Image Signal Processing
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Lecture 81
Fourier Wiener Filter

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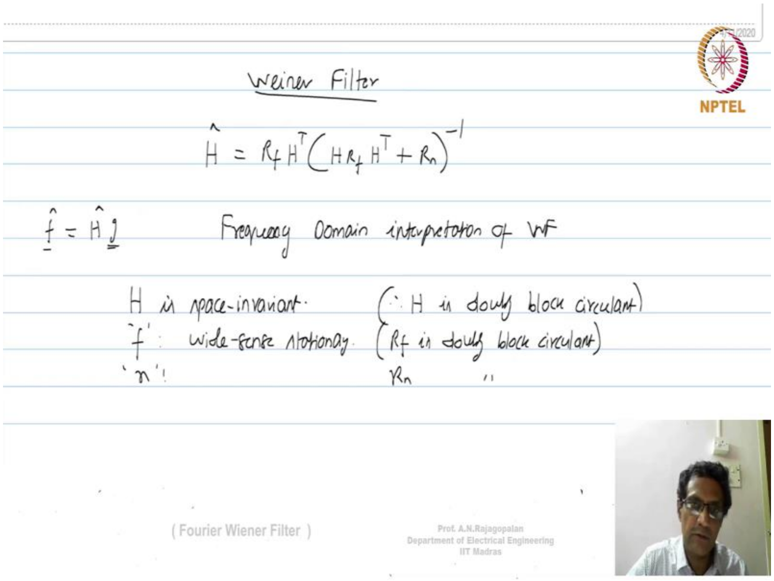

Alright so we saw the spatial domain expression for the wiener filter, which was so the wiener filter if you come back to the wiener filter then what we had was H hat is equal to l H hat, this is a filter and this turns out to be RfH transpose and I also showed another form for it if you see a b c d inversion lambda, wherein you can show that Rf enters, instead of Rn Rf inverse will enter the (())(0:53) that will lend it stability to the solution. Now, this the spatial domain thing is not really that you know insightful except for the prior and things that we can see.

What is more interesting is if I try to look at the, if the frequency domain interpretation of the wiener filter, the frequency domain interpretation of the wiener filter, domain interpretation of wiener filter, then that will actually give us a chance to get a look at its relation with the inverse filter that we saw in a sort of deterministic things earlier and we will be able to relate that I mean we will also be able to see how the wiener filter is able to achieve both the blurring as well as denoising.

Because we saw that we have both blur and noise and for it has to strike some kind of a tradeoff between how much to deblur and how much to denoise and then why, then an optimal way of course. So, all that we can see if we go through the frequency domain interpretation of the wiener filter. But then the frequency domain interpretation for doing this we will have to assume, we will have to make some assumptions the first assumption that we are going to make is H is space-invariant, otherwise you know this frequency interpretation becomes hard.

And it is okay to assume because most of the times blur is space-invariant, therefore it does give you the leeway to actually analyze things for you to make. Now the f process in this case f (())(2:23), we can (())(2:24) assume it to be wide and stationary. So, that the Rf its covariance is actually doubly block circulant, so this means Rf is doubly block circulant. These are the kind of things that we have anyway seen before also, therefore H is doubly block circulant.

So, these two matrices and always remember that when we write f hat to be equal to H hat times g we have lexicographically ordered the image f for the area of f hat that we have estimated is lexicographically ordered, similarly the observed images lexicographically ordered as a vector, so Rf is doubly block circulant.

Now, under these assumptions we can now then Rn same applies to noise and therefore same we can say about Rn too. Now, under these conditions we can make an attempt to go to the Fourier domain and as we know in order to kind of go to the Fourier domain then it will mean that we will have to multiply it by a Fourier matrix.

