Applied Linear Algebra
Prof. Andrew Thangaraj
Department of Electrical Engineering
Indian Institute of Technology, Madras

Week 01
Introduction to the Course

Hello and welcome to this course on applied linear algebra. My name is Andrew Thangaraj. I am a faculty in the Electrical Engineering Department at IIT Madras and I will be handling this course for you.

What is linear algebra? What are its various applications in engineering? Linear algebra is used very very widely. All of you might have taken some course at some point where you solved a system of linear equations and linear algebra is at the heart of that. So the basic idea is - in most engineering systems there are some inputs and there are outputs, and you want a relationship which gives you the output as a function of the inputs. Right? And then you optimize. So you do something. Most engineering systems can be modeled in that fashion and in fact if you look at many of the systems out there, the first cut approximation or the first model is to assume that the outputs are a linear function of the inputs. So linear models for systems occur everywhere in engineering. Definitely in electrical engineering they occur very very widely in electrical circuits and signal processing systems, and even in mechanical systems in more interesting systems like, you know, managing traffic flow, economic models, population models. Any model you take where you have to track some output or some item of interest as a function of inputs, the first and obvious model that people use is a linear model.

So, in fact, it's not wrong to say that when you're in doubt or you don't know anything, you first assume a linear model and then see what happens. So, so linear algebra really dominates systems and studies in engineering and elsewhere, and linear algebra is at the heart of how you do computations with the linear model, how do you solve for some unknowns, how do you design something to achieve a certain output, how do you evolve the system over time, how do you study that, how do you optimize the system in terms of its desired output… So all of this, for all of this linear algebra plays a very important role.

So that's a sort of traditional application. But in today's data driven world, an internet driven world, there are a lot of very interesting modern applications which are very very exciting today, and linear algebra has sort of become the heart of all of that, okay? So to start off, Google's search engine uses a strong linear algebraic foundation in how it was designed. So you can look up this paper which talks about a 25 billion dollar eigenvector which is at the heart of google's page rank algorithm. So that's a very interesting application isn't it? So on a modern internet, how do you do search and how is that connected to linear algebra? It may not be very obvious but there's a very nice interesting connection, and another application I can point you to is these recommendation systems on OTT platforms and even retail platforms you might have seen. How, when you log in, there are suggestions for you on what to watch, what to buy, etc. and it's personalized to you, and how they do that is based on a lot of interesting linear algebra behind it. Okay? So that's also a very surprising and interesting model where linearity is used in a very interesting way to come up with recommendations.

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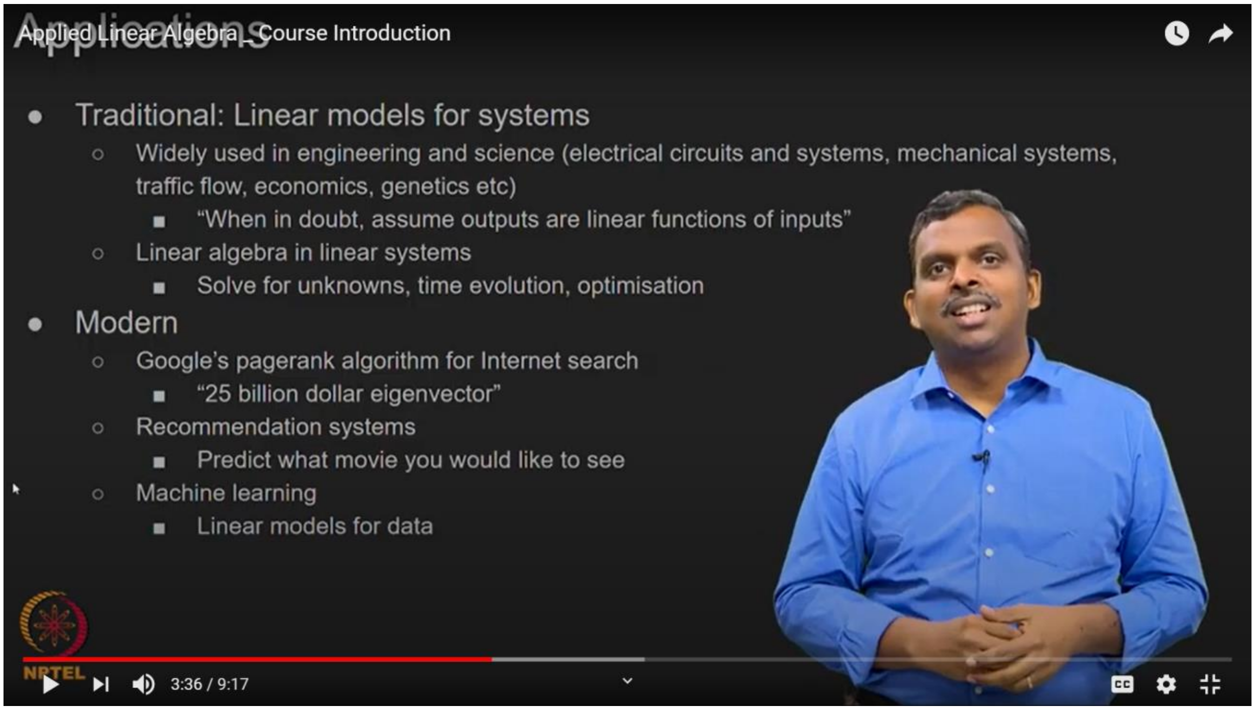

