Image Signal Processing

Professor A.N Rajagopalan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture 64

Histogram Equalization Example

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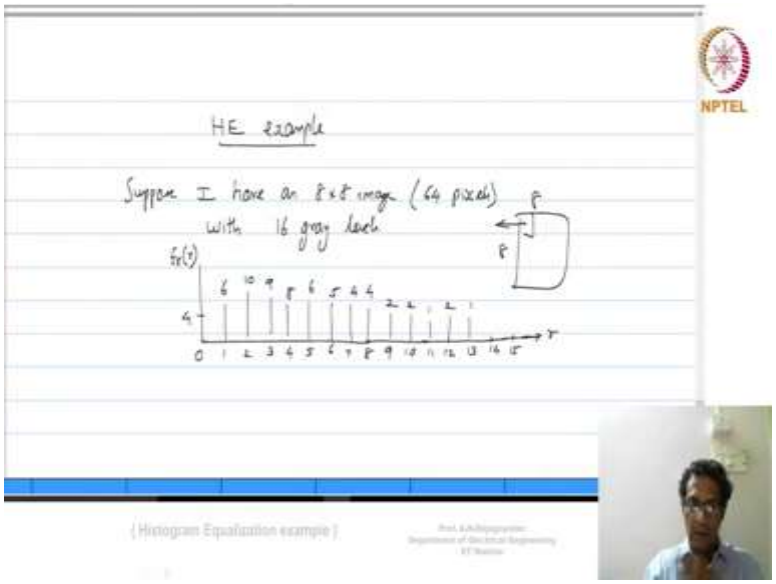

So here is an example, Histogram Equalization Example okay and you can follow the same this one same approach in order to do modification also. Suppose let us say, suppose I have an 8 cross 8 image that is it has 64 pixels in it and with 16 intensities, gray levels okay. So I have this image which is kind of 8 cross 8 okay.

And then each of these could take any value between 0 and 15. Now suppose I plot a FR of R, let me call this with an abuse of notation okay this is so this will be like your probability mass function but they are not going to normalize anything okay. And this is going to take the intensity values as they are because that is the simplest way to get us do this.

So right there so somebody tells you that histogram of this input image looks like this. So 0 occurs 4 times, 1 occurs 6 times, 2 occurs 10 times, 3 occurs 9 times, 4 occurs 8 times, 5 occurs 6 times, 6 occurs 5 times, 7 occurs 4 times, 8 occurs 4 times, 9, 2; 10, 2.

So these heights are not really indicative of the actual values, the actual values I have already put on top, 10 to 11 1, 12 to 13 1, 14 none in fact 0, 15 also none okay, 15 is also 0 okay. So this is your R, okay now let us simply compute the CDF for this.

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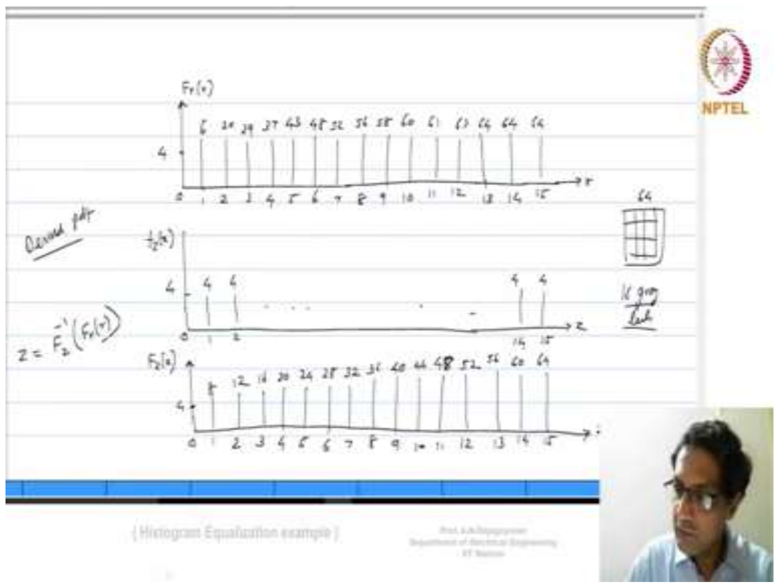

So let us plot what our FR of R will look like. FR of R versus R. So at 0 if you compute the CDF it will be 4 and at 1 that will be like 4 plus 6 which is 10, at 2 it will be 10 plus 10 that is 20 and then you go on.