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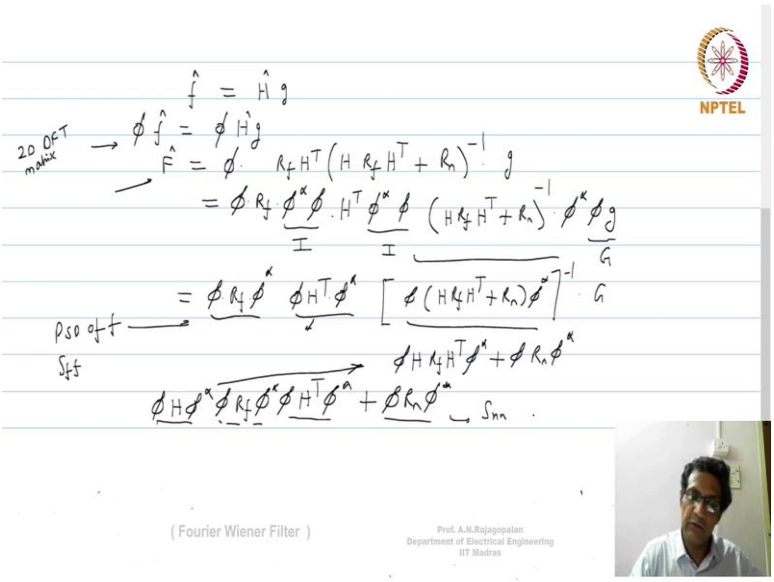

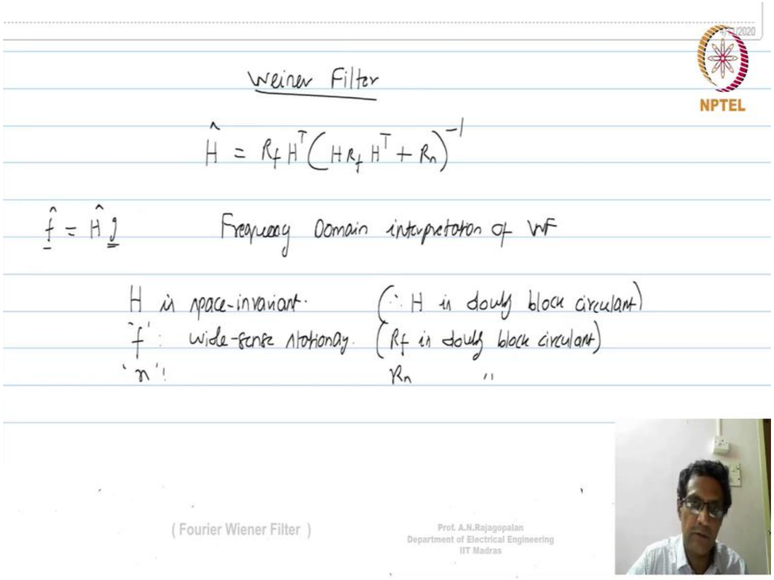

So let us say that I have got a f hat is equal to H hat into g g is my observation therefore I will do phi f hat in order to take me to the Fourier domain, will mean that I have got phi H hat g, of course in this case this has to be a 2D dft, 2D dft of an appropriate size which in this case f hat is m by m then this vector is m square by 1 therefore your 2D dft will also have to m square by m square.

So, this here 2D dft matrix which you are pre-multiplying, now pre-multiplying f hat by phi in order to go to the Fourier domain and this will be this and this takes you to the Fourier domain as f hat now here also we would like everything to come with the Fourier domain but let us first look at phi then H hat itself is what, let us just copy it from here Rf H transpose here we can remove this bracket Rf H transpose and then we got H Rf H transpose, H Rf H transpose plus Rn the whole inverse look at this is your H hat times g.

Now, on the left we have gone into the Fourier domain already but on the right, we are not seeing anything like that so let us play the usual trick let us do phi Rf phi star phi, we know that phi star phi is identity therefore that is not going to change anything if you put phi star phi between I mean between Rf and H transpose.

So, we can put this as H transpose and again now what we can also do is, we can again multiply here, what we can do is we can H transpose and then we can again write this as phi star phi because this is again identity followed by H Rf H transpose plus Rn the whole inverse, let us again put phi star phi g, now phi g will of course take you to the Fourier domain so this will be g so this takes you to the Fourier domain of g.

