

Optical Fiber Sensors
Professor Balaji Srinivasan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture 27
Phase modulated sensors - 7

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Challenges in phase modulated sensors

- ✓(1) Mechanical stability of interferometer → "all-fiber" interferometer
- ✓(2) Role of source coherence → high temporal coherence ($\Delta\nu \sim \text{kHz}$)
 ⇒ phase info from long dist
 → low temporal coherence ($\Delta\nu \sim \text{THz}$)
 ⇒ highly precise local clock information
- 3) Phase fluctuations → Environmental perturbations
 → optical source
- ✓(4) Polarization → Fresnel-Arago law → max visibility when pol are the same
 → zero visibility for orthogonal polarization

Hello, we have been talking about challenges in phase modulated sensors and we looked at various different challenges and we have been also looking at how to possibly overcome them and towards the end of the last lecture we were talking about this specific challenge of phase fluctuations due to environmental perturbations and one way we could potentially overcome that is by using a phase generated carrier technique.

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is required. The system responded linearly to of the input signal between the noise floor of on used (typically 10^{-3} - 10^{-6} rad) and ~ 1 rad. al level (~ 1 rad), distortions were observed due riators overloading from the presence of the signal, thus the dynamic range of the system at els may be increased by 1) lowering the gain ferentiations or preferably, 2) having a sharper cutoff) of the carrier modulation (i.e., a higher order filter). Below this region of obvious distortion, the first harmonic of the signal was typically buried in the noise floor and hence was unobservable, but the ratio of the fundamental to the first harmonic was typically greater than 70 dB (i.e., 3×10^3).

2) As discussed earlier, the operation of the current induced modulated carrier requires the interferometer to use a nonzero path length difference such that the frequency modulator is converted to the required phase modulation. Consequently, this nonzero path difference allows the phase noise to contribute to the interferometer noise. Using the diode laser modulation scheme, the frequency dependence of the mini-

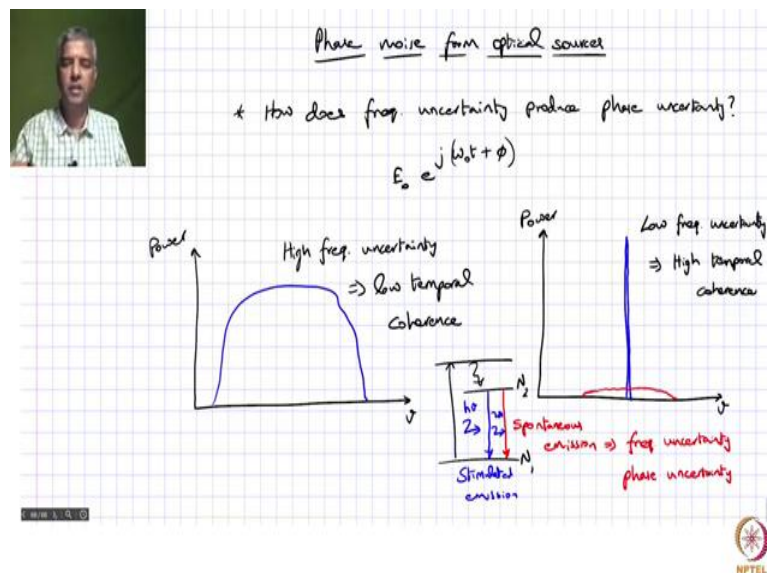
in quadrature using the conventional homodyne detection scheme. As can be seen, the passive homodyne result is approximately a factor of 2 worse than the result obtained with the conventional homodyne. The noise in the conventional homodyne system was determined by the intrinsic intensity noise of the diode laser [8]. Owing to the carrier modulation, the average level of the intensity noise remains the same as with the conventional homodyne interferometer field in quadrature, however, the amplitude of the phase signal is reduced by the magnitude of the relevant Bessel function (5). Consequently, in an interferometer dominated by the laser intensity noise, a factor of ~ 2 difference between the two detection methods is expected for this modulation amplitude.

In an unbalanced interferometer powered by a laser diode, the primary noise source becomes the phase noise due to the fluctuation of the emission frequency of the laser. For a signal-to-noise ratio of 1, the minimum detectable phase shift ψ_m in the interferometer is given by

$$\psi_m = 2\pi D \left(\frac{\delta\nu}{c} \right)$$

But even when we were talking about phase generated carrier technique we were actually looking at this paper by Dandridge et al and we realized that the minimum detectable limit as far as the phase is concerned is going to be determined by this specific factor here which is the uncertainty in the frequency of your light source itself. So that uncertainty is essentially going to be the fundamental limitation as far as phase modulated sensors are concerned.

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So, let us go ahead and try to examine that in a little more detail in today's lecture. So, we are talking about phase noise from optical sources and the fundamental question is why does any change in frequency or any uncertainty in frequency, why does it show up as an uncertainty in phase. How does frequency uncertainty produce phase uncertainty?

So, how do we do that? Of course, we do understand that when we talk about frequency like it is a monochromatic source, then you have a very definite phase for the wave that you are picking up, but if you are looking at polychromatic sources, then the phase is uncertain. So, essentially that polychromatic source is what we are actually talking about as a frequency uncertainty.

But anyway if we look mathematically, what we find is the field corresponding to the optical source can be written as $E_0 e^{j(\omega_0 t + \phi)}$, where E_0 is the magnitude of the field, multiplied by $e^{j(\omega_0 t + \phi)}$, where ω_0 corresponds to the angular frequency, the central angular frequency of your light source.

So, because of the fact that both of these terms appear in the phase of your optical source, even if you have an uncertainty in terms of $\Delta \nu$, you have an uncertainty in terms of

phase of your light source. Of course, it could be vice versa also, any uncertainty in the phase of your light source can potentially give rise to uncertainty in the frequency or it could appear as an uncertainty in terms of the frequency of your light source.

So, if we want to actually understand this a little better, let us just take two examples, let us actually examine the optical spectrum of a light source and let us take the example of one light source where with respect to frequency the optical spectrum looks something like this, it is quite broadband, so we are talking about the optical power as a function of frequency, it is spread over a wide range of frequencies.

And in this case what we are talking about is high level of frequency uncertainty because the frequency can be of the optical source at any particular instant, if you take a photon, the photon can actually have any of these frequencies, so it is a fairly high level of frequency uncertainty and this is the characteristic of what we call as a low temporal coherence, a source with very low temporal coherence.

Now, the same thing we can say instead of actually having your power over so many frequencies which is possibly the characteristic of a multiple longitudinal mode laser, you can say I find a way to make a single longitudinal mode laser, so if you make a single longitudinal mode laser, then of course, you can expect that the line width is fairly narrow, so you basically have low frequency uncertainty and that is because you have very high temporal coherence of the source.