At 3 it will be 29 okay now this may have to be extended further. You may have to go further down, so 3 and then 4 okay you can simply check this 4 is 37; 5 is 43, 6 is 48, 7 is 52, 8 is 56, 9 is 58, 10 is 60, 11 is 61, 12 is 63 and 13 is 64 right since you have 8 cross 8 right. So we end up with 64 as FR of 13 then 14 since 14 is if you do not even occur once even once so 14 you have FR of 14 to be 64 and then 15 again it stays at 64 right, so this is your FR of R.

Now if you look at what is really at Z. At Z PDF okay so a desired one a desired PDF is something right that we want which will be like Z of Z again. This is a desired PDF and we can say that what we want is at zero okay it should be because right you got 64 right of them. So that actually means that so it means that each intensity should occur and since we have 16 gray levels and if each of these grey levels should occur right and now should occur uniformly this is the ratio of occur you know equally it will mean that each of these grey levels should occur 4 times.

So means 0 should occur 4 times, 1 should occur 4 times, 2 should occur 4 times everything should occur like 4 times until get to 14 should also occur 4 times, 15 should also occur 4 times so you got 16 grey levels each one of them should occur equally 4 times. So which then means that if you try to plot FZ of Z right the so this CDF that will be very simple.

It is not exactly CDF I am not really normalizing okay. So that kind of makes our task easy. So 0 is 4, so 1 is 8, so 2 will be so 4 plus 4; 8, 4 plus 4 plus; 12 and then it goes on 3, 16 4, 20, 5 will be 24, 6 will be 28, 7 will be 32, 8 will be 36, 9 will be 40, 10 will be 44, 11 is 48 let us go on. 11 is 48 let me just probably straighten this up a little bit.

So 11 is (40) what is it 48, 12 is 52, 13 is 56, 14 is 60 and 15 is 64 right this is FZ of Z. Now if this was not uniform and it was something else it is somebody gave you some other FZ of Z which need not be uniform then again you would have found out CDF. So which is why I said that the law right it is still the same. So Z is equal to FZ inverse of FR of R would be you know equally applicable whether you are doing histogram equalization or you are doing a modification.

So this modification then there is FZ of Z would not be flat like this it will be something else some arbitrary thing which should be a desired PDF and then rate you would have to compute FZ of Z with respect to that. Now if you see our transformation law what we are saying is taken R so for example if we take R equal to 0 find FR of R which in this case let us say FR of 0 is 4 then see FZ inverse.

So, okay so the point is right so I need to see okay so where does so where does FZ inverse of 4 I need to find out. So 4 occurs here and FZ inverse of 4 is 0 right. So that means so what this means is 0 should be mapped to 0 right. So it means that wherever this 0 occurs that should be replaced by this 0 okay.

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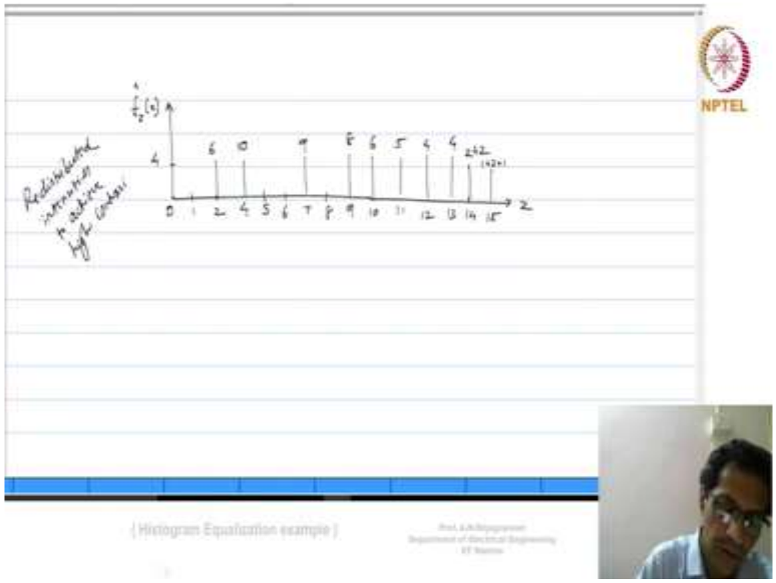

