Image Signal Processing
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Lecture 76
Image Deblurring

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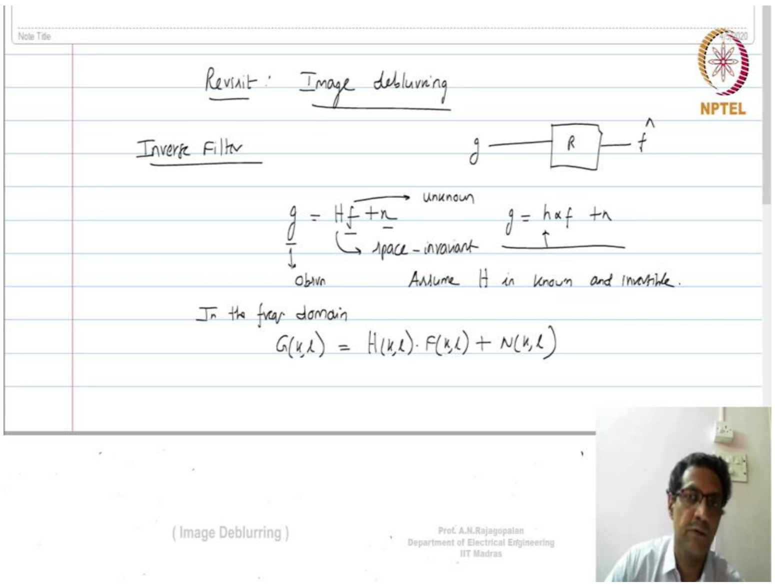

So, to really understand the implication for image deblurring problem, now let us revisit what we started off to really solve which was deblurring problem, along the way we saw so many notions of uniqueness, existence, the ill condition is all that which were not really specifically image processing but in particular but then those are very very general ideas but which are useful across the research areas but now let us come back to what does this all mean for the image deblurring problem, image deblurring is a specific kind of problem is that we would like to gain more insights into.

Now, we like to know what is this notion of ill-conditioned is how does it affect our deblurring, invertibility part the numerical stability part and therefore at how one could probably bring in prior knowledge in order to be able to offset some of fundamental weaknesses or fundamental issues that deblurring inherently has.

Now, in order to understand that let us first study what is an inverse filter, like I said you would need a restoration filter that will actually get you back so we said that we have g which is a blurred image we said we will have a restoration filter R which will then give you an f hat, it may be the simplest filter that you can think of is really an inverse filter, what is an inverse filter and how does it work?

Now, think of go back to our notion of our model, the observation model so which we know is g is equal to Hf plus n, this we know is our observation model, assume for instance that this is phase invariant blur, these are the deblurring model we know, this is the blur and then this is the clean image, unknown and this is the observation.

Now, assume that that H is known, assume H is known that means somebody tells you the impulse response or the points (())(2:22) function, the blurring functions, its space invariant and it is also assumed to be norm. Now, this in the convolution form we know this H convolve with f plus n and then when you write it in a vector, matrix vector form this becomes Hf plus n where this acts as a matrix which will have entries repulsed respondents cited (())(2:46) this becomes a vector now.

Now, I am not specifically underlining vectors but PS in order to see the differentiate between this convolution model and this matrix vector model it makes sense to say this. Now, because of the fact that this is all this space invariant blurred so for the Fourier domain for if you go to the Fourier domain the frequency domain what do we have, we can write effectively, you can write it from the spatial model we can write that G kl some frequency is equal to H kl the Fourier transform of the impulse response, F kl the Fourier transform of the of the image and then let us say N kl.

