I observe something like that and what is special about this $DM - 1$ is that at DM I find I find something like that. Let us say and that and that $DM + 1$. Let me say I find something like this.

Now, do you think it is ok to plot it that way see this is ΔD . So, is this but I said that the measure focus measure may give something higher on the left or lower on the right or may be lower on the left and higher on the right. Do you think I am drawing something wrong or you believe that that is okay?

Should they be symmetric or you feel like it is ok. One will be high and one will be low.

Student: (())(19:13)

Professor: We are going to ΔD away exactly.

Student: (())(19:18)

Professor: Correct true that is true but I am saying irrespective of whether DM is actually the place where the where the frame comes into for the point comes into focus but I am saying when ΔD away from DM and ΔD low on the lower side and ΔD on the upper side. I am saying that you are focus operator seems to sense that on one side it is higher and on one side it is lower.

So, I am asking right. Should I have drawn it like this or should I have drawn it like this so that these two are exactly the same about DM or should I not. Know is it like is it like it will wrong to draw them symmetric.

Student: () (20:01)

Professor: Ok what do you base that on.

Student: () (20:08)

Professor: Ok. But I am looking for something more concrete than that. Think. See the way right I would argue is this I mean if you go back to the blur radius equation it what was it then we had a blur radius equation R_B was some $R \text{ naught } u \text{ naught } 1 \text{ by } \omega D \text{ minus } C 1 \text{ by } D$. Now, the point is right so suppose let us say right suppose you have $R_B 1$ ok which you got by you know by you know being on being on being let us say $1 \text{ by } D \text{ plus } D \text{ plus } \Delta D$.

And then you have got an $R_B 2$ which is like $R \text{ naught } u \text{ naught } 1 \text{ by } \omega D \text{ minus } 1 \text{ by } D \text{ minus } \Delta D$ right. So, you can see that see that right if your frame is actually in focus right then there is no reason to believe that your blur radius has to be symmetric or blur idea should be should be the same on either side of peak at a point of focus. So, about let us say $D \text{ plus } \Delta D$ and then other case it is $D \text{ minus } \Delta D$ right. So, it is not true that it is not true that you sitting here and then and then you know this is the plane of focus. It does not mean that if I am ΔD away from here then I will then under ΔD at the back.

It does not mean that these two will actually introduce the same degree of blur so let us say that we have assume that a DM right it comes into focus then the simplest thing to do would be to have this focus operator run on every frame and that pixel location or around that pixel situated and that pixel it could sense around a small neighborhood and then right and then and then it shows up this value shows us this value shows us this value and then and then and then what you have let us say that right we assume that whatever we see here to be the highest focus measure value and therefore one thing to do would have been simply say that that point comes into focus at this frame.

And therefore this should be my $D \text{ bar}$ ok. That need not be because you know it could be that right we just kind of see over shot right because our ΔD could have been such that we have

quantized it in a manner that we could have just overshot and you know we may we may still not be sensing that where that where that actually \bar{D} is right. It is random because in this figure. See in this figure right so so when you are when you are moving the ΔD it does not mean it does it does not mean that that that peak would exactly come and sit at plane of focus.

You could have just translate a little about because you do not know no you do not know a priority what is 3D surface. You are just randomly moving at ΔD ΔD ΔD ΔD and then you are hoping that everything will come into focus. But you could have just skipped the skipped the point of focus right because that that is your actual \bar{D} . And what this kind of FM plot is giving is a sense of somewhere there. He tries to get this \bar{D} through what is called what is called Gaussian interpolation through a Gaussian interpolation ok.

And the way the way right this is interpreted is that see for example, if you try to try to fit a Gaussian and way if it is a Gaussian right takes these three points. So, he takes a maximum guy ok seemingly what is the maximum and then and then I had have been and then and then around it you have the one on the left and then on one on the right.

It could be the lower and that is high we do not know. But then he fits a Gaussian so in this case what would happen is if you try to fit a Gaussian ok. Let me draw it with another color. So, what it will do is it will try to try to fit something like that so this so this peak will be kind of to the left of left of DM.

If it was the other way then it would have been on the right of DM right if the right was higher. So, he will try to find out where hypothetically where this \bar{D} is and he uses the Gaussian interpolation. Because if you linear and all you can never get a value higher than higher than it is right either of these two peaks, I mean either of these two values. This Gaussian interpolation actually allows you to actually pick somewhat like actually a peak now. Now, there is no great to see theoretical support and all for this.

He just he just does something very simple because he wanted everything closed form and fast. And he uses Gaussian interpolation because in that way right even if there is a peak somewhere to the left or the right, he will try to fit through that and be able to be able to solve for \bar{D} and this is and wherever right and wherever he so the way he solves for is that so let us say let us say

if you call this as this value let me call this as FM minus 1. Let me call this what is that ok so this value is FM and let me call this value as actually FM plus 1. So, one that is there s a DM plus 1.

Let us say that is focus measure output then in this Gaussian interpolation so you see he has actually three equations. FM on one hand so he says as FP something that is not known because he ready to draw do a Gaussian fitting. He raised to minus off let us say DM minus D bar square by 2 sigma square and he does not assume that this should integrate a 1 and all. So, this FP if we need not to be 1 by root 2 pi sigma and also he allows it to be 3. So really the area will be like 1 by root 2 pi sigma times FP.

So, FP is independent of sigma. Some peak that you get there so it does not a Gaussian that is constrained to have an area 1 and then FM minus 1 will be of course FP. He raised to minus of DM minus 1 minus D bar the whole square by 2 sigma square and then FM plus 1. Similarly, is FP he raised to minus of the M plus 1 minus D bar the whole square by 2 sigma square. And since now we can take a take logarithm and all, so the idea is that you can solve for actually D bar ok. So, the actual D bar right which you have here.

So, using these three equations these solves for D bar and not only that ok. He just he does not stop and solving this for D bar, he also finds out FP, he also finds out sigma ok. I will tell you I will tell you what those things actually mean and why those things are relevant might be relevant. So, and then and then and then and then once here once here you can once find this D bar and that is the there is a D bar that will take for that pixel.

Then a nice thing is that you can do all of this completely in parallel. You do not have to wait for one pixel output to start the next one, you can if you have a parallel architecture you know all these all these plots right simultaneously get D bar for every pixel and then and then you can actually plot the surface. So, again just as CEO has issues right this also there are some issues and all which I will talk about later.