And finally I want to generally talk about this area of machine learning which is very exciting and that uses a lot of linear models and a lot of linear algebra in many of its algorithms. So linear algebra is exciting. It has so many, wide range of applications. And also one of the things I haven't mentioned here is typically for engineering students and many students out there, linear algebra is probably the first abstract mathematics course that they do. Right? So it's also an interesting step into the world of mathematics for many people. How to, you know... What is the mathematical theory, how to study that, what are the results that come out of it, theorems and lemmas and all that. So people get introduced to that for the first time in a course like this. So that also, for some people, is exciting.

Okay, so having given you a lot of details on what to look forward to in this course, let me go into the more minute details of what exactly we will be doing in this course. So this course is like a second course in linear algebra. So this probably doesn't come in your first year or so. So in the first year you would do a traditional vectors and matrices kind of course. How to deal with matrices and vectors and how to do computations etc., how to solve linear equation using Kramer's rule or something like that. You might have done a course like that earlier, so this course sort of comes in your third year or so. Third year, second year or something like that and then deals with abstract notion of vector, okay? Instead of just thinking of a vector as a list of numbers or something with a direction and magnitude, we will think of vectors and abstract vector spaces and build it up from basic axioms the way any mathematical object is traditionally defined. So that's what I meant when I said this is sort of the first step for many students into the world of mathematics.

And we'll also study linear maps which are the generalizations of matrices. Right? So we'll go from vector to abstract vector which is a very big generalization. We'll go from matrices to the study of linear maps and look at everything in a proper axiom driven way, define the objects carefully by axioms and develop everything only going back to those axioms, okay?

So what are the prerequisites? It's generally self-contained, but still I would assume basic calculus, you know? Functions, basic things about numbers, adding-subtracting... All of that the basic calculus will assume and also since this is sort of like a mathematical course in some sense, you will have to have comfort with the mathematical notation, mathematical definitions and the basic logic that is used to make inductions, right? So inductive conclusions about, about the mathematical objects we are considering.

