

Optical Fiber Sensors
Professor Balaji Srinivasan
Department of Electrical Engineering
Indian Institute of Technology Madras
Lecture 22
Phase Modulated Sensors - 2

(Refer Slide Time: 00:13)

Michelson Interferometer

Optical Source → 50/50 Beam Splitter → Path 1 (distance d_1) → Mirror → Path 2 (distance d_2) → Mirror → 50/50 Beam Splitter → Optical Receiver

$\Delta\phi = \frac{2\pi}{\lambda}(d_1 - d_2) = \pi$
 $d_1 - d_2 = \frac{\lambda}{2}$

$I_T = 2I_0 \left[1 + \cos \left[\frac{2\pi}{\lambda}(n_1 d_1 - n_2 d_2) \right] \right]$

Graph: I_T vs $\Delta\phi$. Max $I_T = 4I_0$, Min $I_T = 0$, Avg $I_T = 2I_0$. $I_1 = I_2 = I_0$. $\Delta\phi$ axis has marks at $0, \frac{\pi}{2}, \pi, 2\pi, 3\pi$.

NPTEL

Hello, in our last lecture, we started talking about Phase Modulated Sensors.

(Refer Slide Time: 00:22)

Phase-Modulated Sensor

Light Source → $E_1 = a_1 E_1 e^{j(\omega t - \beta z)}$ → Perturbed → $E_2 = a_2 E_2 e^{j(\omega t - \phi_2)}$ → Demodulator → Optical Receiver

$\phi_1 = \frac{2\pi}{\lambda} n \cdot L$

Graph 1: Amplitude vs t . Blue wave with phase shift ϕ_2 and period 2π .

Graph 2: Reference vs t . Green wave.

NPTEL

And we essentially said whenever you want to measure phase, your optical receiver is not sensitive to phase changes. So, you have to use sort of a demodulation mechanism, where any

phase changes can be converted to intensity changes or amplitude changes, which can further be picked up by the optical receiver. And with respect to that we said, we need to know phase is not especially when it is propagating over long distance it is not something that you can measure in absolute terms, so, you need a reference.

And so typically you use a reference which is protected from whatever perturbation you are trying to measure. And then you will be able to pick up any phase change between the perturbed radiation and the unperturbed reference. And in this respect, we said, a common sort of apparatus that you use for picking up these phase changes is an interferometer. So, we are going to move on today to go a little deeper into this.

(Refer Slide Time: 01:53)

* How to achieve phase demodulation?

- convert phase changes to intensity changes
- use interferometers!

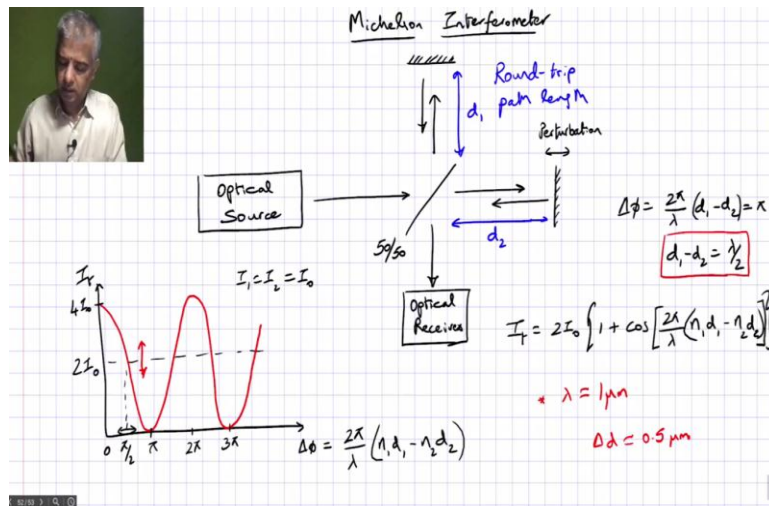
$$I_T = \langle |\vec{E}_1 + \vec{E}_2|^2 \rangle = \langle |\vec{E}_1|^2 \rangle + \langle |\vec{E}_2|^2 \rangle + \langle \vec{E}_1 \vec{E}_2^* \rangle + \langle \vec{E}_2 \vec{E}_1^* \rangle$$

$$I_1 = I_2 = I_0 \quad = I_1 + I_2 + 2\sqrt{I_1 I_2} e^{j(\phi_1 - \phi_2)} + \sqrt{I_1 I_2} e^{j(\phi_2 - \phi_1)}$$

$$I_T = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi)$$

So, we went on to understand interference, the interference phenomena and we went on to look at the output of an interferometer with respect to phase changes, we looked at what is the optimum point for operation.

(Refer Slide Time: 02:14)



And then went on to look at one realization, one example of an interferometer or maybe a relatively simple example of an interferometer is the Michelson interferometer, where the optical source is split into 2 parts by this 50-50 beam filter. And both those parts are reflected back, so they essentially the beams folded back, and then it comes to the optical receiver, where when both these beams are coming together, they are incident on the photodiode.

The photodiode essentially looks at the beating of those 2 fields, and the corresponding intensity that we get, we derived it as something like this. So essentially, if we were to look at this in our delta phi now, with respect to this apparatus, we say, delta phi is going to 2 pi over lambda n1 d1 minus n2 d2, and where d1 and d2 are essentially the round trip path length in this setup. And then we also made this point that this sort of an interferometer is going to be very, very sensitive to path length changes.