So, you say that is one way of making sure that my phase uncertainty is relatively low and and through that I can possibly reduce the minimum detectable limit or I can, in other words pick up phase changes that are really really small, let us say it is in the order of 10^{-7} or 10^{-8} radians, so that would be quite desirable.

But there is a fundamental limitation in terms of what is the lowest line width that you can achieve, what is the lowest phase uncertainty that you can achieve. And that, to understand that you have to go back to laser theory, so you basically say what is a laser? Laser is Light Amplification by Stimulated Emission of Radiation. It is a, laser is actually an acronym. So, what we are relying on is stimulated emission. So, how do you produce stimulated emission?

Well, you would say that I have two energy levels, higher energy level and lower energy level, I have an inversion meaning you have a large number of those atoms in the gain medium at the higher energy level compared to the lower energy level, then if you have say a

photon coming by with the difference in the energy levels, with the photon and energy having the difference, then you can stimulate this transition.

So, I can simply represent this, let us say we are considering a three level atomic system, so we are talking about pumping from your ground state to a higher energy level and then you may have some non-radiative relaxation to another energy level and from there you can basically say you are building up your inversion such that N_2 is a significantly large number compared to N_1 .

Then in that sort of a scenario if you have an incoming photon, with let us say $h\nu$, maybe I will use the same color, where $h\nu$ corresponds to this energy difference then it can actually stimulate and give you a clone of the incoming photon, which has $h\nu$ as your energy and then so you have two photons with $h\nu$. But there is always this possibility that you could have a spontaneously emitted photon also.

So, this is basically what is called spontaneous emission. So, you could have a spontaneous emission event also whenever you have a large inversion and you are not utilizing the inversion with enough number of photons that are coming in to do this stimulated emission. So, this is what we desire stimulated emission, but an undesirable byproduct of that is spontaneous emission.

So when we look at the spectrum, the spectrum, the spontaneous emission can be over fairly large spectrum, and of course, you could find a way to filter that and restrict it to the line width of interest and all that, but nevertheless the point is that you going to have spontaneous emission as an inevitable byproduct of any laser. So, let us examine this a little more closely and then let us see if we can, what is the effect of having this spontaneously emitted photon.

The key point as far as spontaneous emission is concerned, this is actually, not only is producing light over a wide range of frequency, so the frequency of the photon is uncertain, but the key point is that not only the frequency, but also the phase is uncertain because you cannot tell that it is not timed, it is not like at this time you have exactly this many spontaneous limited photons coming out. So, it is actually fairly random in nature and that is actually a significant problem as far as phase modulator sensors are concerned.

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Single longitudinal mode (SLM) laser

Laser Intensity = $|E|^2 \propto \tilde{n}$ (Mean number of photons)

For one spontaneously emitted photon

$$\Delta S(t) = \frac{1}{\sqrt{\tilde{n}}} \cos \theta$$

$\theta \rightarrow$ random variable, uniformly distributed over 0 to 2π rad.

So, let us go into a little more detail about some of these things. So, to understand this, let us go back to our phaser picture, so you are in a phaser picture, you are plotting things in the complex plane. So, the real part, this is the imaginary part of the optical field. So, of course, what we are looking for is this a phaser, which corresponds to your laser, but what we are also saying is there is actually a undesirable spontaneously emitted photon which has this phase angle with respect to the stimulated emission emitted photon.

And that is arbitrarily showing up, so it is basically that angle could be anywhere between 0 to 2π , so it could be this direction, it could be this direction, this direction and so on, but what that means is your actual emission has got a certain uncertainty. The phase of the actually emitted photon has got an uncertainty and that uncertainty; let us call this delta theta. The key point is this can be anywhere between 0 to π that means the uncertainty in the phase can be, it could be the phaser, could be fluctuating around some mean position.

The mean position would be determined by the stimulated photon. So, that is the kind of scenario that we are looking at. So, let us go ahead and try to quantify what is happening over here. So, we know that, so we are considering a single longitudinal mode is called a SLM laser, so that actually reduces the uncertainty as far as the phase is concerned, but even within the single longitudinal mode laser, now we are considering a spontaneously emitted photon.

So, when you consider the laser intensity, we know that is corresponding to the square of the magnitude of the electric field and this is actually a quantity that is proportional to the mean number of photons, so it is proportional to the mean number of photons that means more

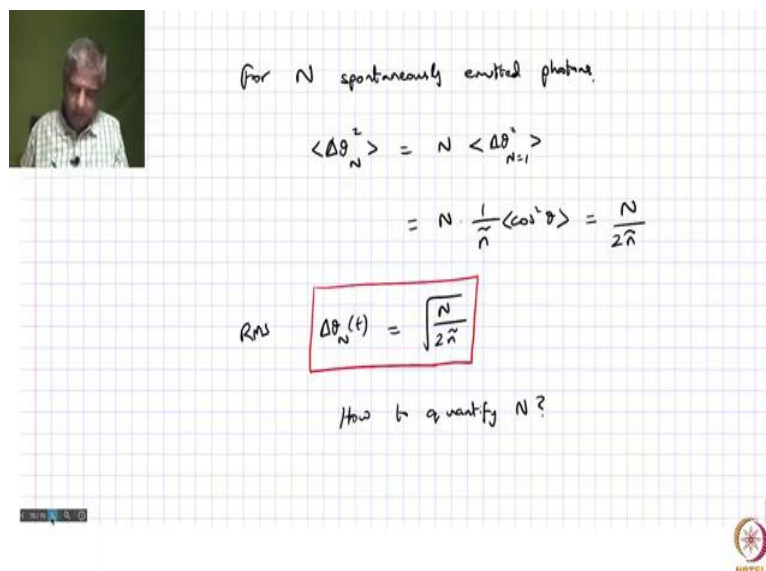
number of photons, more will be the fields, the total field corresponding to that the more will be the intensity.

So, in other words when we talk about E_{naught} here, the magnitude of E_{naught} , it is proportional to root of that mean number of photons. So, now when we consider for one spontaneously emitted photon, when we, what we are trying to quantify is this $\Delta\theta$, so what is the change in the or the variation in the phase angle and I should say that $\Delta\theta$ is not constant with respect to time, it is a varying component with respect to time.

So, if you try to quantify $\Delta\theta$ that is going to be equal to $1/\sqrt{N} \cos\theta$, so because it, if you have more number of photons that is essentially stimulated emission photons, you are going to have less of this effect of this $\Delta\theta$ that the change in the width, the fluctuation in the phase.

And it has a $\cos\theta$ term that is essentially like when we talk about beating of two fields, we have a \cos of $\Delta\phi$ term where $\Delta\phi$ here is the θ , that is the phase angle between the spontaneously emitted photon and the stimulated emitted photon. And this θ is actually a random variable; that is uniformly distributed over 0 to 2π radians. So, θ is actually a uniform that random variable is uniformly distributed over this values of 0 to 2π . So, that is for a single spontaneously emitted photon.

