Image Signal Processing
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Shot Noise
Lecture 5

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Here in short noise is kind of is inherent is basically inherently there as soon as you, as soon as you capture. While you capture short noise automatically enters in, it is not like something it is not like an external phenomena. It is part of the capturing process, a part of the photon capturing process it is a part of the photon capturing phenomenon itself. It is a part of the photon capturing phenomenon. What does that mean?

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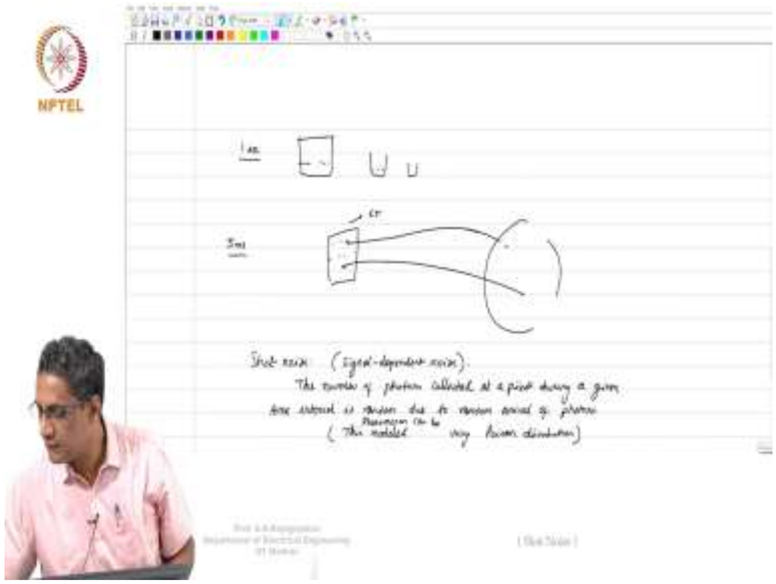

It what actually means that so it means that, you know so it is like saying that if I had a bucket and the let say the rain was falling and suppose I had a large bucket and suppose I waited for 1 second to find out how many water drops I can collect.

Then if I have a large bucket then ofcourse I might be able to say collect something this is kind of reasonable. But suppose I shrink the bucket size make it smaller the same intensity of the rain, and the rain intensity let say roughly the same I can wait for 1 second then you will ofcourse naturally assume that. Because there are many drops that are fallen outside now.

So then which means that you might when actually a volume which is much lesser we try say reducing the bucket size further and so on. Then it means you are going to send it you know, we are going to see fewer and fewer number of droplets. The same thing is what will also happen when you have these photons.

Because what you have is a grid right, on which all your sensors are sitting and here is your 3D scene from where all the rays are impinging. On the sensor, so what really happens is you wait for when you have and if you have seen a camera it has a certain exposure time. So the exposure time unfortunately you cannot make it too high because one it can saturate. If you wait too long you might just accumulate too many things like here, it says saying that if I wait for too long then I might actually fill this entire thing bucket itself.

But that may be okay if you are doing simply doing rain drop kind of thing but in a real application where you have an image which you want to see if you just over expose then what will happen is you will end up actually saturating. If you are ofcourse under expose like I said the other day then ofcourse you may not actually gather enough number of photons.

So normally right you will have a certain exposure for this cameras, okay typically it is some you know, it is in the range of it goes from milliseconds to let say a few seconds depends upon what kind of scene you have. Especially if you have low light you will go for higher exposure time, if you have if you have a brightly lit place then ofcourse you will go for a lower exposure time.

But then the point is this, so when these when these rays are coming and impinging let say red this particular sensor element. Then depending upon the intensity of this point in this is scene if this happens to be a bright a point, then ofcourse you expect to see more number of photons arriving.

If it is some other point in the scene which in this kind of relatively dark then you expect that you may not gather. So probably this is falling somewhere else and then you are gathering fewer number of photons. But the point is this if you wait for the same amount of time that is the even if here I have an exposure time let say I have a file let say a millisecond exposure time which is fixed for this camera.