So, we said, even if the path length changes the order of lambda by 2, you are going to be having a large change, essentially, you will go from maximum to minimum or minimum to maximum whenever the path length changes by lambda by 2. And the fact that, in our case, lambda is in the order of 1 micron, maybe 1.5 micron, whatever wavelength you choose, it is around that order. The fact that we have this thing in order of wavelength, in the order of 1 micron means that delta d is in the order of half a micron and that is to go from maximum to minimum.

So, if you want to actually measure for small phase change just, let us say in the order of fraction of π radians, then that can easily get immersed, even if it is perturbations in the order of 100 nanometers. And that time talking about only in terms of physical, path length changes, but that could also happen because of changes in the refractive index in n_1 and n_2 , of course, we are showing this as a medium where it is all free space.

So, you may say the refractive index does not vary much, but that is again, relative quantity. Even if you are having some pressure changes, because somebody is opening the door of the room where this interferometer is set up, that actually changes some pressure. And even if the changes in pressure causes changes in refractive index in the order of 10^{-6} , refractive index changes of 10^{-6} is enough to destabilize the interferometer movement from maximum to minimum and so on.

So it is, it is a highly, highly sensitive apparatus. So you need to somehow rain in all of these effects and use it to measure only the perturbation that you are interested in. And that is actually a huge challenge. That is one of the biggest challenges as far as using these interferometers are concerned. One possible way of doing that is by going through an all fiber setup, instead of this setup, where it is all alignment sensitive, we need to put the beam slit in the right place, we need to put the mirror in the right place and then folded back, make sure you are getting collinear beams all across.

And so, so, the interference in this arm, the beams are collinear and, and then you need to maintain it that way. And that means that the relative position of all these, especially the 2 mirrors and this beam splitter has to be quite sturdy, like we said, even in the order of fraction of a micron can be supplies in the entire setup. So that actually puts a lot of constraint on, on the setup and to the point that if you were to do a practical sensor, on field sensor, it is going to be extremely challenging to do that in a reliable manner. So that could potentially be addressed by going to an all fiber interferometer. So that is what we will go to techniques.

(Refer Slide Time: 08:17)

Challenges in Phase Measurements

(1) Stability of interferometer

Optical Source

Reference n_1 d_1

50/50

Sample n_2 d_2

Optical Receiver

I_r

$4I_0$

$2I_0$

$I_1 = I_2 = I_0$

$\Delta\phi = \frac{2\pi}{\lambda} (n_1 d_1 - n_2 d_2)$

NPTEL

Michelson Interferometer

Optical Source

50/50

Optical Receiver

Round-trip path length d_1

Perturbation d_2

$\Delta\phi = \frac{2\pi}{\lambda} (d_1 - d_2) = \pi$

$d_1 - d_2 = \frac{\lambda}{2}$

I_r

$4I_0$

$2I_0$

$I_1 = I_2 = I_0$

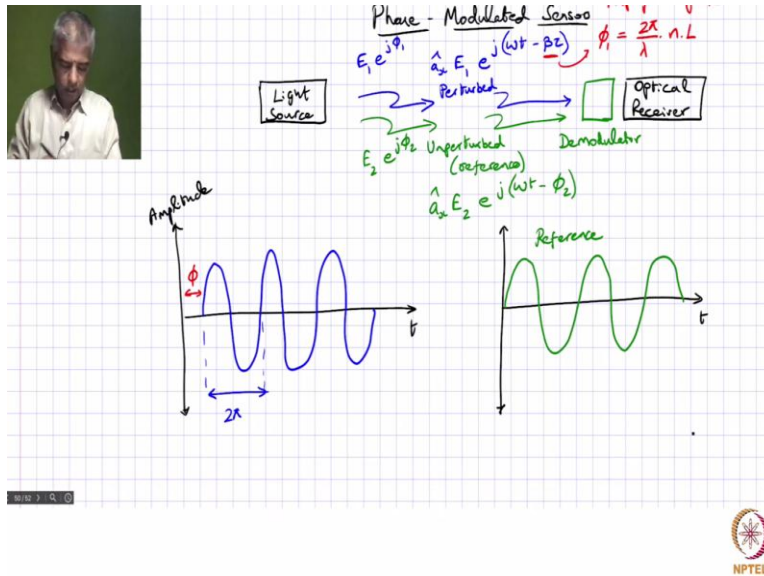
$I_r = 2I_0 \left[1 + \cos \left[\frac{2\pi}{\lambda} (n_1 d_1 - n_2 d_2) \right] \right]$

$\lambda = 1 \mu\text{m}$

$\Delta d = 0.5 \mu\text{m}$

$\Delta\phi = \frac{2\pi}{\lambda} (n_1 d_1 - n_2 d_2)$

NPTEL



So, so we are generally talking about challenges in phase measurements and the first challenge is we are talking about is stability of interferometer. So, you may want to say that, instead of using a free space setup, like what we were seeing before you could potentially use an all fiber setup. So, what is an all fiber setup look like? You have once again this optical source and then the optical source let us say it is fiber pigtail, semiconductor laser or an LED.

So, you have a fiber optic device, which is called power splitter coupler basically and what we are going to use is a 50-50 coupler, which means that 50 percent of the light that is coming through this, incident on that coupler will go in this direction and other 50 percent will go in the either direction. So how does it work? We are not going to spend much time on it right now.