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For N spontaneously emitted photons.

$$\langle \Delta\theta_N^2 \rangle = N \langle \Delta\theta_{N=1}^2 \rangle$$
$$= N \cdot \frac{1}{N} \langle \cos^2\theta \rangle = \frac{N}{2N}$$

RMS $\Delta\theta_N(t) = \sqrt{\frac{N}{2N}}$

How to quantify N ?





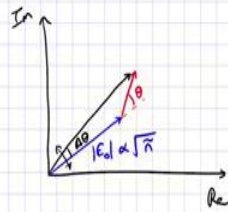
Single longitudinal mode (SLM) laser

$$\text{Laser Intensity} = |E|^2 \propto \bar{n} \quad (\text{Mean number of photons})$$

For one spontaneously emitted photon

$$\Delta\theta(t) = \frac{1}{\sqrt{\bar{n}}} \cos \theta$$

$\theta \rightarrow$ random variable, uniformly distributed over 0 to 2π rad.



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But if you consider, let us say n number of spontaneous emitted photons, so for n number of spontaneously emitted photons what do we expect? Now, when we talk about n photons, each of those is an independent event, it is an independent random event. So, when you have independent random events happening, when you look at the combined picture of that, then it is essentially the variance of the combined event is going to be equal to the addition of the variances or the sum of variances of each of those individual events.

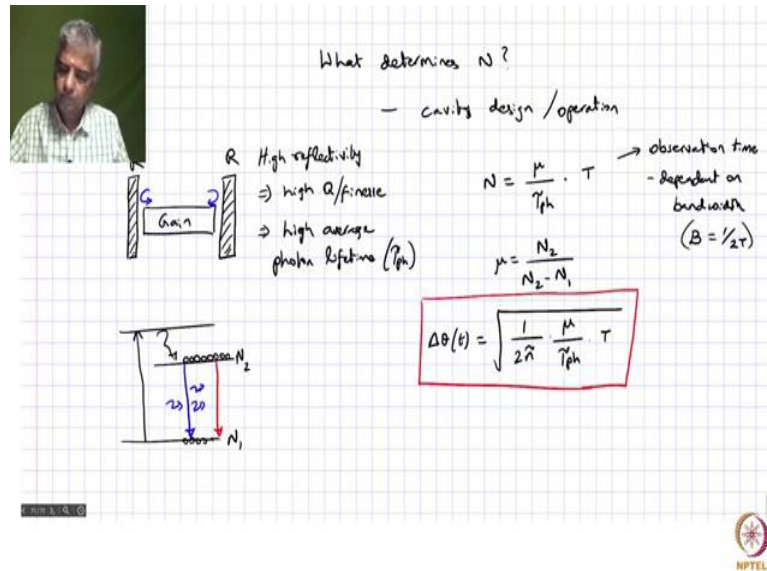
So, if you are talking about the variance that is Δt^2 for n spontaneously emitted photons, so that is going to be equal to n times the variance corresponding to N equal to 1 and this we have already defined, so this is basically this variance, so what we looked at here is actually the RMS value, but if you look at the variance that will correspond to $1/\bar{n} \cos^2 \theta$ and that is actually an average of all those values corresponding to θ .

Now, of course, when you do $\cos^2 \theta$ and you do average over all these possible values of data from 0 to 2π , what do you get? You get a value of half. So, this can be written as $N/2$ times $1/\bar{n}$. So, that is actually the variance actually associated with N spontaneously emitted photons and if that is the variance, then of course, you can actually also find out the RMS value.

The RMS value is just root of that variance, so $N/2$ times $1/\bar{n}$. So, once again what we are saying is \bar{n} should be as large as possible, so that for a given inversion you most of the inversion is spent in actually emitting spontaneous, sorry, emitting stimulated photons and thereby you can keep your $\Delta \theta$ as low as possible.

But of course, that is also mediated by, what is that value of N that you have. So question is – how to quantify N . So, let us actually take this as one key result, but the question is – how to quantify N , the number of spontaneous emitted photon. And specifically the question is – how do you keep N to be a relatively small number?

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What determines N ?

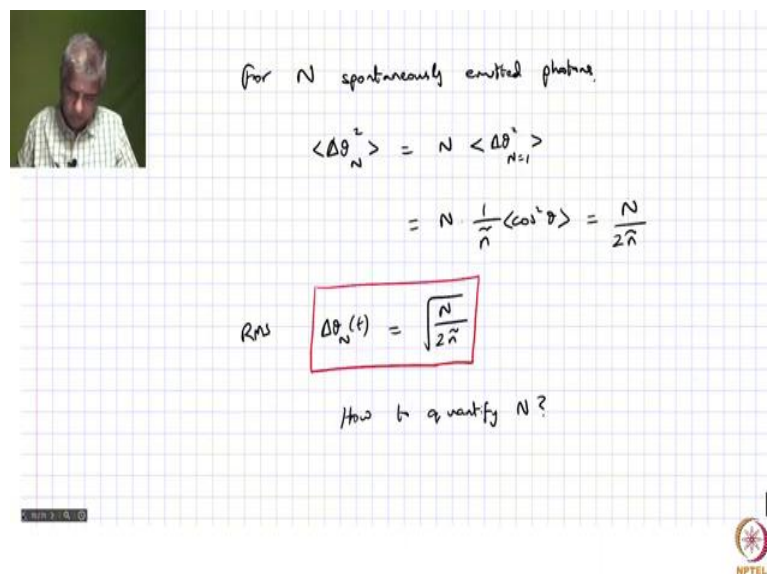
- cavity design / operation

R High reflectivity
 \Rightarrow high Q /finesse
 \Rightarrow high average photon lifetime (τ_{ph})

$N = \frac{\mu}{\tau_{ph}} \cdot T$ \rightarrow observation time
 - dependent on bandwidth
 ($B = 1/2T$)

$\mu = \frac{N_2}{N_2 - N_1}$

$\Delta\theta(t) = \sqrt{\frac{1}{2\tilde{n}} \frac{\mu}{\tau_{ph}} \cdot T}$



For N spontaneously emitted photons.

$\langle \Delta\theta_N^2 \rangle = N \langle \Delta\theta_{N=1}^2 \rangle$

$= N \cdot \frac{1}{\tilde{n}} \langle \cos^2 \theta \rangle = \frac{N}{2\tilde{n}}$

RMS $\Delta\theta_N(t) = \sqrt{\frac{N}{2\tilde{n}}}$

How to quantify N ?

So, to understand that you need to go back to laser theory and understand this process of stimulated versus spontaneous emission and as we, so let us for example, consider a typical laser cavity, let us say it is a fabry perot cavity, so where you have two plain parallel mirrors and then you have a gain medium between them, sandwiched between them, and so essentially you are pumping the gain medium.

