Image Signal Processing
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Lecture No. 65
Image Sequence and Single Image Filtering in Gaussian Noise

So, our next topic under the image enhancement is noise filtering.

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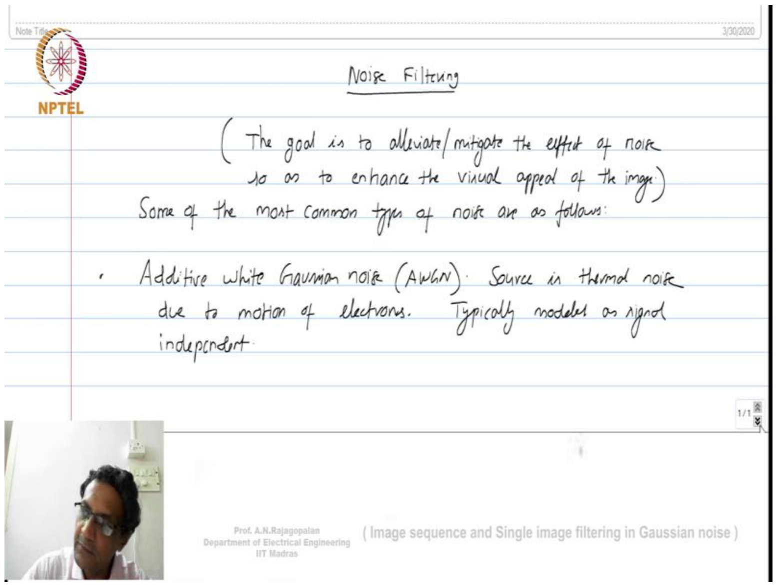

Noise filtering. Filtering, it comes under as a image enhancement, because idea the goal is the goal is to, is to alleviate, or in other words, mitigate the effect of noise, mitigate the effect of noise, effect of noise so as to, so as to enhance the visual appeal of the image, so as to enhance the visual appeal of the image, the visual appeal of the image.

As we all know, as we all know, noise, plays an important role in the sense that you can actually affect the effect the way they do would actually appreciate an image. And there are various kinds of noise, one of the noise that we have already seen is what is called a photon noise which we saw earlier, but there are there are so many of them and we would like to know what kind of simple algorithms that that you can employ in order to be able to alleviate the effect of noise.

That is why that is why it comes to image enhancement, because there is also an objective way of doing it what is called restoration, what is called restoration. And out there later, we would, actually employ a more or more kind of an objective approach in order to in order to reduce the effect of noise. Here, the idea is to simply alleviate reduce some simple methods in order to be able to alleviate the effect of noise or strain and visual appeal.

Now, what this also means that not consider enhancement or a this kind of a subjective notion, simply because of the fact that you know some people like to see images with noise, for example, scanning electron microscopy, and if you take an image of image that you capture, or a SIM, and what typically happens is that it has inbuilt it has inherent noise in it and, its natural appeal is lost, if you try to filter that noise out.

In fact that people do not like to see an old smooth image people would love would rather live at noise rather than overseas an image. So, the whole idea of a noise filtering is that there is a kind of a trade off. Because when you say filter, that means really trying to reduce the effect of noise and that comes at a cost. That comes at a cost because of the fact that when you do that, you are also going also going to end up smoothing them.

So, what level of smoothing is acceptable to what level of noise is something that is totally subjective, which is why we like this particular aspect of noise filtering comes at an image enhancement. Because what let us say what you might think is actually reasonably filtered might look like an over filter or over smooth image for someone else they might say that probably you should have left out a little bit of noise.

Even though even though they are not like to have noise, but some people might say that know that an over smooth image may looks worse than a noise that has some sort of noise and therefore, the kind of methods that we will see right here, they will all be they will all have to be fine tuned to an example. So, that sense that sense, now, a lot easier to fix the size of the size of the filters and so on and then depending upon what size you choose, you might end up over filtering under filtering and so on and therefore, and therefore, again, right it comes under a subjective notion

Now, the point is, there are various types of noise. And we will only lists some of the most common kinds of noise, some of the most common types of noise are as follows some of the most common types of noise are as follows are as follows. One is additive white Gaussian noise, which we can sure you are all familiar with additive white Gaussian noise or what is called what is called AWGN.

This the source is really a thermal sources, source is thermal noise, arising from motion of electrons due to motion of electrons. Typically you know typically modeled as noise independent, signal independent motion of electrons typically modeled as noise independent, signal independent, typically modeled as signal independent that means, that means it would affect, the intensity of this of these signal at a special location irrespective of the strength of the signal.

