Image Signal Processing
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Lecture 30
Shape from focus - Gaussian fitting

So, we were there yesterday, right? At this point where we said, okay, we had to do an interpolation in order to in order to find your d bars more accurately. So you really think about it, think about it as something like this mouse, let us say. So suppose you had this mouse and if you are kind of say translating it, you keep the most on a z stage, and that if you translate it, and then each time you take a picture. Now what will really happen is of course this is more or less vertical, but then imagine something like, like an object, which has a varying depth.

So then what would happen is every time you move by delta d, there will be rim of points, which will anyway hit the focal plane, because some point may have overshot, but there will always some rim that will be in focus, but then the number of such rims if you really look at it is simply the number of frames that you plan to capture.

Now, the rest of the points are all actually out of them in the sense that they have not hit the focal plane at all. So, in that sense, there is really a motivation to look for a d bar which is more accurate, because otherwise what is going to happen is you will simply have end rim endpoint, I mean whatever n such points on the, not endpoints I mean you will have like n such rims, and those are all the points that will likely be in focus in the sense that they are exactly hitting the focal plane, everything else is not.

So that is another way to kind of look at the motivation or really doing this interpolation because for all the other points they have are all kind of either they have undershot, or they have overshot. And so which means that your d bar could be, actual d bar could be more than the thing that you are expecting or it could be just less.

Now, so yeah, so I just wanted to go through this interpolation today. And then if time permits, we should also look at what are the different focus operators that we can use. Alright, so as I said, I had actually started by writing the equations for the F m, F m plus 1 and F m minus 1 which is like a Gaussian fit. So if you just write down L n of f.

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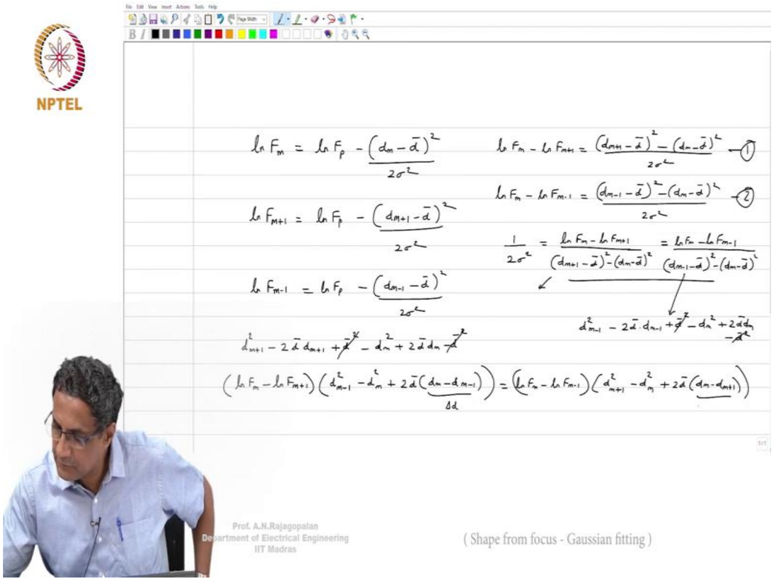

So if you go back and do L n of F m, then we will get L n of F p, these are all very simple things, but we will work through them. d bar on square by 2 sigma square, L n of F m plus one. And doing a Gaussian interpolation other than the fact that it will actually give you this d bar hopefully which is much more, which is actually better than your d m that where you think the maximum is occurring based upon the focus measure. The other thing is that it will also tell you, it will also give you a certain level of assurance about the d bar itself.

Now, there could be cases where you feel that this d bar that you have computed is really not so great and you might want to bank on something else. I will tell you what that means. So, this goes in fitting, it is not only to get your d bar, but it is also to throw some light upon how good is the value of d bar, whether we can actually trust this value, whether we can use it or not, and so on.

So that is also the reason why you get this F p and you also solve for sigma. And then I will tell you how we can actually who can you interpret those. So, L n F m plus 1 is second L n of F p minus d m plus 1 minus d bar by 2 sigma square. And then finally, L n F m minus 1 if you look at that, that is equal to L n of F p minus d m minus 1 minus d bar the whole square by 2 sigma square.

