

Optical Fiber Sensors
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Problem Discussion

Hello, everyone. So far, we have been looking at principles related to the design of an amplitude-modulated optical sensor. We have been looking at some case studies related to gas spectroscopy, then we went to look at a pulse oximeter, and then an optical time-domain reflectometer. So, now let us see if we can put all of these together and whatever principles we learned, we can put that to use to serve a practical application.

Let us say you are an engineer that is taking a requirement from a customer and trying to meet that requirement. How do you go about designing such a sensor? So, that is what we are going to focus on in today's lecture.

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a. Differential absorption spectroscopy is a powerful technique for performing the above gas detection. What is the non-resonant wavelength that you will choose such that the absorption extinction is 20 dB? (3 pts)

b. What type of laser will you use for this application? Assuming InGaAs semiconductor PN junction diode ($n=3.3$, length = 250 μm , internal loss = 20 cm^{-1} , internal quantum efficiency = 0.9, mirror loss of 5 cm^{-1} , threshold current = 30 mA), what is the drive current required to achieve 1 mW output power? (3 pts)

c. If the background losses in the above setup is 90%, same power level at both wavelengths, and maximum gas absorption of 10%, determine the minimum power level expected at the receiver using appropriate mathematical expressions. (3 pts)

So, let us say this is the requirement. So, we previously saw a guest lecture by Dr. Anish Bekal, in which he was talking about flue gases such as carbon monoxide, nitric oxide, sulphur dioxide, and so on. And, and these are pollutants and we need to be monitoring these fairly closely. So, you have been asked to design an optical sensor to monitor the level of NO from an extracted sample at the rate of 10 measurements per second. So, the absorption cross-section of NO is given in this plot. This is the transition cross-section as a function of wavelength for both the NO and CO, but the NO corresponds to these transitions over here, multiple transitions in that region.

So essentially, we have been asked to assume that, you can pick one of those transitions for absorption spectroscopy, and you can assume that it is the transition is a Gaussians shaped transition centered at a wavelength of 1100 nanometre with a spectral width of about 6 gigahertz, and a peak transition cross-section value of .6 and 10 power minus 25 centimetres square. And you are supposed to, it is mentioned this is an extracted sample, so, you can have the sample in a gas cell whose length is 0.5 meters. So, we are to go and design such a system. So, let us go ahead and try to do that.

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The slide contains the following content:

Design of an amplitude modulated sensor

Diagram: A box labeled "Laser" has an arrow pointing to a box labeled "Gas Cell". Above the "Gas Cell" box, there is a double-headed arrow labeled "L=0.5m". An arrow points from the "Gas Cell" box to a box labeled "Receiver".

Differential absorption spectroscopy

① resonant λ
 $\lambda_R = 1100 \text{ nm}$, $P_R = P_{0,R} \eta_{out} \exp(-\sigma_R \cdot n \cdot L)$

② non-resonance
 $\lambda_{NR} = ?$, $P_{NR} = P_{0,NR} \eta_{out} \exp(-\sigma_{NR} \cdot n \cdot L)$

So, what we have been told is, this is actually an amplitude modulated sensor. So, you can start with the laser and that is going to be in a, send through this gas cell to length L is .5-meter meters. And then whatever is coming through the gas cell, you want to pick it up with the optical receiver. Now, this is what we had been looking at previously. And in this case, we said we typically, to pick up this absorption accurately, we use what is called differential absorption spectroscopy. So let us, go ahead and try to look at that.

So, you use differential absorption spectroscopy. So, what do we do in differential absorption spectroscopy? We send a light beam through this gas cell at a wavelength corresponding to the peak of resonance for an absorption event. And then we measure what is the power that is coming through and then you go off to tune the laser. Suppose, you have the ability to tune the laser, so, you tune the laser, so, that it goes off-resonance. And then you make one more measurement.

So essentially, if you are measuring the power that is transmitted at the resonant wavelength, let us call this P_R , that is going to be equal to P_0 of R . So, that is the power that is incident on

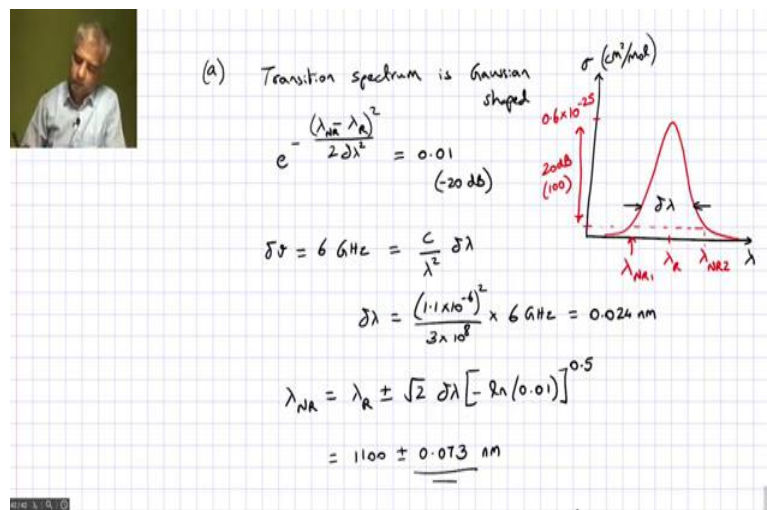
the gas cell. And then you undergo some losses, so only a fraction of that power is going to go through absorption. So, let us actually put all these external losses as η_{external} , and then it is going through absorption on the self that is given by Beer's law. So, you have σ_R is the transition cross-section, multiplied by N is the concentration of the gas, which is what we want to measure multiplied by L , which is the length of the gas cell. And so that is at the resonant wavelength.

So, this is a resonant wavelength, which has been given as 1100 nanometres. So, that is what we have been told that we are interested in picking up nitric oxide in this gas cell. So, we are looking at a resonant wavelength of 1100 nanometres. So then, what do you do? You go off at non-resonance, λ . So, this is we call it, λ_R non-resonance at some other wavelength, we do not know that wavelength yet. But that is one of the things that we need to find out, what is the non-resonant wavelength that you need to go to?

So, let us say we go to this other wavelength, and this is the incident power the other wavelengths, let us assume that you are moving off by such a small enough value of wavelength that the other losses do not change. Within that, then you go minus σ , exponential minus σ_{NR} . That is the transition strength at this non-resonant wavelength multiplied by N multiplied by L . So, that is what we do in absorption spectroscopy. So, let us go back to this question and see what we have been asked to do.

So, what we have been asked to do is, we have been told that you can use differential absorption spectroscopy. And we have been asked to figure out what should be the non-resonant wavelength that you will choose such that the absorption extinction is 20 dB. So, that is what we want to do next. So, let us go back here.