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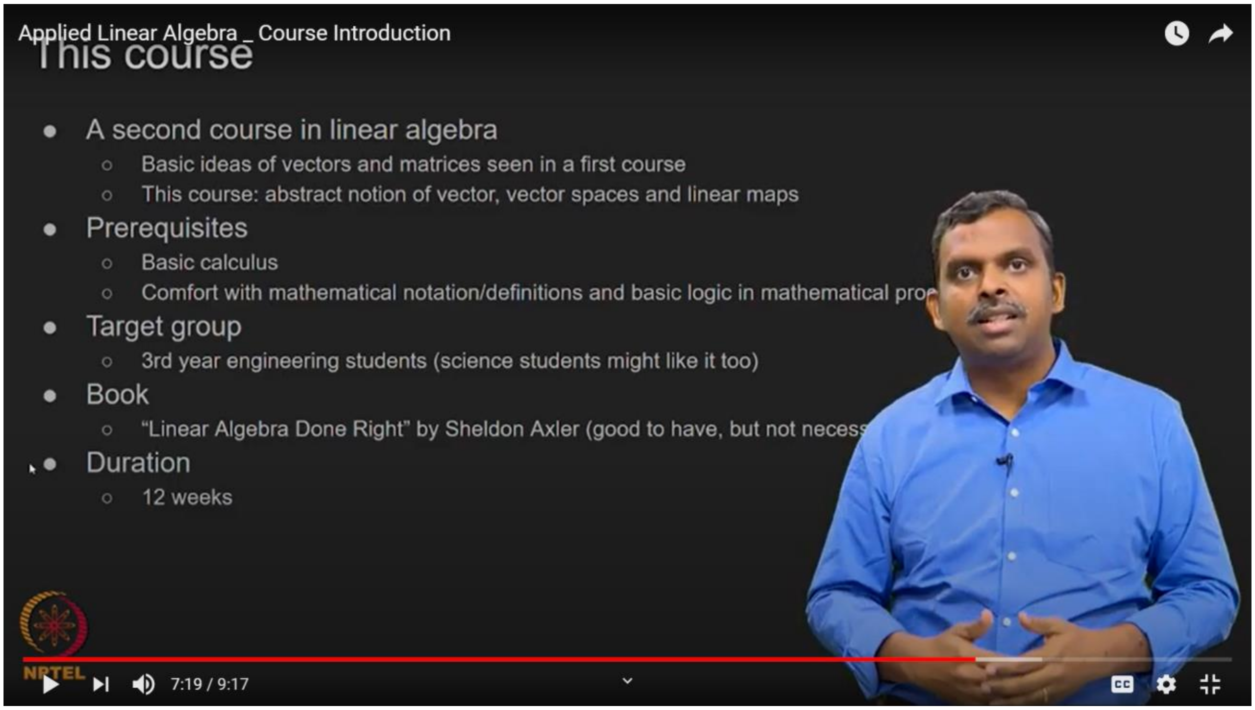

So we will define vectors, vector spaces, linear maps and then we will use the definition very carefully with logic to get more and more better results and theorems and etc. etc. So that's how the course will go. You need a basic comfort with that. And I will take it slow, I'll go slow because I know many engineering students are probably not used to the rigorous mathematical way in which things are done. So we will go slow. But that is at the heart of the course, the basic mathematical way of treating a subject is at the heart of this course. So the target group is generally third year engineering students, but it could be other students as well who want to get used to, get to know about linear algebra in a slightly more modern way.

And the book that I will use is this nice book called Linear Algebra Done Right by Sheldon Axler. It's not necessary for you to have, but it's good to have. If you can buy it, please do go ahead and get a copy and you can follow the course better in that sense. I will sort of closely follow the book and in the way it treats the subject. The flow from one to the other. Of course there'll be some deviations as well, that's based on some... At some points we will deviate from the book as well. Okay?

So the duration of the course is 12 weeks. You know, most of you know how NPTEL courses happen. Every week there is content released, an assignment is released. You have to go through the content, solve the assignments and then at the end you can register for the exam and then write the exam and get a certificate. So that will happen in this course as well.

And just a quick look at what we will be doing in the 12 weeks in case you are interested in more details on exactly what topics are covered in these 12 weeks. We'll begin with the basics of vectors and vector spaces in week 1 and introduce you to the abstract notion of vectors. And then the next three weeks we'll spend studying linear maps. Okay, so this is, like I mentioned, the generalization of the matrices. We’ll point out connections to the matrix world. And then we'll talk about how it's generalized and the various interesting properties and fundamental theorems
that involve linear maps. Okay?

