

**Image Single Processing**  
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**Lecture No. 69**  
**Impulse Noise Filtering**

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The image shows handwritten notes on lined paper. At the top left is the NPTEL logo. The main text reads: "Filtering impulse noise", "Order Statistics filter", "Non-linear", "Median filter (3x3)", and a diagram of a 3x3 neighborhood with a central pixel. At the bottom, there is a small video feed of the professor and a footer with his name and department.

Filtering Impulse noise now as you can clearly see impulse noise is not the same as an AWGN and it can also be sparse what that means is in an image it does not mean that all the pixels are affected. And so, for example as you as you traverse through this image maybe something is affected here or something is affected here or something is affected here but not everything depending upon the situation and, and the other thing is the impulse noise is such that like whatever is this value of value that you get here and that is going to completely change this intensity here it has got will have no relation whatsoever with respect to the original intensity there.

Which then means that typically if you think that pixels ought to take values that are similar within a neighborhood now a pixel that is affected by impulse noise will, will typically not at all follow whatever is around it, because it is been completely changed. And therefore, the usual idea behind doing impulse, impulse filtering is stream is employed what are called Order Statistics filters, Order Statistics filters, these are not the same as the filters that we have seem till now.

What about Rank Order Statistics Filters, in the sense that you rank the pixels according to their intensities and then do some kind of operation, that is the idea and typically these filters are nonlinear. This are non kind of a linear filters an all which we have seen till now, especially with respect to AWG and noise and all what we saw was all linear filter and so on whereas this is whereas this is nonlinear, as far as impulse noise filtering is concerned and at some say very simple filters have been there have been around and these are really, really quite quite say, effective.

And no and this unless otherwise, otherwise said we will assume that know the impulse noise can range anywhere between 0 and 255. Salt Pepper noise when needed, I will explain them I can talk about in some examples only when we know for sure that an image is affected by salt, salt pepper noise, otherwise, we will assume that image has been affected by uniform noise caring a value anywhere between 0 and 255.

Now, when you want to do filtering, one of the most standard filters check that one can use is what is called a Median Filter, you must have heard about it, what is called what is called a Median Filter. This goes by the by the by the idea of what we mean by really median, not median value. Now imagine that imagine that you are actually sitting here and this pixel is what you want to filter you do not know whether it is affected by impulse noise or not, it could be, we are not actually going through going through an approach which is first to detect whether, whether something is impulse affected or not.

And then applying then I am going to go we are going with a very simple filter which is going to act on all the pixels. Now, here what will happen is, so, for example, what it will mean is that even if a filter is not affected by noise, let us say but then you still, you still do some kind of impulse filtering, use some kind of medium filtering, you would not do too much harm because of the fact that these pixels around it are likely to have similar values and therefore, you do not you do not expect too much of the harm to occur.

Now the median filter right think of think of an let us say, suppose you want to you want to think of a 3 cross 3 median filter, what it actually means is that you want to filter this pixel off its impulse noise. And therefore what we will do is we will take this is 3 cross 3, 3 neighborhood and then and then whatever, whatever are those intensities, you are going to write all of them up

then you are kind of either going to put them in an ascending order or you will put them in kind of in, in a kind of a descending order.

And, and then what it will mean is and then once you have it then what is median value after you do this if you pick the median value, then whatever is median value after you do the rank ordering, you pick the median value and that will be the intensity that you will assign in the output array that you will have an output array where for this intensity you will actually copy this median value.

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Median filter

90	100	95
98	200	110
104	120	105

{ 90, 95, 98, 100, 104, 105, 110, 120, 200 }

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So the idea behind the median filter which we employ for filtering impulse noise is as follows Imagine that that you have a bunch of intensities at some location like this. And suppose suppose the central, central this one pixel is the one that is affected by impulse noise, assume that it has a value let us say 200 and the neighboring scene points, will will do not match up, but just because those are the scene intensities let us say that those are much lower something somewhere around 90, 100, maybe 95 maybe 110, 120, 105, 98, 104, 9, something like that.

They know the fact is if, if this was not also affected by intensity by, by impulse noise then of course then, then you would have kind of expected this intensity to be somewhat similar towards, towards neighboring values, but then because of the fact that this has been affected by impulse noise it has become very high it could also have been on the other hand very low depends upon how the impulse has affected it.