But I can tell you quickly that essentially, you have the 2 core, the cores corresponding to 2 fibers, they are brought very close to each other, such as their evanescent field of 1 waveguide is actually overlapping with the evanescent field of the other waveguide. And so, you are essentially able to leak light from one waveguide to another waveguide. And back, of course, but if you get your interaction length right you can achieve the kind of split ratio that you are interested in.

So, those are available commonly available in fiber form, what is called a Fused Biconical Taper, sort of coupler. So, let us say we have one of these couplers, and we can split into two. So why two? Well, now, you can use one arm as your reference arm and other arm as your sample arm. So for example, you can take this arm and you say I am going to put a lens over here, which will

essentially call him in the light that is coming over here, and then I will put my mirror over here, so that I can fold the light back.

And on this side, I am going to put another lens so once again, I can call him in the light, and then I will put my whatever sample that I am interested in measuring. So, this sample could be let us say, a wafer, in semiconductor processing, you want to understand what is the roughness of wafer, you want to check the roughness of a wafer. So, and if the roughness measurement has to be made with, let us say 10s or hundreds of nanometers accuracy you can put that into a through one of these interferometers. And all you do is you basically translate the sample across.

So, you can move the sample across, and then you can actually map out the surface roughness of this sample. So, that could be a potential application. Now, like we talked about, this is going to be your reference. And clearly, in this sort of arrangement, also, your reference has to be absolute meaning that there should not be any phase changes within that reference. So, you need to make sure that this particular portion here it is, it is actually well protected.

And, so it is not, susceptible to external perturbations, because unless you have a very stable reference, you are not going to be able to make very precise measurements of changes in phase. And, of course, you would argue that, why cannot I coat the end of the fiber itself with dielectric coating, so it essentially acts like a mirror. And those are, once again, things that people have looked at before, so they could potentially coat the end of the fiber itself.

And that would be a quite rugged setup, if you do that. But of course, if you have an external mirror, it gives you that opportunity to potentially move that mirror, slightly (()) (14:17) that mirror, so that you can look at a corresponding change in the along the depth of the sample. We will come back to that in a minute. But, but that could be one reason why you may want to actually have an external mirror instead of putting a mirror coating at the end of the fiber, but essentially, you do all your detection using an Optical Receiver.

And that sounds like relatively simple arrangement, yes, you do have a free space part over here. So the maybe some alignment issues, like I said, Here, this can be put in a small capsule. So it is, it is actually occupying much less space. And, and overall, you are taken out all the complexities of the alignment and you have, brought it down to only these regions, which are, which are relatively small, so, it is a much, much better arrangement.

So, you could actually have a much more stable setup for doing this sort of measurements. And of course, what we looked at before actually holds good here as well. So, you could basically use the same sort of analysis and say, the total intensity here corresponds to the beating of the fields from, from these 2 arms and, and then any relative phase changes between those 2 arms.

Like we talked about this phase change corresponds to 2π over λ multiplied by $n_1 d_1$ minus $n_2 d_2$, that could be actually, any changes in that could actually be measured very accurately. And of course, we are interested in measuring changes in d_2 , let us say, this is d_1 now, and this is d_2 those are the round trip path length and in the respective arms, we are looking at changes in d_2 with respect to d_1 , but you have to be in an all fiber configuration, you have to be very vary about this quantity, also, this refractive index of the optical fiber.

Why? I already mentioned that, over here also, even if there are some pressure changes in the 2 arms, that changes the refractive index and, and that could potentially cause perturbations, unwanted undesirable perturbations in your measurement. And, more so over, here because the optical fiber which is made of glass, that glass is actually susceptible to changes in the environment.

Essentially, if there are changes in let us say, strain or temperature of the environment, that can actually vary, change the refractive index, the dn over dt , the, what he called the thermo-optic coefficient is in the order of 10^{-6} for glass, and similar order for dn over $d\epsilon$, which is essentially what is called a strain optic coefficient. So, small changes in those quantities can, can actually perturb this thing. So, you still have that issue, but let us just say for a minute that we have some way of dealing with that, we will come back and see what is that way, but let us just let us just go with that.

Now, clearly, this sort of setup is much more easy, much more rugged and so much more easy to package into instruments so that he put the entire thing into a compact box, and you just have, and well, this one, the reference is still within the box, but this particular point where the fiber probe is coming out with a lens and that is you just have a (λ) (19:15) beam available. That is the only part that of this optical setup that comes out and then that actually probes the external environment, In this case, like we talked about, it could be a solid silicon wafer that you are trying to find the reference of, all good here.

I do want to make a point about the optical source at this point. So let us just see what, what, what sort of optical source would you use. We made this argument that we have been all along we have been talking about phase with respect to in a we quantified phase a 2π over lambda multiplied by n multiplied by L, and then we do this waveform and all that to represent the wave.

But when we draw a waveform like this, what we are essentially saying is, it has a very specific frequency. And then it corresponds to a very specific lambda. And so essentially, what we are alluding in all this discussion, it is a monochromatic source. So does it have to be a monochromatic source? Does it always have to be a monochromatic? So as well the there is of course, there is no such thing absolutely as a monochromatic source, you are so you are essentially talking about highly temporally coherent sources, which are essentially having very, very narrow line with maybe in the order of kilo hertz.

