Image Signal Processing
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Lecture No. 37
Image Transforms-Motivation

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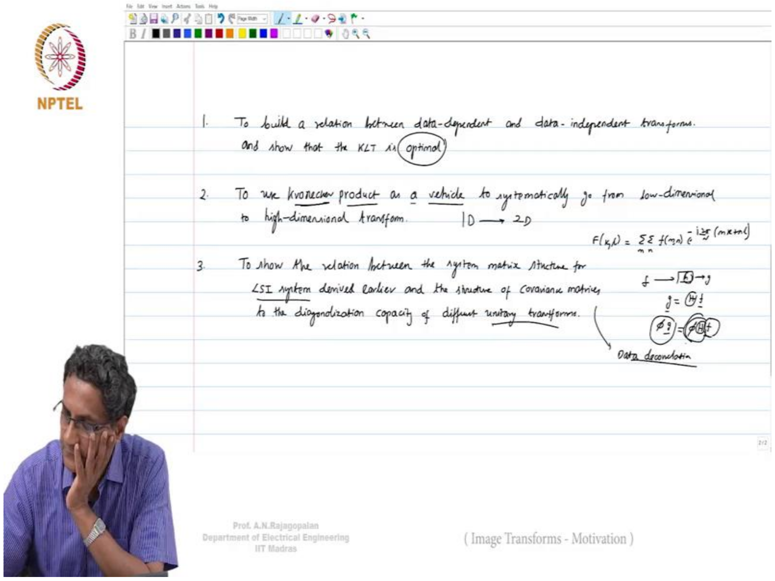

So, the so the way we go about doing this is the 3 heads and even if it is not clear right now it is. But after we read as we scroll through the whole thing, you can actually come back and look at these 3 points and you will notice that notice that whatever whatever we do will revolve around these 3 things. The first thing is this. So, what we want to do is to build so, so as we walk through this whole thing, on image transforms we want to build relation between between data dependent between data dependent and data data independent transforms.

So, this is that we do not want to treat them in isolation. Initially, we will treat them as an isolation, but then at some point in time, we will try to create a bridge between the two data dependent and data independent transforms, independent transforms. And show that the KLT so it actually means that, that instead of a data independent basis, if you were to construct a basis coming out of the KLT, that would be optimal.

I can show that that KLT is optimal. So, what optimal in the sense that optimal, I will talk about it later. It is a terms of two things and one is D correlation of data. And second thing is how much of energy can it pack.

Let us say rate of any if you if you if you going to look at the first m coefficients or when then the first m coefficients, how much energy can some transform pack, and the more it can pack in fewer coefficients, the the the the many would think that had that kind of a transform would actually be actually be better. So, the KLT is optimal, KLT is optimal, okay. So, all this notion will come. We will come to this notion later.

So, what is this notion of optimality? Then 2, to use a tensor product to use this one a Kronecker product, a kronecker to use the a kronecker products. A kronecker is for matrices typically, but then a tensor product is what is generally called when you go to higher dimensions. Use because we are just interested in going from 1D to 2D so I am writing it as a kronecker product. Kronecker product as vehicle as a vehicle to systematically go so the whole idea is we are going to say systematically go.

We do not want to suddenly throw something in and then say that I that is what it is and then we simply have to accept it, to systematically from low to high dimension from low from a low them from low dimensional transform to high dimensional transform. So, for us it is like, and from low dimensional. So, why is this important to high dimensional form.

Why is this important because as I said, instead of instead of directly starting with a 2D transform, what we will do is, what we will do is we will start with 1D transform that we know. We will first show how to go from go from 1D to 2D, using using this as a vehicle a Kronecker product as a vehicle, which then means that you know, that will kind of that will at least now, tell you that this is then possible, even when you want to do go for higher dimensions and so on.

But we will stop it 2D. We are not going to go beyond that. But but then you will be able to appreciate the fact that I that this kind of progression from 1D to 2D is automatic it is kind of natural. So, so it is like saying that suppose I write down F mn out over F kl if it is a if it is a 2D DFT, so I did as whatever F mn summed over m and n e power minus j two pi by n m k plus nl now, one way to say that say that you extend 1D DFDA to 2D DFT is this.

But then you would lose all the all the all the underlying sort of a structure that exists simply say that except this is a this is a 2D DFT you will accept, but then the point is you will not see that you know that this can this can get this can naturally come from a 1D you do not looks like an array there is some j minus j 2 pi by n m k that is what you use for 1D,but it looks like either there is something but then that structure is not obvious.

We look at it this way. So, which is the way which is the reason why I avoid doing that. It will be simple. In fact to do that, just to introduce DFT That way or an 2D that way, but then unfortunately read it does not reveal the the underlying structure. So, we will rather go through this and then show that how this is a kronecker product and It will comes into the picture to in order to help us go from a lower dimension because then you will automatically know how to go to higher dimension.