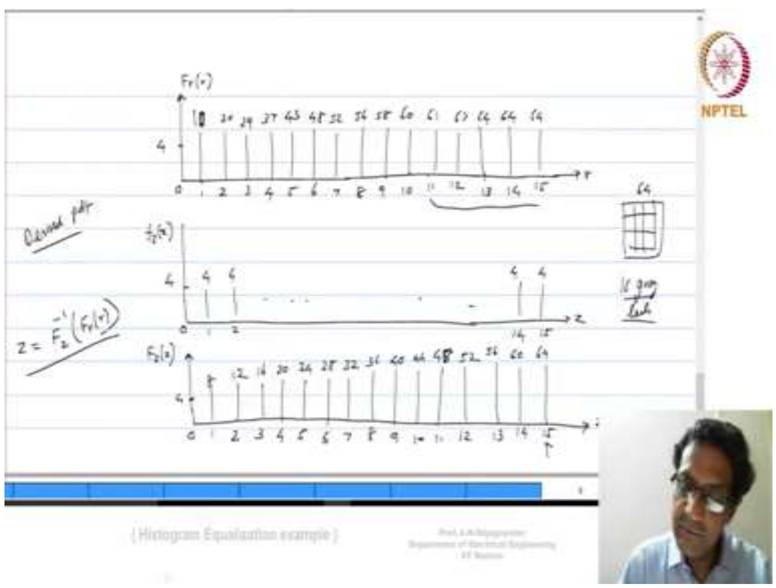

So the output histogram if you try to plot what will be the PDF or the output histogram what will that look like; let us put that as F hat Z of Z because we need because we may not get the exact FZ of Z that we want. So you will have at 0 right we will have 4 that means 0 will occur 4 times.

Now next R equal to 1, so FZ of 1 (sorry) FR of R is 6, FR of 1 is 6 and then you do not have any 6 here so you kind of, so you sort of look at what is next highest value right the nearest highest value to which this can be mapped that in this case is 8 okay which is why I said right when you do a discrete case right I mean a discrete approximation will not be the same as doing the continuous case.

But then whether the law is this, so 6 maps to 8 and 8 in turn FZ inverse of 8 is 1. Therefore what did I have here 0 1, so FR of R okay now this should have been 10 by the way right because I think this was not 6 FR of R should have been 10 because we had 4 and then 6 okay. So we should have added the two therefore this should have been 10 not 8.

4 then 4 plus 6; 10 and then after that I think it is fine 29, 37, 43 48 (())(08:31). So what we have to look at is 10. Now 10 right the when the nearest value that maps to 10 the next highest value is 12 okay. And so FR of 1 is 10, FZ inverse of 10 that means we have to go to 12 and then look at what it maps to. So it maps to intensity 2. So basically 2 will occur the same number of times that 1 occurs.

Now 1 occurs how many times? If you go here 1 occurs 6 times and therefore 2, so 1 will not occur in the output image at all, 2 will occur 6 times. Let us go on further right let us look at R equal to 2 okay because we want to check every intensity in the input image where it will map to the output image. So if you have 2 right so 2 maps 20. And then 20 ofcourse maps to 20 here.