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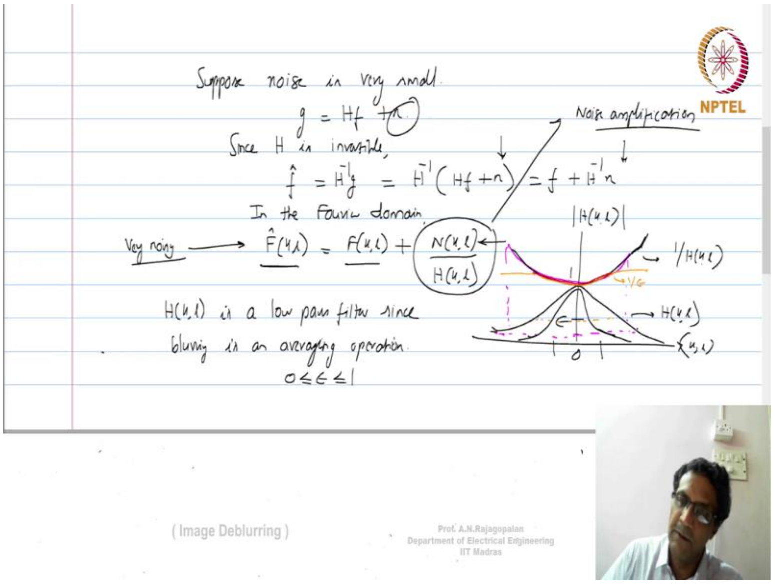

Now suppose, noise is very small that means in my observation the noise is very, very small tiny teeny noise, so g is equal to Hf plus n, now so you would be tempted to do the following if noise is very small then maybe what I would like to do is I would simply say because of the fact that H is invertible, since H is invertible and I might just say let the approximate f hat as H inverse g, I can do this because my noise in the observation very small let us ignored it, I mean it is there I know but then I just ignore it when I compute my solution, I simply say H inverse g but then H inverse g is nothing but H inverse but g is my observation model that cannot change, that has noise in it and therefore this is how it will become, this will become f plus H inverse n or in other words in my Fourier domain I can write equivalently that F f hat of kl is equal to F kl plus N kl upon H kl, where this Fourier transform of the impulse response.

Now if you notice this is called the inverse filtering because all that we have done is we multiplied g with the inverse of H because H inverse is what you would like to, H inverse is what you need to undo the effect of blur, so it is called the pure inverse filter. Now, you have the N kl by H kl now that means your solution, the estimate that you are getting of F is away from F by this number.

Now, if this was is N kl we would have been happy because we knew for a fact that noise is very small, therefore if the solution had been F kl plus N kl we would have been happy, unfortunately it is not that it is N kl by H kl the fact that even if noise is small but the noise exists because that is where g is (())(5:51) Hf plus n had to be brought in because noise is not 0, so therefore, some noise is there and therefore some small amount of noise is there but then when you are kind of deriving the solution what is going on is that you got like N kl divided by H kl and for us H kl is not something arbitrarily H kl is really a blurring function, what does that mean?

That means that let me draw the right hand side, so that means that H kl is going to look like that, this is kl and that this is my H kl and as we know that the magnitude of H kl at 0 comma 0 should be 1, you can think of this is the magnitude of H kl if you really wish pt this as magnitude because we know that the impulse response sums up to 1, therefore h at 0, 0 should be 1 and then as your (())(6:42) increases, so it start to fall, falls because of the fact that H kl is a kind of a low-pass filter, any averaging operation, blurring is an averaging operation, is a low-pass filter since blurring is an averaging operation, blurring is an averaging operation, when you say that your blurring an image you are doing some kind of an averaging, weighted averaging perhaps, whatever it is so it will have to be a low-pass filter, your (())(7:09) become blurred and all that if it is a low pass filter.

Now, doing 1 by H kl amounts to saying that you have something like that, so this is you are going up like that and that is 1 by H kl, this is the H kl, then your 1 by H kl would look like that. Now, the issue with this is that when you are doing 1 by H kl and then so if you see 1 by H kl keeps increasing as kl decreases, therefore for small amounts of noise it would not matter because H kl and all will be close to 1 but then as noise increase k comma l increases then for higher K comma l H kl is actually falling or 1 by H kl is actually increasing, therefore there is an amplification of noise, noise amplification going on here at higher frequencies which that means that if you simply blindly try to do 1 by H kl for all frequencies then I think that it is blurring like this.