Then when I take when I take a certain sort of some picture with that 5 millisecond assume that the scene has not changed, same scene nothing has changed, the sensor had not moved nothing we just keep the entire apparatus fixed. And suppose I take an image and suppose I record the intensity here let us say on a scale of 0 to 255 grey levels it records let us say 65 as the intensity.

Now if I after about after a while if I again do the same process assume that the scene has not changed it is all under our control. We have not moved at all, the same exposure time 5 millisecond if I wait and suppose I see what do I get you may not get exactly 65. You may get 62, you may get 67 you cannot guaranty that say number of photons that you gathered with respect to which you counted a certain density will always be the same.

Because the arrival of photons is actually a random phenomenal, okay. So which is why what happens is and this uncertainty right that you have in terms of the number of photons which you can gather that uncertainty will be its the effect of that uncertainty it is going to be higher and higher as your signal strength falls. Which means that you get a darker scene you will that this uncertainty is going to have a larger effect.

Let us compare to the case when you have a scene which is actually brighter, okay. So which is something we would like to analyze in a kind of say little more detail. So I am going to write down a few things. So as far as short noise is concern, so as far as short noise is concern what we can say is it is actually a signal dependent noise. This is unlike your AWG and all where typically assume that the noise is independent of the signal it is identical in nature and all of that.

There is normally something that we keep doing often. But as far as the light the image formation process is concern short noise is not of that kind. Short noise is signal dependent noise which is what actually makes it harder to even remove.

For example if you had a signal short noise affected image and suppose somebody run an algorithm so that we can at least clean it up a little bit and so on. It is bit hard signal dependent noise signal dependent noise, there are various kinds of noise by the way this is one such type signal dependent.

And then what we want to say is that the number of photons the number of photons, the number of photons collected at a pixel collected at a pixel during a given time interval which is typically our exposure time. When we say given time interval that is typically our exposure time is random, this is because the arrival of photons due to random arrival of photons. And therefore, this can be modelled this phenomenon can be modelled using Poison distribution, this phenomenon can be modelled using a Poison distribution.

Now let me just, so if you just see the map behind this we will just do little bit of map just to understand just to have a formal understanding of what is going on.

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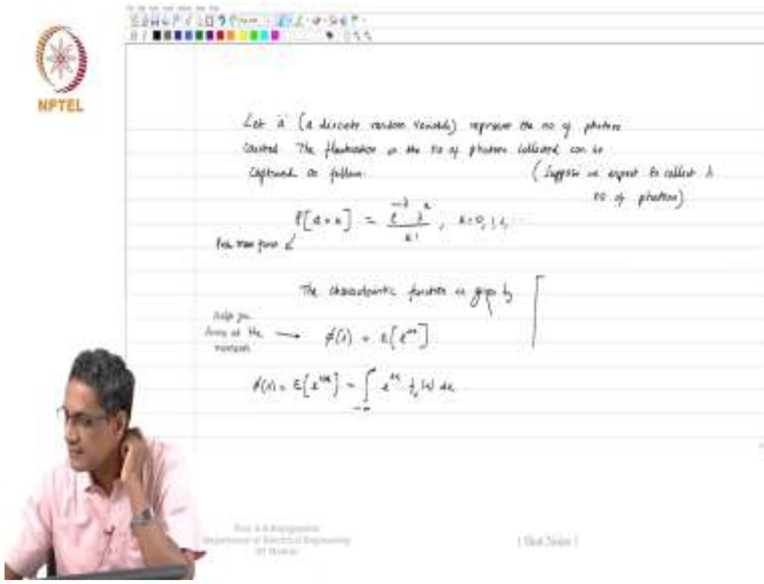

So let a, right a discrete random variable let a, a discrete random variable I may just say rv from now on discrete random variable represent the number of photons counted, counted, collect whatever you want to say. Then the fluctuation or the uncertainty, the fluctuation in the number of photons collected can be captured can be captured as follows.