Do you really need that sort of order, so that you can actually have this interference happen? Or can you tolerate with broader light sources like a Fabry Perot laser, for example, as got it is not kilo hertz, it is in order of gigahertz or maybe hundreds of gigahertz, can you still get reasonable interference with that sort of source. So, that is our next challenge. So, let us actually examine that a little more closely; what is the role of source coherence in all these phase measurements?

(Refer Slide Time: 21:45)

(2) Role of source coherence → temporal ?
→ spatial //

$$I_T = I_1 + I_2 + G_{12} + G_{12}^*$$

$$= I_1 + I_2 + 2 \operatorname{Re} \{ G_{12} \}$$

Inherent Refel $0 \leq |g_{12}| \leq 1$
Degree of Coherence

$$= I_1 + I_2 + 2 \sqrt{I_1 I_2} \operatorname{Re} \{ g_{12} \}$$

$$= I_1 + I_2 + 2 \sqrt{I_1 I_2} |g_{12}| \cos(\Delta\phi)$$

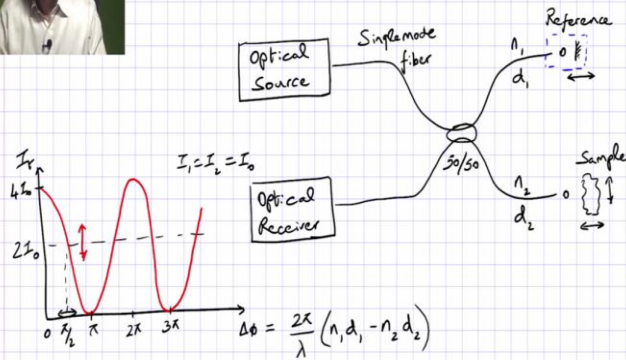
$$I_T = 2I_0 (1 + |g_{12}| \cos \Delta\phi)$$

The graph shows intensity I_T on the y-axis and phase difference $\Delta\phi$ on the x-axis. The x-axis has markers at $0, \pi/2, \pi, 2\pi, 3\pi$. The y-axis has markers at $2I_0$ and $4I_0$. A red curve oscillates between $2I_0$ and $4I_0$. A blue curve oscillates between $2I_0$ and $2I_0$. A vertical double-headed arrow indicates the amplitude of the red curve is $|g_{12}|$. Text on the graph includes $|g_{12}|=1$ and $I_1=I_2=I_0$.



Challenges in Phase Measurements

(1) Stability of interferometer



© 2004 MIT



* How to achieve phase demodulation?

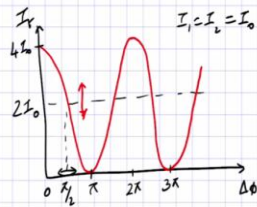
- convert phase changes to intensity changes
- use interferometers!

Degree of correlation
 $g_{12} = \frac{G_{12}}{I_1 I_2}$

Mutual coherence function
 $G_{12} = \langle E_1 E_2^* \rangle$

$$I_T = \langle |\vec{E}_1 + \vec{E}_2|^2 \rangle = \langle |\vec{E}_1|^2 \rangle + \langle |\vec{E}_2|^2 \rangle + \langle \vec{E}_1 \vec{E}_2^* \rangle + \langle \vec{E}_2 \vec{E}_1^* \rangle$$

$$I_1 = I_2 = I_0 \quad = I_1 + I_2 + \sqrt{I_1 I_2} e^{j(\phi_1 - \phi_2)} + \sqrt{I_1 I_2} e^{j(\phi_2 - \phi_1)}$$



$$I_T = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\Delta\phi)$$

© 2004 MIT



So the second thing that we want to investigate is role of source coherence. So of course, when we talk about coherence, we can actually characterize it as either temporal coherence or spatial coherence. So spatial coherence, as you know, is how well, 2 points across a wave front are correlated. And if you are talking about a light actually coupled through, let us say, all of these fibers, so all of these fiber is single mode fiber. So, it is only one mode in that and then the fundamental mode, in an optical fiber, as you probably know, it is close to a plane wave, essentially. So, so, it is a highly, highly spatially coherent, it is one of the best spatially coherent sources that you can get. So, that is not an issue.

So, this one you say, we have taken care of just by going to an optical fiber, fiber based setup. So, the question really is, what about temporal coherence, what is the role of temporal coherence in our interference? Now, let us just go back and look at this picture here, we said the total intensity of this interferometer corresponds to beating of these 2 waves even whose fields correspond to E_1 and E_2 .

And when we talk about beating of the 2 waves, you are essentially looking at the, what is called the this, this beating corresponds to what is called the mutual coherence function. So, this is G_{12} and similarly, this can be called as G_{21} or you can just call it G_{12} conjugate so, this is actually what is called the mutual coherence function. And so, that is actually quantity with units in the order of intensity, but if you want really look at the degree of coherence.

So, you say, on a scale of 0 to 1, 0 being incoherent and 1 being absolutely coherent you want to, you essentially want to characterize the coherence of the source, you could use what is called a degree of coherence. So, instead of the mutual coherence function, you can actually define quantity called degree of coherence g_{12} such that it is this capital G_{12} divided by the basically the total intensity corresponding to the 2 waves.

So, because E_1 multiply by E_2 is going to give you an intensity, so, if you divide that by the intensity, so, you are normalizing the mutual coherence function by the intensity, then you can actually get fractional value a value between 0 and 1. So, essentially what we are saying coming back to this, when you are looking at when you want to actually account for the role of coherence, you need to now write this expression the total intensity of the interferometer is going to be given by I_1 plus I_2 plus G_{12} plus G_{12} conjugate, where G_{12} corresponds to the mutual coherence function.