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And let us go to a fresh page. So, what do you have been asked to do is, you have been told that when we look at this transition at the wavelength of 1100 nanometres. So, we are looking at sigma, which is typically expressed in centimetre square per molecule. We have been asked to assume that it is a Gaussian-shaped transition. So, it is going to go look something like this. And we have been told that this wavelength is the resonance wavelength. And at that resonance wavelength, this is being given as 0.6 multiplied by 10 power minus 25 is the strength of the transition. So, that is the transition cross-section.

Now, you have to go to some non-resonant wavelengths. So, such that we are seeing a drop of 20 dB in the absorption trust section, so we say, go down 20 dB over here, that is a factor of 100 and that where that meets this transition curve here, that is going to be our lambda NR. So, it could be either over here or on the other side. So, you can call it lambda NR 1 or 2 or whatever. So, those are one of the wavelengths that we could pick to go to, and then we will essentially do the non-resonant.

So, what we have been told is that the transition spectrum is Gaussian shaped. So, is shaped as a Gaussian. So, we can write the spectrum, it can represent as E power minus lambda lambda minus lambda R the whole square divided by 2 times delta lambda square, where delta lambda would correspond to the RMS width of this transition. So, this here would correspond to delta lambda. So, we have been told that this non-resonant wavelength is picked such that lambda NR minus lambda square, if we do this, then this will actually be corresponding to a factor of 0.01. So, that is, corresponding to 20 dB in terms of absorption. So, that is, 20 dB down.

So, if we do the math, so we can rearrange all these terms and try to get an expression for λ_{NR} , λ_R , we know, $\Delta\lambda$ is something that we have been told. So, we have been told that $\Delta\nu$ according to the question is 6 gigahertz. So, if you know $\Delta\nu$ is equal to 6 gigahertz, can you figure out what is $\Delta\lambda$? Well, you can say that $\Delta\nu$ would correspond to C/λ . So, $\Delta\nu$ corresponds to $\Delta(C/\lambda)$. So, we are not interested in the sign, but we are interested in only the magnitude. So, we can just write it as $C/\lambda^2 \Delta\lambda$ that is given us 6 gigahertz.

So, you can rearrange terms and you can say $\Delta\lambda$ now corresponds to λ^2 , which is 1.1×10^{-6} the whole square divided by C , which is 3×10^8 multiplied by 6 gigahertz. So, that will correspond to $\Delta\lambda$, which is in nanometres. So, if you do the math, what you will find is this corresponds to .024 nanometres. So, this $\Delta\lambda$ this width, the spectral width in terms of wavelength is given us .024 nanometres. So, which we can plug into this expression here and, and get an expression to get a value for λ_{NR} . So, to do that, we will have to just rearrange some terms.

So, if you take the inverse of this will be, this value equal to $1/\lambda$ of .01. So, you take the negative of that, so negative goes over there. And so, then you take the numerator and move it over there so that corresponds to $2 \Delta\lambda^2$, multiply by $1/\lambda$ of .01, the whole negative, and then you take the square root of that and then move the λ_R on the other side. And that is how you can get λ_{NR} . So, if we do all of that, what we will find is λ_{NR} equal to λ_R . And so, it could be plus or minus, it does not matter because either one of these values, so λ_{NR} , plus or minus $\sqrt{2}$, $\Delta\lambda$, multiplied by $1/\lambda$ of 0.01 the whole root, so that is, to the power of .5.

So, you can do this and then you can find out what is the shift-in wavelength that you need to do, to get to the non-resonant wavelengths, such that the absorption drops by a factor of 100. That is what we are trying to get to. So, if you evaluate this, you will find is this λ_R been told is 1100, so 1100 plus or minus, if you do the math, you will come to about plus or minus 0.073 nanometres, rather 73 Pico meters is what you need to move so that you can get down by a factor of 100 as far as the absorption and transition cross-section is concerned. So, that is essentially one thing that we have been asked to find out. So, let us go back and see what else we need to do.

So, that is the non-resonant paper, then you have been asked to have this question, what type of laser will you use for this application? And then you have been given certain parameters, and you have been asked to figure out, what is the drive current required to achieve 1 milliwatt output power. So, this we discussed previously. So, if you want to do differential absorption spectroscopy, you need a source which is much smaller compared to this delta lambda.

So, you ideally need a delta function, but let us say it is of a finite width, but as long as the width is much smaller compared to delta lambda, then that will be good for this application. So, delta lambda we found out is 0.024 nanometre which corresponds to 6 gigahertz in terms of spectral width. So, you need something, you need a laser whose line width this much less than 6 gigahertz.

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- Differential absorption spectroscopy is a powerful technique for performing the above gas detection. What is the non-resonant wavelength that you will choose such that the absorption extinction is 20 dB? (3 pts)
- What type of laser will you use for this application? Assuming InGaAs semiconductor PN junction diode ($n=3.3$, length = 250 μm , internal loss = 20 cm^{-1} , internal quantum efficiency = 0.9, mirror loss of 5 cm^{-1} , threshold current = 30 mA), what is the drive current required to achieve 1 mW output power? (3 pts)
- If the background losses in the above setup is 90%, same power level at both wavelengths, and maximum gas absorption of 10%, determine the minimum power level expected at the receiver using appropriate mathematical expressions. (3 pts)
- What type of photodiode would you use for the above application? Assuming anti-reflection coated facet, photo-electron conversion efficiency of 0.95, absorption coeff of 10^4 cm^{-1} , what should be the absorption width to achieve a responsivity of 1 A/W? (3 pts)
- Identify and quantify the possible noise sources in the above setup. Assume that you have OPA656 (noise characteristics provided above) for the transimpedance amplifier, and a 10-bit ADC with a range of 0 – 1 V. (4 pts)
- What scheme would you adopt to achieve the best possible minimum detectable limit? Provide schematic diagram of setup and appropriate mathematical expressions. (4 pts)

So, we can safely. So, let us look at the B part. So, we say, we will use a laser whose line width is far less than 6 gigahertz. So, one standard laser that we know whose line width is much less than that, that essentially works with the single longitudinal mode, what laser is that? That is the distributed feedback laser. So, you can go for a distributed feedback laser, in short, it is called a DFB laser. So, what are we talking about just to refresh our memory? We are talking about semiconductor laser with PN junction and then, so, this is something that looks like this.