Now, the idea behind the behind that or taking the median values to make sure is that you kind of knock this outlier out and you should be able to remove this outlier and somehow replace it with let us say one of the neighboring values because neighboring intensities we would kind of see believe that those kind of belong to the, to the scene intensity and therefore, if you were to replace with one of those neighboring values and you would have you would have kind of arrived at the right value for, for this for this pixel location right value for the intensity at this pixel location.

So, when you read before you compute the median, you of course not put pixels or put these intensities in return an order, it does not matter whether you were whether you put it in descending order or in an ascending order. Suppose, we just put these pixels in an ascending order, then we will have some like 90 95 98 then I think we have 100 then 105, 104 is there 104 here, then 105 then I think it is 110 and 120 and 200.

Now if you see an outlier, which, which would not fit the surrounding intensity goes off to the extreme and therefore when you actually pick up the pick the median value that you would end up picking the medium values 104 therefore, this pixel intensity will eventually get replaced by 104 after you do the median filtering. Now, if you had done it descending manner in this ruler, it would have come on the left extreme again when you actually pick the median value you would have knocked it out.

So, the whole idea behind the point median filter for impulse noise is to simply make sure that the outlier will not, not be chosen after filtering assuming that the that the surrounding intensities that have not been affected, affected and dishonoring intensities are the ones that actually represent the scene intensity. That is the idea behind the median filter.


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$x(m,n) = \begin{bmatrix} 10 & 12 & 15 \\ 1 & 7 & 8 \\ 14 & 20 & 22 \end{bmatrix}$  → will get replaced by 12 in the filtered image.

$\{1, 7, 8, 10, 12, 14, 15, 20, 22\}$   
↑  
median

3x3  
5x5



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Let us take one example in order to illustrate this better. Now, let us say that we have  $x$  of  $m$  comma  $n$  to be this array which is 10, 12, 15, 1, 7, 8, 14, 20, 22. So, we are so, we would like to so, so, the idea is that we have we have we have a pixel here, which we want to replace it is 3 cross 3 neighbors are these. So, now, what, what should we do? So, now, we first end up no, so, if you write down we first write down this array, but and let us write down in some order either in an ascending order or kind of descending order.

Let us say that we now we now we put these intensity values in an ascending order that will be near like 1, 7, 8, 10, 12, 14, 15, 20, 22. So, we have actually nine matters 3, 6 9. So, the median is at least 4 on either side and then and then whatever stays in the center is the median and this this is. So, so, then what will happen is so, the 7 will get replaced will get replaced by 12 in the in the filtered image replaced by 12 in the filtered image, by this by this is median value.

In case you choose to have some kind of an even filter it is very, very rare. If you have even an even number of elements here, then the median value is simply the average of the central, central two values but normally that is very uncommon. So, we just go with odd size filters so, you can have 3 cross 3 you can have 5 cross 5 but typically you know beyond this is uncommon. So you will have medium filters of the size 3 cross 3 or 5 cross 5.

You can also show that no it should be straightforward for you to show that a median filter is not linear that I that that I would I would leave it as an exercise to show that it is non-linear. Now there are there is there is also another so this median filter.

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Max filter  $g(x,y) = \max_{(s,t) \in S} \{ F(s,t) \}$

Find the brightest points in an image  $\rightarrow$   $3 \times 3$  neighborhood centered at  $(x,y)$   
 $5 \times 5$

Such a filter is useful to deal with pepper noise if the impulse noise is unipolar.

Min filter  $g(x,y) = \min_{(s,t) \in S} \{ F(s,t) \}$

Finds the darkest points in the im  $\rightarrow$   $3 \times 3$  or  $5 \times 5$  nbd of  $(x,y)$

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Now among the Order Statistic filter some of the some of the filters that are very common, what is called a Max filter, all these are rank order based. So, that why where they are all called order statistics filters. What this does this does this this intake  $g$  of  $x$   $y$  if you call this is the filtered image, this will be max of the max of the max of the noisy image at the location  $f$  of  $f$  of  $s$   $t$  where  $st$  that is all those all those locations belong to the neighborhood of let us say  $s$  of  $x$  comma  $y$ .