So, in the previous case, we assume that it is absolutely, perfectly coherent source. So, we did not worry about writing out g_{12} that is the small g_{12} separately, because small g_{12} was equal to 1, but now, we want to actually understand the effect of coherence. So, we will just write it out explicitly like this and G_{12} is actually a complex value. So, that the addition of a complex number with its conjugate is going to give you 2 times the real part. So, you will get I_1 plus I_2 plus 2 times the real part of G_{12} .

And, and of course, you can write this as $I_1 + I_2$ and you bring out, you write G_{12} in terms of root of I_1, I_2 multiplied by small g_{12} which is the degree of coherence, so, you can say this is going to correspond to root of $I_1 I_2$ multiply by 2 and the real part of this degree of coherence which we said is G_{12} is a value between 0 and 1, 1 being perfect coherence and 0 corresponding to incoherent or you can call it perfectly incoherent light.

So, in this and G_{12} mind you as got an amplitude, magnitude as well as phase in fact, what we are talking about here is actually the magnitude of G_{12} is a value that is between 0 and 1. So, if you want to write it out clearly, you will say that $I_1 + I_2 + 2 \sqrt{I_1 I_2}$ and then you write this as magnitude of G_{12} and then the phase term and the phase term is once again this is a complex $e^{j\phi} e^{j\Delta\phi}$ term

And so real part of that is going to give you once again \cos of, so \cos of $\Delta\phi$ and if I_1 equal to I_2 equal to I naught as we both the intensities are equal. So, you can just write this as $2I$ naught $1 + G_{12} \cos$ of $\Delta\phi$. So, essentially, what is different about this, this is our total intensity now, is the fact that we got this additional term here this magnitude of g_{12} which is actually corresponding to the degree of coherence of our source.

So, let us actually see what is the effect of that. So, previously, we were assuming that this g_{12} magnitude of g_{12} equal to 1 and that is what gave us this sort of thing, if it is if G_{12} is absolutely incoherent if G_{12} equal to 0 then this beat term goes out and then you just have a flat line at $2I$ naught and that does not really help anyone. Then if it is absolutely incoherent, then you are not going to be able to use this as a phase modulated sensor.

So, that is a certain need for coherence and in this case, what we are talking about is we are not talking about spatial coherence, because that is already taken care of by the optical fiber, we are talking about temporal coherence. So, now, we are saying we need a temporally coherent source so, that you can get an interference. Now, if you want to really look at what is the effect of that then we say for g_{12} which is actually magnitude of g_{12} which is between 0 and 1. Let us say around 0.5 then you are looking at a waveform or dependence that looks like this.

So, it is not going to the high of the highs absolute maximum or absolute minimum, it is somewhere in between and this visibility of these fringes with respect to $\Delta\phi$ corresponds to magnitude of the degree of coherence. So, higher the degree of coherence more temporally

coherent the sources, more will be the relative change in intensity when you have a change in phase, you can say that it is more sensitive as a phase modulated sensor. So, that is all fine. But, but let us actually dig a little more deeper and understand what it means as far as the so, what type of source are we quantifying now?

(Refer Slide Time: 33:53)

* For a partially coherent light source ($|g_{12}| < 1$) Why?

Autocorrelation function $G_{12} = \langle E^*(t) E(t+\tau) \rangle = |G_{12}| \exp(j2\pi\nu_0\tau)$