Now, the only thing we have in a DFB laser is that you have some periodic corrugation near the active region, such that it provides distributed feedback all along the length of the laser, and then that is giving us emission from this, so you have a highly reflective mirror. So, you have a highly reflective mirror over here and a partially reflecting a mirror on another side, so then what you get out is light from this aperture over there. So, that is what we saw before.

Now, when it comes to the output power from such a structure, we saw that the output power is determined by the rate at which you have these carriers that are injected inside into the structure. And what matters is the number of carriers that you inject beyond a certain threshold value. So, that is what contributes to stimulated emission in the cavity. So, we saw previously, so, P_{out} corresponds to $h\nu$ over Q multiplied by there could be something called an internal quantum efficiency, which means, that is the fraction of recombination events that results in the emission of a photon. So, that is what we call us at the η_{int} multiplied by $I - I_{th}$.

So, whatever current the rate at which we are sending in charges basically, I over Q . So, but the current or beyond this threshold value is what is going to contribute to this significant output power. So, we are looking at P_{out} as a function of current and then that is going to be going through spontaneous emission up to a certain point. And then if you start having stimulated emission, and you start overcoming the losses in the cavity, and then you can get this output. So, we have been asked to figure out what is the current at which we are able to achieve 1 milliwatt of power. So, we have been told that P_{out} , we should take this 1-milliwatt η_{int} is given. So, if we go back here.

So, we have been told that the internal quantum efficiency is 0.9. And then you have also been told that the internal loss corresponds to 20 per centimeter and a mirror loss of 5 per centimeter. So, let us see if we can put all of those together. So, η_{int} is given us, 0.9, the threshold current is given us 30 milliamperes. So, that we know and then, of course, that is

the power, that is generated within the cavity, but the power that is escaping, fraction of power that is escaping from the cavity is given us into η_{external} which is given by the output mirror loss α_M divided by the internal losses plus the total mirror losses. Where this back mirror is almost 100 percent reflective, so, we do not expect much losses there.

So, as far as the cavity is concerned only this mirror is providing the losses. So, you can write this as α_M over $\alpha_{\text{int}} + \alpha_M$, in the denominator denotes the total losses in the cavity. So, this is given us 5 over $20 + 5$. So, if we do the math is 5 over 25 is 0.2 . So, we have all these values and we know what newest corresponding to C over λ , and λ is supposed to be 1100 nanometre as far as this application is concerned.

And $h c$ over Q is a constant so you can write it as $h c$ over Q multiplied by 1 over λ . And $h c$ over Q is a constant that is given by a value of 1.24 if you make the calculation, if you hitches Planck's constant C the velocity of light and free space, and Q is the charge of an electron. So, if you plug all this in, so you will find that this corresponds to 1.24 divided by λ as long as λ is expressed in terms of a micron. So, we have all these values. So, now, we can plug everything back into this expression, and then we can find expression for I .

So, if you just rearrange all these values, you will find this corresponds to I equal to $I_{\text{threshold}} + P_{\text{naught}}$ which is 1 into 10 power minus 3 multiplied by λ in micron, so, that is 1.1 divided by 1.24 that accounts for these terms and then 1 over η_{int} which is $.9$ η_{external} which is 0.2 . So, if we do all this calculation, what you will find that this corresponds to 30 plus, roughly it will come to about 5 milliamps. So, this will correspond to about 35 milliamps. So, you need a drive current of 35 milliamps. So, that is somewhere over here, let us say this is 35 milliamps so that you can get an output power of 1 milliwatt at 1100 nanometres, which is what we are going to use in our setup.

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Design of an amplitude modulated sensor

Laser

$\xrightarrow{\quad}$

$\leftarrow L=0.5m \rightarrow$
 Gas Cell

$\xrightarrow{\quad}$

Receiver

Differential absorption spectroscopy

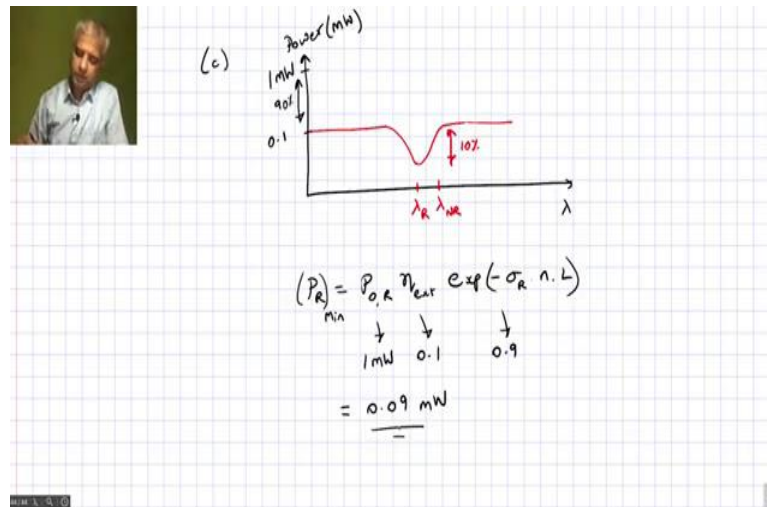
(i) resonant λ
 $\lambda_R = 1100 \text{ nm}$, $P_R = P_{0,R} \eta_{out} \exp(-\sigma_R \cdot n \cdot L)$

(ii) non-resonance
 $\lambda_{NR} = ?$, $P_{NR} = P_{0,NR} \eta_{out} \exp(-\sigma_{NR} \cdot n \cdot L)$

So, this is what we are going to use to probe this (i)(28:47). We will assume that we are able to get the same amount of power. So, this is given us 1 milliwatt, we will assume that this is also 1 milliwatt, there is not going to be a significant change in power when you are tuned from one wavelength to another. And let us say this laser were able to tune that much by just changing the current or temperature. At which it is operated. So, we are able to get both the resonant and non-resonant wavelength, with the same laser. So, that is how we are going to do this spectroscopy.

So, let us go back here and let us find out what else we need to be doing. So, the next thing you have been told is that the background loss in the above setup is 90 percent. And we are asked to assume the same power level that is what we talked about just now was just one milliwatt. And the maximum gas absorption is 10 percent. So, we have been asked to assume it is a maximum gas absorption of 10 percent. So, if that is the case, then the question is, what is the minimum power level expected at the receiver using an appropriate mathematical expression. So, we already looked at some of those expressions. So, let us just go back here and let us see how this is going to work out.

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So, this is Part C, maybe I will just use the color. So, part C, this is what you have been told, let us just try to visualize what we have been told. So, let us say this is a function of lambda, you are looking at the power that is coming through the gas cell. So, we know that you are initially starting with 1 milliwatt of power from the laser. But we have been told that 90 percent of that is lost to external, whatever attenuation mechanism, so, 90 percent of that let us say over here, so 90 percent is lost. So, what we have remaining is only 10 percent of that. So, that is 0.1, let us see this is expressed in milliwatts. So, we have a value of point 1.