So, you have, like we talked about previously you are pumping the gain medium and then you build an inversion into with respect to N_1 and hopefully lot of your photons will go through a stimulated emission but like we said you always all have the possibility of this spontaneous emission happening as well.

So, if we examine this cavity and we want to look at the dependence of N ; so we ask the question what determines N ? So, that is actually is determined by the cavity design and the cavity operation. So, what do I mean by this? What do I mean by the cavity design first? Well, I have these cavity mirrors that if you go for a high value of R , which means if you go for highly reflective mirrors, then you have essentially what is called a high Q cavity, high finesse cavity

And if you have a high finance cavity what can you say about the photon that is could be present inside this cavity. The photon is going to keep bouncing back and forth over a relatively long time, so when we talk about the average photon lifetime, this means it is, there is a high average photon lifetime. So, let us call this τ_{ph} . So, you have a fairly high average photon lifetime, if you have a high finesse cavity with highly reflective mirrors.

So, how does that impact? If you have a large number of photon build up through feedback, so whenever there is feedback from these mirrors that is going to actually cause more and more of the stimulated emission or in other words if you have a longer photon lifetime that means there is less stimulated emission, so you can write an expression for the stimulated emission, which actually depends on this.

It has got a inverse dependence with respect to the photon lifetime because larger photon lifetime corresponds to high finesse cavity in which more of stimulated emission is happening, so you have lesser probability of spontaneous limited photons. But it also depends on the inversion that you have and specifically the inversion such that, so when we talk about inversion we are talking about lot of these atoms sitting in the higher energy level compared to the lower energy level.

We are also talking about the utilization of the inversion, that utilization of the inversion is something that can be represented by this factor μ , where μ is given by N_2 over N_2 minus N_1 . So, N_2 is actually the number of atoms in the excited level with respect to the total inversion. If you have a large number of atoms that are the excited level with respect to the inversion and essentially you do not have as many stimulated, as many photons that can stimulate this this transition, then that is going to give rise to more of spontaneous emission.

So, it is proportional to μ . So this is μ divided by τ photon multiplied by your observation time, so let us say the observation time window is given by t , so what we are talking about, this is the observation time, so in a practical sensor what determines the observation time that is dependent on the receiver bandwidth. So, the receiver bandwidth can be written as $1/2$ times t , t corresponds to the observation time.

So over that observation time you might actually have n spontaneously emitted photons and where N is determined by this inversion as well as your photon lifetime inside the cavity. So, now I can substitute this over here, we have quantified n , so I can just say your $\Delta\theta$ of t now corresponds to $1/2$ times n where n is the number of mean number of photons and multiplied by μ over τ photon multiplied by t .

So, what we are saying is, let us go from this side, so larger, if you have low bandwidth receivers you are integrating all these events over a longer time and that means there is clearly more possibility of spontaneous emission and more spontaneous emission means more fluctuation in terms of the phase more uncertainty in the phase. It is inversely dependent on the root of the τ photon.

Well, why do we talk about root because the variance is inversely proportional but when we are talking about the RMS value there is a root that comes into the picture, but it is inverse proportional to τ photon, which means that if you want to reduce $\Delta\theta$, the phase uncertainty, you have a larger photon lifetime, which means you go for a higher finesse cavity.

And then it is also dependent on μ , which corresponds to how effectively use of the inversion if N_2 is, if all your atoms are the excited state, so μ can actually be as slow as 1, because N_1 becomes 0, then you have μ equals to 1. So, that is one way of, well, in that case you may not have as many photons stimulating this transition, so that is actually the, so that can give rise to a lot of. Well, if you, let us rephrase that, if N_2 , if the inversion, the highest value of inversion could be N_2 by itself.

So, if you have more photons in the ground state, then that means $N_2 - N_1$ is a value that smaller than N_2 and that means μ is a much larger value and when μ is much larger that means there is actually a much more spontaneously emitted photons and then because of that you have much more uncertainty in the phase of your optical field. So, let us actually put all of this together and let us see what it means from an interferometer perspective.

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Differentiate & cross-multiply

③ $-B G J_1(\phi_c) \cos[\phi(t)] \frac{d\phi}{dt}$ ③ x ② $\rightarrow B^2 G H J_1(\phi_c) \cos[\phi(t)] \frac{d\phi}{dt}$ $J_2(\phi_c)$ -⑤

④ $+B H J_2(\phi_c) \sin[\phi(t)] \frac{d\phi}{dt}$ ④ x ① $\rightarrow -B^2 G H J_1(\phi_c) J_2(\phi_c) \sin^2(\phi_c) \frac{d\phi}{dt}$ -⑥

⑤ - ⑥ $\Rightarrow B^2 G H J_1(\phi_c) J_2(\phi_c) \frac{d\phi}{dt} \Rightarrow$ Integrate to extract $\phi(t)$

So, let us do a quick recap on what we have been talking about, so we have been looking at how to extract this phase changes phi of t, and of course, we realize that we are going to have to do this differentiate and cross multiply scheme to extract phi of t, let us just talk about how to optimize this phi of t. So, to optimize this we need to take care of all these factors J1, J2, B, G and H and all of that, so let us just look into all of this one by one.

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$J_1(\phi_c)$
 $J_2(\phi_c)$

$\phi_c = 2.6 \text{ rad}$
optimum bias point

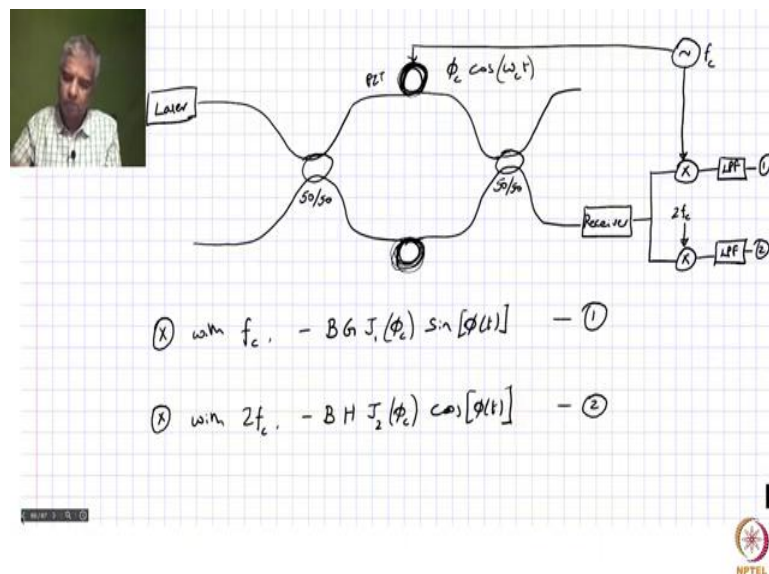
G & H are chosen that they help fill the AEC
(J_1 & J_2)
 B has to be chosen such that they do not
cause significant shot noise @ receiver

Let us first look at the Bessel components. So, what do these Bessel components mean? Well, you can draw this Bessel function J of phi c as a function of phi c, so if you draw the different Bessel functions, so let us say you start with J naught, J naught is actually going to be an

oscillatory component with the exponential decay of the envelope and then you look at J1, J1 is going to start from 0, and then so J naught actually starts from a value of 1.