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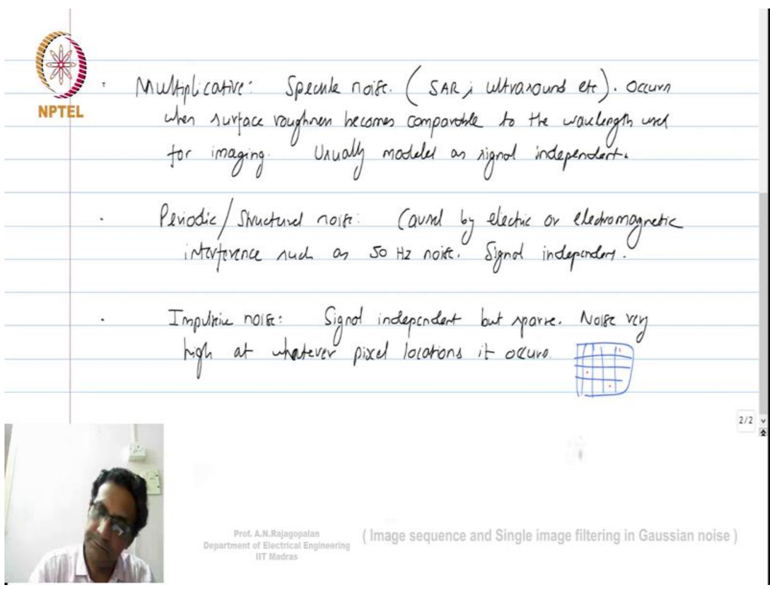

The other kind of noise is what is called multiplicative noise. The first we saw was actually additive in nature. The other one is multiplicative. And the standard example under this is what is called the speckle noise, speckle noise is not those are really common in optical imaging, but it is very common in synthetic aperture radar imaging, SAR ultra sound and so on here it is very, very common in these in these areas where also the imaging that then offers of sort of a different kind.

Now, so, as this occurs and the sources and the sources of noise is when occurs when occurs occurs when, when surface roughness so the surface that you are imaging, surface roughness becomes comparable comparable to the wavelength used for imaging. So, what that means is that they are going to be multiple scatters from the same from at different points and these could you know interfere in a destructive or in a constructive way leading to what we call this is speckle.

This is usually modeled as usually modeled as signal independent, usually modeled as signal independent, and it is multiplicative in nature. Then the other one is periodic or structured noise. This occurs because of because of standard fluctuations mean and around us. Let us say cost by interference electromagnetic interference electrical electromagnetic interference electric or electromagnetic interference such as such as 50 hertz noise which is our supply noise 50 hertz noise.

And, and this kind of noise has a regular pattern that it periodic and so on. You can a single door. Then the other thing that could affect you is what is called impulse noise or impulsive noise because of noise or what is called impulse noise and this ever been this is also signal independent. In the impulse noise also signal independent but it is sparse and one of the one of the key things that separates such from others is that it is sparse signal independent but sparse, in this in the sense that it will not effect an immediate not all the spatial locations.

But wherever it occurs it will hit those locations pretty hard in the sense that but sparse but noise strength very high but noise very high where wherever it occurs at whatever locations at whatever pixel locations it occurs. So, it actually means that you could have an image, this raw picture or you could have an image but lots of intensities out there but then this noise impulse noise could be affected maybe at some locations here perhaps on here and so on. So it is going to be sparse but then wherever it occurs it will really change the change the intensity value there significantly

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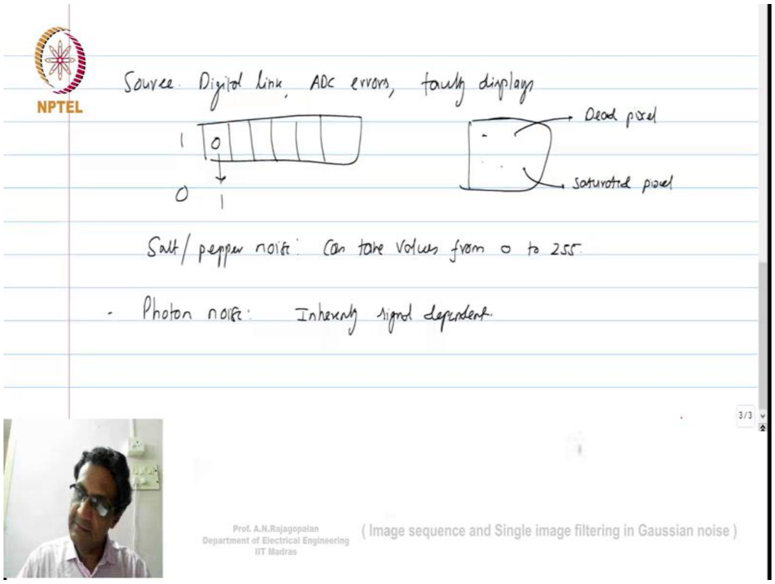

And the source of this is more of a digital link errors in a digital link, digital link ADC errors so, what these things mean is for example, you could have you could have you might be going to see transmitting some information and let us say the MSB ought to be 0 but then because of transmission error or whatever I mean this becomes 1. Which then kind of changes the signal strength completely and independent of the signal value or this could be 1 and then it could actually make it 0.