And within that, what we have been told, so that is the power level that we have at the non-resonant wavelength. So, the non-resonant wavelength we have negligible absorptions so we can say that the non-resonant wavelength the output is going to be corresponding to .1 milliwatt, and so that takes the background. And then as you approach the resonant wavelength, it is going to go through and absorption. So, this is lambda R. And we have used some other wavelength here as our non-resonant wavelength that which almost, there is negligible absorption.

Now, what we have been told is this absorption corresponds to 10 percent. Maximum absorption corresponds to 10 percent. So, let us try to express that. So, we have been told that P_R we know as $P_{\text{naught } R}$, and then η_{external} multiplied by E exponential of minus sigma R , n into L . So, this factor of exponential minus sigma and into L maximum of that is going to be 10 percent. So, it is at that maximum absorption, you are going to have the minimum power. So, what we have been asked to figure out is that what is the minimum power that we

are expecting at the receiver so, that is going to correspond to the resonant wavelength. So, let us now see if we have all these values to figure out.

Now, we know this is 1 milliwatt, that is what you are starting with. And η_{external} is the fraction of power that is available for absorption. So, 90 percent is lost. So, the fraction of power that is available for absorption is 0.1. And this is, some people find it a little tricky to understand. So, we say 90 percent is lost, that is what the question is saying. According to the question, we are losing 90 percent of the power, 90 percent is background losses. So then if we say that that is a background loss, people say that η_{external} is equal to 0.9, which is wrong because 90 percent is lost. So, what is remaining is 10 percent of the incident power.

So, you have to be very careful about that part. And then, of course, we have been told that the maximum absorption corresponds to, 10 percent of whatever power is available. It is 10 percent off 0.1 milliwatt in this case, and so, that is no, it is not 10 percent of 0.1. So, this is another important thing. So, this is 10 percent is the deviation that we are having from the power level of 0.1 milliwatt. So, this factor corresponds to 0.9. So, 90 percent of the power is remaining after this maximum absorption.

So, if we just do the math there, what you will find is this corresponds to 0.1 milliwatt and 90 percent of that is 0.09 milliwatt. So, that is what we are looking for. That is the minimum power that we expect, at the resonant wavelength, that is the minimum power that we expect at the receiver. So, now we need to build a receiver such that it is capable of picking up this minimum power level. So, let us now move on. And let us see what else, I think we are now ready to design our receiver.

So, let us just go back and see what we need to be doing next. So, what type of photodiode will be used for the above application? And then, of course, you are given certain parameters for the photodiode, what should be the absorption width to achieve responsibility of 1 ampere per watt? So, let us look at that now. By the way, we are talking about picking up 90 microwatts of power. And we are saying is, the responsibility needs to be only the order of 1 amp per watt. So, that already tells you that you do not need an APD photodiode because you have so much power already. So, you can go for a PIN photodiode but then, the wavelength that we are using is 1100 nanometres. So, what material would you use to pick up 1100 nanometres?

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(d) To detect 1100 nm, we can consider ~~Si~~ (band edge)
 InGaAs PIN photodiode

$R = \frac{\eta q}{h\nu} = \frac{\eta \lambda (\mu\text{m})}{1.24}$ where $\eta = (1-R_f) (1 - e^{-\alpha w})$
 $= 0.79$

$0.7 \text{ A/W} = \frac{\eta \times 1.1}{1.24}$

$\eta = \frac{1.24 \times 0.7}{1.1} = 0.79$

$w = -\frac{1}{10^4} \ln(0.17) \text{ cm}$
 $= 1.78 \mu\text{m}$

And when we look at that material, we find that to detect 1100 nanometres, we can consider silicon, because silicon has responsivity, from typically from 400 nanometres to about close to about 1100 nanometres, but that is right at the bandage, that is the job the absorption. So, it falls off in terms of responsibility very sharply around that wavelength. So, we will say that maybe that is not as suitable. So, what we can go for instead is indium gallium arsenide, which exhibits fairly good responsibility from about 0.7 or 700 nanometres all the way to 1600 nanometres. So, that is what we are choosing, we are choosing an indium gallium arsenide PIN photodiode for this particular application.

So, now, the question is, if you are using a PIN photodiode let us draw the structure here. So, we have a very thin P region a very thin N region, and a relatively long i region and we have electrodes on either end of that and then we are looking at optical power being incident on this. According to so, what we have been asked to do is, figure out the width of this absorption region. So, this width double U such that you can support the responsibility of 1 ampere for what, actually 1 ampere per watt is not possible in indium gallium arsenide at 1.1 microns, I will tell you why. So, that was a mistake there. So, we should correct that.

Let us look at responsivity, responsivity is given by eta q over H nu. And that can be, of course, we saw before can be written as eta times lambda as well as lambdas is expressed in microns divided by 1.24. The same thing, it is hit C over q, corresponds to a value of 1.24. As long as lambda is expressed in microns. Where eta itself is given by 1 minus Rf, that is the fraction of power in entering into the structure, the structure can have a facet reflectivity of Rf.

And then it has a photoelectron conversion efficiency is given by η multiplied by the amount of power that is absorbed, which corresponds to $1 - R_f - \alpha L$. α corresponds to the absorption coefficient in the media multiplied by the width. Now, we have been asked to assume that RF is antireflection coated. So, R_f is 0. We can assume R_f is 0 and η . We have been asked to assume that for this material, it is 0.95. And we have also been given α at this wavelength, we have been told it corresponds to 10^4 per inverse centimeter.

So, we can plug all those and find out the η . But η , where do you find that from? η will have to get it from this expression because we know λ , λ is 1.1 micron. So, if you try to do that, and if you try to get 1 ampere, per what responsibility. So, it multiplied by 1.1 divided by 1.24. If you do the math over here, you will find that η is 1.24 divided by 1.1, which is a value greater than 1. Now η cannot be greater than 1, cannot be because this is a fraction, η is a fraction and this is also a fraction, which has a maximum value of 1. So, η has to be a number that is less than 1, which is not satisfied over here.

So, contrary to what is asked, we will assume that we have been asked to get a, take a response at a 0.7 and per watt. So, if you say, that is our target responsibility, then η going to be given by 1.24 multiplied by 0.7 divided by 1.1 which is a value of 0.79. Now, this we can take over here. So, this entire thing has to be equal to 0.79.