Power spectral density \leftrightarrow Autocorrelation function

FT

Weiner-Khinchin Theorem

Lorentzian $\tau_c = \frac{0.32}{\Delta\nu}$

Gaussian $\tau_c = \frac{0.66}{\Delta\nu}$

$S(\nu) = \int_{-\infty}^{\infty} G(\tau) \exp(-j2\pi\nu\tau) d\tau$

$G(\tau) = \int_{-\infty}^{\infty} S(\nu) \exp(j2\pi\nu\tau) d\nu$

Cohherence length $l_c = c\tau_c$

Challenges in Phase Measurements

(1) Stability of interferometer

Optical Source

Singlemode fiber

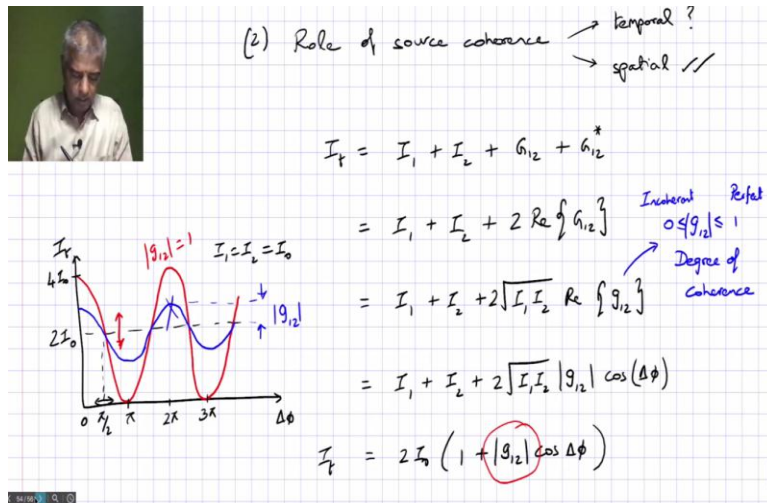
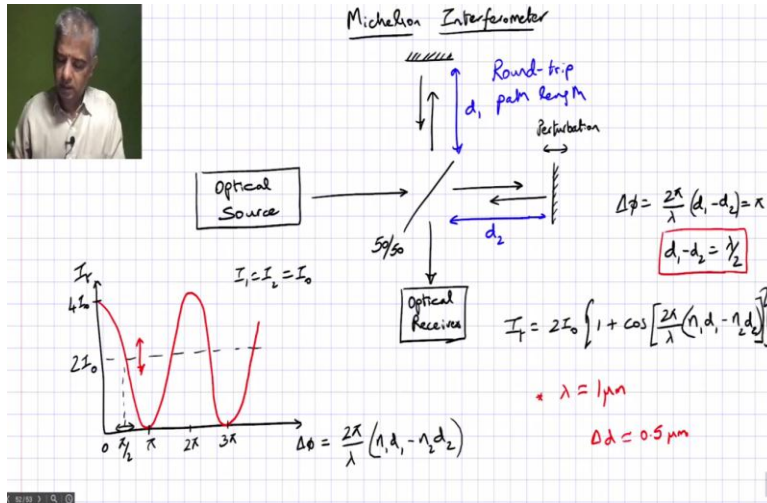
Reference

Sample

Optical Receiver

$I_1 = I_2 = I_0$

$\Delta\phi = \frac{2\pi}{\lambda} (n_1 d_1 - n_2 d_2)$



So, the question is the, the point that we are making is for partially coherent light source, talking about temporally coherent light source g_{12} is less than 1 and the question is why? How do you how do you get g_{12} level? So, what, what is the source characteristic that defines an partially coherent source? And to do that you need to look at you know, you need to look specifically at what, what are the waves that we are interfering. In this sort of a setup we are splitting source wave into two.

So, we have this may split into 2 and the same is actually coming back and then we are looking at what we are picking up the receiver. So what we see at the receiver is 2 delayed versions of

the same source, because we just have one source here, and then we are just splitting into 2 and then recombining.

So, when we look at this we understand that we are looking at beating of delayed version of a particular wave. So, what does that correspond to? Well, that corresponds to the Autocorrelation function of the source. So, you can write this out as an Autocorrelation function basically, what we are saying is the autocorrelation function which may be defined in, so in this case, that is what the mutual coherence we are looking at. So, G_{12} is nothing but E conjugate of t multiplied by correlate correlated with a delayed version of that.

And, and of course, when we talk about E of t it is a time varying function. So, there is basically correspond to $e^{j\omega t}$ or E conjugate corresponds to $e^{-j\omega t}$, whereas this one the second term has $e^{j\omega t}$ multiplied by $e^{j\omega\tau}$. So $j\omega t$ because we are beating this complex fields $j\omega t$ goes out and what we essentially are observing if we are making a measurement in the in the basement is you can write this as magnitude of G_{12} multiplied by exponential of $j2\pi$, let us say ν_0 is the center frequency. So, ν_0 multiplied by τ where τ is the delay between the 2 arms.

And where is the delay coming from the delay is coming from the fact that you are able to physically change the path length of one arm with respect to the other. So, essentially a Michelson interferometer is measuring the autocorrelation of any given source. Now, the key to understand this what does it mean in terms of the spectrum of the source? So, how do we associate the autocorrelation function to the spectrum of the source and that is where we can go to what is called the Wiener Khinchin theorem which says that the power spectral density of a source and autocorrelation of a source or the autocorrelation function of a source are essentially Fourier transform pairs.

So, let us say $S(\nu)$ is the power spectral density. So, $S(\nu)$ can be written in terms of $G(\tau)$ as exponential of minus $j2\pi\nu\tau$. And so, that is integrated over all possible values of τ . And similarly, once you know if you measure $S(\nu)$ you can find out $G(\tau)$ as well. So, $G(\tau)$ is going to be given by minus infinity to plus infinity an integral that you do over all frequencies $S(\nu)$ exponential of $j2\pi\nu\tau$.

So, so essentially, if you know the, let us say the power spectral density, let us say you use a spectrum analyzer and you are able to figure out the power spectral density. And let us say the power spectral density is something that looks like this, then based on this, you can actually take a Fourier transform in this case, inverse Fourier transform will go from frequency domain, back to the time domain, so you can get G of τ and G of τ now will look like this.

So, I made it look like it is sort of Gaussian shape, but it could be any arbitrary shaped spectrum and you can get similar corresponding Fourier transform you can take and you can get the other spectrum. So, the interesting part here is, if you look at the spectral width, let us say the full width half maximum of your spectral width that will have some relationship with the coherence time the full width half maximum of this autocorrelation function.

So, that is actually is inversely proportional to $\Delta \nu$. So, essentially what we are saying is, if you have larger spectral width, you have smaller coherence time. And so, what is the implication of this? Well, the implication of this is when you go to this sort of a setup, we made it rather maybe it is even more apparent in this, we said we can keep moving this mirror d_2 with respect to d_1 and you keep can keep going up to whatever path length difference that is d_1 minus d_2 could be whatever value it could be in the order of meters, kilometers or 1000s of kilometers and so on.

And you would still have this sort of beating happening, that is the kind of picture that we projected initially. But now, what we are saying is not only is your, this particular interference cannot happen over such a long distance, you will essentially have this interference only over a certain, certain distance. So, it is not over infinite distance, but now, it is that that interference can happen or it is measurable only within this region. So, the true picture to look at, when it comes to understanding the interference is something like this.