So what is that mean so suppose we say let P of probability of a equal to k suppose let say. Now suppose we say that suppose we expect to collect let me say suppose we expect to collect lambda number of photons on the average expect to collect.

Okay so it that means that for that particular sensor element for that particular pixel suppose we assume that we expect to collect on the average lambda number photons then P of, P of a equal k so P is really a probability mass function in this case, okay this is a probability mass function. So P of a equal k as you know will be given by a Poison distribution that is e power minus lambda, lambda power k by k factorial. And k going from this is a counting process so 0, 1, 2 and so on.

Now so it actually means that on the average you expect to collect lambda but then you are trying to compute the probability that what will be a equal to k where let say k is some let say k is number a number of photons. Probability that you will actually you will actually collect k number of photons when you expect to collect lambda number of photons in the average. That is what this expression is telling.

Now if you look at the characteristic equation or the characteristic function is given by I do not why that happen it is given let say let call this phi of s which is what is a standard sort of notation which is expectation e power sk. If you had really a continuous case as soon as sight if you had a continuous case let us if you had phi of s is equal to let say some expectation e power sx then this is will be a capital.

Then you would have e integral minus infinity to infinity e power sx fx fx dx this what you would have. Now in this case what we have really you know really a discrete case. And also if you look at and ofcourse you know and when you have a characteristic function like this it also helps you to arrive at the movements, if they ofcourse exist arrive at the movements may helps you.

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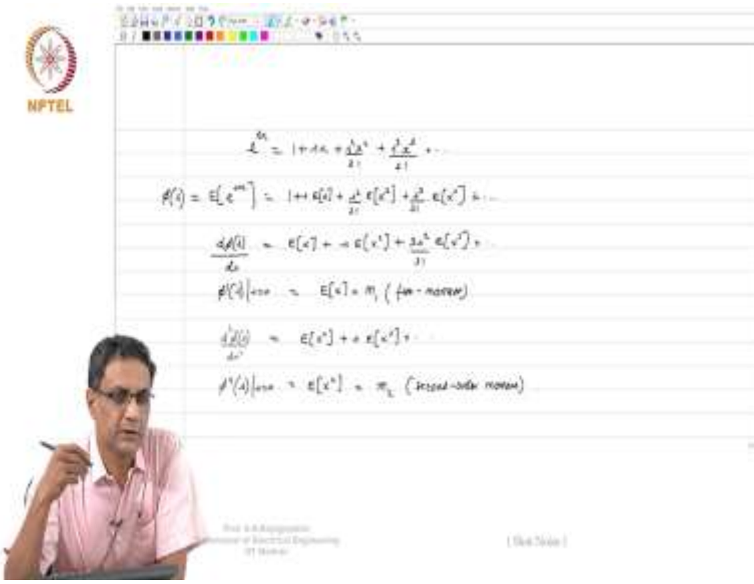

So in this case so for example in general if you had something like why do we say that, okay that it will help you arrive at the movement so whether because if you do e power sx. And when know that a Taylor series expansion of this is 1 plus sx plus x square what it is x square by 2 factorial plus s cube x cube by 3 factorial and so on.

Now if I take the expectation of this then that small (())(11:24) this x should be ideally random variable should be capital. So 1 plus s into expectation of x plus x square by 2, now everywhere assume it is really a capital x because I have written it as small x. So expectation of x square plus s cube by 3 factorial expectation of x cube and so on.

This is your phi of s this is what we call as your phi of s. And you can actually clearly see that if I do d phi by ds, right if I do d phi s by ds what do I get? So the first is a constant goes away this is s so I get expectation of x and then I will get the cv like what is it s times e expectation x square.

This will be 3 s square expectation of x cube by 3 factorial and so on. Now if you evaluate phi dash which is d phi by ds at s equal 0 then you get. Then the second term will drop off all the subsequent terms will drop of because they have either s or higher orders of s. Therefore what you get is expectation of x which is really the first movement of this variable of the cc random variable so first movement.