Therefore you do 1 by H kl over all frequencies you will end up producing and if they are a F f hat of kl which is very noisy, which should be unacceptable to you, therefore in the inverse problem what is normally done is you try to write small little trick, what you do is you fix up some (())(8:30) think of it as some kind of a slider which is a number between

0 and 1 and then you limit the amplification factor because that making the, taking the whole H kl as it is that creates problems because at 1 by a large kl, 1 by H kl becomes really big and that amplifies your noise in a high manner, this is what we mean by the ill-conditioned mess, we have assumed that H is invertible that is the reason why we could divide by H.

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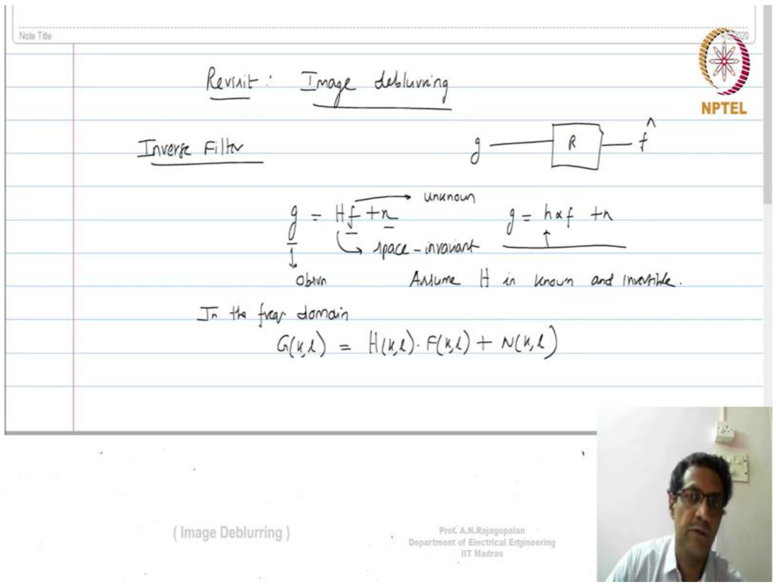

Look at these we assume space invariant in fact if you see here we said H is known and actually invertible in fact, that also we have assumed that means that we have assumed a very ideal condition, in a real situation we may not even be able do a H but we have assumed something that is that can be as easy as we can make it but despite all of this there is still trouble brewing because of the fact that that you are doing in N kl by H kl and 1 by H kl is blowing up, therefore what is done is you limit, so the 0 is less than epsilon less than or equal to 1, you keep a slider and then what you do is.

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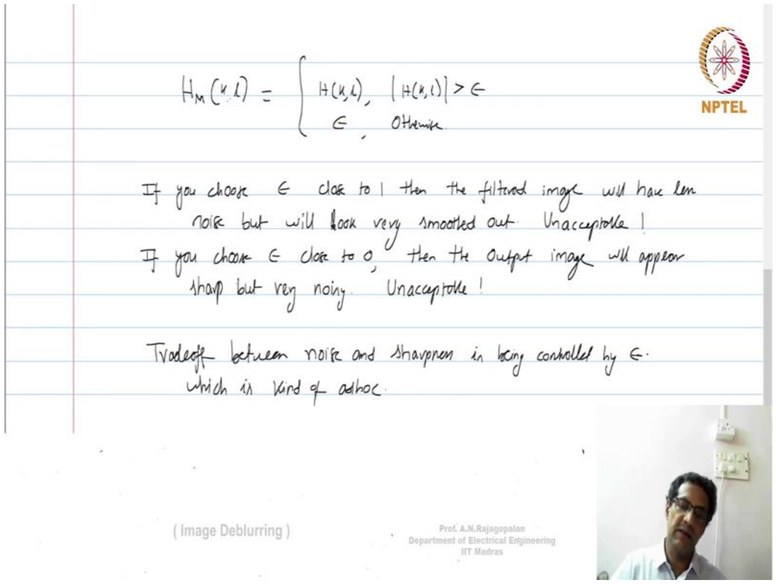