So, this could be a 3 cross 3 neighborhood about neighborhood center at  $x$  comma  $y$  for example, or it could be even be 5 cross 5 centered at  $x$  comma  $y$  oh sorry at  $x$  comma. So, so, what is Max filter does is an among these intensity values it will pick only the only the max value rather than the median value this max filter this is again an order statistics filter will pick the max value.

But why do you think why do you think something like that makes sense? This is useful get such a filter is useful to handle such a filter is useful to deal with pepper noise, to deal with pepper noise, pepper noise? If the if the impulse noises unipolar, if the impulse noises unipolar, unipolar in the sense that in the in the in the sense that we know that we know that we know that that is that the image is affected only by this kind of pepper noise and not by salt might salt pepper will

kind of make it a bipolar noise. So, it says unipolar is in such unipolar noise, it makes sense to use a max filter.

So in a sense, what you are trying to do is you are trying to find the brightest points in the image because you are using the max value to finding out the brightest points in an image. Now similar to this you can have what is called really a min filter and the min filter is such that it does exactly the opposite of what the what the max filter does is equal to min of  $f$  of  $x$  comma  $t$  all the things  $x$  still remain the same or what we have said earlier.

So, this min is calculator overall pixel locations  $x$  comma  $t$  the intensity is pixel location  $x$  comma  $t$  that belong to the neighborhood of  $x$  comma  $y$  3 cross 3 or 5 cross 5 neighborhood, neighborhood of  $x$  comma  $y$  and, and what this ends up doing is it finds the darkest points in the image finds the darkest points in the image and as you can see darkest points in the image. And why would, why would something like this be useful?

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A min filter is useful to reduce salt noise if the impulse noise is unipolar (eg. saturated pixels)

Alpha-trimmed filter: Suppose you have  $N^2$  pixels. Rank them according to their intensities. Delete  $\alpha$  lowest and  $\alpha$  highest gray levels. Let  $F_r(x, y)$  be the remaining  $N^2 - 2\alpha$  pixels.

$$g(x, y) = \frac{1}{N^2 - 2\alpha} \sum F_r(x, y)$$

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This is useful when min filter is useful when the noises unipolar filter is useful, useful to reduce salt noise, to reduce salt noise salt noise. When if the impulse noise is unipolar again, if the impulse noise if the impulse, if the impulse noise is unipolar obviously. So, salt noise example saturated pixels and so on saturated pixels. Now, there is also something called the Alpha Trimmed filter which is somewhere array in between is somewhere in between an averaging filter and a median filter, this is called the Alpha Trimmed filter. You know what does this?


Suppose you have suppose you have suppose we suppose you have  $n$  square pixels let us say that let us say that it taken an  $n$  cross  $n$  window suppose you have  $n$  square pixels, then what did does is, now suppose order them. So, first order, order them according to their intensities according to their intensities, rank order them basically rank order them according to their intensities then what do you do then you delete then you delete the  $d$  by 2 lowest and  $d$  by 2 and  $d$  by 2 highest gray levels, highest gray levels where  $d$  is something which we need to choose and let  $f_r$  be the residual intensity is  $f_r$  of  $s, t$  be the remaining be the remaining that is why it is called a Trimmed Filter

Remaining alpha alpha now the think of  $d$  as alpha be the remaining  $N$  square minus  $d$  pixels excels. From these remaining, remaining  $d$  pixels  $x, y$  minus  $d$  pixels you, you kind of a filter such that  $g$  of  $x, y$  so the filtered image at some location  $x$  comma  $y$  is equal to 1 by  $N$  square minus  $d$ , you take the average of these are these remaining intensity values,  $s$  comma  $t$  were, were of course. So,  $s$  comma  $t$  far of  $s$  comma  $t$  will be simply the remaining  $N$  square minus  $d$  pixels.

So, what does actually means is that you are at some  $x$  comma  $y$  whatever yours at some  $x$  comma  $y$  and, and suppose let us say taken a 5 cross 5 neighborhood which means that you have got say 25 pixels. You order all these intensities and then, then you pick a  $d$  whatever you want to do whatever  $d$  you wish to choose and then and then in the ordered pixels, when you have ordered them you can knock off the ignore the the lowest  $d$  by 2 and then the highest  $d$  by 2 and then whatever remains is to see a  $f_r$  of  $s$  comma  $t$  just, just, just take the average of these pixels.




