Image Signal Processing
Professor A.N Rajagopalan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture 63
Theory of Histogram Equalization and Modification

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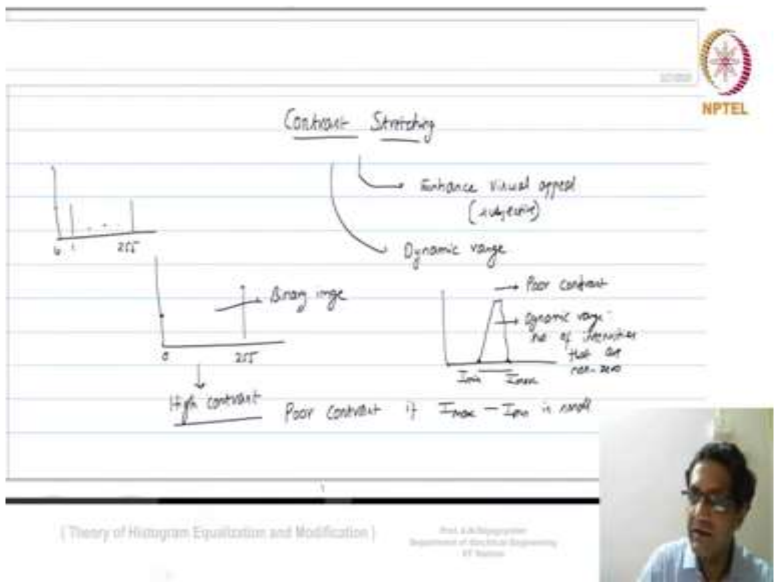

So the next topic and the image enhancement is what is called Contrast Stretching. Okay we all have some idea when we say that you know we want to improve the contrast but we would like to know what exactly it means in the image processing language. Now contrast is again done to enhance visual appeal. Now how much contrast should we increase or decrease in order to be able to normally a contrast stretching means improving the contrast or increasing the contrast so to in order to enhance the visual appeal.

Now how much should you do is again a subjective matter, some people may feel that something is the contrast is being increased too much whereas some people might be happy with average contrast and so on. And again this is a subjective issue which is why its right comes under image enhancement. Now there is a difference between contrast and dynamic range, these two are not the same, okay.

There is a difference between the two and this can be explained by examining the histogram as an image. If you compute the histogram as an image and suppose it turns out that it is very narrow. Let us say what that means is if you have a max intensity Imax and if you have a min intensity which is Imin okay, this is Imax that is Imin, then you say a poor contrast right, we will conclude that you have a poor contrast if Imax minus Imin is small, if Imax minus Imin is small okay.

On the other hand right suppose you have an image that just had let us say right two gray levels something at 0 and something had to see 255 then we would say that such a thing has a high contrast, this would have a high contrast. Let us know I suppose to this has a low contrast but then the dynamic range of this is higher, this has a higher dynamic range as compared to the one on the left okay simply because so dynamic range is the number of intensities that are actually non-zero is in the image.

Okay so ideally that if you had an image that would occupy all the all these values right when suppose you have an image that would occupy everything down from right from 0 to 1 to 255 right then such an image would have a very high dynamic range as well as a high what is contrast which would be probably the most ideal thing which we would like to have. But right this would be something like a binary image.

So a binary image we would say has a high contrast but then in terms of dynamic range it has a very-very limited dynamic range. This could have a reasonable dynamic range but then right this histogram indicates a poor contrast okay. Now in order to understand what we really what these things, now what is in translates to okay I am going to show an example.

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Okay let us just look at the situation where in let me see okay when you see this image you will understand right what we are trying to say. So if you the first picture right this image is considered to have a contrast that is low, poor contrast. If you plot the histogram where you see that most of the values are in and around this range whereas okay, so in that sense the contrast is very limited.

The dynamic range probably is not bad but then I mean it has a contrast you know that is really poor and the whole idea behind contrast stretching is to be able to take this image and apply some kind of a transformation on it. So as to be able to right you know get an image that would probably look like this. Now in this case what has been done is what is histogram equalization.