Similarly right if you are interested in the second movement then what you will do you will do d square you will do the the second derivative d2 phi s by ds square which is. So expectation of x is a constant so that is not dependent on s that will drop away we will get expectation of x square and then here you will get 2 into 3 therefore that will cancel of your 3 factorial you will get s into expectation of x cube.

And then ofcourse some other terms, okay for which you should have found a derivative. Again evaluate phi 2 dash of s at s equal to 0 that will give you this guy will drop of, so all the higher order terms and then you will get expectation of x square which is nothing but m2 which is the second movement, second order movement and so on. Which is also the reason why we call this is an MGF a movement generating function. Because once you know phi of s then you can, now if this movements exists then you can actually compute them.

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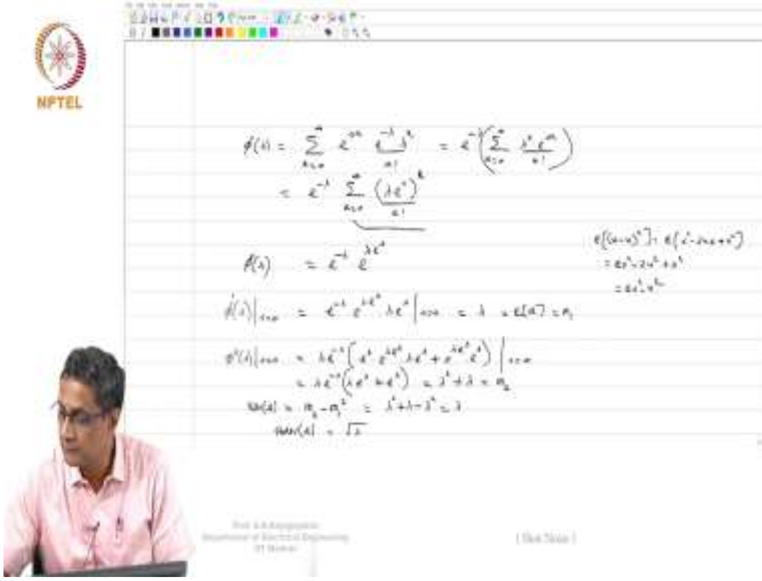

Okay now in our case since it is really a discrete case let us go back and try to work out phi of s for our situation which is a short noise situation. So there it will be a summation we do not have an integral because we have k which takes this discrete values. And therefore k goes all the way from 0 to infinity and then we have what is it e power sk and then e power minus lambda, lambda power k by k factorial, this is what you will have.

And lambda ofcourse is independent of k so therefore we can pull out this e power minus lambda factor and we will be left with k equal to 0 to infinity this is a multiplying what is this e lambda k e power sk by k factorial. And or in other words we will have like e power minus lambda summation k equals 0 to infinity.

Then we can write this is lambda e power s the whole power k by k factorial. Which then means that which then means that you have a form it is like k equal to 0 you will get 1, k equal to 1 you will get lambda e power s, k equal to 2 lambda e power s the whole square by 2 factorial and so on. So again so this entire thing will converge to e power x. So you will end up with e power minus lambda e raise to x in this case will be lambda e power s.

So e raise to lambda e power s is what you will get if you do. So this is the entire sum here will simply give you the term it will be a close form simply e raise to x where x is lambda e power s. so this is your phi s. Up to this point it is okay? So now if you are interested at the first moment then ofcourse what we can do, is we can do the same thing as we have said, we will do phi dash of s and evaluated at s equal to 0.

So if you do phi dash of s first of all you will get e power minus lambda and then e raise to lambda e raise to s into lambda e raise to s and this needs to be evaluated at s equal to 0. So what you get here e power 0 is 1 so this guy is 1.