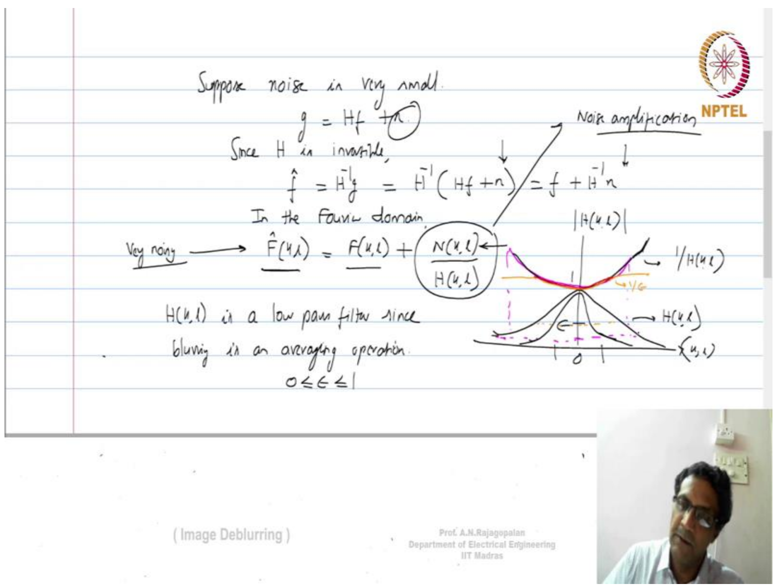

So instead of using H kl as it is you modify it to something called HM kl, where HM kl will be equal to H kl provided magnitude of H kl is greater than epsilon and may epsilon otherwise. So, what this means? Means is that so as long as your magnitude is fairly high that means right till this point right here you continue to deblur and then the moment it falls below value of epsilon you kind of restrict this H kl to epsilon which means that you kind of limit inversion to 1 by epsilon, so this is like 1 by epsilon.

So, means that your only kind of, so what this means is that you are able to deblur only in this region, up to this frequency after which even though the actual blurring might have happened, it is going way down but then that you are not deblurring all that you are only deblurring up to this frequency.

Therefore, what you can say is, so if you choose epsilon close to 1 then the output image, the de filtered image will have less noise because of the fact that you have hardly declared deblurred it, less noise but it will be too smooth but will look very smooth out or blurred so unacceptable.

You do not like just because it does not have noise it does not mean that it was okay. Now, if you choose epsilon close to 0 then the output image will be sharp because of the fact that you have been you are deblurring because then you are going like all the way, so you are going like suppose you choose epsilon here then it means that you are going to deblur all the way right up to this.

Let us say right here is your, so we are going down so that is the epsilon then you are actually deblurring a lot more and therefore then the output image will look sharp, will appear sharp but very noisy, but will be very noisy again unacceptable, unacceptable therefore what this means is that one needs to find out what may be the optimal epsilon for this image my deblurred output looks okay.

Now, this also means that if you have heavy blur then your ability to deblur will actually go down because heavy blur will actually mean this curve will become even more narrow, that means that your magnification will in fact start much earlier than for example, when you have a less blur and so on. Therefore, your control so how much you can deblur it is like a trade-off between how much you can deblur versus how much of noise you can accept.

So, this kind of a trade off, so trade-off between noise and sharpness and output sort of sharpness, the quality or deblurring quality between noise in the deblurring quality, here we are we doing it in a loose sense in the sense that epsilon is sort of trade off is being controlled by epsilon, it is controlled by epsilon that is what is happening which is ad-hoc there is a systematic way to actually do this, we might be able to do a much superior job as compared to a simple inverse filter, I mean such a theory that allows you to bring in prior knowledge in order to be able to regularize your solution in order to be able to get a judiciously guide, guide the algorithm to the correct solution and not to a noisy one is called regularization theory which is what we will see next.