ADC errors in fact it can also occur in faulty kind of displays what is actually means is that now you could have you could have a display wherein some pixels are faulty what that means is that these could be there either a dead pixel which means that the respect of whatever signal you input this will always remain dark or you can have a saturated pixel which which irrespective of the normal the input intensity the melt input signal strength remains that saturated pixel.

So, it also occurs because of so, so, even if you have a display that is faulty that that kind of an image you can again through an impulse noise the other kind of thing that you have to deal with is what is called photon noise and photon noise is you have already seen is inherently is inherently signal dependent, inherently signal dependent and so on. This is also called short noise. So, so, we have in various kinds of noise that the sources also change and so on.

And our goal would be to get a look at simple algorithms that can actually alleviate mitigate the effects of these noise on images so as to be able to enhance the visual appeal of these images. Now, we will not be looking at all of them the ones that we will be looking at there are the important ones which is like AWGN what kind of filters can we have for this and then followed by periodic structure noises.

This is actually simple to eliminate using what is called the notch filter and then and then we will also look at impulsive noise, photon noise and multiplicator you will not be studying this particular course as these are beyond the scope of this course.

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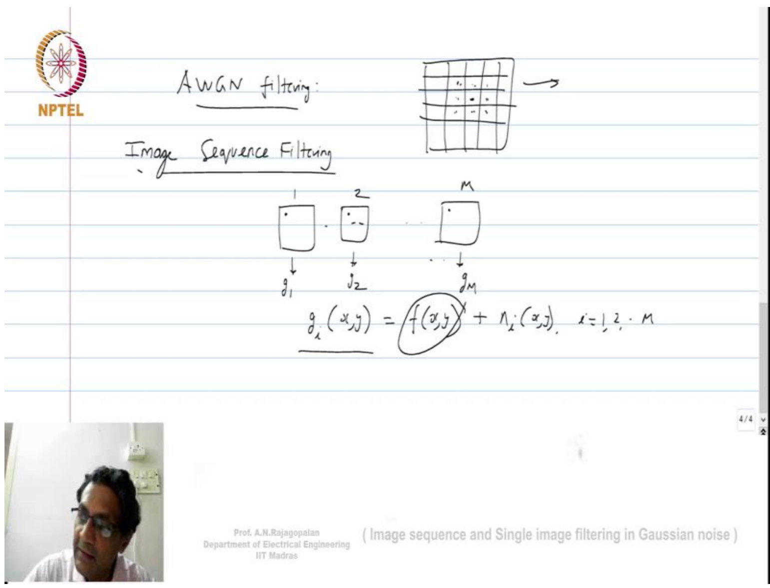

Now having said that let us first kind of look at what do we mean by it by AWGN filtering. Filtering and as we all know, what we would what we would typically like to do is you know, somebody gives me gives me an image, which is just one image, I normally have just one image and I would like to I would like to do some kind of filtering. You would have already seen that, people try to use what is called what is called a, what is called a uniformly weighted filter or what is called mean averaging, wherein you just simply average all these intensities.

Suppose you want to know, the intensity at this location, you want to filter the intensity here, you simply take an average of all the, all the, all the pixel intensities in and around its neighborhood, and, and, and, simply call that as default value, and then the same thing when you do a turn on another location and so on. Or you could do some get a weighted average, which would, which would be some kind of a Gaussian filter, where you would probably have no attach to more weight to the central guy, less weight to these neighbours, and so on.

So, it is not clear as to why what is the motivation by doing something like this? Why would it filter noise and so on? So, in order set the background for this we we are aware of such filters, but then in order to understand why these things make sense, I am going to introduce what is called email sequence filtering first before we even look at single image filtering. So, image sequence filtering.

Now, in image sequence filtering, what we will assume is that we have we have an we have a bunch of images of a scene which we which we capture at rapid succession. It actually means that let us say that let us say that this is the first frame, the second frame, and then, these are some m number of frames, that we captured at rapid succession.