Now, we would like to club things phi Rf phi star, we know we will diagonalize Rf because Rf is doubly block circulant therefore it will be diagonalized by the 2D dft matrix then phi so now we will de-couple this and then take phi onto H transpose phi this will give you a complex conjugate of your dft. If it was phi f phi star it would have been your dft coefficient but because it is H transpose it will give you H star that is the complex conjugate of a dft coefficient, then this whole thing that you have here this we can push the inverse inside and call this as phi H Rf H transpose plus Rn phi star the whole inverse and here it is g.

And you can see that this is still correct because this is like a b whole inverse which is b inverse a inverse phi star inverse is phi, which is sitting here at the inverse of this is inverse into a inverse into phi inverse this inverse is here phi inverse is phi star and therefore this is all phi. Now, this we can further simplify as you can push phi star from there we get phi H Rf H transpose phi star plus phi Rn phi star.

And this you can further simplify so this guy gets simplified as phi H phi and same trick phi star phi Rf phi star phi H transpose phi star plus phi Rn phi star. Now, we are able to go into the Fourier domain for all the terms, so now let us write this down so what this means is individually if I take up the k lth coefficient and remember that phi H phi star is a diagonal, matrix phi Rf phi star is diagonal, phi H transpose phi star is diagonal, phi Rn phi star is diagonal because every one of these matrix is doubly block circulant.

Therefore, the inverse is also diagonal because you it is just 1 by, simply there is a simple core of all the diagonal entries therefore it makes it easy. Now the Fourier transform of the auto correlation function gives you the power spectral density psd of f. Let us indicate sf suppose we indicate that it is sff so that will be the power spectral density of f.

Now this will be H star of kl which will be the dft coefficient of P impulse response the complex conjugate of the dft coefficient of the blur which in this case is space invariant this will be the dft coefficient of the point spread function itself, this will be sf, this will be this will be the complex conjugate of the dft coefficient of the spread function, the point spread function this will be the psg of noise.

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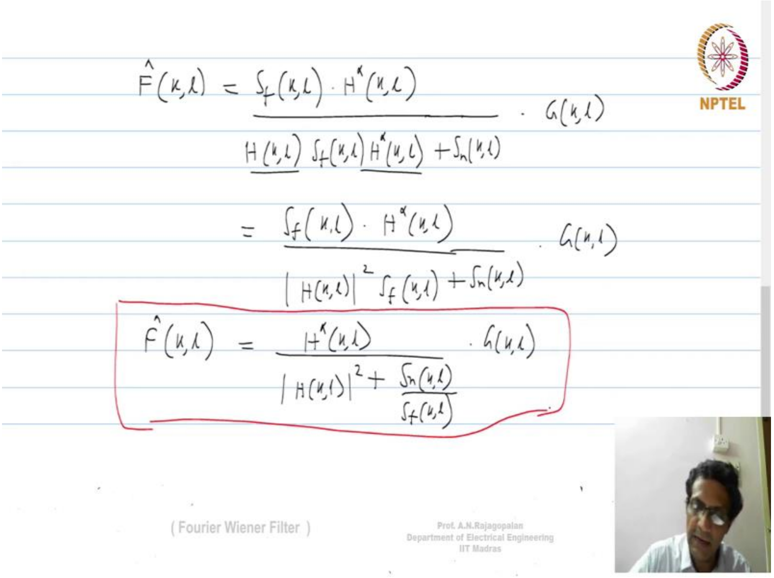