So, this actually may not be the true picture, because all said and done, when $\Delta \phi$ equal to 0, that is actually the autocorrelation of the wave when the delay is 0, so, that 1 wave just correlated with another wave. So, that should actually give you maximum correlation, even if the degree of coherence is very small. So, this picture is actually little, I would say misleading, so, I would not rely on this. Rather I would go to this other picture where I would say, let us say this is my, the total intensity that I am measuring, let us say and this go to a different color here, so it is easier to pick this up.

So let us say this is τ , and you are looking at, let us say you are looking at I of t the total power. And we know that the average if I_1 equal to I_2 to equal to I naught, that average is going to correspond to 2 times I naught and the maximum will correspond to four times I naught, but now what we are saying is, since you are looking at a partially coherent source, you are looking g^2 less than 1, then you are not going to have interference forever.

So, you would essentially have interference only over this region. And so, that actually defines the envelope of the center front that corresponds to this envelope over here and within this, you will have this changes in phase. So, actually too many lines over here, so let me just go back and redraw this. So, we are saying that the envelope looks something like this, and the actual fringes will look it looks something like this.

So, those are the fringes that we expect. So, of course, if you go for quite narrow line width when $\Delta\nu$ is very small, then that means τc is very large and then this will just sort of get flattened out. So, you will see these fringes going from minimum to maximum over larger values of τ or in other words, we can actually represent this in terms of $\Delta\phi$ now. So, if larger changes in phase also, a larger path length differences, you can still get very good interference.

On the other hand, if your source coherence is limited, you can get coherence only over these interference fringes only with over some limited range of $\Delta\phi$. So, once again, this corresponds to τc . Now, how does τc relate to $\Delta\nu$? You can just say if your, it depends on the shape actually. So, the shape of the spectrum, so, if you have a rectangular shape spectrum, then τc will be $1/\Delta\nu$.

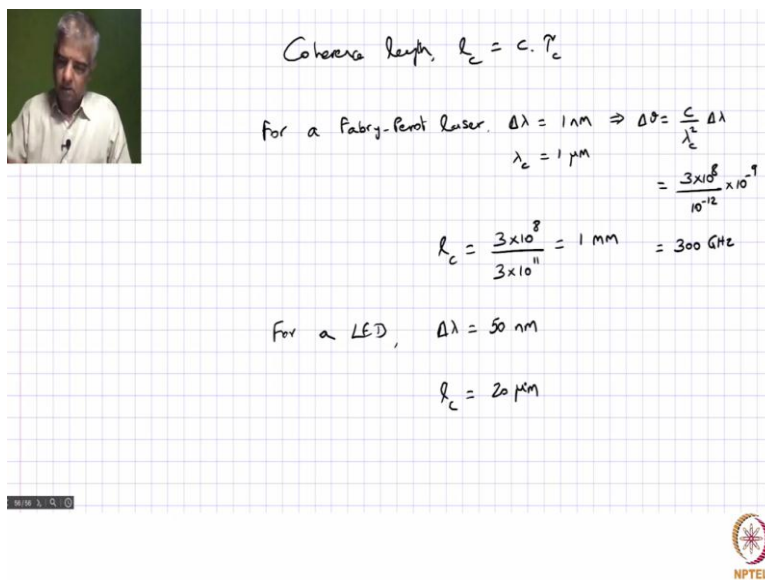
But if it is let us say a Lorentzian shape, which is common seen shape for the spectrum then τc is given by about $0.32/\Delta\nu$, and if it is Gaussian, so if it is a Gaussian shaped spectrum then τc equals $0.66/\Delta\nu$ or maybe. So, that is the relationship between τc and $\Delta\nu$. So, this actually tells you that maybe this we started with this question as to what sort of source you should use.

So for long path length differences, if you want to make a measurement, let us say over 10 kilometers. So, that is something that that we are working on now, we are developing a sensor, which can actually pick up perturbations at a distance 10 kilometers away. So, the sample arm is

extended to 10 kilometers now, we are looking at an OTDR type of setup. So, we are looking at a sample arm that is 10 kilometers away and then some perturbation there, we need to pick up.

So, you need to have you need to support path length differences of 10 kilometers. So, you need to have correspondingly high coherence source. So, that is one thing we have not done so far. So, let us see over here, how does this coherence time correspond to coherence distance, well, that is simple. You basically say coherence length l_c is just the velocity of light in free space multiplied by tau c. So, let us actually take that up in a separate page.

(Refer Slide Time: 52:38)