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If  $d=0$ , we get the mean filter.  
If  $d = n^2 - 1$ , then we get the median filter.  
Such a filter is useful for combined Gaussian + Impulse noise.

Weighted median filter (WMF).  
Suppose  $x = [x_1, x_2, \dots, x_N]$



$W = [w_1, w_2, \dots, w_N]$   
↳ positive integer



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Now, now, the idea is that so if  $d$ , now, this Alpha Trim filter leads to leads to two extreme situations, if  $d$  is equal to 0, then of course, what you have is simply the averaging filter I mean because  $d$  equals 0 that means you are taking an average of all the, all the say all the all the  $N$  squares intensities,  $d$  equal to 0 we get the mean filter, we get the mean filter. On the other hand, if  $d$  is equal to  $d$  is equal to  $N$  square minus 1, that means these will  $N$  square minus 1 then, then the output is simply the, the median value. So, which means that what you end up with is simply the median filter.

Therefore, therefore, so, for, when other than these two extreme situations and you pick a pick a different  $d$ , then then such a filter is useful when you have a combination of is useful for a combination of for images that have combined, combined Gaussian or combined Gaussian and impulse noise, because we know that spatial averaging is good if you have a Gaussian noise, we know that median filter is good medium for impulse noise, and therefore, certain Alpha Trim filter is good for a combination of Gaussian and impulse noise.


Now, the other thing that one might want to ask us when we did average in filters for AWGN, and we had a notion of spatially weighting them like for example a weighted averaging a Gaussian for example, we said that Gaussian could, could assign more weight to the central pixel and less weight as you go outside and so on. So, one might want to ask is there is, is there a

similar thing like that one could do with mean who would that one could do with median filters also, can we talk about a weighted median filter? The answer is yes.

So, the other kinds of filters that are also around okay among them are a few, which I am going to explain what is called a weighted median filter or a WM as it is called as a weighted median. How does how does how does a weighted median filter work? That is as follows suppose, suppose you have  $x$  equal to let us let us say  $x_1$  comma  $x_2$  all the way up to  $x_n$ . So, it basically means that we have taken you have taken a window about, about some pixels that you would like to filter and these values are like an  $x_1$   $x_2$   $x_3$  and so on,  $x_n$ .

Now choose  $w$  to be equal to  $w_1$   $w_2$  all the way up to  $w_n$  where, where this  $w$  consists of all each  $w_i$  is simply a positive integer, it is a positive integer that means you can take values 1 2 3 4.

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$$y[n] = Wm(x)$$

$$= \text{median}(w_1 \triangle x_1, w_2 \triangle x_2, \dots, w_n \triangle x_n)$$

where  $w_i \triangle x_i = \{ \underbrace{x_i, x_i, \dots, x_i}_{x_i \text{ repeated } w_i \text{ times}} \}$

Eg  $x = [12, 6, 4, 1, 9]$   
 $\text{med}(x) = \text{med}\{1, 4, 4, 9, 12\} = 6$   
 $w = [1, 2, 3, 2, 1]$



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Now, now, let us let us let us let us compute the output as  $y$  of  $n$  at that location  $n$  let us say  $y$  of  $n$  will be, will be of course the weighted median filter of  $x$  the weighted median  $x$ , which we are not going to write as median of this is we want to simplify to median of  $w_1$  triangle  $x_1$ ,  $w_2$  triangle  $x_2$  all the way up to  $w_n$  triangle  $x_n$  where, where this where this operator the triangle what this means is  $w_i$  triangle  $x_i$  is simply equal to  $x_i$  repeated  $w_i$  times.

Simply repeat.  $x_i$   $w_i$  times.  $x_i$  repeated, repeated  $w_i$  times that is what it means repeated, repeated  $w_i$  times. So, now you can see that you are trying to wait it in a sort of in a sort of a different manner when it comes to a weighted median filter, you are repeating that intensity value rate depending upon what you give us give us  $w_i$  for that particular intensity value.