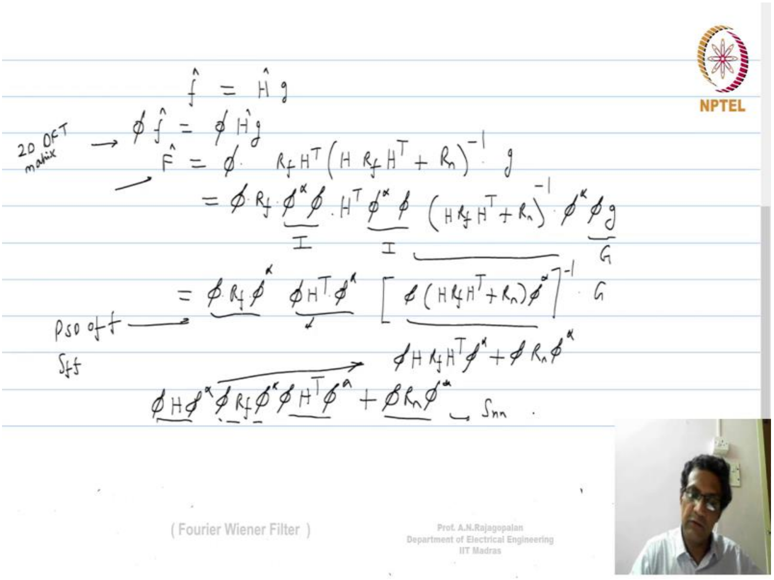

Let us indicate this as snn therefore for every k comma l because all these guys are diagonal so it is easy to write them down instead of a matrix form we can write this down as for a Fourier coefficient k comma l we can write this as see at the top we have got like sf this is sff of k comma l or let us write this as sf of k comma l, this is H star of into H star of k comma l the whole divided by because it is a diagonal and you are taking the inverse of the whole thing divided by phi H phi star will be H star of kl into this is sf of kl into this is H star of kl.

Now this is H, this is not H star because the first term is just phi H phi star this is phi H transpose phi star is H star of kl and then plus Sn of k comma l the whole into g of k comma l which is the dft of the blurred image, blurred or noise image or this can be in turn written as sf k comma l into H star k comma l upon magnitude H kl square because of the fact that you got H into H star sf kl plus Sn kl or into g kl of course or this can be written as if we divided by sf kl assume that kl is sf in not 0 anywhere by magnitude H kl square this is the most standard form plus Sn of kl by sf of kl, this is your wiener filter in the Fourier domain.

And now this into of course g kl, which not forget that we have to multiply it with the observation in order to see the deblurred image. So this f hat is your deblurred and denoised image. Now, this filter you can have interpretations for this filter which can throw some light on what is going on, the interpretation follows like this interpretation because that is what we want to do.

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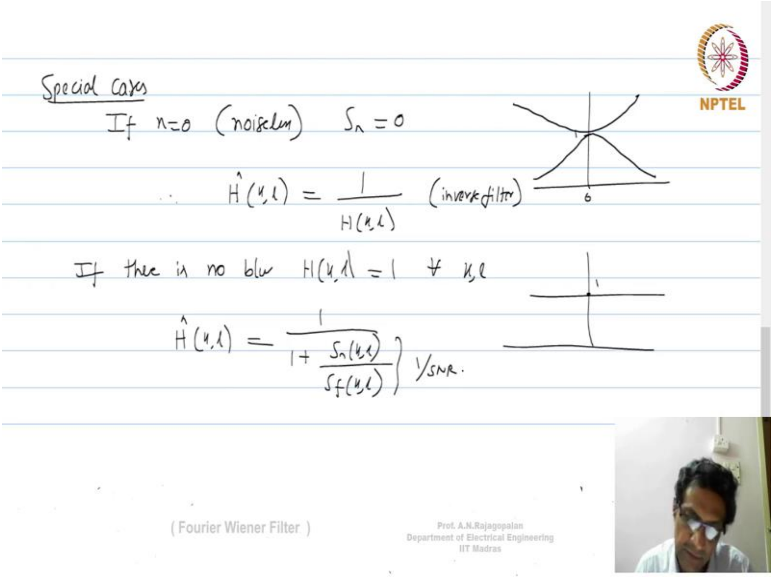