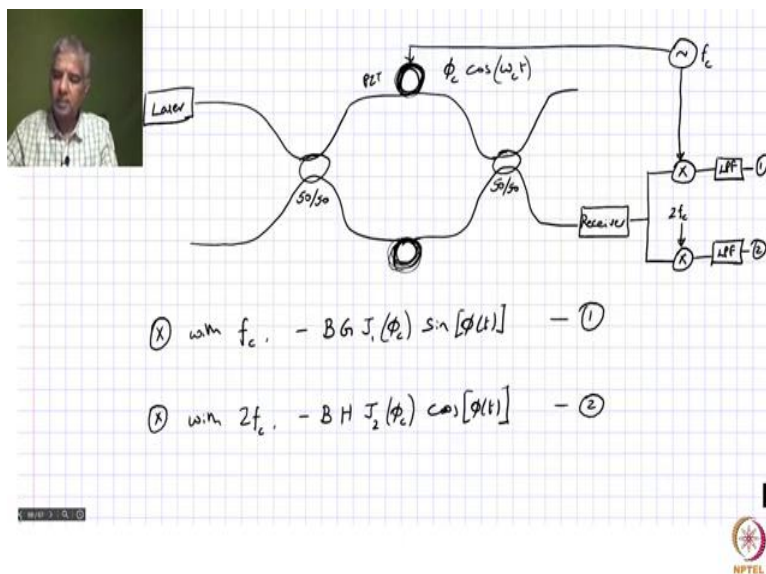
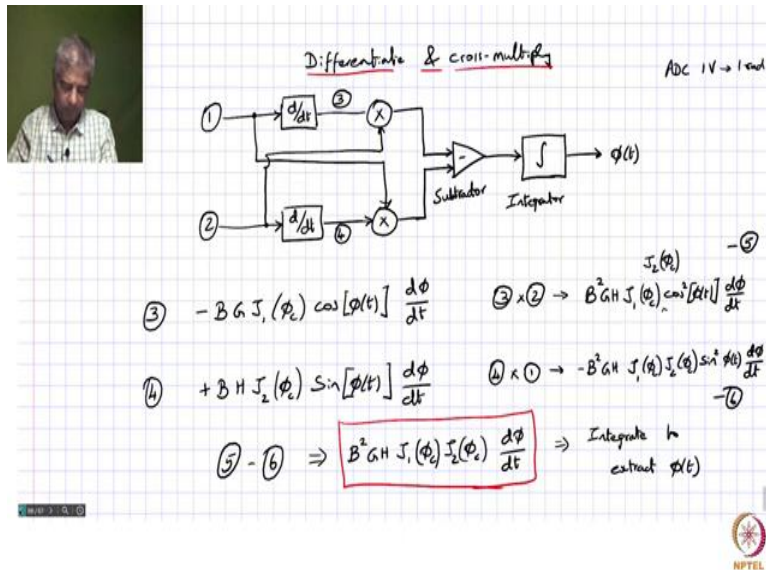
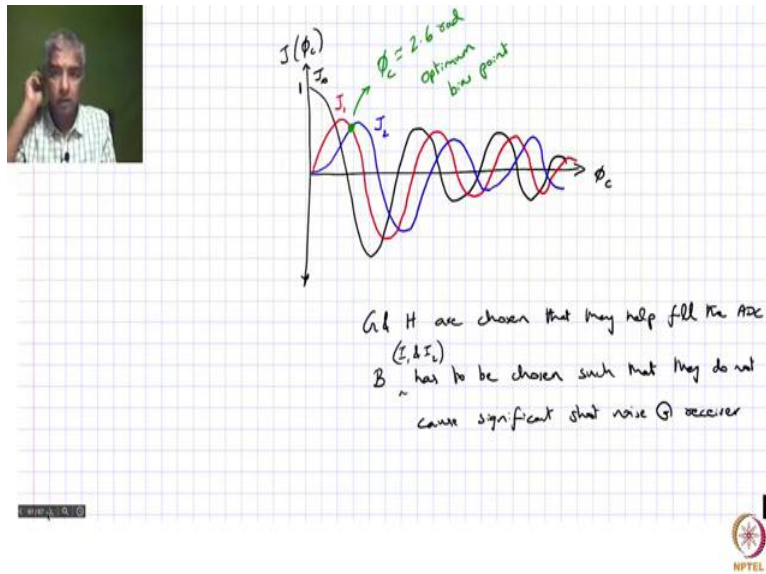
Whereas J1 starts from a value of 0 and then once again it it is oscillatory, but it does not swing as much as J naught does. And then J2 is another oscillatory function, but that does not go up as much as J1, so it is relatively smaller in amplitude, but it is, that is also decaying and oscillating. Now the question is what is the optimum value of phi c that you can choose.

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Remember phi c is something that is up to you, because you actually send a voltage from your signal generator to drive your PZT and the voltage should be such a value like, it can call this v_c, so such value that it incorporates a certain phi c through this PZT, So, then the question is what should be that phi c?

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Now, the ϕ_c should be such that when you have small changes in ϕ_c , let us say it is, although you give a very specific voltage to get a very specific phase point, the voltage might have some small changes, like it might go from 2.59 to 2.61, it may be fluctuating by about 0.01 radian, if there is any such changes, it should not cause a big change in the phase that you are trying to extract.

So, you choose a point such that if it goes less than that value, let us say it goes 2.59 J1 is increasing J2 is decreasing, so when you are looking at the product of J1 and J2 that is constant and similarly if this ϕ_c value goes up to 0.01 higher, it goes to 2.61, J2 is increasing J1 is decreasing, once again the product is not going to change as much. So, you look for these crossover points between J1, J2.

So, there is one crossover point here, another crossover point here, another crossover point here and so on, so you can conveniently choose this crossover point, so that will be your optimum bias point, so that any small changes in small fluctuations in ϕ_c should not change the overall ϕ value that you are trying to pick up and then it comes to optimizing the value of G and H.

How do you optimize the value of G and H? Now, when you think about the final power value that you are reading this ϕ of t you are reading, ϕ of t should typically design for a certain range, there is a maximum ϕ value for which you design this sensor, let us say that maximum ϕ value is 1 radian, so it should be such that this final ϕ of t that you are getting is actually a voltage value.

So, that voltage should fill your adc, so if your adc is say, adc is one volt, then that one volt should correspond to one radian, if one radian is the maximum phase that you are trying to pick up. So, you need to fill your adc, so to fill your adc you can use, you can actually change the value of G and H. What does G and H represent?