So, let us take an example to understand this, let us say that  $x$  equal to 12 comma 6 comma 4 comma 1 comma 9, if I had done median of  $x$ , what would that be? That will be that is a median of we could just put this in an ascending order 1, 4, 6, 9, 12 and therefore, the median value will be this so which is equal to 6. So the so the, so the median filter value of this of  $x$  will be 6.

But suppose you want to do a weighted median, in which case I have to tell what is this  $w$  matrix now corresponding to each intensity would give this as 1, 2, 3, 2, 1. So, what this means is that 12 is going to repeat 1 time 6 is going to repeat twice, 4 is going to repeat thrice, 1 is going to repeat twice, 9 is going to repeat once.

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$y[n] = w_m(x) = \text{median}(12, 6, 6, 4, 4, 4, 1, 1, 9)$   
 $= \text{median}(1, 1, 4, 4, 4, 6, 6, 9, 12) = 4$

Center weighted median filter (cwm)  
 Gives more weight to the central pixel.

$y = \text{median}(x_1, x_2, \dots, x_{c-1}, x_c \Delta w_c, \dots)$


Therefore, the  $y$  of  $n$  if you say is equal to weighted median of  $x$ . That will be equal to median of, now, we will have a look repeat well once 6 twice, 4 three times 4 4 4, 1 two times so 1 comma 1 and 9 one time this would be median of now, now, that is a sort these numbers, let us say in an ascending order 1 comma 1, 4 comma 4, comma 4, 6 comma 6, 9 comma 12. And now if you look at the median value there are 2, 4, 6, 8, 9 therefore you leave 4 here leave a 4 here the median is 4? So, the value is no 4?

And as you can see, instead of if you are taking a simple median, you would have ended up with a value of 6, but then because of the fact that it took a weighted median, and if you look at it, this weighted median actually put more emphasis for the intensity fourth. And therefore, the output you also see that the median value has turned out to be for the weighted median value, okay, so that is it, you can also have a have a weighted median center, which can, which can actually do things similar to the way that we would expect a weighted averaging filter filtered through in the sense that we can you can start emphasizing certain intensities more depending upon what you want to want to read how you how you actually how you actually construct the weight vector.

Now, along the same lines, there is also something, something called Centrally Weighted or Central Weighted Mean filter or Center Weighted. Let us write it a Center Weighted, Weighted Median Filter what is called CWM Center Weighted, Weighted Median Filter what is also called CWM Okay, this gives more weight gives more weight to the central pixel more weight to, to the to the central pixel.

This is somewhat similar to to the fact that you would want to give the pixel under consideration more weight, while you are doing the filtering process. So, like saying that  $y$  is equal to median off so so this will turn out to be so you will have you may have  $x_1$   $x_2$  all the way up to let us say  $x_{l-1}$ , and then or let us say  $x_{c-1}$ . If you want to call the central pixel a  $c$ , then  $c-1$  where  $c$  is an integer. Then you have  $x_c$  repeated some  $w_c$  times. Well, we are writing it as  $w_c x_c$ ? How are we are we writing it?

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$$y[n] = \text{wm}(x) = \text{median}(12, 6, 6, 4, 4, 4, 1, 1, 9)$$

$$= \text{median}(1, 1, 4, 4, 4, 6, 6, 9, 12) = 4$$

Center weighted median filter (CWM)  
Gives more weight to the central pixel.

$$y = \text{median}(x_1, x_2, \dots, x_{c-1}, \underbrace{w_c \cdot x_c}_{\text{center}}, x_{c+1}, \dots, x_n)$$

When  $w_c = 1$ , you get median filter. When  $w_c \gg N$ ,  $y = x_c$ .