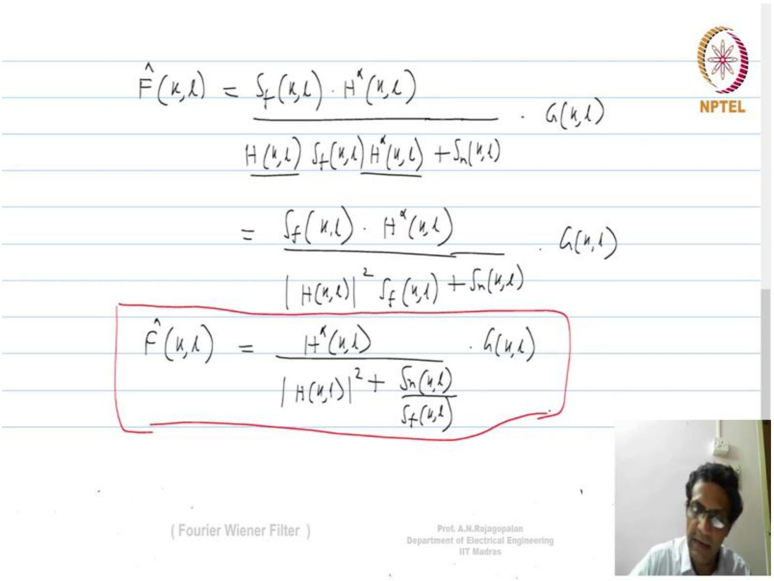

Now, suppose examine special case, special cases if n equal to 0 that means noiseless if there is no noise therefore which means sn equal to 0 that means your sn is 0 therefore what will happen is in this expression you will get H star by magnitude H kl square which is simply 1 by H kl. So therefore, the wiener filter H hat of kl becomes 1 by H kl which is simply the inverse filter.

And as you can see because it says that you have no noise therefore that it is willing to do for example in this case you have got this blur and there is no noise and this value we know is 1 for the forward blurring operator, therefore it says that I will go ahead and kind of deblur just as an inverse filter, that would do.

That is the way you have to work if you have a special cases n equal to 0. I mean next case is if let us say if there is no blur there is no blur or in other words we have H kl is equal to 1 for all kl that means you have only noise in this case which actually means that you have a blur which is simply a constant but then you do not have any kind of noise and for this situation if you examine what kind of H hat you get therefore your domain interpretation you will get. Now in the earlier equation substitute H star as 1, H everything is 1 therefore you will get 1 by 1 plus sn kl by sf kl and this sn by sf in a loose sense this you can look upon this as 1 by signal to noise ratio because signal to noise ratio is sf by sn therefore in a loose sense this is like 1 by signal to noise ratio.

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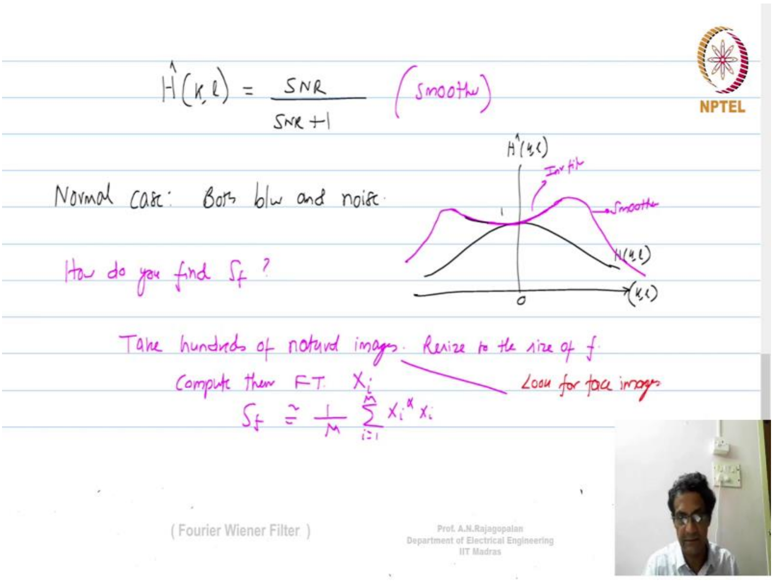