Why rapid succession? Simply because of the, because just to make sure that the scene does not change, by the time we capture the m frames, and also, we can assume that, that, that all these frames are absolutely aligned and accurately aligned, they are already they come aligned in the sense that if there is x y, then there is also x, y it is also x, y and so on. So, these so these pixels are all staying at their, at the same place across the frames.

Let us call let us call this observation as g1, let us call this observation is g2 and so on. And let us say this is gm. Now, what we know is each of these frames, as we observe each of these observations has been influenced by something noise. And let us call that the gith frame that we have observed is let us say the original intensity that we would like to see is f at x comma y plus some noise which is eita i at x comma y or going all the way from see 1 to m.

So, the idea is that we would like to we would like a recovery f, ideally f is what that was that was noise free, that is all clean image, we would like to recover it from these observations, which are actually gi going all the way from 1 to m.

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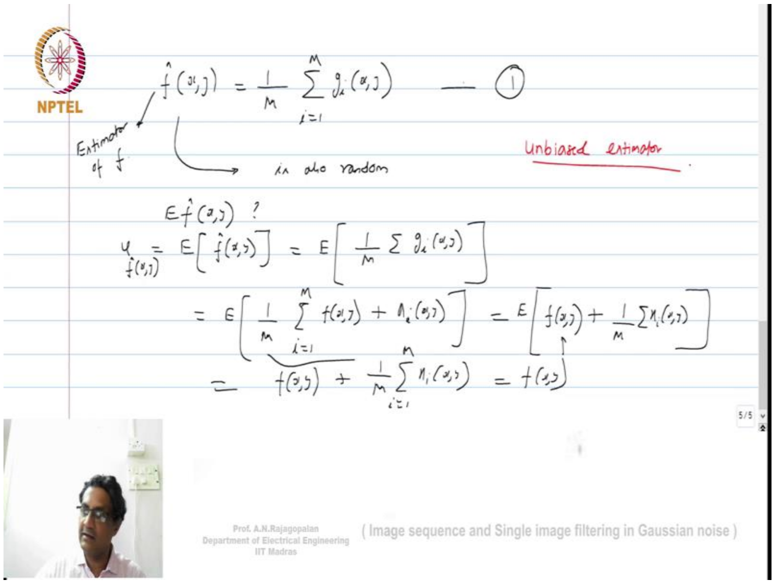

Now a standard estimator for this in image sequence filtering is given by f hat of x y is equal to 1 by m summation gi of x, y, i going from 1 to m. Now, because of the fact that g i is random, we know that gi is random because even though F is actually a deterministic quantity, see ideally, f could also be random. But no, but in this case is for simplicity, we will assume that you know, f is actually a deterministic quantity, which means it is a constant. And eata i is a random quantity, and therefore gi is random.

And because this hat comes as an average of gi, therefore f hat is also random. It is also random. Now, and and in this case, is an estimator of, f. This is an estimator of f. And the idea is that when you have such an estimator, one of the things that he would like to ask us what is expectation of f hat of x y or what is the mean value of this estimator? Because it means value would actually give you a sense for a sense of how much you can rely on this estimator.

Now, what is this value? We can actually evaluate this by doing the following. Let us say that expectation let us call that as mew f hat of x y. Let us call this as the mean value of this estimator that will be x expectation f hat of x comma y that in turn is expectation f hat by itself is 1 by m summation gi of x y, because we know that f hat is simply this from equation 1 above. But then, we know that the observation model clearly says that gi itself is nothing but f of x y plus eata i of x y there.

Therefore, and also i going all the way from one to m, and therefore right what we can do is this. So, this also this quantity inside because f is a constant this becomes expectation of so, will get m times f x y by m therefore does f of x y plus 1 by m summation eata i of x comma y, this will be equal to because of the fact that these are constants of the simply comes out f of x y and because the expectation is linear operator, we can actually push that inside summation eata i of x comma y i going from 1 to m.

And since we assume noise to be 0 mean, this becomes equal to f of x y. That means the mean value of, of f hat is in fact f of x y, such an estimator is called an an unbiased estimator, which is actually a desirable property of an estimator. It is an unbiased estimator, that means its mean value is equal to the equals be equal to the unknown, which you are trying to estimate. If f was actually a random quantity, we would have asked for mu hat or mu f had to be equal to expectation of f.

But in this case, f is simply a deterministic quantity. That is how we have assumed it. And therefore, what we are what we are simply saying is a estimator unbiased? The answer is yes. But that by itself is not enough. We also need to know how much is a variance of this value, because the variance is very high, then it means that our trust on the estimator will then be less. The other thing is, we would also like to know whether the whether the variance is a function of the number of observations or not.