And then we will just do a small sort of manipulation here, so just do L n of F m minus L n of z F m plus 1, so that will knock off your F p, and you will get d m plus 1 minus d bar the whole square minus d m minus d bar by 2 sigma square. And let us also do L n Fm minus F m minus 1. L n F m minus L n F m minus 1. So that will give you the m minus 1 minus d bar the whole square minus d m minus d bar the whole square by 2 sigma square. Or in other words what this means is that okay, we will also knock off sigma now because right now, we are only interested in d bar.

So, 1 by 2 sigma square is equal to L n of F m minus L n of F m plus 1 upon d m plus 1 minus d bar whole square minus d m minus d bar square. And this we know is also equal to So, if you look at whatever it call this equation 1 and 2 or whatever and from 1 and 2, then clearly 1 by 2 sigma square equals L n F m minus L n F n minus 1 upon D m minus 1 minus d bar square minus d m minus d bar square. So, if you see finally, so we can even knock off sigma now. So, all that we have, so now we have this equation which involves only d bar so, both sigma and F p we have got an out.

So, now if we simply expand the lower one, it is simply this one denominator. So, what do you get? So, if expanded you get d square m plus 1 minus 2 d bar d m plus 1 plus d bar square minus d m square plus 2 d bar d m minus d bar square. So, this denominator, d bar square sort of gets cancelled out and you are left with this. And similarly, if we take this denominator, we will have d square m minus 1 minus 2 d bar d m minus 1 plus d bar square minus d m square plus 2 d bar d m minus again d bar square.

So, here also d bar square it will cancel off, therefore what we are left with is really L n of F m minus L n of F m plus 1 into if I cross multiply, so what we get d square m minus 1 minus d square m plus 2 d bar d m minus d m minus 1, then this side is equal to L n F m minus L n F m minus 1. Here, on this side it will be d square m plus 1 minus d square m plus 2 d bar d m minus d m plus d minus. Here it is d m minus d m minus 1, this is d m minus d m plus 1. So, we know that d m minus d m minus 1 is simply Delta d, d m minus 1 is m minus 1 delta d, d m is m delta d so delta d, d minus d m plus 1 is minus delta d.

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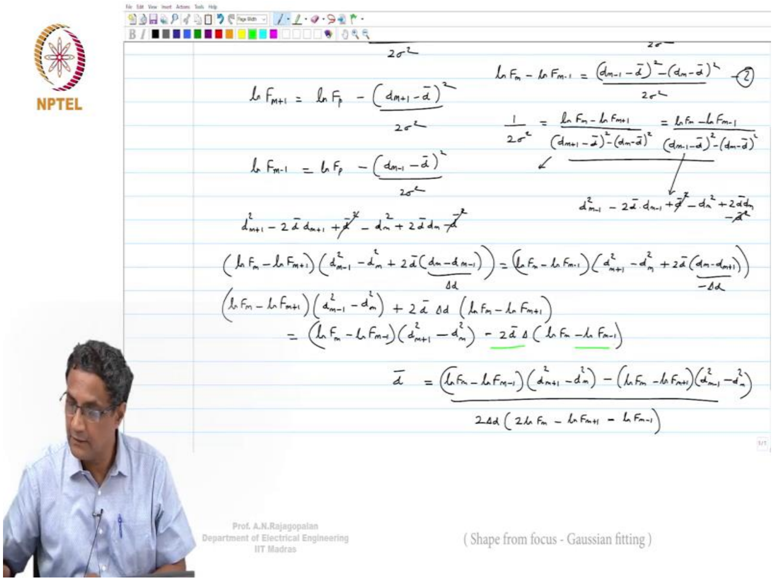

Therefore, what we have is L n F m minus L n F m plus 1 into d square m minus 1 minus d square m plus 2 d bar Delta d into L n F m minus L n F m plus 1 is equal to, on the right hand side L n F m minus L n F m minus 1 d square m plus 1 minus d square m minus 2 d bar because that is minus delta d so this will become minus 2 d bar delta d L n of F m minus L n of F m minus 1. Therefore, we just clubbed terms that are involving d bar. So, in this if you think about it, all these quantities are known, F m we know, F m plus 1, d m, d m minus 1, everything we know actually delta d everything we know except d bar.