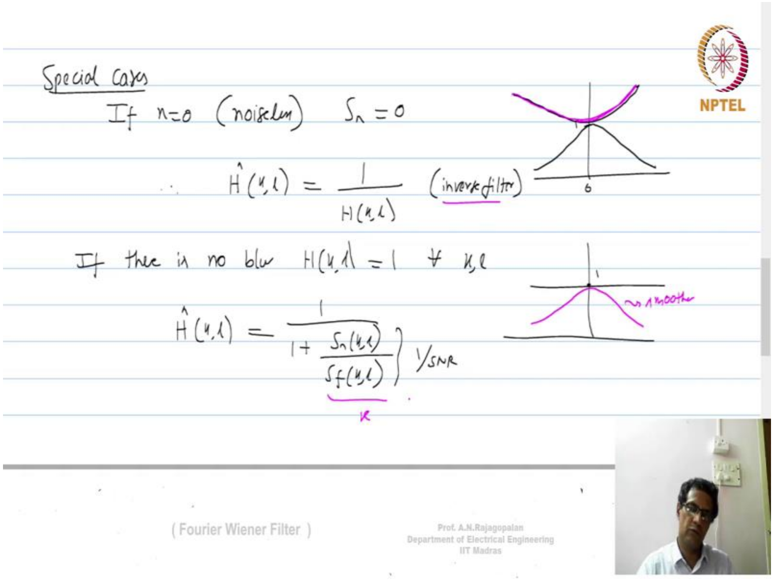

And therefore you can further write this as for the case when there is no blur when there is only noise no blur and there is only noise, so you get SNR because 1 by 1 plus SNR 1 by 1 plus 1 by SNR so it is SNR by SNR plus 1 so what you have is SNR by SNR plus 1. Now if you see this what it means is that at lower frequencies, if you see lower frequencies your SNR, SNR is likely to be high and because of this it will be overwhelmingly larger than 1 and therefore these 2 will cancel off and then you will get a gain of roughly 1.

And then at higher frequencies, SNR is likely to be very low and therefore your gain will be roughly equal to SNR itself which means that it will start to fall therefore if you look at so in this case the inverse filter, the wiener filter behaves like an inverse filter in the first case when you had noiseless situation and in the case you have only noise and there is no blur it will try to act like a low pass filter.

So, this is like a smoother that means it is just going to smooth out noise and because at higher frequencies it wants to put lesser and less emphasis on noise and therefore it will try to give a gain which is much less than 1 for noisy, for noisy values in order to handle noise. Therefore, at higher frequencies the gain will go down, therefore if you look at it this guy acts like a smoother when there is only blur and in this case it acted like an inverse filter in the other case.

Now, what you typically will have is the normal case is when you have both blur and noise and when you have both blur and noise what the wiener filter does is it kind of strikes a beautiful tradeoff between being inverse filter and then being the smoothing filter because those are the two extremes.

Therefore, if you think about it when you have blur this is your blur this is your Hkl and what the wiener filter will really do is, if you see the expression of wiener filter when you have both blur and noise what it will attempt to do is the following initially this gain is 1, the gain at 0, here is your k comma l, this is your frequency and this is your H hat of k comma l, this is your Fourier, this is the frequency response of your wiener filter.

Now, at lower frequencies it will behave more like an inverse filter because it knows that at lower frequencies it can afford to actually invert the blur therefore it goes like that at lower frequencies it goes like that, it behaves like an inverse filter at lower frequencies because that is how it is supposed to behave. And at higher frequencies it kind of begins to acts like a smoother because it knows that beyond a point behaving like an inverse filter does not make sense, because then it will amplify noise.

Therefore, beyond a point it will start to reduce the gain and similarly beyond a point it will start to reduce a gain. So, it has some kind of hump here and in this region it behaves like an inverse filter and after some point it behaves like a smoother and after what point it should take over from being an inverse filter to a smoother it depends on the equation itself it is automatically done, we do not have to choose this.

It will automatically do it depending upon the values of your, of the power spectral density of noise power spectral density of the signal and rate how much of blur you have in the image. So, in that sense it will get to choose its value optimally, so this transition will happen in a optimal way because your whole wiener filter itself is an optimal filter and therefore this is how it achieves the deblurring as well as denoising.