The idea is that strictly speaking theoretically speaking, histogram equalization actually means that you take this input image and transform it in a manner that the histogram of the output right will occupy all the levels right it will have all intensities in it and all of that should occur equally. Now in this case we see that when that may not be the case but then definitely the histogram is now been stretched so the contrast definitely has improved, okay at the outpt.

Now the rate between actually a discrete approximation when we take this and then a continuous theoretical case that there is going to be a distance which I will explain down the line but for the time being just look at the image you definitely appreciate that something like this is definitely better than actually seeing an image like that right so you have improved the contrast.

Now the output is histogram whether the output of histogram is flat whether it occupies all the intensity levels or not why that is not so and all right we will see later. This is all because of a discrete approximation.

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Now following this the histogram specification or histogram modification is not the same as histogram equalization. In histogram modification what we say is suppose we take Lena image, image of Lena we know that this is an image that will say one of us appreciates and then we all know that it has a good contrast right and it has nice features good contrast and so on. Therefore if you plot the histogram of Lena it looks somewhat like that. Now if you take the same landscape image that we had earlier which had a histogram like this.

We could either equalize it which is what I showed in the earlier slide or you could say that change this image such that the new histogram would look like the histogram of Lena. Okay so basically that this modification. So it may not be Lena sometimes where you might even have some variance but whatever image that you have a reference image you have a reference histogram which you think is good.

And therefore right you want to modify the, modify your input image in a manner that after modification the new transformed image that is what is being shown here would have a histogram that would kind of that would obviously mimic the histogram of Lena in this case right. So this is called histogram modification. Now both of these problems right are kind of are very tightly coupled.

And therefore now we will try to see propose a method it is going to be which is a unifying method that would that you could simply use it for either equalization or for (())(06:43) or for the modification purpose okay. Now how we do this I mean how should we go about doing like this.

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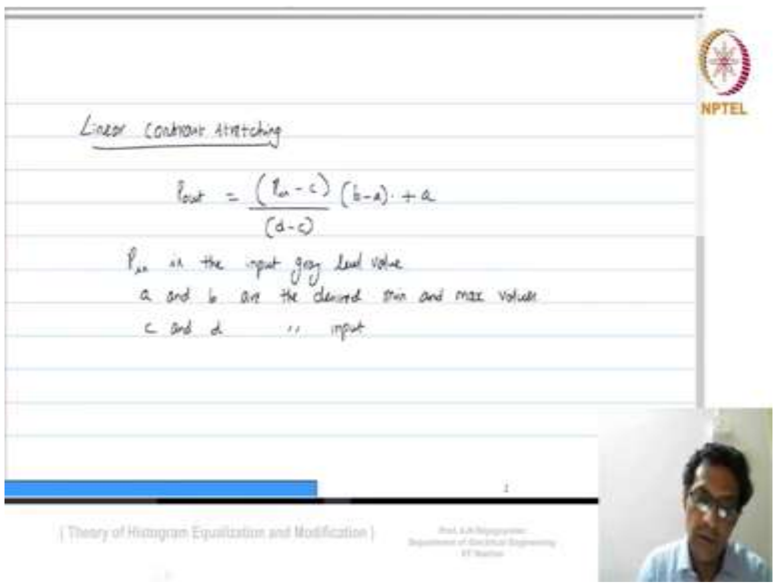

Now before we actually plunge into histogram equalization let me just mention that okay let me just mention that there are also other ways of doing into histogram stretching what is called some simple techniques before you even plan to write and try, before we even talk about how exactly should we do histogram equalization and histogram modification.

There is something called linear contrast stretching. These are very simple methods but then they are not as effective as let us say histogram equalization or histogram modification. So in a linear contrast stretching what do we do, we say that the output (itself) so the input intensity should be mapped to on output intensity Pout such that Pout is equal to Pin minus C upon D minus C plus into B minus A plus A right.