Because ideally, I would, I would kind of tend to believe that I should have an estimator actually improve as I keep improving, as I keep increasing the number of observations, the more observations I have, then my trust on the estimator should go up or my trust on the estimated value, should it ideally go up what is called really a consistent estimate. So we want to ask whether, whether this estimator way we have defined it as a consistent estimator.

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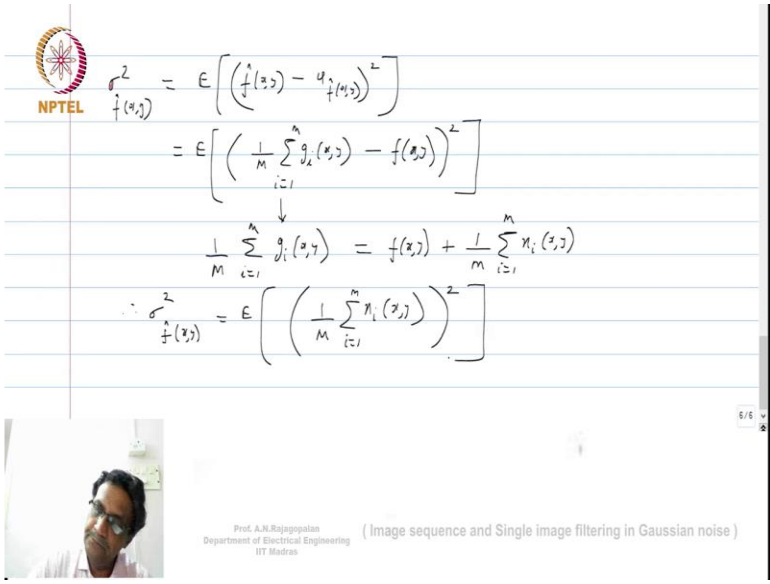

In order to check that, what you have to look at is the variance of this guy. So, let us kind of look at look at variance of f hat of x y, which we know is simply expectation f hat of x y, f hat of x y minus mu f hat of x y the whole square. Now this we know is the expectation or f hat of x y is is simply 1 by m summation gi of x y i equals 1 to m minus mu f hat this f of x y because it is an unbiased estimator and therefore what we have is this.

Now, if you simply examine the first quantity here, this is 1 by m summation i equals 1 to m gi of x y, which we already saw is nothing but f of x y plus 1 by m summation eata i of x y. We saw this already i equal to 1 to m. Therefore, sigma square if had to x y is going to be equal to expectation and then this minus this so, this f of xy will get knocked out and we will get 1 by m summation eata i x comma y, i going from 1 to m the whole square.

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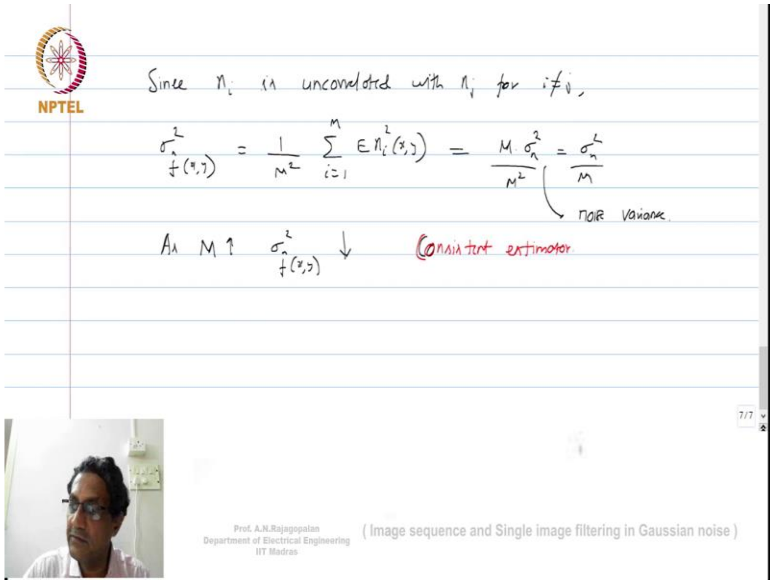

Now, since the noise realizations are assumed to be in correlated since eata i is uncorrelated with you know eata j and they ought to be 1, why should noise in one frame would be have a correlation but noise in the other frame typically they can be modeled as uncorrelated. This uncorrelated with eata j for i not equal to j and and since noise is 0 mean what this means is that sigma squared f hat of x y it will reduce to 1 by m square what did we have here 1 by n square 1 and summation expectation eate i squared of x comma y i going from 1 to m.