So, if we rearrange the terms, then we can find out L which is going to be given by the inverse of the α , negative inverse of α which is 10^4 , and then you have a L of 0.79 divided by η . So, that is and there is going to be a $1 - R_f$ factor so, that will be L of 0.17 and this is in centimetre because α is expressed in terms of centimetre. So, 10^4 centimetre is micron. So, you can just evaluate this work out to be 1.78 microns. So, that is the width of the absorption region, that is necessary to support the responsivity of the PIN photodiode of 0.7 amp per watt.

Now, let us just go back to the question again and see what else we have been asked to do. So, you have been asked to identify and quantify the possible noise sources in the above setup. Assume that you have OPA 656 for the trans-impedance amplifier, and you also have a 10-bit ADC with the range of 0 to 1 volt. OPA 656 noise characteristics are given. So, e_n is the voltage noise current density that is given by this curve over here, it is higher at low frequencies, this is the $1/f$ noise, and then it becomes white noise. Whereas, i_n is flat

across so that that just given us 1.3 Fenton amp per root hertz. So, we are supposed to use this for building our trans-impedance amplifier. So, let us go back.

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(c)

0.09-0.1 mW

Max. power @ receiver = 0.1 mW

Corresponding photocurrent = 0.07 mA

Transimpedance gain = $\frac{1V}{0.07 \times 10^{-9}A} = 14.3 k\Omega$

Bandwidth requirement = 10 Hz - 400 MHz

Total noise current = $\sqrt{5 \times 10^{-23}} \sqrt{B}$

$= \sqrt{5 \times 10^{-23} \times 6.67 \times 10^4}$

$= 19 \times 10^{-9} A$

$(f_{3dB})_{TIA} = B = \sqrt{\frac{GBW}{2 \times R_f \times (C_f + C_o)}}$

$= 66.7 kHz$

Let us take a fresh page once again. And let us try to do a noise analysis. So, to do the noise analysis, we need to say, let us get a picture of our receiver's circuit. So, you have a certain power level that is incident. And we already figured out what is the minimum power that we are getting. So, the minimum power, what we determined earlier corresponds to 0.09 milliwatt, what is the maximum power that we are going to have that corresponds to this value, the non-resonant the value that we will get at the non-resonant wavelength that corresponds to 0.1 milliwatt.

So, we can say that this corresponds to 0.09 to 0.1 milliwatt that is the kind of input powers that we are expecting over here. And then we will take this photocurrent that is generated over here, I_P corresponding to this input power, and then we will go through the transimpedance amplifier and then we will take the output of that and go to analog to digital converter, and then it is available as digital samples for further processing and display. So, that is the receiver design that we are looking at. Now, the question is what sort of transimpedance gain you need? And then what is the corresponding noise that you accumulate at the receiver.

So, to do that, we need to understand that this ADC, it is given us 0 to 1 volt. So, with the maximum power that you have here, it should be such that it filling the ADC. So, that is what we want to first find out. So, the need to have a transimpedance gain, such that this power that is falling on it is incident on the photodiode, it is generating a certain photocurrent. So,

let us look at this maximum power at receiver is 0.1 milliwatt at 0.1 milliwatt, what is the photocurrent that we are expecting, so the corresponding photocurrent is at a rate of 0.7 amps per watt, that is what we looked at the previous section. So, that correspond to .07 milliamperes.

And this photocurrent now needs to be converted to a voltage so that it fills the ADC. So, the trans-impedance gain that we require in this case is going to be 1 volt divided by 0.07 into 10 power minus 3. So, that is going to be, so that is amperes. So, that is if you do the math and take the inverse of this, this will work out to be about 14.3 kilo-ohms. So, that is the trans-impedance gain that we need and 14.3 kilo-ohms is small enough that we could support with 1 TI stage itself. Especially, it is not just the trans-impedance gain but also the bandwidth requirement.

So, in our case our bandwidth requirement, we are required to do only 10 measurements per second, that is the requirement. So, we do not need much more than 10 hertz the information that we want to capture is not beyond 10-hertz type of frequency. So, we can, that is low enough bandwidth requirements. So, we can possibly get all this trans-impedance gain in 1 OPA 656 stage itself. So, we will say that we have a trans-impedance amplifier with 14.3 kilo-ohms as the feedback resistance. If we have that, then the question is what are the noise sources that we expect over here? And, and can we quantify those noise sources?

Now, first of all, we do realize that when light is incident on a photodiode the arrival time of the photons are random in nature. So, we have what is called the shot noise, that is poised on distributed arrival time. So, we have what is called the shot noise that is present over here. So, and then while it is getting amplified, it will essentially accumulate thermal noise in the receiver and then also the Op-Amp whose noise characteristics we just saw, then Op-Amp will also contribute to the noise. So, those are the major noise source, and of course, we can also have quantization noise in the ADC. Let us just neglect that for a second now and let us look at all these other noise sources.

So, to look at the other noise sources, we can say this the noise is EN_{RMS} at the output. I think we are using E_N for the voltage noise current density of forward of the Op-Amp. So, let us say this corresponds to EN_{naught} and what we are interested in the RMS voltage there. So, EN_{naught} we previously saw if we just add all those, each of those as an independent random variable, so the variance of each of those, the noise power correspond to each of

these processes, those are going to add, and then we will, and then we will have to take the square root of all the summation of all of those. So, that will give us the RMS value.

So, we saw that this would correspond to R_f multiplied by root B, the whole root of 4 KBT divided by R_f is the feedback resistance of the T plus we are using PIN photodiode. So, we do not have any of these noise contributions to avalanche processes. So, it is simply $2 q I_p$ multiplied by I_p and then you have noise. So, we are essentially capturing the noise current density. So, then we have the noise current density corresponding to an Op-Amp. So, the variance is $i_{oa} \text{ Op-Amp square}$ plus, you have $e \text{ Op-Amp square}$ divided by $R_f \text{ square}$, because we are once again, tracking the noise current density. So, let us try to quantify all of these numbers.

So, R_f , we been told is, we just figured out it is 14.3 kilo-ohms. So, we substitute that over here. So, we will get a value of something the order of $1 \text{ into } 10 \text{ power minus } 24 \text{ watts}$ for the thermal noise component, and then we look at the shot noise component, that is, if you evaluate this will be $2.24 \text{ into } 10 \text{ power minus } 23$. And then, i_{oa} corresponds to the current noise density that is provided by the Op-Amp, that is in the order of $1 \text{ Fenton amperes per root head}$. So, if we evaluate that, that will come to about $1.7 \text{ multiplied by } 10 \text{ power minus } 30$. So, that is quite negligible compared to all these other numbers and then UAA. So, this is an interesting point that we need to keep in mind.