What do these things indicate? So Pin so out here Pin is the input gray level value, this input gray level value. Now A and B are the desired min and max values, min and max values that means after you histogram equalize you would like you know A and B to be the min and the max values respectively C and D on the other hand right are the input, the min and max values of the input min and max values of the image.

Okay that means whatever is the min and max value of the input image, if your Pin is equal to C then ofcourse Pout maps to A which is the minimum value of the output image. If Pin is equal to D then D minus D and D minus C cancel off and you get Pout is equal to B which is the max value of the output image. So one could do something like this it is a simple method okay it will give you some result.

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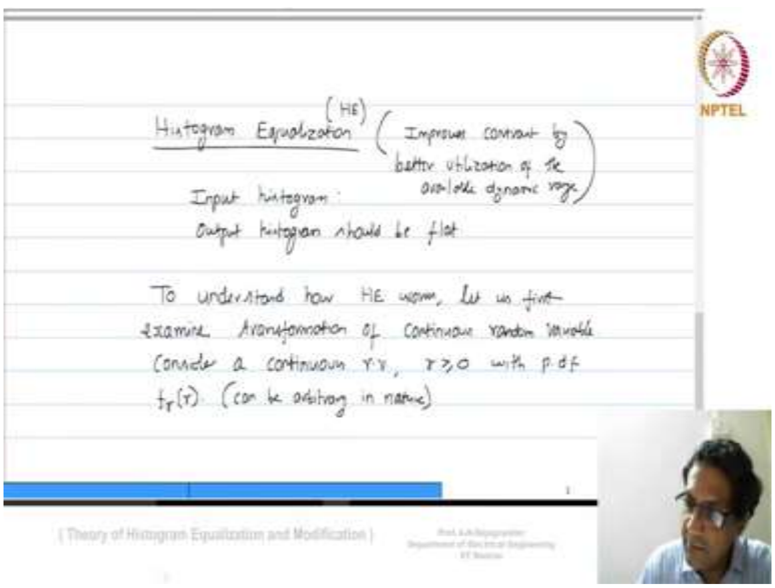

But then the most standard way right in which you want to do contrast enhancement is what is called histogram equalization. The other one is ofcourse histogram modifications. So we will start with you know histogram equalization okay equalization. Now what exactly is this notion of histogram equalization? So what it says is that you know your input histogram could be whatever it is.

So your input histogram could have whatever levels it has and that could have whatever intensities it has. The output histogram so given the input histogram the output histogram should be flat, should be flat that means it should have all intensities in it and then it should have all the intensities occurring equally okay.

Now the whole idea behind this is it improves contrast so great we can kind of in a sense right say that it improves contrast by better utilizing or for better utilization of the available dynamic range. I will tell you what really I mean by this okay. This will become more clear okay as we go on. Now to start with it will take out, we will take up I will take up a continuous case to understand how this works.

Okay to understand so let us call this as HE which is histogram equalization. To understand how HE works histogram equalization works let us kind of let us first examine. Let us first examine transformation of a continuous random variable, examine transformation and summation of a continuous random variable okay. And we will come from there and then we will see right what will happen if you do a discrete approximation.

We will take a continuous case because in the continuous case when we can establish the conditions under which this should be done and then when you do a discrete approximation you may not really end up with the same result that the continuous case it would have led to but then your discrete method it is basically grounded on the this one continuous theory okay.

Now consider a continuous random variable let us say consider a continuous random variable let me write this as RV a positive random variable R greater than or equal to zero because since we are dealing with intensities we have this kind of a positivity condition with probability density function PDF given by FR of R okay. This is a reference PDF okay. This can be arbitrary in nature okay this can be arbitrary in nature right.

We do not have any kind of a condition on this, it can be arbitrary in nature of course it will satisfy all the conditions that a PDF is expected to satisfy otherwise it can be arbitrary right and what is what are we seeking?