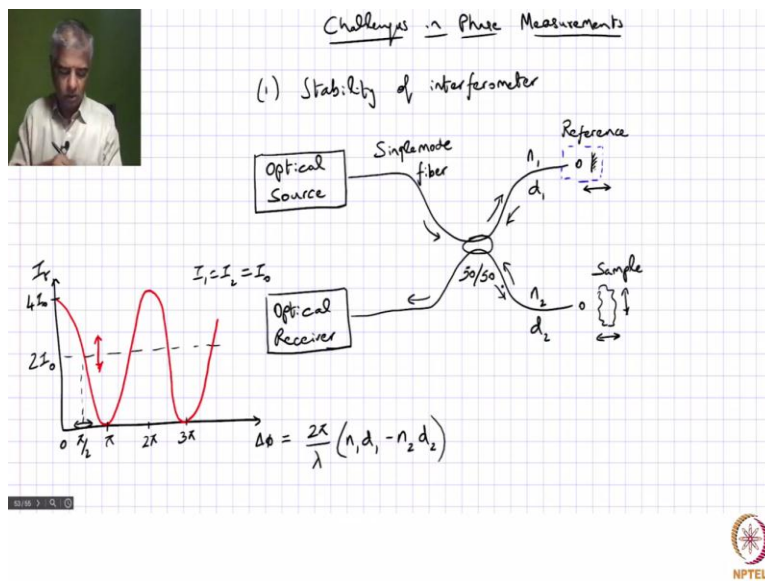


Coherence length, $l_c = c \cdot \tau_c$

For a Fabry-Pérot Laser, $\Delta\lambda = 1 \text{ nm} \Rightarrow \Delta\omega = \frac{c}{\lambda^2} \Delta\lambda$
 $\lambda_c = 1 \mu\text{m} = \frac{3 \times 10^8}{10^{-12}} \times 10^{-9}$

$l_c = \frac{3 \times 10^8}{3 \times 10^{11}} = 1 \text{ mm} = 300 \text{ GHz}$

For a LED, $\Delta\lambda = 50 \text{ nm}$
 $l_c = 20 \mu\text{m}$



Challenges in Phase Measurements

(1) Stability of interferometer

Diagram: An Optical Source feeds a Singlemode fiber that splits into two paths. The upper path is labeled 'Reference' with refractive index n_1 and length d_1 . The lower path is labeled 'Sample' with refractive index n_2 and length d_2 . Both paths recombine at a 50/50 beam splitter, which is connected to an Optical Receiver.

Graph: Shows Intensity I_r versus phase difference $\Delta\phi = \frac{2\pi}{\lambda} (n_1 d_1 - n_2 d_2)$. The intensity oscillates between $4I_0$ and 0 . The average intensity is $I_1 = I_2 = I_0$.

* For a partially coherent light source $|g_{12}| < 1$ Why?

Autocorrelation function $G_{12} = \langle E^*(t) E(t+\tau) \rangle = |G_{12}| \exp(j2\pi\nu_0\tau)$

Power spectral density \leftrightarrow Autocorrelation function (FT)

Wiener-Khinchin Theorem

Lorentzian $\tau_c = \frac{0.32}{\Delta\nu}$

Gaussian $\tau_c = \frac{0.66}{\Delta\nu}$

Coherence length $l_c = c\tau_c$

NPTEL

So, we say that the coherence length l_c equal to c multiplied by τ_c . Now, let us actually try to figure this out for different values of source spectrum. So, for Fabry Perot laser which has $\Delta\lambda$ in the order of 1 nanometer, if for a Fabry Perot laser $\Delta\lambda$ equals to 1 nanometer and if you say this is 1 nanometer $\Delta\nu$ equal to c over λ^2 and $\Delta\lambda$.

So, λ let us say, this is λc let us say that the center wavelength is 1 micron. So, this is 3×10^8 divided by micron square. So, this is 10^8 minus 12 and this is 10^8 minus 9. So, that corresponds to 3×10^{11} hertz, so, that is basically 300 Gigahertz. So, $\Delta\nu$ is 300 Gigahertz. So, what is the corresponding coherence length l_c equal to so, l_c equals to 3×10^8 divided by $\Delta\nu$. So, this is 3×10^{11} .

So, that is, that corresponds to coherence length of 1 millimeter, so, that would be in the order of 1 millimeter. So what does that mean? It means that even if you are path length changes, Δt is in the order of 1 nanometer, sorry, 1 millimeter, you are going to have the interference fringes go down by a factor of half, because coherence length corresponds to the length, path length difference where the, this envelope up falls down to half the value. So, the fringe contrast should have now gone down by a factor of 2.

So, on the other hand, so this is for Fabry Perot laser, and of course, we have assumed that you know, τ_c equals to 1 over $\Delta\nu$, for simplicity that we assume that is a rectangular spectrum in reality, it is not a rectangular spectrum, it is more like a lorentzian spectrum. So, there is a

factor of 0.32 also that comes into the picture. So, it would be more like 0.32 millimeter, but just neglecting that for a minute.

So, for LED, if you use a semiconductor LED, $\Delta\lambda$ equals to 50 nanometers. So, then this will be even smaller, instead of 1 millimeter, you will see that the coherence length would correspond to 1 millimeter divided by 50, which corresponds to 20 micron. So, if you use a broadband source, you can actually have this interference fringes only for a very short distance. Whereas, if you use a narrow band source, I mean, you can go to even 1 megahertz is the DFB laser, the 1 megahertz you can do the scaling and you will find that, that will already correspond to hundreds of meters.

So, if you want to have path length differences of hundreds of meters, you will go to 1 megahertz if you want path length differences, if you want to pick up phase perturbations from distances as high as 10s of kilometers, then you may need to go to kilohertz type of line width. So, that is the key part in a phase modulated sensor, where you want to what sort of path length differences you want to pick up your perturbations, according to that you need to have a corresponding source spectral width that supports that, like I said, long path length differences, small source spectral width.

On the other hand, if you want to have path length differences only in the order of micron, one application of that is optical coherence tomography, which we will probably talk about the next lecture, we will see that there you need to have very low coherence, so maybe an LED would do the job for you. So let us stop right there.