So, when we talk about voltage noise current density, we see that we need only 10 hertz from a trans-impedance amplifier. If we look at that, we will essentially get about something the order of, it is 75 nano volt per root hertz. So, that is something that we need to keep in mind if you are able to go to a larger frequency. Especially, if you are able to go beyond 10-kilohertz type of frequencies, then you can possibly reduce this if you just because you need only 10-hertz bandwidths. So, if you shift your detection to higher frequencies you could possibly get incur much lesser noise, how do you shift the frequency of operation?

Well, one common method that we saw in one of the period previous lectures is using lock-in detection. So, instead of doing all these measurements at DC, what if you are able to do this measurement by with modulated wave light instead of doing with continuous-wave light, if we can do intensity-modulated light then especially at a frequency say 10 kilohertz or about, then you can possibly bring down the voltage noise density. We will come back to that at a later point but at this point, we are assuming that we are doing it in baseband which means that we want to look at the value at 10 hertz, which is 75 nanoamp per root hertz.

So, we have plugged that into this and divide that by R_f , we will get a value of 2.75 multiplied by 10 power minus 23 . So, among these, we clearly see that these are your primary noise contribution. So, it is, first of all, it is a shot noise-limited type of receiver, it is not thermal noise limited receiver, because the amount of power that we are expecting is relatively on the higher side, if the same thing was in the order of nanohertz, then the shot noise component will be very low, you will have a component because of this thermal noise and you may have to even go for an avalanche photodiode in that case.

But this is a case where we are looking at shot noise limitation, but what we are also seeing is because of the 1 over F noise because of the frequency at which we want to do this detection is the low-frequency side, you are actually having a relatively high contribution from that. So, that is actually from the Op-Amp itself. So, those are the kind of things to consider. Now let us just go back and see what else we have been asked to figure out. So then, finally, we have been asked to figure out, what scheme would you adopt to achieve the best possible minimum detectable limit? Provide a schematic diagram of setup and appropriate mathematical expressions. So, let us try to do that.

So, while we are at this, let us also try to figure out what is the total noise power that we have in this case. And to figure out the total noise power, so we need to evaluate all these components, first of all. So, we can just say that the total noise current density that we have here it corresponds to the square root of all these additions of all these things. So, this is corresponding to square root of, you add these two components that corresponds to 5 into 10 power minus 23 . So, it is square root of 5 into 10 power minus 23 and square root of B . So, where B is the noise bandwidth of your receiver, so, since we are multiplying that by root B , you do not call this the noise current density, that is the total noise current itself.

So, now the question is what is B ? So, B the requirement is only 10 hertz, but when you do a design using a trans-impedance stage, we know that B corresponds to the $f_{3\text{ dB}}$ of your trans-impedance amplifier. So, if you look at the $f_{3\text{ dB}}$ of your trans-impedance amplifier, which is corresponding to the noise bandwidth of the receiver that we are looking at, that corresponds to the root of the gain-bandwidth product divided by $2\pi R_f$ and then there are multiple components capacitance contribution, so, C_f plus C_D plus $C_{\text{Op-Amp}}$ and all that. So, that is the capacitance corresponding to all of this.

Now, C_D $C_{\text{Op-Amp}}$ and all that, let us say that corresponds to about typically in the order of 1 Pico farad, so, that is fine. So, what about C_f ? Now, in this case, we want to limit the

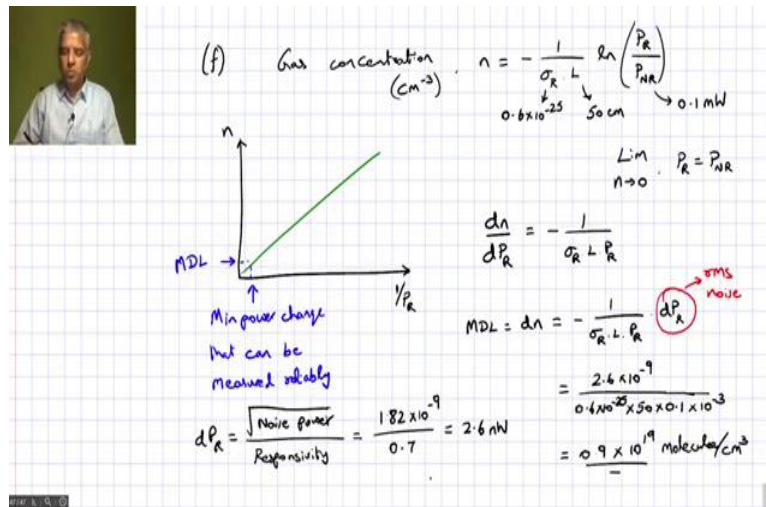
bandwidth. So, we want to limit the bandwidth we already know that f corresponds to 14.3 kilo-hertz. So, we need to constrict the bandwidth, so, that you do not accumulate a lot of noise. So, you need to low pass filter your receiver, whatever the received signal is. So, you want to limit that, and when it comes to limiting this, C_f if you look at typical values of C_f that you can pick up. So, that you can put this into sort of like PCB, you will find that this corresponds to about 1 microfarad. And we know that the gain bandwidth of OPA 656 is about 400 megahertz.

So, if you plug in all these values, you will find that B corresponds to a value of 66.7 kilohertz, and your requirement is only 10 hertz, but you are having to deal with a B or value of 66.7 kilohertz. That is primarily because, of course, you can increase R_f , so that you can constrict B , but you do not have an opportunity to do that because you already filled your ADC with this value. So, you cannot go any more beyond that. So, the only choice is to increase C_f but, when you are getting it as a circuit in a PCB, that may be limited. So, that is where we this limitation we are considered as 1 microfarad is concern.

If you, for example, go to 1 millifarad then you can get to a much smaller value but to get to 1 millifarad, you need a fairly bulky capacitor. So, it is not compatible typically with a PCB. So, that is the constraint that we see. So, because of this when you look at, this is the total noise current that we have looked at. And so, if you look at, if you evaluate this, you will find that this is $\sqrt{5 \times 10^{-23} \times 6.67 \times 10^4}$ if we do the math, so that corresponds to 30, and it is another 3, so the what 33×10^{-19} . So, you can take it as 10^{-18} , so, you can get 10^{-9} out and multiplied by roughly about 340. So, square root of 340 is going to be something 19×10^{-19} . So, that is in amperes.

So, that is the total noise current that we are expecting. And, of course, if you want to look at the voltage corresponding to that, you have to multiply that by R_f and so that, if you do multiply by R_f , which is 14 kilo-ohms, then you will probably get something in the order of microvolts. So, we will come back and do that later. So, that is the total noise that we are picking up from this. Now, in the last part, we been asked to figure out, what is the kind of minimum detectable limit that we have. So, let us look at that.