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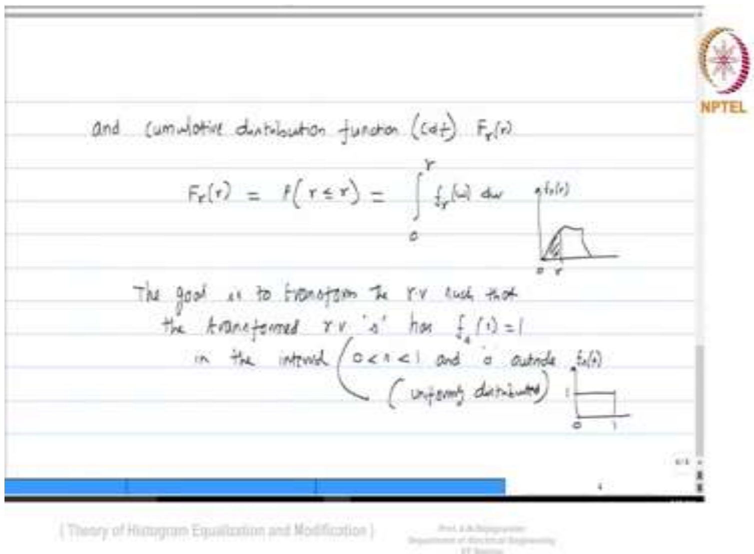

Okay let us say that and the know has a cumulative CDF right that is a cumulative this one a distribution function. Let us call it let us represent it as CDF, C D F capital F R okay. So where right so we know that FR of R simply means is equal to probably that R takes up the value less than or equal to R or this can also be obtained as integral 0 to R, FR of W BW right.

So this equals the area, the area under your is FR of R. So if your FR of R integrates to 1 and suppose you want to find out what is capital FR of R then you would kind of indicate the area under this PDF which is FR of R and right and then we would find out what is FR of R.

Now what is the goal? The goal is to apply a transformation, the goal is to apply transformation or is to apply goal is to is to apply a transformation such that is to apply a transformation or let us say you know goal is to transform the random variable such that the new random variable such that the transformed variable transformed random variable let us say S has FR of S, has a PDF let us call it FS of S is equal to 1 in the interval 0 less than S less than 1 and 0 outside okay.

So which basically means that if you like this to be uniformly distributed. So in a sense what we would like it to be is to go like 0 to 1 and 1 okay this is what you ideally would like your say FS of S to be and (())(15:15).

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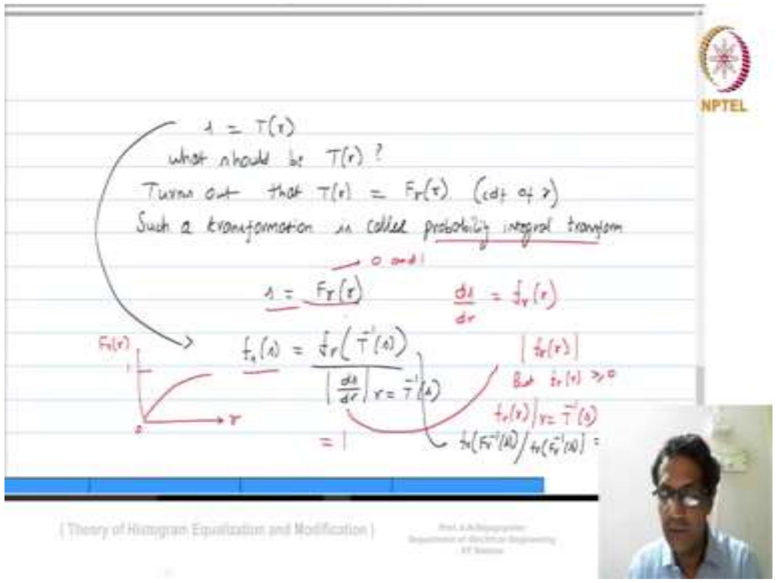

Okay now what should be that particular say transformation which we need to apply in order for this to happen. So you can think of you know S to be equal to some T of R and the and what should be T of R such so that what should be T of R so that you know so that FS of S is 1 right in the interval 0 to 1 and it is 0 outside. Now it turns out that TR the T of R is in fact equal to FR of R okay which is say CDF of R okay.