So d bar, so if you club so d bar, okay 2 delta d okay d bar if we take that out as common, and then if you pull this L n F m to the left, then you will get to L n F m minus L n F m plus 1 plus L n F m minus 1, right? If you pull everything onto the left side, so 2 L n F m because this L n F m will come to the left. And then plus L n F m minus 1 minus L n F m plus 1, this is equal to L n F m, you can simplify this further but that is okay, d square m plus 1 minus d square m, then let us push that out here minus L n F m minus L n of F m plus 1 into d square and minus 1 minus d square m.

Or in other words, we can say that we can get your d bar by simply dividing this whole guy by 2 delta d. If I bring this thing down, so 2 delta d 2 L n F m minus L n F m plus 1 plus L n F m minus 1, and if I remove all of this so, so 2 L n F minus L n F m plus 2 delta d. So if I remove this and shift it all to the left then what I am left with is really the d bar equation and which looks like that. I mean, straightforward, no great surprise is there. L n F n minus d square m minus 1 minus L n F m minus L n F m plus 1, I am just checking if everything is okay, 2 L n F m plus, Oh there is a minus L n F minus 1, there should be minus L n F m minus 1 because this minus and minus plus, so this should be minus because here this 2 delta so we have this guy know 2 Delta d, so this minus and minus plus therefore we are pulling it to the left so that will become minus.

It is correct know? It should be minus L n F m plus 1 minus L n F m minus 1. Is that okay? So except for that sign, everything else is okay. And then which also means that one thing is that d bar and delta d will not get knocked out because even if you do sort of by distributed thing and all d m plus 1 plus d m d m plus 1 minus d m or even if you write delta d will not get knocked out, it will still remain. And one might also be interested like I said right, one could also be interested in sigma square, so because that also gives you, s I will tell you what you might want to use sigma square for.

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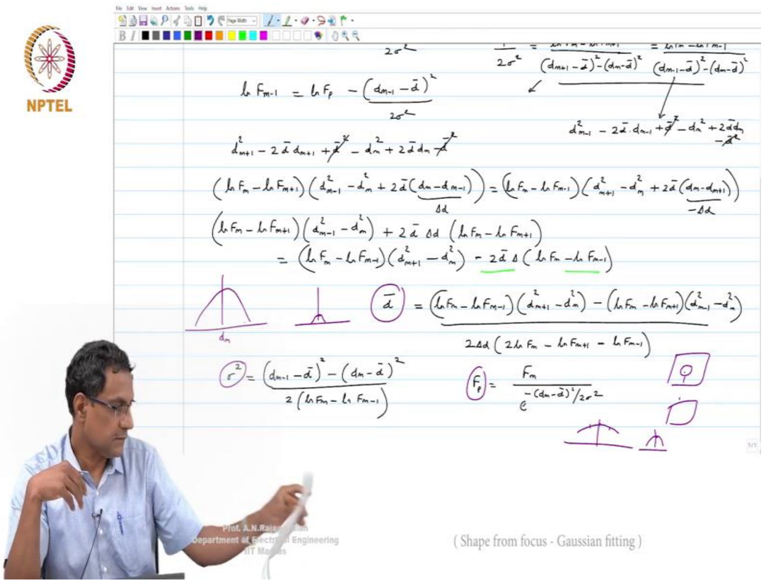

So, the expression for sigma square I will simply write it down here, okay, you already have it there. So simply following from there so sigma square is equal to d m d bar square minus d m minus d bar square upon 2 times, you can use either of the two, I am just writing one of the L n of F m minus 1 and then you can also use the very first expression that you wrote for F m in terms of F p, use that to get your F p. F p is equal to F m upon e raise to minus of d m minus d bar square by 2 sigma square.

So, what this means is that we can get so we will first we can solve for d bar first and then we will also look for what is the value that we get for sigma and what is the value that we get for F p.

Now, the way these other two things are actually used is as follows. So, sigma is something like a spread, so we try to fit this Gaussian and it is giving you a sense of the spread. Now, what would it mean if you had a sigma that was large, how would you kind of interpret that? Let us say that it gives out a sigma that turns out to be large.