So, then the third point is all that we talk about will resolve around these 3 things this to show the relation between between the system matrix so again see at that time when we talked about a system matrix it as for an LTI or an LSI these are an imaging system. We just saw some structure. We saw that if it was LSI, it was actually a W block circular matrix.

If it was 1D, then we said it should be a circular matrix. But then we stopped with that. We did not go any further. But now in the light of this image, transform, that are there, instead of just kind of looking at these transformers, something that act on images, you can also kind of a, let look upon how these how these things play a role.

When you have to go from let us say, 1 domain to another domain in the sense that, let us say, you know, instead of thinking the same way that you interpret, interpret, let us say, a convolution to the to these domain as a product, now you can kind of know you can take a signal take its Fourier transform and talk about what happens or you can take the take a convolution between two signals and then and then kind of apply the Fourier transform and talk about the output as a product of the transform.

So, so in a similar way we want to see how the system matrix, what is its relation? So, for example, so for example, when I write a sample, suppose let us say I have a simple sort of a 1D on the LTI system, the H let us say whatever and let us say input as x or input as f and then outcome is G. Now we know that we know that I can I this as write g is equal to Hf, it is a vector and then we know that we can write this in as a as a circular matrix.

Now, now, this operation is simply a convolution of if you want a what of what a Fourier interpretation for that, then what happens is if you if you kind of see think about applying the Fourier transform now, what will you do? You will take this DFT matrix multiply you know acted on g and then you will say, if I applied phi on the left then I need to apply phi on the right side also I'll get phi Hf.

But then beyond this point looks like looks like I am stuck because because I want a I want a relation that will relate the DFT of g with the DFT of not this H, but then the small h, in this capital H of course has entries which are which are filled with the smallest and it has zeros.

And then and then I want to be able to say relate the DFT quotients f of f. Now, this is not really an image transform, but then it is again the role of a transform that is coming in and therefore what is this phi is relation to this guy h now, right, so, this phi must be something special for the h, which will then get a us enable you to get up enable you to relate the DFT quotient on the left as a product of the DFT quotient of the filter with small h and and the DFT quotient of f. t.

So, so it is not immediately obvious. So, so so when I am writing this h, this is what I mean. So here, it is not about taking an input signal and then trying to apply a transform, it is more about more about, having a system matrix coming in now, which is that h guy, and then trying to see what what what kind of h has a relationship with what kind of transform. So, there is, so again it certain transforms have certain affinity to the structure

h. which also reflects later when you have a covariance, which could have similar structure.

So, so thw so the way this kind of this sort of rubs off on data as well as a system is kind of somewhat similar. This will all become a little more clear at after we go through all this, but I am just saying that the same action that that the same affinity that has had that it has for H similar affinity It will also have when you show it a covariance that has a structure, similar to let's say, H, but then H can generally will not be symmetric and all that. So, so so that is why I am not I am not making a general statement.

But I am just making a loose statement that that the action on h that you see in terms of the system matrix, that is why if you if you look at what I am going to write next relation between system matrix structure for LSI LSI systems derived earlier derived earlier. And the structure of the covariance matrices, that is what I meant.

So, this is not just for h, system matrix is one such example. It will also have implications for covariance matrices on which you might actually wish to act covariance matrices okay. So, so in a sense right these two these two are together, the system matrix structure for the LSC system and the structural covariance matrices to the diagonalization property to the diagonalization property to the diagonalization property of diagonalization (capa) diagonalization capacity of different unitary transforms.

See, once we set the stage, then then it is better rather than start doing one by one it without without sort of having a feel for why we are doing it. So, what this actually means is, so on the one hand, what was this, what is this unitary transforms affinity to a particular system matrix in terms of in terms of diagonalization and both are in terms of diagonalization.

But then when we when we when we when we look at diagonalization here, this is like saying this diagonalization throws light on how let's say a convolution becomes a product in the Fourier domain. The other diagonalization of a covariance matrix has to do with a data in a D correlation. So, how you interpret it will change. So, there we will not interpret it as as kind of a system, there we will look at how this how does a particular transform D correlate data.

So, so if you if you actually diagonalize a covariance matrix, what it means is that you are choosing a new set of basis where, where things become uncorrelated, and unknown, or unknown, or D correlated data is actually nice to nice to handle because there is a lot of this correlation. So, it is like saying that say, if y and x have a high correlation, why should I send both? I may lose or lose, you know, lose a little bit but then I may be okay with it. So, I may simply send x because then if I send you x when, you know roughly what y could be like.

Whereas, whereas if, all that you can do provided you are able to move to another domain where this kind of de correlation happens. So, so in that sense in that sense as you see this that say this again there is a unitary transforms have have a role to play. So, so they have they have a role in a sort of a general sense.

If you could have read theses 3 points. Having said that now let us look at now that we have set the stage. So, as you can have as we go through, you can always walk back to this to this slide and you will know that we are somewhere anything that we talked about we will be in one of these 3 points.