Now, the one sort of question that remains is how do you find, I mean so how do I find sf? Now, you can imagine that you can go to an image and then find the homogeneous region that means find the roughly a very smooth region and then compute the variance of the image in that region in the observation and that will give you a sense for noise. Because in all of this we are assuming that we would know the blur, how do you find sf could be a question.

Now, how do you find sf but of course in this case we can even assume that if it non-blind deblurring, if it is non-blind then we will assume that because it is what we are assuming till now, we will assume that the point spread function is known, we may even assume that spectral density of the noise is known, but what is not clear is how do you get sf because we have only a single image and where is your auto correlation function and so on.

So therefore, there are the common way to do it is, to simply assume is, one of the way is to kind of do it is to simply assume this to be a constant k and find out for what value you have k, is your, you see deblurring turning out to be best visually. So, simply this is value k and you simply vary k, all the way from 0 to upwards. So, when it is 0, it is like a, you know it is like a instead of k, it will be kl is equal to 1.

And then you can take these ratios as the constant and then for that you have, you do an inversion and then kind of you change your value of k and for each value of k you compute inverse filter and then you compute the deblurred image and try to see which one of them is most appealing.

But then more common, for example MATLAB allows an implementation like this, but what is more common and more sensible to do is, to take a bunch of, take hundreds of natural images around you, and from you in fact if you just go to a data base where there are so many data bases available these days, therefore we could simply look for natural images, data bases natural images.

Take hundreds of natural images, compute the Fourier (transform), of course change whatever, resize them to the size of the image that you have on hand, resize them and then what you do is, so resize to the size of f, which is your, or the size of f or g that you have with you and then compute, compute Fourier transform, compute the Fourier transform, let us say Fourier transform, let us say each of these images have a Fourier transform Xi.

And the approximate Sf is 1 by M summation, if I you know write Xi, if it is a matrix Xi is a matrix now, Xi it is a kind of dft, it contains dft coefficients of that natural image. So do Xi star Xi, this is a matrix, element i is a matrix multiplication, that i goes to 1 to n. So, such an averaging of the magnitude square of the Fourier coefficients of natural images can be taken to be rough estimate of Sf, because in the absence of any other knowledge.

If you knew a little bit more about Sf, suppose let us say now somebody told you that Sf is actually a face image, then I can do a little better, instead of using natural images what I will do is, I will probably go look for faces, look for face images and take many of them, and then again do the same thing, hundreds of face images, compute Fourier transform, take the magnitude square and then average it over all those images and that can be taken as Sf.

And if g is blurred face in it, then this Sf will be actually much better than taking arbitrary natural images because again this is some kind of a prior that you know, because if you look at an image, you will know whether it is a face or whether it is something else. Now, that prior can again be utilized in order to kind of to bring in more (())(21:09) solution.

So, that is why I said right at the beginning that the prior can come in any form and you can bring it in the algorithm, you can bring it in whatever ways, that you can actually bring that information and that is going to lend it stability, numerical stability. Therefore, instead of using arbitrary natural images, if you try to use only face images, given that g is actually a blurred face image, then in that case, if you try to compute Sf using only face images, then that estimate will give you a better deblurred image than the one that you would get with just arbitrary natural images.

With that we conclude the wiener filter, and you would, and just one last comment that the wiener filter of course has its Fourier kind of an interpretation and all but as of today if you try to see what kind of deblurring algorithms are out there, where are kind of most sort after and which are most used are the ones that are still in this facial domain, especially the kind of constraint least square kind of solution where I told you that, we have an observation term.

And then you have prior, the prior is of the forms some gamma times norms of Qf, norm of Qf and the norm itself could be L1, norm could be L2, Q could be a laplacian, Q could be first derivative along X as first derivative along Y, some of those gradients, the L1 norm, because those are the things, because the wiener filter is nice in the sense that you have close form expression.

But then because you are assuming space-invariant you are assuming some knowledge as the spectral density and all of that, this could still, this will not still match up to the quality that you would get through spatial optimization, therefore spatial optimization method has still more general, more accommodators as I said right at the beginning, they are more general, they can deal with more general situations, that priors can also be far more powerful than the inclusive priors something like a wiener filter has.