And assuming that the noise variance is sigma square n, this will mean you will get m times sigma squared n by n square is equal to sigma square n by m where sigma square n is the is the variance of noise. Therefore, you see that the variance of this estimator is turning out to be sigma square n by m that means, as m increases, then increases your sigma square if hat goes down at some x comma y is going down. So, which is why, which is why this is a consistent estimate.

This is a consistent estimator which is again a desirable property. So, so this whole thing so this image sequence filtering approach what it is saying is you have a bunch of frames and then as long as your frames are aligned, you can simply average the phrase in order to arrive at a final frame, which will have noise much less than any of these frames, and it carries and it is supposed to carry the information about the about the clean version of the image, which is f.

It might still be affected by some mode of noise because of the fact that there is only going down as sigma square n by m but as you keep increasing n we can hope to see a more and more clean version of the scene of the image or the scene. Do the idea of image sequence concentric? Now, it is this idea that as we borrowed over to the case, when you do single image filtering, the idea is exactly the same, except that to what extent this will get fulfilled that remains to be seen.

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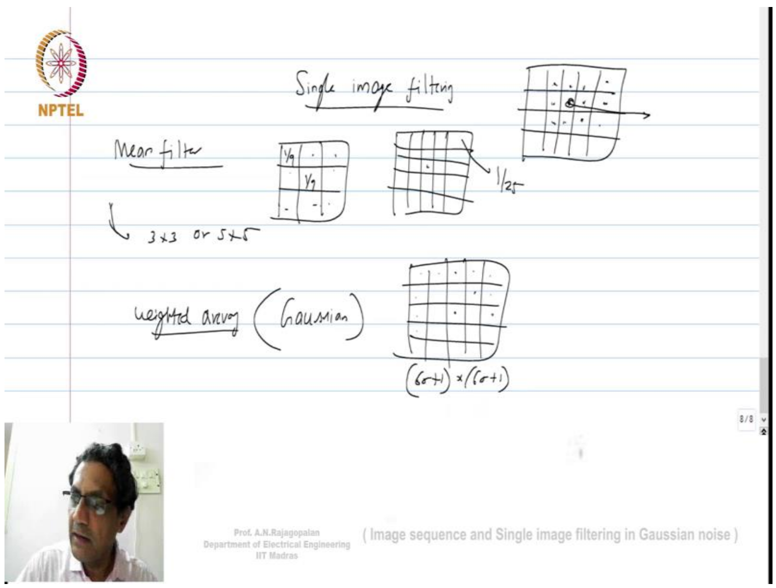

So, when you do single image filtering, when you go to single image filtering because in most cases, you will have just one image, say you will have a bunch of frames for what you would typically want to do is what is called single image filtering. If you have this one image with you not a sequence, and you would like to filter noise from this image, so as I told you, people will typically do what is called spatial averaging.

So, if you want to filter the noise here, you will take this intensity, multiplied by some weight take this, take this, all these will multiply with some weight, and then the weighted average is what you will say is the final filtered output value for this intensity. Now, the most common filters, as we have seen are what is called what is called the mean filter, which means that it has a filter, you have to fix the size of this filter, you choose it to be 3 cross 3, then all the weights will be say 1 by 9, all the way each one of these is 1 by 9, so that they all sum up to 1.

The idea now, you know, when you try to relate it to the main sequence filtering problem that we that we saw, you can actually relate the two now, you will either you understand the significance of doing something like this. So, when you apply a 1 by 9 weight all for for all these pixels, what you are trying to tell us, the implication, or the sort of or the idea behind doing this is follows.

Because these pixels are right next to this guy spatially, so this called local averaging, we are hoping that these intensities or little intensities, this pixel would have been roughly the same as this says pixel whose filter value you are trying to estimate. And if that is the case, then you can assume that all these have been affected by noise again, AWGN, and they are independent of each other. And then therefore, when you average them, you would get the same kind of an effect that you got with respect when you did in a sequence filtering.

So, you are hoping that the more number of pixels you average, then you are the filtered output will have that much less noise. So, you might say why stop at 3 cross 3, why not I go to 5 cross 5? 1, 2, 3, 4, 5; 1, 2, 3, 4 so you have got 5cross 5, your center is here. Now each weight now becomes 1 by 25 so that they all add up to 1, all these weights when you add them up, you can add them up to 1 and now instead of choosing a 3 cross 3 neighborhood you can choose a 5 cross 5 neighborhood to average.