So e power minus lambda e power lambda will cancel off and then you will be left with lambda, which is why I said right at the beginning that if you are expecting on the average lambda number of photons because the mean of this random variable a is actually a lambda.

So which is another way to say that expectation of a is lambda this is nothing but your expectation of a, okay or in other words for our problem, right in our own physical interpretation what it means is on the average we expected to see, expected to be able to see lambda number of photons to count, we expected to count lambda number of photons. But then probability that you will end up counting k number of photons is that e power minus lambda that expression.

And then if you are interested in phi 2 dash of s that also you can find out equate evaluated at s equal to 0. So it is like lambda e power minus lambda we can pull it out and then inside will be e power s. And then e power lambda e raise to s into lambda e raise to s plus what is it e raise to lambda e raise to s into e raise to s and evaluate this at s equal to 0.

If we do that then what you will end up with this lambda e raise to minus lambda and then this is e raise to lambda because it is all my s is not very clear. But now, this is e raise to s and therefore you will get a lambda also. Lambda e raise to lambda plus here this is 1 e raise to lambda. Which then means that you will get, what you get lambda square plus lambda that is your m2, okay that is the second moment.

And we all know that if you are interested in the variance of a, so variance in terms of the first and the second moment is what m2 minus m1 square, you know that right. So it is like saying that expectation of suppose we had some random variable x minus if the mean was mu then this is we know our expectation x square minus 2 mu x plus mu square.

This is like expectation x square minus 2 times, so it will be mu square because you will get expectation x into mu expectation x is already mu plus mu square. So this is expectation x square minus mu square. So which is expectation x square is m 2 mu is m1, so m2 minus m1 square. So what was, what is our m2 in this case is lambda square plus lambda minus m1, m1 was lambda, this is m 1.

And therefore, you get minus lambda square which is equal to lambda or in other words the standard deviation that means your deviation about the mean value will then be let us say standard dev of a is equal to root lambda.

So it actually means that, means if your signal strength is lambda then your see noise strength is actually square root of lambda. So what it means is if you are actually expecting to let us say.

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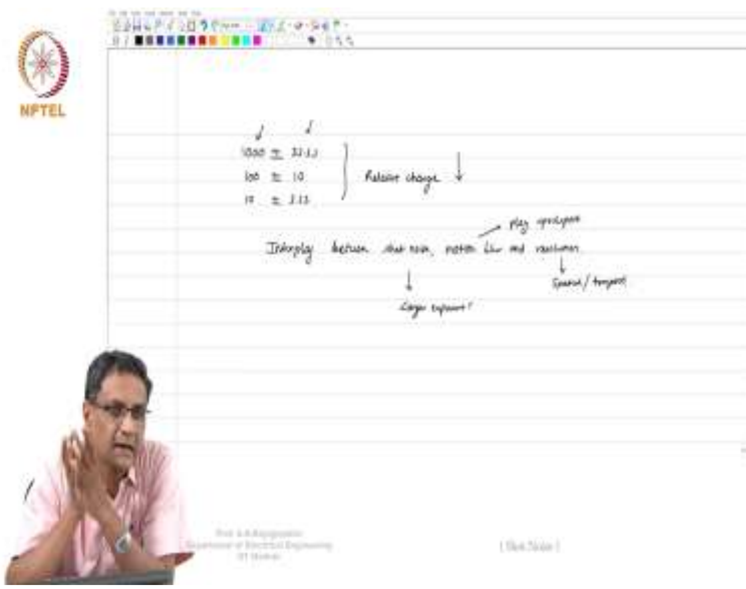

So if your lambda is expected to be around say thousand, suppose let us say on the average for a particular location, those all like location dependent. Because it seem characteristics to be varying all over. So at a location if you are thinking that you are expecting to gather 1000 photons very likely that you will gather something like 1000 plus minus root of 1000 which is like 33 what is that 33 something, so 33.33 whatever.