G represents the strength of the signal that is coming here, H represents the strength of the signal that is coming here. So, you can actually change those voltages so that you can increase the signal that your, final signal that you are getting, such that it fills the ADC. So, that is how you go about choosing G and H. And finally you want to choose B carefully.

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Overcoming Environmentally-Induced Phase Noise

- Phase Generated Carrier
Dandridge et al
IEEE JQE, 1982

* Design of a hydrophone

Reference $I_0 \cos \omega_c t$

Shot noise
Thermal noise

Detector

Measurement

I_0

Laser

Phase

Time

ϕ_s

ϕ_c

f_s

f_c

$\sim 100 \text{ Hz}$
Acoustic signal

B

$$I_f = I_1 + I_2 + 2 \sqrt{I_1 I_2} J_0(\phi_c) \cos(\Delta\phi) + \dots$$

$$\phi(t) = \phi_c \cos \omega_c t + \phi_n(t)$$

Differentiate & cross-multiply

ADC 1V \rightarrow 1rad

① $\frac{d}{dt}$ ② \times

③ \times ④ $\frac{d}{dt}$ ⑤ \times

Subtractor

Integrator

$\phi(t)$

③ $-B G J_1(\phi_c) \cos[\phi(t)] \frac{d\phi}{dt}$

④ $+B H J_2(\phi_c) \sin[\phi(t)] \frac{d\phi}{dt}$

⑤ $\Rightarrow B^2 G H J_1(\phi_c) J_2(\phi_c) \frac{d\phi}{dt} \Rightarrow$ Integrate to extract $\phi(t)$

③ \times ② $\rightarrow B^2 G H J_1(\phi_c) \cos^2[\phi(t)] \frac{d\phi}{dt}$

④ \times ① $\rightarrow -B^2 G H J_1(\phi_c) J_2(\phi_c) \sin^2[\phi(t)] \frac{d\phi}{dt}$

Normal. $I_f = A + B \cos[\phi(t)]$

PGC $I_f = A + B \cos[\phi_c \cos \omega_c t + \phi(t)]$

$A + B$

$\phi(t) = 0$

$\phi(t) = A \cos \omega_c t$

f

f_c

$2f_c$

$3f_c$

$4f_c$

$$= A + B \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \cos \phi(t)$$

$$- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \sin \phi(t)$$

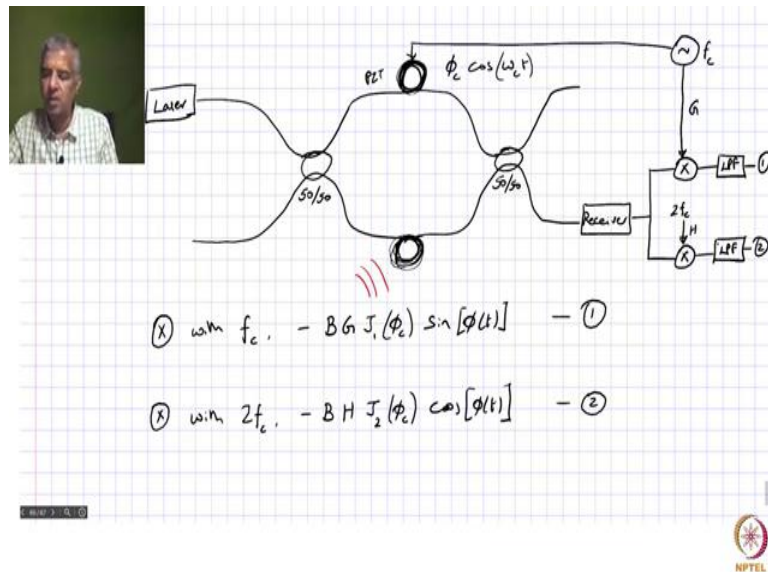
$$\phi(t) = \phi_c \cos \omega_c t + \phi_n(t)$$

$$\cos \phi(t) = \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \cos \phi_n$$

$$- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \sin \phi_n$$

$$\sin \phi(t) = \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \cos \phi_n$$

$$+ \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \sin \phi_n$$



So, let us actually see what B represents, to see that you need to go back and look at your, the original where we all, where it all started, we said we are looking at this beat function, the strength of that beat is what we are calling as B, so that is what we used in these expressions, this B is representing this value. And when you look at this closely, it consists of the degree of coherence, which you, of course, try to increase as much as possible.

You try to choose a laser which is relatively narrow line width, so you can get a fairly high degree of coherence close to one, if possible and then you have the relative values of I1 and I2. So, how do you choose I1 and I2? Well, to start with you in an interferometer to get maximum contrast, you try to get even to be matching I2, so you can adjust the intensities in the two arms by adjusting the losses in the two arms so we can balance I1 and I2.

So that is one thing we do and now overall it is controlled by the intensity from the laser, so you can actually, this if you call it I naught, you can adjust I naught, so that you can adjust I1 and I2 as well. But what should those values be? Well, we understand that in a receiver you have what is called shot noise. So, what is shot noise due to? It is because of the random arrival time of photons at the receiver.

And that actually scales with, the shot noise variance scales with the input power or the input intensity of light that is incident on the detector. So, you need to reduce the intensity I1 and I2 such that you can reduce the shot noise component. But if you reduce this to too small a value, let us say you go down to nanowatts of power, in that case you need to actually boost up your signal by having a large gain for your receiver.

And in that case you have a thermal noise component that comes into the picture. So, you need to essentially adjust I_1 and I_2 , so that you can balance the shot noise and thermal noise, so that you can achieve as high a signal to noise ratio as possible. And by adjusting I_1 , I_2 you are adjusting B and through that you are essentially making sure you get a relatively clean signal, so you can extract this ϕ of t in a reliable manner.

Having said that we do have to be careful about this ϕ of t as we defined a little earlier, it corresponds to $\phi \cos \omega t$, this is the signal that we want to extract, but there is still noise incorporated in that, we are reducing the noise as much as possible through this lock-in detection and low-pass filtering that is something that I did not explicitly mention previously, when you do this beating you also need to incorporate this low-pass filter before you extract this one and two.

So, by low-pass filtering to essentially include the essential components, like if you are trying to pick up something at 100 hertz, then you do a low pass filter around that, so that you not actually accumulating a lot of noise from higher frequencies, so you need to use this low pass filtering judiciously. On the other hand if that your acoustic signal that is incident on this.

If the acoustic signal that is incident on this hydrophone, if you know it is going to be at 100 hertz let us say, then instead of a low pass filter you can actually use a band pass filter and then just pick up that 100 hertz component or even better, you can beat it instead of f_c , you can beat it with f_c plus 100 hertz or something like that.