Therefore 2 maps to 4 and 4 will occur the same number of times the 2 occurs and 2 occurs 10 times. Therefore 4 will occur 10 times. Let us go let us do it one more time. FR of 3 is 29 okay and 29 maps to the nearest value next highest value is 32, 32 maps to 7. And therefore 7 will occur the same number of times that 3 occurs and 3 occurs 9 times right.

So we have like 5 would not occur in the output image, 6 would not occur, 7 will occur 9 times that is the same number of times where actually 3 occurs okay. And then right if you try to get go on like this we try to do it for every value okay you will get till the point right. I am going to draw this so 7 then you will have nothing at 8 okay I will leave it to you to check all this. So 9 will repeat 8 times, 10 will be 6 times all this we can check to verify yourself.

11 will be 5 times, 12 will be 4 times, 13 is 4 times okay. Now let us go and check okay. Now so this 13 is in fact occurring the same number of times at 8 occurred. So 8, so see 8 that is 56 and therefore right 56 maps to 66 here therefore we say 8 right. So 8 maps to 13 and 13 will occur the same number of times that 8 occurs. So 8 if you see here occurs 4 times and that is why we had a value 34.

Now if you look at the next value after 13 here, after 50 so we finish 56. Let us look at 58; 58 maps to 16 okay that means 14 will occur the same number of times that 9 occurs and 9 occurs 2 times right the 14. At 14 you will have 2 but then it would not stop there because if you see, so we did we did 58, so 58 went to so 9, so 9 was mapped to 13 if you look at 60 right, 60 will also map to 14 okay.

So the intensity 10 will also map to 14 therefore 14 will also occur the same number of times that this so that this kind of say 10 occurred. So wherever 10 occurs, so now 10 occurs 2 times and therefore and you will have 2 plus 2 here which is so right these are the changes that occur when you do a discrete implementation. Now let us go to the, let us go higher up again.

Now let us go to 61, so see right 61 will map to 64 that means 15 will occur the same number of times that 11 occurs but then 63 will also map to 64, 64 will also map this. So all the values right starting from 11 and upwards will all map to intensity 15 okay so that means 15 right so 11, 12, 13, 14, 15 wherever they occur will all be replaced by this value okay, grey level value which is 15.

Now 11 occurs how many times let us see. So 11 occurs 1 time, 12 occurs 2 times, 13 occurs 3 times, 14, 15 do not occur at all. Therefore for 15 what you will have is 1 plus 2 plus 1 okay and because you have got like 11 occurring here right 11 occurring 1 time,12 occurring 2 times, 13 occurring 1 time, 14 and 15 do not occur at all.

Therefore right this will be the final histogram as you can see right as we can see this is not flat definitely not in fact what it has done is redistributed the rate intensities, redistributed intensities in order to get a higher contrast to get a to achieve higher contrast and that is why we said that you know your say dynamic range.

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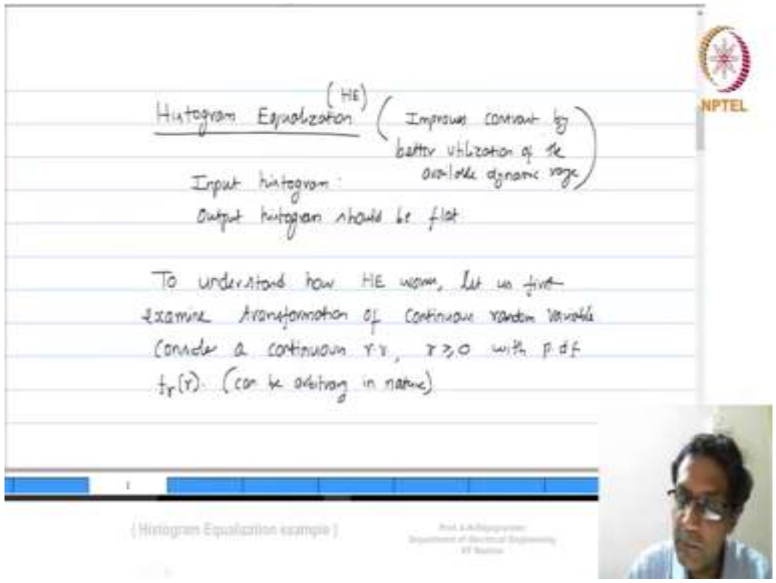