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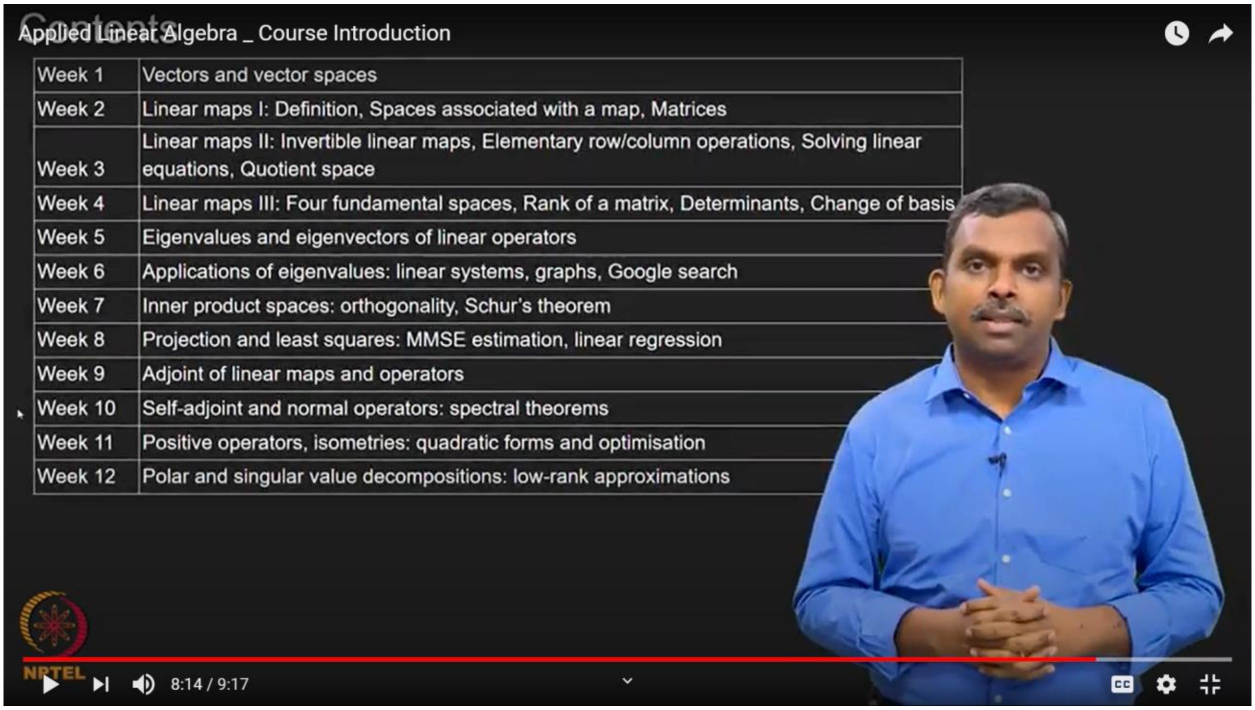

Week 5 and 6 are associated with eigenvalues, eigenvectors and their role and importance in various applications and in theory as well, okay? The second half of the course deals with inner product spaces. So, the first six weeks we will not talk about the inner product or the dot product that you may be familiar with. In the next half of the course we will introduce inner product and study inner product spaces. So inner products bring with them a lot of intuitive feel for what vectors are and what vector spaces are. And we'll do that and then we'll study projections, adjoint and the most important spectral theorems concerning self adjoint or symmetric matrices,
and normal operators, okay? And then we'll conclude in the last two weeks. We'll look at positive operators, isometries, some quadratic forms, properties of that and finally in week 12
we will study singular value decomposition. In today's world of machine learning, singular value decompositions are extremely important. And we'll point out something called low rank approximation which is very useful as well. Okay, so this is a picture of what we will be doing
and I hope you like these topics, and I hope you register for this course, and I hope you have a good time. All the best.