(Refer Slide Time: 71:44)



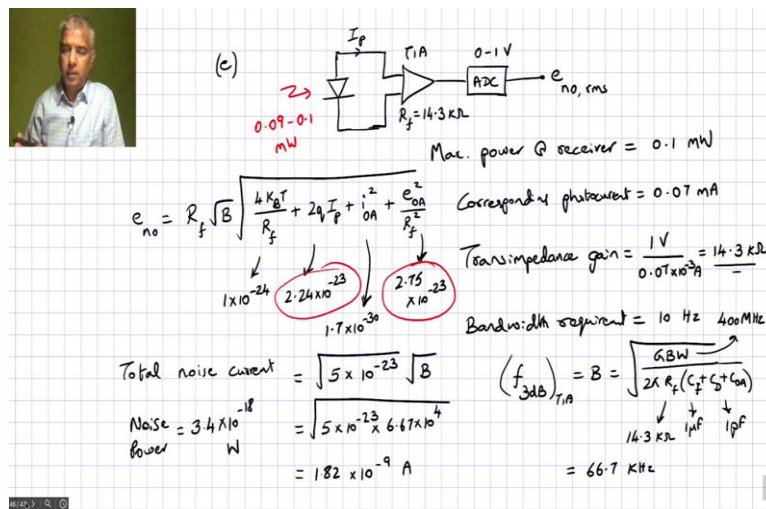
(f) Gas concentration (cm^{-3})

$$n = -\frac{1}{\sigma_R L} \ln\left(\frac{P_R}{P_{NR}}\right)$$

0.6×10^{-25} 50 cm 0.1 mW
 $\lim_{n \rightarrow 0} P_R = P_{NR}$

MDL \rightarrow Min power change that can be measured reliably
 $\frac{dn}{dP_R} = -\frac{1}{\sigma_R L P_R}$
 $\text{MDL} = dn = -\frac{1}{\sigma_R L P_R} dP_R$ (rms noise)
 $= \frac{2.6 \times 10^{-9}}{0.4 \times 10^{-25} \times 50 \times 0.1 \times 10^{-3}}$
 $= 0.9 \times 10^{19} \text{ molecules/cm}^3$

$dP_R = \frac{\text{Noise power}}{\text{Responsivity}} = \frac{1.82 \times 10^{-9}}{0.7} = 2.6 \text{ nW}$



(c)

$0.09 - 0.1 \text{ mW}$ I_p 1 A $0 - 1 \text{ V}$ ADC $e_{no, rms}$
 $R_f = 14.3 \text{ k}\Omega$ $\text{Max. power @ receiver} = 0.1 \text{ mW}$

$e_{no} = R_f \sqrt{B} \sqrt{\frac{4k_B T}{R_f} + 2q I_p + i_{OA}^2 + \frac{e_{OA}^2}{R_f^2}}$ Corresponding photocurrent = 0.07 mA
 Transimpedance gain = $\frac{1 \text{ V}}{0.07 \times 10^{-3} \text{ A}} = 14.3 \text{ k}\Omega$
 Bandwidth requirement = $10 \text{ Hz} - 400 \text{ MHz}$

Total noise current = $\sqrt{5 \times 10^{-23}} \sqrt{B}$
 Noise power = $3.4 \times 10^{-18} \text{ W}$
 $= 1.82 \times 10^{-9} \text{ A}$

$(f)_{3dB} = B = \frac{\text{GBW}}{2 \times R_f (C_f + C_{OA})}$
 $= 66.7 \text{ kHz}$

How do you define the minimum detectable limit? Now, from our expressions before, we can figure out what is N as a function of PR. So, if we just rewrite that expression, so, the gas concentration which is the number of molecules per cubic volume that is N, N can be written as when we rearrange all those terms, (they are) N can be written as minus sigma R multiplied by l and then Lon of PR, that is the transmitted power measured at the resonant wavelength over the transmitted power that we measure at the non-resonant wavelength. So, that is, it is true that we get this gas concentration.

If you plot this, let us say we are plotting this, we are trying to figure out what is the concentration based on a measurement of PR, if you look at this, all of these are like constant values. So, PNR for example, for our case, we said is 0.1 milliwatt and sigma R we know that

is 0.6×10^{-25} L we know is 50 centimetres, this is 50 centimetres, that is 0.6×10^{-25} at the resonant wavelength. So, all these are constant and only PR changes, PR changes according to you know, changes in, in the gas concentration. So, when you have a higher concentration, the value of PR is lower. And conversely, if it is low concentration, PR is higher, how high can PR be?

Well, you have the limit of N tending to 0, then PR will become PNR. When that essentially goes back and look at this, we say if there is no absorption, then this will just be a flatline. So, PR is going to be equal to PNR. And then of course, we are saying, as concentration increases PR decreases. So, it is an inverse relationship. So, I can write it, I can plot it with respect to the inverse of PR and if we do that, then what we see in this case it should be a sort of linear relationship. So, the question is what is the minimum detection limit that we have? So, the minimum detection limit would be somewhere over here. Let us call this minimum detection limit, and what is that limited by?

Well, that is limited by the uncertainty in your power measurement. So, this is the minimum power change that can be measured reliably. So, that will essentially limit the lowest concentration that you can pick up that limits that determines the MDL value. So, let us to determine this, we will say that, we can that corresponds to ΔPR change or ΔN change with respect to ΔPR . So, we can differentiate this expression. So, if you differentiate that expression, you say $\frac{dN}{dPR}$, what is that value? Well, this is $\frac{1}{PNR}$ or $\frac{1}{PR}$, but PNR is a constant. So, when you do that differential, you will find that it is got, it comes to $\frac{1}{PR}$ so, this will be nothing but $\frac{1}{\sigma_{RL} PR}$ multiplied by PR.

So, if we want to now find out the minimum detection limit MDL, MDL corresponds to $\frac{dN}{dPR}$, that is the smallest value change in the concentration that we can pick up, that corresponds to $\frac{1}{\sigma_{RL} PR}$ multiplied by the dPR . So, now, the question is what is dPR ? dPR corresponds to the uncertainty which is essentially the noise power that we have. So, this is the RMS noise, that we have, that is incident, noise equal, and power that we consider this incident on the photodiode.

