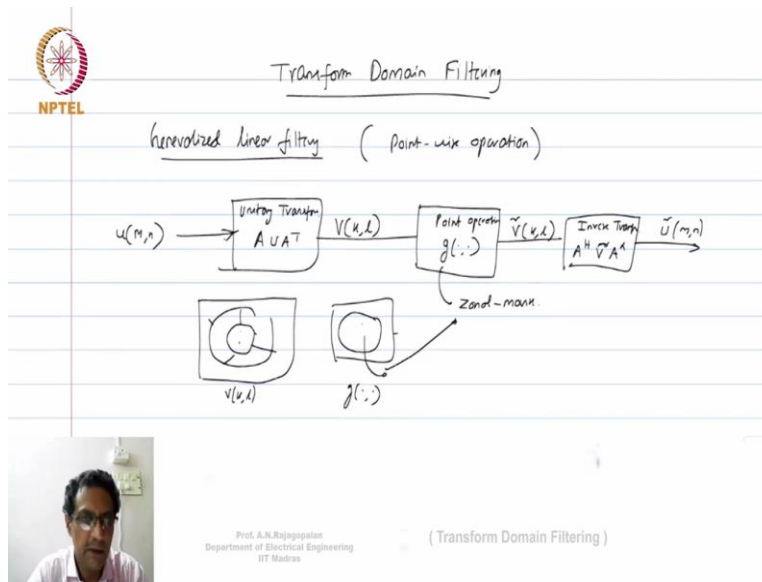


Image Single Processing
Professor. A. N. Rajagopalan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture No. 70
Transform Domain Filtering

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We will actually look into Transform domain filtering and the thing to remember is that transform domain filtering which means that instead of operating in the spatial domain that you are going to operate in these transform domain and transform domain that typically means some kind of a unitary transform that you would want to apply and go to a transform domain.

And but then it is not very common simply because of the fact that spatial domain filters give you so much flexibility for example, in elaborate does not have a Fourier counterpart, so, so spatial filters are much more adaptive, they are much more much more easy to read them in and all their data much more effective. And, and represent which is the reason why transform domain filtering is not really all that common.

But then for certain kinds of noise for example, periodic noise and all this is very useful. And the way and we call it generalized linear filtering. Because, because of the fact that we call a generalized linear filtering, because transform domain, domain filtering operators are typically

done using a unitary transform. And, and Let us say, it is a point, point wise operation. It is a point wise operation. Why? Because of the following reason.

So, the transform domain looks like this. So, so you have an image, let us say u of m comma n , then what do you do you compute a unitary transformation, you kind of apply a unitary transform on this, let us call this UT or, let us just write it down. So, you act some unitary transform on it, depending upon the situation, typically, this would be a DST, if you do that, then you have AUA transform.

So, we know that we need to do a AUA transpose where let us say A could be A would be the unitary transform corresponding to be corresponding to 1D. And therefore, in order to get a get a 2D unitary transform of u will do AUA transpose, that let us say gives you transform coefficients v_{kl} . Now what you do is, you do a point operation now that is why that is why it was written as point operation, where do you use some kind of what is called a zonal mask? So, the zonal mask is something that will do a point wise multiplication, let us call this as g . Let us call this as the zonal mask. So, it is like a point wise operation in the in the indicative transform domain.

So for example, if you had if you had a Fourier, if A was a Fourier transform, then what it would mean is your zonal mass, now we know that your v_{kl} suppose we assume it to be a centered spectrum, we know that the low frequencies are all here, the high frequencies are, are away from here, and what you could do is you could have a zonal mask which might actually like quick measure, which will actually I know attenuate the higher frequencies and, and simply let the lower frequencies pass and so on.

And so accordingly, you could have a filter here, which is called which is a zonal mask, which will do a point wise multiplication operation here and this point wise multiplication operation will change your v_{kl} , which was here it is your g . So, once you multiply g with v_{kl} point, point place manner, you end up getting what is called what is called \tilde{v}_{kl} . You have you have your transform coefficient have now been no been changed, some have been attenuated and some will be passed and so on. And this will then be read inverse transform.

So, you do and they do an inverse transform operation, invert Inverse transform, which will mean that you do A Hermitian v A star this way also \tilde{v} A star because which is, it should be done on \tilde{v} and outcomes \tilde{u} of m comma n if it is now back in the back it originally gets v

domain. So, u tilde mn has now been transformed. But so this transform domain filtering, it is not to say it is not as effective as the spatial, spatial domain filters, because spatial domain filters are more accommodative.

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The slide contains the following handwritten text:

SVD filtering

$$S = A \Sigma B^H = \sum_{i=1}^r \sigma_i \underline{a}_i \underline{b}_i^H$$

$$\hat{S} = \sum_{i=1}^p \sigma_i \underline{a}_i \underline{b}_i^H$$

$p < r$

Notch filtering for periodic noise 50 Hz.

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Prof. A.N. Rajagopalan
Department of Electrical Engineering
IIT Madras

(Transform Domain Filtering)

I mean, you can also you can also do a transform domain such as such as SVD filtering, it is also possible to do SVD using SVD also, then you can do some kind of filtering, which is, which also comes into the comes under consider transform domain filtering. What does it mean is if you had S equal to some a sigma b Hermitian. And now this let us say, we could write it down as sigma i , $a_i b_i$ Hermitian like we did before.

