Image Signal Processing
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Lecture - 44
Extending 1D Unitary Transform to 2D - Summary

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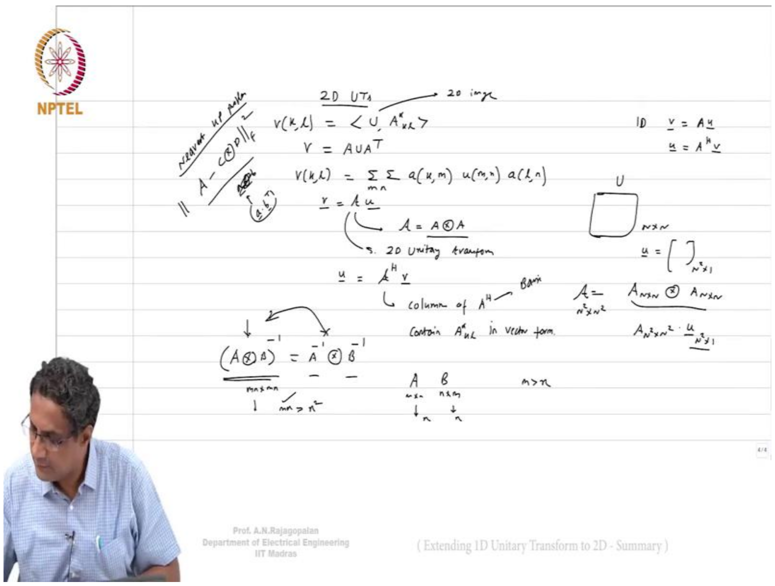

So this week the, and so there are alternatives forms as you have seen already. And so each one, all of them are equally valid. So you can compute V k, l as inner product of U with A star kl, so in 2D unitary transforms.

So we can actually compute this transform matrix V k, l for every coefficient as inner product of U with, where U is image, of course, U is your 2D image. This is also the same as V. This, another way to write it is V is equal to A U A transpose, that is also equally valid where you get your all the V, k, ls through a matrix multiplication, that is also valid.

What is also valid is V k, l is equal to double sum m n going from 0 to n minus 1. What was that? a k,m; u m,n; a l, n; this is also correct. They are all equally valid. So some books may use one over the other, but then all of them are equally valid forms; but the one that I like the most is this one where we wrote down v is equal to script A times u, where script A is A Kronecker A. And this form, this form I feel sort of the most elegant because it kind of leads the way from 1D to kind of say, 2D in a seamless manner.

This looks like just one walk away, whereas the others look like they are kind of, kind of say, roundabout; these are like straight. Just as in 1D we had v is equal to A u, now going to a 2D simply entails us to use, use a different unitary transform, this is what we called as a 2D unitary transform. And this script A, this is a A Kronecker A, and as I said yesterday if you take an image and then you stack it up as this vector V, U, suppose you have an image which is of size let us say N cross N, so you stack it up, this is your image U, make it as an N square cross 1 vector, you can convert it in an N cross 1 vector.

In order to get a 2D unitary transform, you have to, you have to look at your A which is N cross N, which is coming from 1D, take its Kronecker with A N cross N, and this becomes a script A which is N square by N square and operate this script A on U.

So, do not take A N square by N square, because A N square by N square is also there, that is also available but you cannot multiply this with U, which is N square by 1, and claim that I have actually I have 2D transform, that will be a 1D transform corresponding to a sequence of length, corresponding to this sequence which is of length N square by 1, that would be, that would be wrong; that would be incorrect.

So a 2D transform comes us a natural sort of a progression from 1D, simply as a Kronecker of the 1D unitary matrix with itself. And the whole idea is that it once, once you have set this all up then we do not have to spend additional time after each one of these, because then simply a matter of replacing DCT by an appropriate matrix. So we keep replacing A. If it is DFT, then we replace it with 5, if it is DCT, we replace it with something else. If it is whatever, a slant, we replace it with something else and so on.

And we also saw that just as we had here, u is equal to A Hermitian v, which made our entire task very simple because taking the inverse is like just to compute the, compute the Hermitian of the, of the forward transform. Similarly, here, yesterday we showed this. We showed that u is script A Hermitian. I hope we showed this yesterday, where script A Hermitian is simply the inverse of script A.

So all of this works out fine and then we also saw that the columns of this guy will contain columns of A Hermitian, contain the basis image. So A star kl in a vector form. Vector form in the sense that because you also ordered your image right in that form, therefore these images are also sitting in that order. And therefore, it is simply matter of taking, suppose you had an N square by 1 column, this we can imagine that is an N cross N image. So we just take the first N entries, put it as the first row; second N entries, put it as the second row, and so on. And then that gives you back your image.

So even though this may look like a column but you know that the basis images are all sitting as columns of A Hermitian. So all of this, if it has sunk in well, then we are kind of move forward.

Student: Do you have frequency response for 2D case also?

Professor: Same thing, same thing. So there also, just as you have a basis image, if you look at A, you still have not taken a transform. If you look at the DFT matrix, only after you apply it on some signal, you get as a spectrum, similarly here.