There is also called probability okay such a transformation it has this is a standard thing such a transformation is also called such a transformation is referred to as a probability integral transform, okay. So such a transformation is referred as probability integral transform so then what this means is that S equal to FR of R okay.

And then in general right if you had S equal to T of R then we would have said FS of S is equal to FR of T inverse of S right, we are looking at a scalar random variable divided by DS by DR modulus evaluated at R equal to T inverse of S right. Now given that S equal to FR of R right. So DS by DR is simply FR of R right and therefore this term becomes mod of FR of R however but FR of R is always greater than or equal to 0.

Therefore this mod has no implications here in this case therefore what you will get is FR of R evaluated at R equal to T inverse of S right, now provided you choose S equal to FR of R, provided ofcourse that you choose S equal to FR of R okay provided you choose this otherwise not okay therefore which is okay.

So when you do this then what you get is FR of T inverse of S and then you again get FR of T inverse of S or in this case it is basically FR inverse of S and therefore this is equal to 1. And because of the fact that S is FR of R and FR of R will always have values between 0 and 1 right. So FR could be something that grows 0 all the way to 1 okay this will be your FR of R okay.

If you plot it versus R and therefore right what this clearly shows is that you have an FS of S that is equal to 1 and then S lies S takes values in the range 0 to 1 okay which is what we wanted. So this probability integral transform is what we need to apply right in order to be able to say equalize the histogram or in order to be able to arrive at a final grade of a PDF that is going to be flat uniform.

Now this would be this would be right at theoretical result, this would be a theoretical result well if you wanted it to be even more clearer then you can in fact write this as FR of FR inverse of S divided by FR of FR inverse of S okay which is equal to 1 because T inverse in this case when you choose the special case of S equal to FR of R this what you will end up.

Now in reality, so in reality ofcourse whether you get a flat histogram or not when you have a discrete image, discrete set of intensity values what will happen if something that you would like to wait and see.

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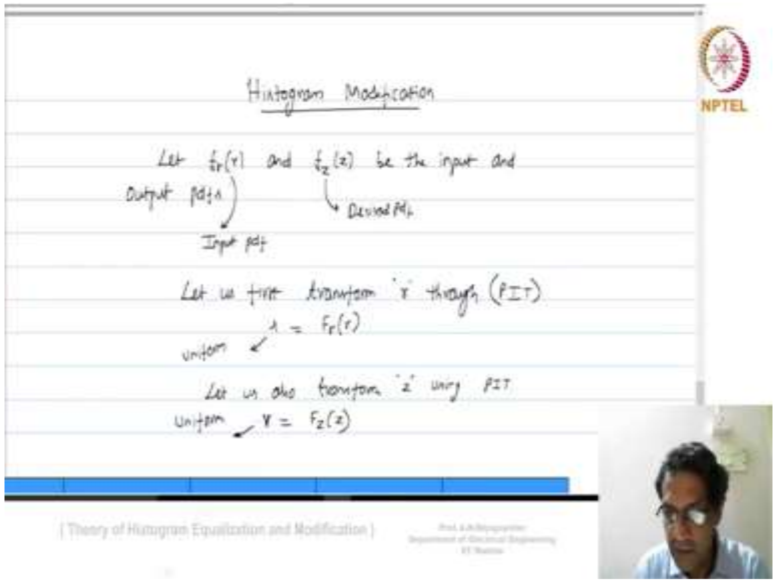

Now instead of histogram equalization suppose what you want to do is histogram modification right that is just a small step away. So histogram modification you can again write along the same lines and we can in fact have a unified kind of right now a method which we could use for both histogram equalization as well as histogram modification. Now modification what does this entail? It says let FR of R and FZ of Z be the input and output PDS that means FZ of Z is simply a desired PDF.

Whereas this FR of R is your input PDF so you want to again transform it you want to transform random variable R to random variable Z such that Z has a PDF FZ of Z. Now in order to do this transformation what we will do is the following. Let us first write a transform R through PIT, this is first transform. Transform R that is the input random variable through PIT which is simply probability integral transform.