So, let us go ahead and determine dPR . So, dPR is going to be given by root of the noise, the total noise power divided by your responsivity, and the total noise power if we calculate that will correspond to, what we have over here 5×10^{-23} multiplied by B. So, I made a mistake here, so, this should be 1.82. So, that is the total noise current, that is the

RMS noise current, but when we talk about noise power that will be square of that, so, that will become 3.4×10^{-18} watts. So, when we want to do the calculation of the equivalent optical power. So, this is the electrical power.

So, if we want to look at the optical corresponding optical power then we can say DPR, we already put it there, this is we have the corresponding electrical power root of that. Why are we taking a root? Because we are interested in the current noise because we are what is happening in the detectors, the optical power is current getting converted to a photocurrent. So, we need to take the root of that to take the noise current and divide that by the responsivity. So, that is what we have here this is 1.82×10^{-9} that corresponds to the noise current divided by the responsivity which is 0.7 amp per watt.

So, that will correspond, that will give us the equivalent change in minimum change in power that we can detect. So, that will be if you do the math, it will correspond to about 2.6 nanowatts, so now we can plug this on over here. So, we know all the other values, so this is 2.6×10^{-9} divided by you have 2.6×10^{-25} multiplied by L which is 50, multiplied by PR which is in the limit of N tending to 0 because that is the limit that we are interested in. It just corresponds to PNR which is 0.1 just verify that that is PNR is 0.1×10^{-3} . So, if you do that math, it will come to, so this is 50 times 0.1 is 5 multiplied by 0.6 is 3. So, 2.6 divided by 3 is about 0.9, 0.9 multiplied by 10^{-28} . And that is 10^{-9} . So, it is 10^{-19} .

So, that is 10^{-19} molecules per centimetre cube. So, that is the minimum detectable limit. And so, what does it correspond to in terms of the actual concentration? So, if you do DN over N, if you do then you can figure out what is the actual concentration, the minimum concentration in terms of parts per million and this will work out to be roughly, if you take the ratio of 10^{-9} and 10^{-3} that corresponds to something in the order of parts per million. So, that is the kind of level that we are able to achieve with this technique. Now, the question is, how can we improve this? What can we do to improve this? Well, we know already that one way of improving it, that is what we discussed previously.

If by this potential, we can reduce this component how do we reduce this component, we can go to the higher frequency of operations from doing this in DC, you can go to a higher frequency.

(Refer Slide Time: 85:38)

$(10 \text{ kHz}) f_m$
 Laser \rightarrow [] \rightarrow Receiver \rightarrow \times LPF \rightarrow 10 Hz
 f_m

* MDL can be further reduced through lock-in detection
 $f_m @ 10 \text{ kHz}$, LPF cut-off @ 10 Hz

* MDL can be further reduced through
 Wavelength Modulation Spectroscopy
 $f_m \rightarrow$ detector @ $2f_m$

f_m f

(a) Transition spectrum is Gaussian shaped

$$e^{-\frac{(\lambda_{NR} - \lambda_R)^2}{2\delta\lambda^2}} = 0.01 \quad (-20 \text{ dB})$$

$\delta\sigma = 6 \text{ GHz} = \frac{c}{\lambda^2} \delta\lambda$

$$\delta\lambda = \frac{(1.1 \times 10^{-6})^2}{3 \times 10^8} \times 6 \text{ GHz} = 0.024 \text{ nm}$$

$$\lambda_{NR} = \lambda_R \pm \sqrt{2} \delta\lambda [-2.3 / 0.01]^{0.5}$$

$$= 1100 \pm 0.073 \text{ nm}$$

So, instead of in your setup, we say we have this laser, we need to drive some current. So, if you drive it, let us say we are driving it with some, we are driving the current at some frequency f_m , then, we have this going through the gas cell and then whatever is coming back, we are taking it to into the receiver, this receiver output now, before you just coming off the trans-impedance amplifier, you can mix it with the reference frequency. And then you can do lock-in detection, then you can beat it down because this one is you are expecting some signal around f_m , let us say f_m we pick it about 10 kilohertz, why 10 kilohertz?

Well, if you just go back to what has been given for our application, so, we have been told that you are using an Op-Amp that is having a white noise beyond 10 kilohertz. So, if you go to this value, then you can bring down the noise by quite a lot. So, that is what we are looking

at. So, if we do that, you can reduce the noise that you are accumulating at the receiver. And that is what we are seeing over here, you can reduce this component significantly by two orders of magnitude, potentially.

And then you can also through that you can bring down the noise that you are accumulating and then you can improve. So, that if you bring down the noise that you are accumulating there, then you can reduce dPR, the minimum change in power that you are able to detect from the receiver, and through that you can improve your minimum detection limit. So, what we are saying is MDL can be further reduced through lock-in detection at fm in this case fm could be in the order of 10 kilohertz, which means that in your receiver, you can still, whatever bandwidth over which you are accumulating noise that noise you will be able to reduce by using low pass filter over here.

So, you have the low pass filter with a cut-off of 10 hertz. So, you do the locking detection with fm of 10 hertz and low pass filter cut-off at 10 hertz, because that is all the information that you want to get. So, in frequency, what we are doing is we have fm, and through this lock-in detection process, we are beating it back to baseband. And then we are applying a low pass filter at 10 hertz and, that is all the noise that we pick up. So, the noise power will also significantly come down here, when we were calculating noise power, multiplying this by this value here.

Now, you do not have to use this value you can use only 10 hertz, and if you use 10 hertz, that is a factor of 10 power for less. So, your noise power will be a factor of 100 because it is coming well so that the corresponding noise power will also be reduced by that factor. So, that is clearly one way of improving the minimum detectable limit. And another way of improving it further is, so, this is one way and the other way you can improve it even further is MDL can be further reduced through what is called wavelength modulation spectroscopy. That is, instead of just doing intensity modulation, what if you are able to modulate the wavelength itself such that.

In this case, so, you are doing wavelength modulation spectroscopy like we talked about previously, if you are able to do this at some value fm, and then you are able to do this detection at 2 FM frequency. So, you have a corresponding intensity variation at 2 FM frequency. So, if you lock into that, then you can have a 0 background. So, essentially you do wavelength modulation spectroscopy where modulation is done at fm and then you are detection at 2 fm. So, if you are doing that there is negligible background. You do not have

any other, spurious absorption coming into the picture, then you can possibly get even better improvement even better value as far as the minimum detection limit is concerned.

So, that hopefully sort of gives you a feel for all the things that you could do with amplitude modulators sensors. So, that is just a realistic example. So, this is what you need to do. You need to take the requirements from your customer, you need to see what are all the things that you could possibly do. It all finally comes down to, how you can handle noise. And what we saw was using these lock-in principles is a very nice way of handling noise and achieving fairly low values, fairly high performance, fairly low detectable values. So, let us stop right there.