


**Image Signal Processing**  
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**Lecture 41**  
**Alternative forms of 2D**


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$$V(x, l) = \sum_m \sum_n a(x, m) u(m, n) a(l, n)$$


$$V(x, l) = \langle U, A^k_{x,l} \rangle \quad Y = A U A^T$$

$$A^k_{x,l} = \underline{a}_x^k \underline{a}_l^k T = \begin{bmatrix} a^k(x, 0) \\ a^k(x, 1) \\ \vdots \\ a^k(x, n-1) \end{bmatrix} \begin{bmatrix} a^k(l, 0) & a^k(l, 1) & \dots & a^k(l, n-1) \end{bmatrix}$$

$$= \begin{bmatrix} a^k(x, 0) a^k(l, 0) & a^k(x, 0) a^k(l, 1) & \dots & a^k(x, 0) a^k(l, n-1) \\ a^k(x, 1) a^k(l, 0) & a^k(x, 1) a^k(l, 1) & \dots & a^k(x, 1) a^k(l, n-1) \\ \vdots & \vdots & \ddots & \vdots \\ a^k(x, n-1) a^k(l, 0) & a^k(x, n-1) a^k(l, 1) & \dots & a^k(x, n-1) a^k(l, n-1) \end{bmatrix}$$


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$$V(x, l) = \sum_m \sum_n a(x, m) u(m, n) a(l, n) \quad 0 \leq l \leq n-1$$

$$Y = A U A^T$$

$$A^k_{x,l} = \begin{bmatrix} a^k(x, 0) a^k(l, 0) & a^k(x, 0) a^k(l, 1) & \dots & a^k(x, 0) a^k(l, n-1) \\ a^k(x, 1) a^k(l, 0) & a^k(x, 1) a^k(l, 1) & \dots & a^k(x, 1) a^k(l, n-1) \\ \vdots & \vdots & \ddots & \vdots \\ a^k(x, n-1) a^k(l, 0) & a^k(x, n-1) a^k(l, 1) & \dots & a^k(x, n-1) a^k(l, n-1) \end{bmatrix}$$


$$V(x, l) = \langle U, A^k_{x,l} \rangle$$

$$= u(x, 0) a(x, 0) a(l, 0) + u(x, 1) a(x, 1) a(l, 1) + \dots + u(x, n-1) a(x, n-1) a(l, n-1)$$

$$+ \dots + u(n-1, 0) a(n-1, 0) a(l, 0) + u(n-1, 1) a(n-1, 1) a(l, 1)$$

$$+ \dots + u(n-1, n-1) a(n-1, n-1) a(l, n-1)$$

$$V(x, l) = \sum_m \sum_n a(x, m) u(m, n) a(l, n)$$

$$u(m, n) = \sum_{x, l} a^k(x, m) u(m, n) a^k(l, n) \quad 0 \leq m \leq n-1$$


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Ok so right let us just move to this one. So, yesterday I was telling that I was telling and so I said that you know VKL, another way to write. So, all these are right equivalent forms. Some over MN AKM UMN which is your image In right. And I will just leave it you as an exercise to show that now V is equal to au transpose. If you if you try to substitute for I have been inside a. If you

put a 0 0 all of that right and then if you could have take this product, I will leave it to you to show that these two are in fact equivalent.

What I wanted to show was I mean right. To what probably is not is not immediately apparent is no why are these two equivalent in a product to you with a star Kl will this not show this and then and then we could have move on to a Kronecker. So VKL is right in the product you with A star Kl. So, what is A star Kl in general will show it for a for a general case. So, A star Kl is A k star Al star transpose. So, for an n by 1 vector right. What will this mean so a star mean what is this A k star are what are the entries A star K 0 A star K 1 all the way upto A star k n minus 1 ok. That is that Kth column of a Hermitian.

Then Al star transpose will be Al star a by 1 star a star L 0 A star L 1. All the way upto A star l n minus 1. Look no so, the rank of A star Kl will be what will be the rank of this game. Not that it is very very one exactly. Now, if you simply multiply the two right. So, what you will get this A star K 0 A star l 0 A star K 0 A star l 1, all the way upto A star K 0 A star l, n minus 1. So, really A star K 1 star l 0 A star K 1 A star L 1 A star K 1 A star l minus 1 and then finally red dot dot dot dot will go all the way down A star K 0 A star l 0 A star k 0 A star lL1. All the way upto A star. These touches off may be that is a problem.

A star K 0 A star l n minus 1 then all the way down to A star what is that K n minus 1 right. A star l 0 A star k n minus 1 A star l 1 all the way upto A star k n minus 1 A star l n minus 1. So, this guy is coming from A k star A l star transpose. So ok now if you look at the inner product right, what is that VKL VKL which is a new product u with A star Kl will then be u 0 0 so the right. If you look at the first entry of u is sort use it 0 0 0 1 and so on right. U 0 0 and then you need to take the complex conjugate of the first entry, so that will be A k 0 A l 0 plus u 0 1 A k 0 A l 1 plus plus plus.

So, you where are we U 0 comma n minus 1 or did I finish it. A k 0 A l minus 1 and then we will just go all the way down plus plus plus now then u n minus what is that first entry is n minus 1 0 A k n minus 1 A l 0 plus u n minus 1 comma 1 comma 1 A k n minus 1 Al 1 plus plus plus plus u n minus 1 comma n minus 1 A k n minus 1 Al n minus 1 and then all of this right have assumed a square image but then it can also be a rectangular image in which case as the sum will change.

Instead of going from  $A_{n-1}$  when and what is that  $u_{n-1}$  you will get  $u_{n-1}$  something ok. So, that will be the only sum the range over which the sum will be taken will change ok that is all. But this what we make it simple. So, now if you see this rate if you are guy compared to compare it with this guy you can clearly see that this is the same as summing over an  $m, n$  so on the left we can have  $A_m, A_k, m$  then  $u_{m,n}$  and  $A_l, n$  because for example right if you if you see ok if you freeze your  $k$  and  $l$  right on the left suppose we freeze  $k$  and  $l$  and let and suppose you keep them equal to 0.

Let  $n$  run from 0 to  $n-1$  so all that will be the first row because  $m$  equal to 0 so so  $A_k$  which is 0 which is what you find here a  $m$  equal to 0 and then right  $n$  goes from 0 to  $n-1$  right. So,  $n$  goes from 0 so  $l, 1$  all the way upto  $l, n-1$  and similarly  $u$  will go like  $0, n$ . So,  $0, 0, 0, 1$  all the way upto  $0, n-1$  that is the range over which will go to sum  $l$  and then and then right all the way down and then when you when you take what you call  $m$  is  $m$  as  $n-1$  then  $n$  again goes from 0 to  $n-1$  right so here where is this so here  $M$  is  $n-1$  and then  $n$  goes from  $0, 1$  all the way upto  $n-1$  and that is what will so so  $A_k$  of course right.

So, this is  $m$  so this remains the same right everywhere only this  $\Delta l$  will go from  $A_l, 1$   $A_0, A_l, 1$  all the way upto  $l, n-1$  right so. So, the alternate form is also ok. Some people do not like to use that and other thing is ok so this so this I leave it to you show that  $A^* K_m u_{m,n}$  all this we would not derive minus. All this you can show now but here  $m$  will run from 0 to  $n-1$ . Here  $k, k$  and  $l$  will run from 0 to  $n-1$  ok.