But then where is the problem there you could have chosen image sequence filtering you could arbitrarily increase the M increase M and you could have average as many frames as you wanted, but here that you cannot do because of the fact that as you increase the size of the filter, you are now hoping that a pixel here has an intensity similar similar to the pixel here which may not which may or may not hold good and most likely it will not because of the fact that as you go farther away from from this pixel of interest, the likelihood that this guy will have the same intensity as this is going to be lesser and lesser.

And therefore, people will not go beyond 5 cross 5, 3 cross 3 or 5 cross 5 is typically common you do not go like 7 cross 7 or 9 cross 9 because then you will end up smoothing the image because he will start filtering out edges, you will start smoothing the edges and therefore it will just not really and really I think that he would like to have.

Which is why I said that this kind of subjective because how much filtering to apply before we end up over smoothing the image and when it becomes annoying in the sense that they may just look too smooth rather you would rather see tolerate some amount of noise than kind of tolerate an over smoothed image is a is a subjective matter.

That is why is that is why this comes in that image enhancement this kind of filtering. So, in one way to kind of avoid this is what is what is what to do is to kind of do a weighted averaging, rather than give all of them equal weight, you could do a weighted averaging filter, weighted average such as simply as a Gaussian, a Gaussian filter, where you can actually take let us say, a Gaussian filter, you choose the sigma whatever you have, let us say a filter that is a size 6 sigma plus 1 by 6 sigma plus 1 if that is a filter.

Then what you would do is act a pixel under consideration you will get maximum weight because it is a Gaussian and then as you go outwards, you will start to give give lesser and lesser weight to the to those pixels that are farther off the idea being that now you will reduce the amount of smoothing because the guy who is very different will get a less weight and therefore therefore, therefore, the ability to smooth will become less and therefore, even though you will be filtering out noise, but then at the same time your say edges will not your edges will not will not tend to be over smooth.

Because of the fact that we are going to give less weight for those pixel entities that are farther off. This is called a weighted averaging.

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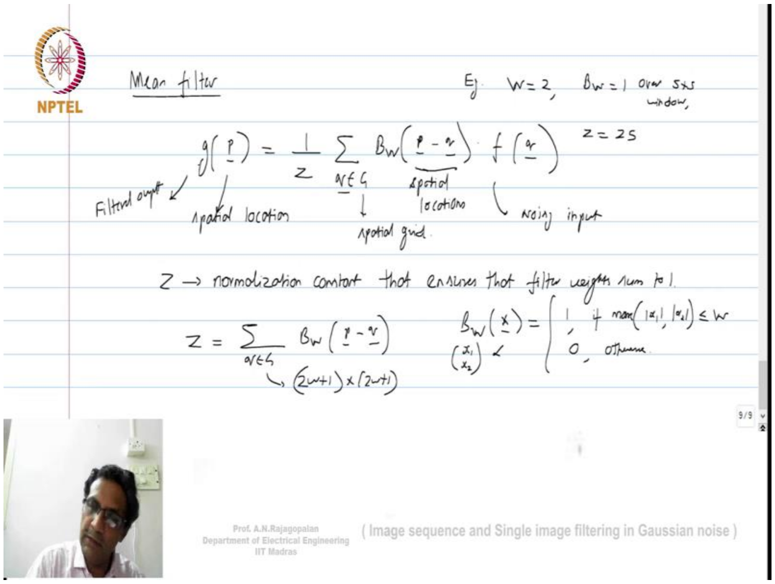

These two filters are no are typically you know given in a more formal way can you write them in a more formal way can the way you would write them as follows let me just write that down. So, in the more formal setting, you would write it as follows. So, the mean filter, so the mean filter it looks like this.

So, g is a filter output at a pixel location P is given as 1 by z, where z is a normalization constant and a box filter, what is called p minus q into f of q, where f of as f is the noisy image. So, p is a spatial location, it is actually a 2D vector which indicates the spatial location in the filtered output. This is the filtered output which is supposed to be noise free it would be completely noise free, this is the noisy input at location q, p and q are spatial locations, and this average is taken over a cube belonging to g.

Let us say g is the spatial grid of locations. This is your q here and you are trying to estimate at p intensity at p this p minus q, spatial location or spatial grid, let us say it is called the spatial grid and and what you have now, z is a normalization constant which ensures that the weights sum up to 1 for the filter, it is a normalization constant that ensures that filter weights sum to 1, that ensures that filter weights sum to 1, sum to 1. And z is given by q belonging to g B w p minus q that means, the sum of all the weights p B w p minus q and and what is this B w itself.