Now if you are expected to gather only 100 photons that means you have a slightly darker scene now because you have only about a 100 photons again for the same time this all time is kept fixed. Suppose we expect to gather only 100 photons then it means that you are uncertainty will be 100 plus minus root of 100 which is 10.

If you still have a much more darker scene then it means that you are looking at some like an average of 10 photons let us say this is just numbers.

10 then plus minus 3.33, what is this indicate? This actually indicates that the if you look at the relative change that you are expect to see is actually a relative change is lowest here and then quite keeps increasing as you go down. Lowest because here I mean on 1000 a 33 change does not appear to be so very significant.

But then a 10 on 100 looks a lot more significant and then and our 3.33 on 10 looks even more significant. That is why on the one hand you have the signal strength here and then on the other hand you have the noise strength in this root of lambda and you can see that the, which is why we call it a signal dependent noise.

Because depending upon the signal strength your noise strength keeps varying. If your signal is very powerful we have a very brightly lit scene then very unlikely that you will find you will have issues. Because you are already taking in a lot of photons there may be some uncertainty right about that value but that is okay you can actually live with it.

But if you have a very dark scene then you will end up with and especially in dark scene I forgot to tell you yesterday that when I showed that the flower from the flower vase I had shown it said that an also especially if you want to notice noise you should go for homogenous regions.

Should look at regions right where there is smoothness, because when there is a lot of activity it is hard to make out what is going on. But then if you go to places where things are smooth okay then it is like saying you now if there was no noise you would have found that to be a kind of a uniformly nice region, just like a wall for example. You might just want to see that all intensities are roughly the same.

But then if there is low light then what will happen is when he tries to estimate those values I will be this uncertainty because of low light and there you will see the noise activity much more rather than looking at regions that have a higher activity. Now that is the notion of the, now there is actually there is a nice interplay by the way between there is a nice interplay between short noise, motion blur and the resolution.

What this actually means is so for example I mean if you say that you have an issue which hard night let us say your scene is dimly lit, so therefore short now is an issue. Now you might say why not it is just wait large wait for a longer time. So which means that can I kind of go for a go for a larger exposure can I use a larger exposure if you are lucky you can. The sense that if the scene does not there is nothing moving in the scene or if your camera is perfectly still not or when both happen the camera is perfectly still and then your what do you call the scene is like unchanged.

Then in that case we can wait we can wait for a longer time wait for say more number of photons to come and therefore right given a larger time you can integrate you can really expect to see more number of photons. But what will play spoils sport is this guy, this is one that will play spoil sport because what it will do is if there is some motion of any kind.

Then what it will do is it will so this so you will your image will start to look blurred you may be gathering intensities. But then because of the fact that for that long a scene may not remain constant it could or may be your hand is not able to hold the camera still for that long.

Therefore, whenever that is why night photography is always stuff because you have to wait longer and then during that time you are really hoping anything do not dynamically change. So and the other factor is that so and then the other factor if you feel that if I want to go for in fact the spatial resolution is not the only thing that gets affected by short noise. This resolution both spatial as well as the temporal they both are actually affected.

Because of the fact that temporal may be if you want to go for a higher frame rate let say I have a camera which does a 30 frames per second I want to go for 1000 frames per second. That means the amount of time that I am waiting to capture has already come down no, because in 1 second we have to now 1000 frames. That means amount of time that I am waiting to capture the photons is already much smaller now.

Which then means that is why rate if you watch actually a video most of these cameras rate when they when you capture a still picture it looks very great. But then the movement you capture a video and especially if it is a eye frame red camera we will start seeing that your images are not any more all that good. That is simply again due to short noise nothing else, fundamentally there could be other factors I am not saying there is nothing.

There could be other factors in terms of other kinds of noise I can play are all coming from inside the system. But mainly your own ability actually to take in a given number of photons will kind of drastically come down. And that actually increase the uncertainty.

So anyway so this interplay rate we will again later as we move along but as we our saying that later when we talk about super resolution and all those things we will come back to this not now but after long time. So just for the time being I just wanted to say this.