Let us say that we get so suppose we do S equal to FR of R so we know that S is uniform and so on and that we have already shown. Let us also transform Z using PIT which then means that let us say that we get some other variable let us say V, this is not R, this is V and let us say V is FZ of Z . So now V is also uniform since you have applied the PIT on Z okay. So you get V is also uniform, S is also uniform.

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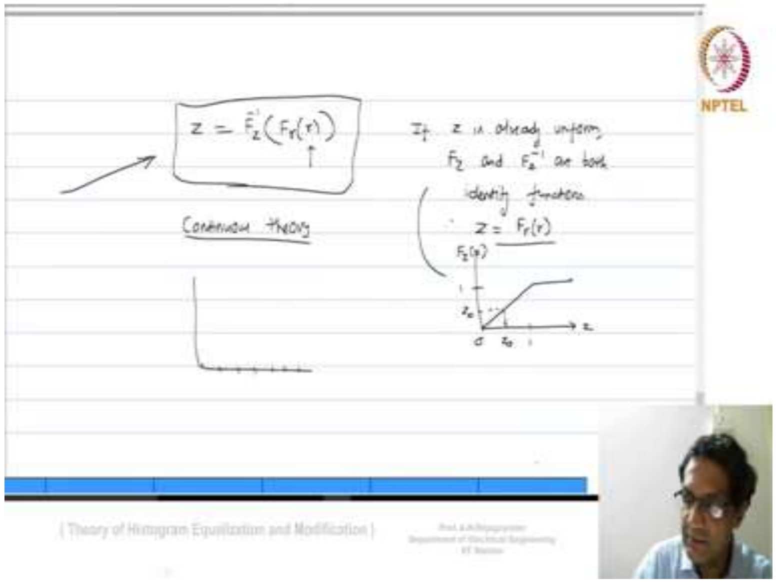

So what this means is that now if I want to know what transformation should apply in order to get Z, so Z is equal to FZ inverse of V but then since V, since both V as well as S are uniform instead of substituting, so instead of putting V, I might as well put FR of R right, I might as well put S which is FR of R okay.

So it means that if I start with an input random variable R, I should do FR of R, the CDF and then I say do perform FZ inverse on that in order to be able to get this transformation right. So if you want to do a modification this would be the rule right that you would have to apply. Now in case it so happens.

Now why I said that both histogram equalization and histogram modification could be done under the same rules is because if let us say if it turns out if FZ is already uniform okay Z is already uniform in the sense that you know that means to start with, this is already uniform then we know that FZ and FZ inverse are both the identity are both identity functions.

And therefore your Z which should be your transform variable should be simply FR of R which is what we said we have to do if you want to do histogram equalization. So that means when so the point is so right so the point is if your final Z is actually uniform, if it is already if you are not asking for something else and if you simply asking for a uniform random variable then the say transformation which you seek in order to kind of get FZ that will be uniform will then be FR of R.

So yes right this is easy to show because if you see the CDF of the uniform random variable right then we will go like that and then it will hit one right. And therefore you have got like 0 to 1 and then this will be 0 to 1 and therefore anywhere you pick a value let us say Z naught then that will take a value Z naught here and therefore FZ of, so if you look at it as, if this is FZ of Z then versus Z then FZ of Z naught is Z naught and then FZ inverse is Z naught is also Z naught.

And therefore this is an identity mapping which is why Z will then be. So the transformation that you need to apply so as to make so as to transform R into uniform random variable will be simply FR of R okay this is what you are see, this is what a continuous theory says. But then in reality what we have is actually a discrete grid but we have only a histogram which exists only at certain on the also we have a discrete set of intensities. And therefore a discrete approximation is what will happen.

Now because of that certain things will not happen the way you would actually ideally like them to happen. But still the implementation when you have a histogram implementation that follows this law but then at the same time right you have to make certain approximations in order to be able to arrive at the final result okay, I will take an example in order to explain this is, okay this is best explained.