So, B w of some x let us say where x is let us say x1 x2, this is the form of x then this is equal to 1 if max of mod x1 mod x2 is less than or equal to some would, 0 otherwise. So, so so as an example if you want as an example, suppose I take I choose W to be equal to 2 and what this means is that is that this z will now be over a window of 2w 1, 2w plus 1 cross 2w plus 1 so, this filter size it says 2w plus 1 by 2w plus 1 and all those weights will be added.

And so, you so, so this filter size so, the B w itself will be a 1 over 5 cross 5 window, because w is 2 so a 5 cross 5 window so the filter is 5 cross 5 size and 1 z, because you going to add all the weights over 5 cross 5 because z will be equal to see 25 so that the affective filter the they going to apply it will have weights all uniform 1 by 1 by 1 by z 25. And so, you will have a 5 cross 5 filters, will be 1 by 25 and that will multiply each of these intensities f of q within this neighborhood of 5 cross 5 and that will be the result in intensity at at p in the output image g.

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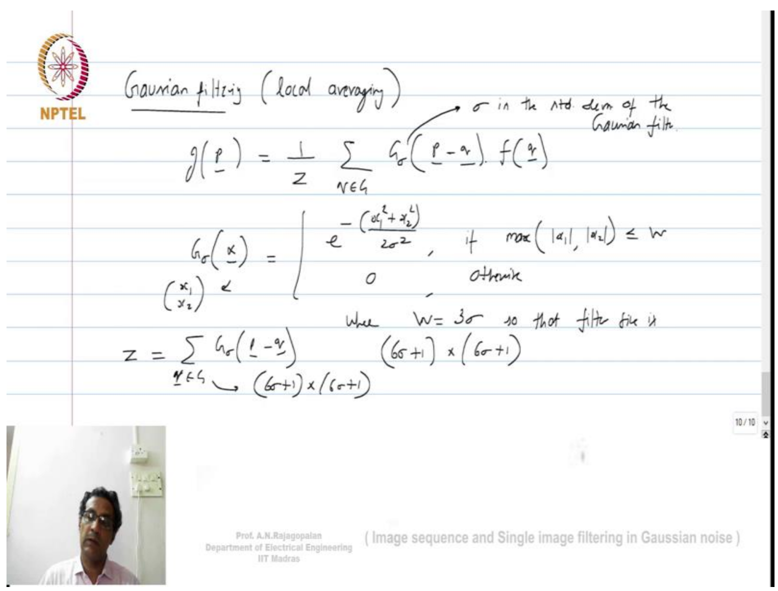

Instead if you did a Gaussian kind of filtering which will be a weighted filtering, then this equation will get modified slightly. And the modified equation and all these are of course local averaging, this is also local that is also local, we will see later what it means to do non-local averaging, but these filters are all local these are the most common and most simple to implement.

So, what this will do is this will do g of P to be equal to 1 by z again, everything will look very similar to what we did earlier except that now it will it will involve a Gaussian instead of a box kind of function. It will be p minus q and then this will this will wait the intensity at q in the say noisy image and all these have this pair that are the same this one definitions and the only thing is g sigma of x let us say if I where it says x1 x2, then this will be equal to e raise to minus x1 square plus x2 square by some 2 sigma square by the sigma is the standard deviation of this filter.

Which is again up to up to you to choose standard deviation of the Gaussian filter. This is something that basically we have to choose of the Gaussian filter and e bar minus provided is max mod x1 x2 is less than or equal to would, 0 otherwise and this w itself will be where w we know is going to be 6 sigma or 3 sigma w be will be 3 sigma and so that so, that your filter size is sigma 6 sigma plus 1 by 6 sigma plus 1.

So, the filter size so, so, for this so, that filter size becomes 6 sigma plus 1, filter size is as we all know 6 sigma plus 1 if it is a Gaussian, this is what we would like the filter size to be 6 sigma plus 1. And sigma is something that let us say we need to choose and z will again be a sum over all the weights the sigma of p of whatever and so, so want to write this is p minus q, you can write it as p minus q where q belongs to g, and, this, will be summed up over over 2w plus 1, or in this case, because it is a Gaussian you can directly write it as summed over 6 sigma plus 1 plus 6 sigma plus 1.

And therefore, this will again ensure that these weights sum up to 1 except that because of the fact that you are trying to use a Gaussian filter, now more weights will be assigned to be to to to the central pixel and then as you move away from that, you will give less and less weight. So, this is how a Gaussian filter works these great both of these filters are called local filters, we will eventually see something that is going to be a powerful method what is called non local means filtering.

The issues with these filters is that the local filters that they can end up smoothing on adjust. We did something which is undesirable. Therefore, it reason non local means it does accomplish a very, very effective like we will kind of talk about them subsequently.