So script A contains, contains that if you look at A Hermitian, this is containing the basis images. So if you think of it as really a DFT, a 2D DFT then that will have, it will have a complex basis. I mean, if you go like above 2, then that basis is not going to be real but, but even in 1D that is true, you have a complex basis. And then again, you can kind of think about a magnitude spectrum, you can think about a phase spectrum just like you have in 1D. All that is the same the way you have it in.

Student: (())(06:35).

Professor: For an image, what is the interpretation? Oh, the interpretation is that how does your intensity change with your spatial coordinates. In time, we say how does the signal strength change with time. That is what we mean by, that is the interpretation of frequency in 1D. In 2D, there is no time notion, there is only X and Y. So it is like saying along the spatial coordinates, how does your intensity change. If it does not change anywhere, then it means it is like a kind of a DC image, if you want change it all. Suppose it is simply a constant all over, then it means it has, it has only a DC component.

If it has, if it has some variation, suppose you make it vary as a sinusoid, then it will mean that it will have a single tone, if it is something that is a combination of sinusoid then it will be same thing, whatever you do in 1D same thing. Just that the interpretation is, here it is varying with spatial coordinate. If you had a video, then you would say how does it vary? Not only with respect to spatial coordinate but also, but also with respect to time. So we will have omega, mu, and one more axis, psi.

Student: What will co-axis (())(07:38).

Professor: What you do?

Student: What will the co-axis (())(07:41).

Professor: Where in the?

Student: After the Fourier transform, (())(07:44).

Professor: Well, we have not even reached the Fourier transform yet. The Fourier transform, so the omega, suppose you talk about it over omega comma nu, so omega will tell along X-axis what is a variation, nu will tell along let us say, Y-axis what is the variation of the say, intensity as a function of Y. And the omega will mean what is the variation of intensity as a function of X.

So, so see if you have a signal, for example, maybe I can show you some example on that, that will make it more clear. We have not even hit, hit actually DFT yet. Of course, I can see that you are inquisitive because that is something that you can map on to.

I mean now, simply as an aside, let me just ask this since, since we are on there is A Kronecker thing. In general, if I, yesterday I showed this property, we said A Kronecker B inverse is A inverse Kronecker B inverse. If A and B are not square let us say. If A is, A is actually a rectangular matrix and so is B, can A Kronecker B be square? A is rectangular, B is rectangular, can A Kronecker B be square? It can be, no. What kind of matrix will that be? A will have let us say, A is, A is not square so what, M cross N and N cross M. So something like that.

So M cross N, this will be N cross M. So now, suppose, so I am just trying to, trying to ask you can this be, can this be independently valid? I mean to say because if A is rectangular then, of course, you cannot takes its inverse. But A Kronecker B it looks like it is a square now if I choose like that. Now, will this mean that I can say invert A Kronecker B?

Student: No.

Professor: No. Why? What will be the rank?

Student: Minimum of (())(09:44).

Professor: Minimum of, exactly. So the rank of this guy is going to be M N, I mean max of M N. We will say A Kronecker B is of size mn cross mn, so it can only have a maximum rank of m into n.

This guy's rank, suppose you assume that, suppose we assume something, m is, let us say larger than n, then this guy can have a maximum rank n, this guy can have a max rank n, B. So the rank of A into rank of B is n square, whereas mn is, of course, a number which I have, now which is actually, wait a minute. So what happened? So mn, mn is number which is actually larger than n square and rank of A Kronecker B can only be rank of A into of B, it can only be n square, it cannot be m into n; m into n because m is greater than n, it is definitely greater than n square, and therefore you cannot invert.

So, therefore, this condition, I just, just saw it. You might just want to kind of, I mean it has various implications. If you just look at it a little more seriously, it is an interesting property.

And by the way, some people, for those of you who might be interested there is also something called, what is that? It is an, so they solve it as an optimization problem, it is called, for those of you who are interested it is called, I think it is called a Nearest Kronecker Product, Nearest Kp problem.

So it is like saying suppose I give you, anyway, I mean this is not a part of this course and all. I am just saying, suppose I had a matrix A and suppose I wanted to, wanted to do a decomposition such that norm of A minus let us say C Kronecker D, I want to, I want to find out matrices C and D such that the Frobenius norm of the square of this is as small as possible.

These are the kind of problems that these people actually solve. I mean, is there a unique way to do this? In fact, singular validity decomposition and all of these tests be will typically use. In fact, people who are rank 1 approximation, if you instead of a matrix, if you had a vector here, then it will be like a rank 1 of approximation. Yesterday we said that A star kl, somebody said it is rank 1.

So you can, in fact, have instead of a matrix suppose you had a some, some A Kronecker B, where that is A, now suppose you said that if was a vector, so we had A into B, B sort of transpose. Then you can ask if you do something like this then, this is a rank 1 approximation. So you can use SVD, you people know that SVD can be used for a rank 1 approximation and also some people, so around this, there are so many other things for those of you are interested. So we will move on, we will take up a DFT now.