So, how would you interpret that? So it will actually mean that instead of having a very sharp curve going like, this was more or less not flat, but it has a large sigma. So what it would mean is your ability, see ideally what you would like is a sigma that is small because then it would mean that something is coming to sharpness and then immediately as you go to the left or to the right, it is falling off, right?

That means your ability to say that that is where things are focus is quite good, then you know that is exactly it speaking and then on either side, it is falling off pretty abruptly. Whereas, if something is like flat or close to being flat and is slowly varying then it means that your own ability to tell exactly in which frame it is coming into focus, I mean you may still get some value.

Now for some frame where you find that it is high, it is highest, but that does not mean that you are very sure no, because it would simply mean that your texture itself is such as that. So if you go back to that stack, where that we will be drawing, so what it would mean is that you are probably looking at a region which is not really so active or we just probably underlying image intensities are not really that very active.

Because of which when you sort of translate and then read you capture these multiple frames, it is saying that the variation the blur, you are not able to sense it much and therefore, your own ability to tell. So what it would mean is that having some kind of a curve like that, versus some kind of a curve like this, that you would rather go for this, because here things are falling off pretty quickly that is here, right?

When this looks like this is a peak, but then how do I know. So, it could also be that, this is also probably quite close to the peak. And therefore, sigma is something that we look for and then if we find the sigma is again it where we kind of where we draw a line and say that the sigma is high enough depends on the experiment.

But if we find that sigma for some points it is relatively coming out to be much higher than for let us say sigma at other points, then that is an indication that the image is probably lacking enough information in those particular locations. And therefore, you could either go with d bar that you have computed from here and still hope that you are not probably going too wrong. Or the other way to do it is you kind of look around, and then see kind of ignore this completely and just look around and see what d bar have come around it. And if you are lucky, and let us say, right, if let us say this region is bad, but then the next, the adjacent regions are not so bad, therefore, you have a more sort of a confident d bar there.

Confident again, in the sense that those signals are not really that big and so on, and therefore you just want to maybe replace this guy with that d bar because anyway, we going to make an assumption that things are locally smooth. I mean, we do not expect of course you could go wrong if you are sitting on an edge and so on.

But because you are not going to be always sitting on an edge, so you could simply borrow on the fact that neighbors should look alike. And therefore, instead of using this, I would still bank upon just picking some d bar. We can take the median of the neighbors or take the average of the neighbors or something more sophisticated can be done, I will talk about it later what is called a tensor voting.

But let us say the simplest thing that you could simply do is take the average of the median of the neighbors and just copy that d bar at that location. And F p is again another thing that is also good because what can happen is imagine that right at most locations you get a nice kind of bell curve.

So it is like this your d m, and then about which you have this curve. Now, at some places you may still get this kind of a curve, but then it could be something like that. Now, which means that your peak itself is very low relative to let us say, the kind of F p that you have been able to get elsewhere in the majors.

There are some portions where F p is turning out to be very small, your sigma may be still okay, but then F p itself is very small. In which case again, you may not want to probably take that and you would then go look for scout for some d bar in the neighborhood and simply use that.

So this in that sense, the sigma and F p are actually useful. So when you actually copy that d bar value, so each time when you do the copying, you are sort of more sure that what your code is copying is hopefully a good value. Okay, because these can be noisy, right? See, the whole strength of this particular method is that, it can be done simultaneously, it is a parallel, kind of a scheme.

So it is a parallel scheme in the sense that it you can operate at one pixel independently or the other. But then that also means that you could have trouble because you are not enforcing some kind of smoothness. We are not saying that if this guy carries a value d bar then the other guy should have a value close to it, we are not saying that.

So which means that we can certainly see some spikes in your plot that is simply because of the fact that the image is lacking some texture underneath and so on and therefore you could would think that I mean generally nature does not have whatever scenes like that, where you have a sudden spike coming out of a kind of a 3D scene.

So you can get, so you can do out do a heck of a post processing or else, we can take this depth map and then do some kind of no post processing, but all those are kind of ad hoc there (())(20:22) post processing you do also. So hopefully if you have a textured image, you will not have any issues but then that you cannot guarantee in a real scene. Anything that that you want to ask prior to going to those focus operators, which are supposed to do this job now.