And then you can extract the specific component corresponding to and $2f_c$ will correspond to $2f_c$ plus this 100 hertz or 200 hertz those components and then you can extract the specific frequencies of interest for us. So, those are some of the modifications you can do to reduce the noise further, but that is essentially what this phase generated carrier method is all about.

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Passive Homodyne Demodulation Scheme for Fiber Optic Sensors Using Phase Generated Carrier

ANTHONY DANDRIDGE, ALAN B. TVETEN, AND THOMAS G. GIALLORENZI, SENIOR MEMBER, IEEE

Abstract—A method of homodyne demodulation using a phase generated carrier is described and experimentally demonstrated. The method has a large dynamic range, good linearity, and is capable of detecting phase shifts in the microradian range. The detection scheme obviates the phase tracker resetting problem encountered in active homodyne detection schemes. Two methods of producing the carrier are presented, one employing a piezoelectric stretcher, the other using current induced frequency modulation of the diode laser source. These two methods are compared. The origins of the noise limiting the system are briefly discussed.

1. INTRODUCTION

RECENTLY, there has been considerable interest in using optical fibers as the sensing element in devices such as hydrophones, spectrophones, magnetometers, accelerometers, and ac current sensors [1]. One of the configurations which

differential drifts between the arms of the interferometer. The drift causes changes in the amplitude of the detected signal (signal fading), as well as distortion of the signal (frequency up-conversion).

Several detection schemes are currently available: passive homodyne, active homodyne (phase tracking), true heterodyne, and synthetic heterodyne. Each of these techniques has both advantages and disadvantages. The current state of these detection schemes is reviewed in [1]. At this time, only the active homodyne system has reached a level of high performance (10-10⁻⁶ rad sensitivity with good linearity and low harmonic distortion), packageability (<24 cm³), and low power consumption. In order to achieve this high level of performance, the technique requires relatively large piezoelectric phase modulators and fast reset circuitry. Large modula-



dynamic range, good accuracy, and is capable of detecting phase shifts in the microradian range. The detection scheme obviates the phase tracker resetting problem encountered in active homodyne detection schemes. Two methods of producing the carrier are presented, one employing a piezoelectric stretcher, the other using current induced frequency modulation of the diode laser source. These two methods are compared. The origins of the noise limiting the system are briefly discussed.

1. INTRODUCTION

RECENTLY, there has been considerable interest in using optical fibers as the sensing element in devices such as hydrophones, spectrophones, magnetometers, accelerometers, and ac current sensors [1]. One of the configurations which has shown high sensitivity is that of the Mach-Zehnder all-fiber interferometer. In this configuration, there are many methods of detecting relative optical phase shift between the signal and reference fibers. The design of the detection scheme is made nontrivial by the presence of low frequency random temperature and pressure fluctuations which the arms of the interferometer experience. These fluctuations produce

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Several detection schemes are currently available: passive homodyne, active homodyne (phase tracking), true heterodyne, and synthetic heterodyne. Each of these techniques has both advantages and disadvantages. The current state of these detection schemes is reviewed in [1]. At this time, only the active homodyne system has reached a level of high performance (10-10⁻⁶ rad sensitivity with good linearity and low harmonic distortion), packageability (<24 cm³), and low power consumption. In order to achieve this high level of performance, the technique requires relatively large piezoelectric phase modulators and fast reset circuitry. Large modulators are undesirable in multielement sensors since they increase the active sensor's size and decrease its reliability. Additionally, the need for the sensor circuitry to reset itself every time the environmental noise drives it past its dynamic range adds additional noise. In this paper, a passive homodyne technique which obviates the two problems discussed above is presented. Unlike other passive techniques previously reported [2], this technique has been shown to offer a very high level of performance with a linear dynamic range of ~10⁷. This large linear dynamic range allows both small and large amplitude signals commonly encountered in applications to be observed with excellent fidelity. Two methods of utilizing this approach

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The authors are with the Naval Research Laboratory, Washington, DC 20375.

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The present approach outlined below offers improvements in terms of detection accuracy and simplicity of construction. The variation in the light intensity detected at the output of the interferometer may be written as

$$I = A + B \cos \theta(t) \quad (1)$$

where $\theta(t)$ is the phase difference between the arms of the interferometer. The constants A and B are proportional to the input optical power, but B also depends on the mixing efficiency of the interferometer. If a sinusoidal modulation with a frequency ω_0 and amplitude C is imposed on the interferometer, then (1) becomes

$$I = A + B \cos(C \cos \omega_0 t + \phi(t)) \quad (2)$$

where $\phi(t)$ includes not only the signal of interest, but environmental effects as well. Expanding (2) in terms of Bessel functions [5] produces

$$I = A + B \left[J_0(C) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(C) \cos 2k\omega_0 t \right] \cos \phi(t) - \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(C) \cos (2k+1)\omega_0 t \right] \sin \phi(t) \quad (3)$$

From this expansion it is clear that when $\phi(t) = 0$, only even

multiple of ω_0 and low-pass filtering to remove the terms above the highest frequency of interest.

For the carrier frequencies considered in the experiment, namely 0, ω_0 , and $2\omega_0$, the output signals after mixing and filtering are

$$\begin{aligned} A + BJ_0(C) \cos \phi(t) \\ BGJ_1(C) \sin \phi(t) \\ -BHJ_2(C) \cos \phi(t), \end{aligned} \quad (5)$$

respectively, and where G and H are the amplitude of the mixing signals for ω_0 and $2\omega_0$.

In order to obtain a signal that does not fade as a function of undesired fluctuations, two signals, one containing the sine $\phi(t)$ and the other cosine $\phi(t)$ are utilized. The time derivative of the sine and cosine terms are cross multiplied with the cosine and sine terms, respectively, to yield the desired sine and cosine squared terms [1]. The process will be illustrated by considering the output signals for ω_0 and $2\omega_0$. The time derivative of these are obtained from (5) and are given by

$$\begin{aligned} BGJ_1(C) \dot{\phi}(t) \cos \phi(t) \\ BHJ_2(C) \dot{\phi}(t) \sin \phi(t). \end{aligned} \quad (6)$$

Multiplying this by the signal for the other frequency produces

$$B^2GHJ_1(C)J_2(C) \dot{\phi}(t) \cos^2 \phi(t)$$



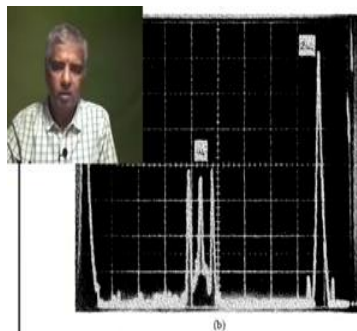
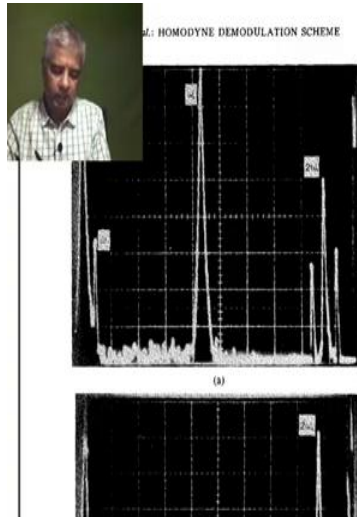