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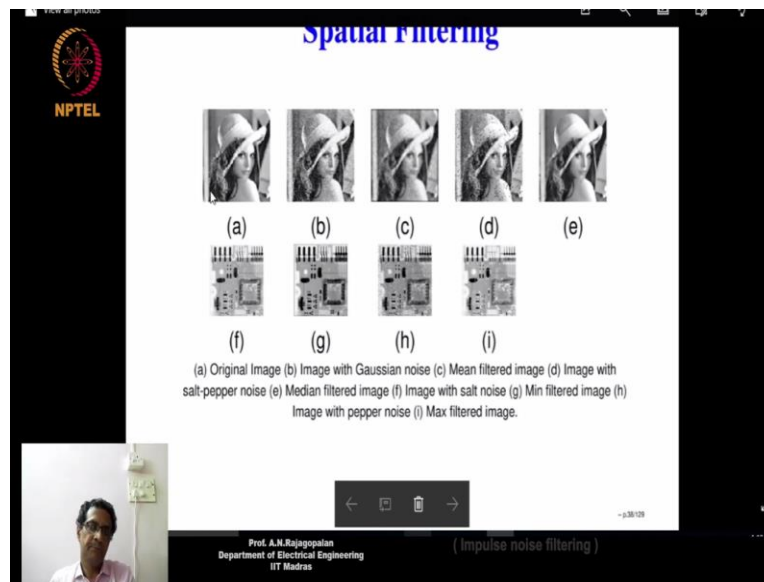
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we use so let us follow the same kind of notation that we have been doing before. So we use  $x_c$ , so the  $x_c$  repeats  $w_c$  times, then you have  $x_c$  plus 1 all the way up to  $x_m$ . Now, in this what you have done is you have kind of repeated, so essentially weighted center weighted because you are weighting the center by, by, by, by a certain weight  $w_c$ . Now, you can also show that when  $w_c$  is equal to 1 when the  $w_c$  is equal to 1, you get you get the median filter.

And, and when  $w_c$  is greater than or equal to  $n$ , you will get  $y$  to be equal to  $x_c$  itself? Which means that it is like an identity filter. If you want if you if you choose if you simply want to see things that intensity itself, you could also do that, or using a central or using a center weighted median filter, these are two extremes again, whereas you might want to choose to  $w_c$  somewhere in between. Having said this, let us kind of look at a few examples of how these are at how this filter works the median filter.

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Here is here is an example of how a Median filter output would look like. Now, this one is simply Lena. We just we just synthetically created we synthetically added noise. Now, this is the original Lena image and then when you add a Gaussian kind of noise and if you so you get this so Gaussian noise of course we cannot use a medium filter at all so you would use a simple averaging filter and then as expected there is some cleaning up of noise but there is loss of edges and so on.

If you had if you had if you had image with salt pepper noise it was all salt pepper noise because, because it almost looks like you have sprayed or sprinkle this image with, with salt and pepper. So, you can see the salt pepper noise here and then if you clean it up with a median filter what you get is this and this definitely looks reasonably sharp and that is the advantage of of the this one medium filter even though you have played it all over the image.

It need not apply the median filter selectively, we have applied it all over the image because the noise is reasonably dense here. Now this is a case f for example, it says an image that is affected by salt noise okay that means it has lots of saturated pixels think of it as some 40 ac display or whatever and now now what you could do is you know you could actually could actually apply apply what is called a what is called a Min Filtered in order to be able to handle this kind of unipolar noise salt noise.

And therefore, what you get is this image which is filtered output but again looks pretty good. In fact it should we should watch out for things that actually emerge which are not so clear it you know in the, the original image when when it is affected by salt noise. This is the one which is affected by actually, this one, this one a pepper noise that means this kind of makes me look more dark because it is affected by this kind of pepper noise.

If you use a Max Filter then of course many of these features emerge now and you are also able to then handle this kind of noise. So, it is very simple to implement, this kind of, this kind of image, yes of course they do not, they do not have a fast implementation because convolution does not work these are non linear, but, but still these are, this are still you can say heavy use these kind of filters whenever whenever one wants to be with and impulse kind of a noise.

And and to kind of a to this conclude this topic on the topic of noise filtering, let me just if you just mentioned the fact that, fact that may be not talked about let us say Multiplicative Speckle noise. Speckle noise is typically dealt with using wavelets and so on. And Photon noise also, we have not talked about photon is, is, is a signal dependent noise. And, and no, and again, it is not something that is actually easy to handle.

And typically, one of the standard basic ways to do it is kind of applying and anscombe transform and making it into a bit of a Gaussian random variable and then going through AWGN and so on. But that again is kind of beyond the scope of this course and therefore, therefore to be just have, we just have one more one more thing to actually finish before we wrap up, the wrap up this is a Noise filtering and that is what is called Transform Domain Filtering.