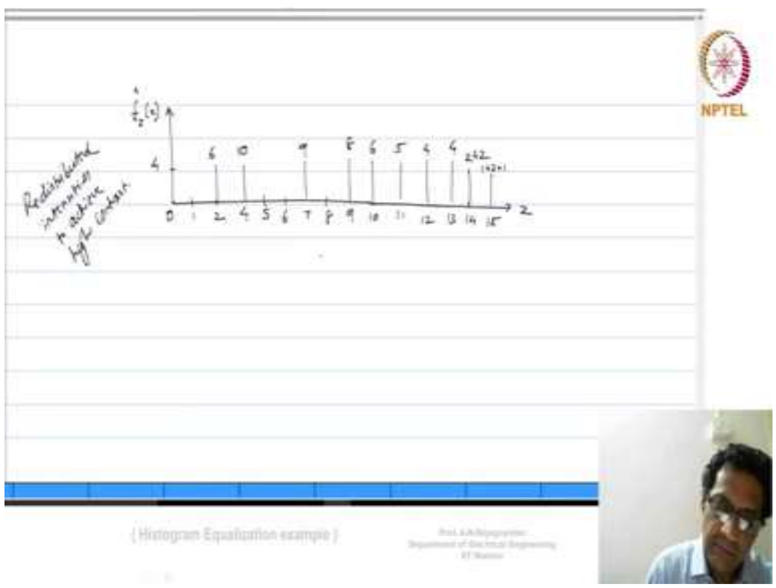

If you go, if you kind of go back to the first statement that I made here when you do histogram equalization so what we do is we improve contrast by actually using by doing a better utilization of the see available dynamic range okay (())(13:39) use of the available dynamic range. So that sense so you realize that the histogram is not flat but then right this indeed stretched. The flatness is not occurring simply because of the fact that you are doing you know a discrete implementation.

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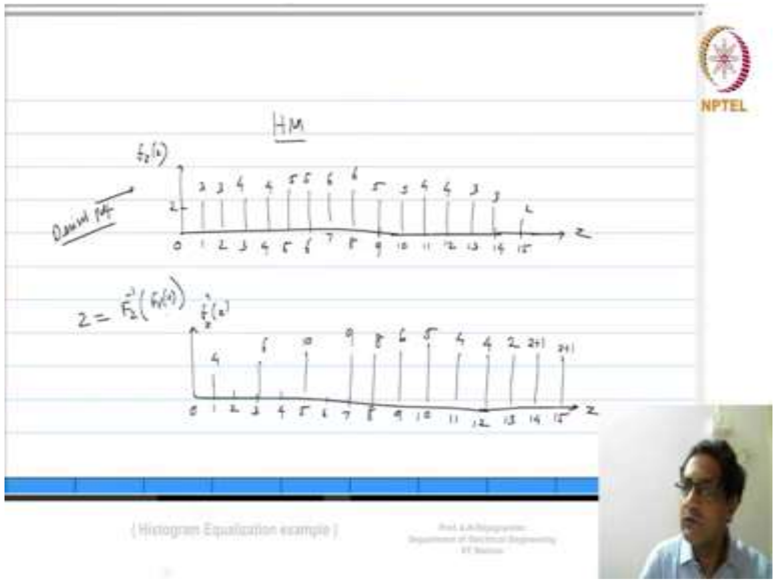