And suppose it has r single vector singular values which are, which are non 0, then they go all the way up to i equal to 1 to i , in order to reconstruct as exactly, what we could do is in order to filter S , we could create an S hat, which we need to sum only from sigma i a_i, b_i Hermitian where we kind of restrict get the summation to i is equal to 1 to p , it is a p is p is less than R . So, you can ignore the highest single values, believing that those probably contain noisy basis, basis images and therefore that you do not want to we do not want to take contributions from there that simply limit the contribution that that that helps you arrive at as had to the summation from i equal to 1 to p .


So, so SVD can also be used for useful noise filtering, not very commonly done, but yes or no because we are talking about transformed domain this is also one way to one way to do it. So, it

does not have to be necessarily Fourier or something. And one way one, one important right application of so let us say the Fourier kind of filter is not is in terms of notch filtering. And, and especially okay notch filtering for let us say periodic noise know if you can, if you recollect it in the right in the start of noise filtering, I mentioned that that you can have a 50 hertz kind of a periodic noise because of electromagnetic interference and so on.

And, and such a noise rate is very easy to is very committed to filter using Fourier domain, because, because it would appear as a spike in the Fourier domain at exactly those, those that that frequency and therefore, if you simply suppress that and reconstruct the image, you would have you would have eliminated, eliminated the noise. So, that period for certain kinds of noise, I mean, doing it in the spatial domain is not going to be easy at all, whereas doing it in the in the, in the transform domain makes up makes a lot more sense, if one way to one way to do this.

And, and therefore, even though transfer domain is not very common, but then does not mean that it is not used or something. So, there are situations that transform domain filtering can turn out to be very useful.

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Fourier-Based Filtering


- **Low pass filtering:** De-emphasizes high frequencies (edges, noise).

$$G(u, v) = H_{lp}(u, v)F(u, v)$$
- **Ideal LPF**

$$H_{lp}(u, v) = \begin{cases} 1 & \text{if } |u|, |v| \leq D_0 \\ 0 & \text{if } |u|, |v| > D_0 \end{cases}$$
- **Gaussian LPF**

$$H_{lp}(u, v) = e^{-(u^2+v^2)/\sigma^2}$$

(Transform Domain Filtering)



Prof. A.R. Rajgopal
Department of Electrical Engineering
IIT Madras

Here are some transform based filtering examples. We have seen already so, how to do it, we call this generalized linear filtering. And I just want to show you a few examples under this.

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The slide features the NPTEL logo in the top left corner and the slide number '-p42/23' in the top right. The main title is 'Low Pass Filtering' in blue text. Below the title are three grayscale images of a woman's face wearing a hat. The first image is labeled 'Image', the second '+ Gaussian noise', and the third 'LPF output'. The 'LPF output' image shows reduced noise but is slightly blurred. At the bottom left is a small video inset of a man. At the bottom center, it says 'Prof. A.N. Rajagopalan, Department of Electrical Engineering, IIT Madras'. At the bottom right, it says '(Transform Domain Filtering)'.

So, this all now, the first one is low pass filtering the Fourier domain, you have an image and then the second one is the image adequate noise is a Gaussian noise. And then that when you do a low pass filtering in the Fourier domain, and you would get something like this, you can see that the noise level is kind of come down in the in the output, but then there is also concerned that the images become a little blurred.

This is natural to expect, there is a reason why I said that spatial, spatial filtering technique, citizen element, bilateral filter, and all are far more effective. But in but in any case, one should know as to how these algorithms perform.

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Then, look at this Notch filter. And here, you will find that, you are doing a Fourier kind of filtering actually makes a lot of sense. So, here is the image and then whether when you add noise, single tone noise, what you get is this kind of regular pattern all over the image. Now, removing this kind of noise, using spatial domain filtering later would not appear to be very straightforward.

Where it is doing the Fourier domain makes a lot of sense, because we know that there is a single tone, it will appear at some spike in the Fourier domain, and if you know the frequency at which rate is occurring, because typically some electromagnetic interference, then what you could do is you could just go to the Fourier domain, and it is suppressed that, that, that particular frequency along with it. Of course, the image also has some component data that also gets suppressed, but that is after you reconstruct, you get, you get back the get, get the get the filter output.

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The slide is titled "SVD Filtering" in blue text at the top center. On the left side, there is a vertical dark grey bar containing the NPTEL logo. The main content area features two side-by-side grayscale images of a person with a camera on a tripod. The left image is labeled "Noisy Image" and shows significant salt-and-pepper noise. The right image is labeled "Top 30 singular vectors" and shows a much clearer reconstruction of the same scene. Below the images, there is a small inset photo of a man in the bottom left corner. At the bottom center, the text reads "Prof. A.N.Rajagopalan, Department of Electrical Engineering, IIT Madras". To the right of this, it says "(Transform Domain Filtering)".

Now, you can also do what is called a SVD based filtering of noise. So, here is a camera man image, which is a standard dimension, and that, and it has been affected by noise, we have added noise to it. Again, transform domain filtering, like I said, it does not have to be always fully a base, you can also do some big SVD filtering. So, what you do is you just take, let us say, a few of these singular, singular vectors do not take all of them.

So, when you when you just take, let us say, you know, in this case, it is a it is a 30 top singular vectors that have been taken out through in order to reconstruct this image, you see that? See that? It has been able to mitigate the level of noise a little bit, but then, it is, it is not great. That this result, I do not know is not so great, but it just helps to know that the SVD is very, which we did earlier for other applications. It also finds application like for let us say filtering for filtering noise.