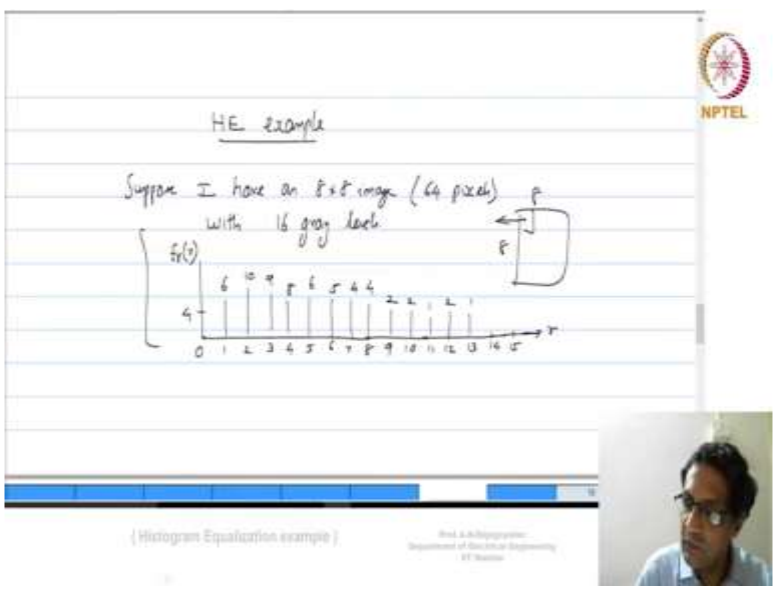

The same thing can be carried over to do the histogram modification, histogram modification can be done in the same way except that FZ of Z will now be will not be flat now, somebody will specify what that is. For the same problem I am going to give you an FZ of Z and I would like you to verify what you get as output okay.

Now suppose I tell you that my FZ of Z looks like this. Suppose I am going to give suppose I tell you that FZ of Z is this which is not flat, this is a desired kind of a histogram. 1 is 3 okay 2 is 3, 3 is 4, 4 is 4, 5 is 5 this turns out that somebody gives you something like this 6 is 5, 7 is 6, 8 is 6, 9 is 5, 10 is 5, 11 is 4, 12 is 4, 13 is, 14 is 3 and 15 is 2. If you had FZ of Z this is supposed to be a desired PDF and then and now this is a desired PDF and you know and the input PDF is still the same right whatever we had here.

Suppose I gave you the input to be the same as what we have for the earlier example. Now do a histogram modification now what will you do, you will again apply the same logic FZ inverse of FR of R right. You will of course first compute FR of R which is in fact the same then your FZ of Z will now change because here because this PDF has now changed.

Therefore the FZ of Z for this will change and therefore your final histogram modified result will also change. I will just give you the answer and I would like you to check that let us say this is what you get. So FZ of Z looks like this. At the end what you have is 0 nothing, at 1 what you have is 4, at 2 you have nothing, at 3 you have 6, at 4 you have nothing, at 5 you have 10, at 6 you have nothing, at 7 you have 9 okay.

So right as we can see 4, 6, 10, 9 and so on right these are exactly the same intensities that you had here 4, 6, 10, 9 and so on right so all those are getting redistributed now 6, 7 then 8 is 8 okay. I will just leave it you to verify that now this indeed is the answer, 9 is 6, 10 is 5, 11 is 4, 12 is 4, 13 is 2, 14 is 2 plus 1 okay this is again something I would like you to check 15 is again 2 plus 1 right which is equal to 3 okay. So here FZ of Z should I it will look like. So those two examples that I showed were actually implemented in this manner.

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So one was this histogram equalization result which I showed here. This is again an equalization implementation where we said that the output should the output histogram should be flat as we can see this is not flat right. So this implementations simply does that kind of a redistribution and then what you get is a high contrast image.

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Similarly the other one that I showed you earlier which was an histogram modification. So here the sort of a desired histogram was this and then the input histogram was then transformed in order to be able to match this histogram and then they would eventually end up with a histogram which again is not really identical to Lena but then it uses the law right which we used but you know because of because of what we do what we because of the fact that we have done it is straight approximation.

We cannot really expect the same as we what would happen in the continuous case but then the continuous case is the case that actually gives the foundation to better go ahead for the approach itself right and then therefore this is how histogram modification and histogram equalization are actually implemented.