Fig. 1. Spectrum analyzer trace of the output of the interferometer driven with a large amplitude signal at ω_c and small signal at ω_s . (a) In quadrature $\phi(t) = \pi/2$ rad. (b) Out of quadrature $\phi(t) = 0$. Vertical scale is linear, horizontal scale is 2 kHz/div.

noise ratio, the value of the amplitudes G and H should be as large as possible without overloading the electronics. The factor B depends on the optical power and the mixing efficiency in the interferometer and is the most difficult to control. Any variation in the output of the beam splitters or a

ing higher order filters (a simple second order in this experiment). The rolloff at low frequencies to the presence of a bleeder resistor across the value could be increased if a flat response is required. The system responded linearly to of the input signal between the noise floor of on used (typically 10^{-3} - 10^{-6} rad) and ~ 1 rad. al level (~ 1 rad), distortions were observed due to the differentiators overloading from the presence of the residual carrier signal, thus the dynamic range of the system at high signal levels may be increased by 1) lowering the gain prior to the differentiations or preferably, 2) having a sharper cutoff of the carrier modulation (i.e., a higher order filter). Below this region of obvious distortion, the first harmonic of the signal was typically buried in the noise floor and hence was unobservable, but the ratio of the fundamental to the first harmonic was typically greater than 70 dB (i.e., 3×10^3).

2) As discussed earlier, the operation of the current induced modulated carrier requires the interferometer to use a nonzero path length difference such that the frequency modulator is converted to the required phase modulation. Consequently, this nonzero path difference allows the phase noise to contribute to the interferometer noise. Using the diode laser modulation scheme, the frequency dependence of the mini-

The experimental configuration employed to demonstrate this homodyne scheme was that of a bulk optic Michelson interferometer. This system (described in detail elsewhere) [6] allowed accurate control of the optical path difference and could readily be isolated from environmental noise sources in our setup. Typically, the interferometer noise of this system was 2×10^{-7} rad (at 1 kHz). Both mirrors of the Michelson interferometer were mounted on piezoelectric cylinders so that both "real" and drift signals could be produced. The bulk Michelson interferometer was chosen rather than the fiber system to allow greater flexibility in control of the optical path difference. The fact that a bulk interferometer was used is not essential to determining the usefulness of this technique since all-fiber interferometers (powered by diode lasers) of sub microradian performance have been built and operated routinely [1]. Consequently, the results of this work are directly applicable to these systems. In this experiment a Hitachi HLP 1400 laser was used as the source.

The implementation of the demodulation scheme requires that a high frequency carrier signal be produced in the interferometer. The following two methods were employed: 1) piezoelectric cylinder, and 2) modulating the emission frequency of the laser diode by modulating the laser drive current. The first method has the advantage that a zero optical path difference may be used in the interferometer. The second

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The implementation of the demodulation scheme requires that a high frequency carrier signal be produced in the interferometer. The following two methods were employed: 1) piezoelectric cylinder, and 2) modulating the emission frequency of the laser diode by modulating the laser drive current. The first method has the advantage that a zero optical path difference may be used in the interferometer. The second method has the advantage of eliminating the piezoelectric cylinder as well as allowing the interferometer part of the fiber system to be removed from electrical components. This scheme has the disadvantage that to convert the current induced frequency shift to a relative phase shift, a nonzero optical path difference is required, thus the phase noise [6] can become the factor limiting the sensitivity of the sensor. This limitation will be discussed in detail in the experimental performance section.

A carrier wave phase shift of the desired amplitude (i.e., 1.4 or 2.2 rad) was easily obtained using the piezoelectric cylinder at the fundamental and first harmonic of its longitudinal resonance which occurred at 50 and 100 kHz, respectively. To produce the cosine and sine terms, the circuit shown in Fig. 2 was used. For the case illustrated in Fig. 2(a) the fundamental of the signal and the sideband of the fundamental of the carrier were used, whereas the sidebands of the fundamental of the carrier and the first harmonic of the carrier were utilized in the implementation shown in Fig. 2(b). The use of the two configurations will be discussed in greater

The results of the three experiments performed above are summarized in Fig. 9, where the solid line in the upper curves represent the average level of the phase noise. The lowest curve was obtained for the interferometer held manually in quadrature using the conventional homodyne detection scheme. As can be seen, the passive homodyne result is approximately a factor of 2 worse than the result obtained with the conventional homodyne. The noise in the conventional homodyne system was determined by the intrinsic intensity noise of the diode laser [8]. Owing to the carrier modulation, the average level of the intensity noise remains the same as with the conventional homodyne interferometer field in quadrature, however, the amplitude of the phase signal is reduced by the magnitude of the relevant Bessel function (5). Consequently, in an interferometer dominated by the laser intensity noise, a factor of ~ 2 difference between the two detection methods is expected for this modulation amplitude.

In an unbalanced interferometer powered by a laser diode, the primary noise source becomes the phase noise due to the fluctuation of the emission frequency of the laser. For a signal-to-noise ratio of 1, the minimum detectable phase shift ψ_m in the interferometer is given by

$$\psi_m = 2\pi D \left(\frac{\delta\nu_s}{c} \right)$$



Of course, I want to acknowledge that most of this material is coming from this 1982 paper that is in Journal of Quantum Electronics, in 1982, is when this scheme was first proposed and by Dandridge et al and they were looking at these picking up phase values in the presence of low frequency random, the temperature and pressure fluctuations, so that is what they want to deal with.

And they are targeting 10^{-6} radians to 10^6 radian type of range and to achieve that they are essentially proposing this phase generated carrier technique, which involves using this piezoelectric phase modulators and this is all the math that we went through previously, so all that comes from this this paper here and once we have this $\cos \phi t \sin \phi t$ terms, then you do this cross differentiate and cross multiply scheme.

First you do the beating and you get the f_c and $2f_c$ components and then you do the cross differentiate step and then the cross multiply step and then your subtraction and then finally you integrate to get the ϕ of t term. And here are some some sample waveforms that they acquired experimentally at that time, this is actually for a condition where your f_c component is dominant compared to this $2f_c$ components so this is closer to quadrature.

And this is actually a case where your $2f_c$ and 0 , the dc components are dominant compared to the f_c component so this corresponds to a case where ϕ is closer to 0 . So, of course, they talk about this beating at the f_c and $2f_c$ to get their those beat components and then they talk about how to do this differentiate and cross multiply scheme to extract ϕ of t .

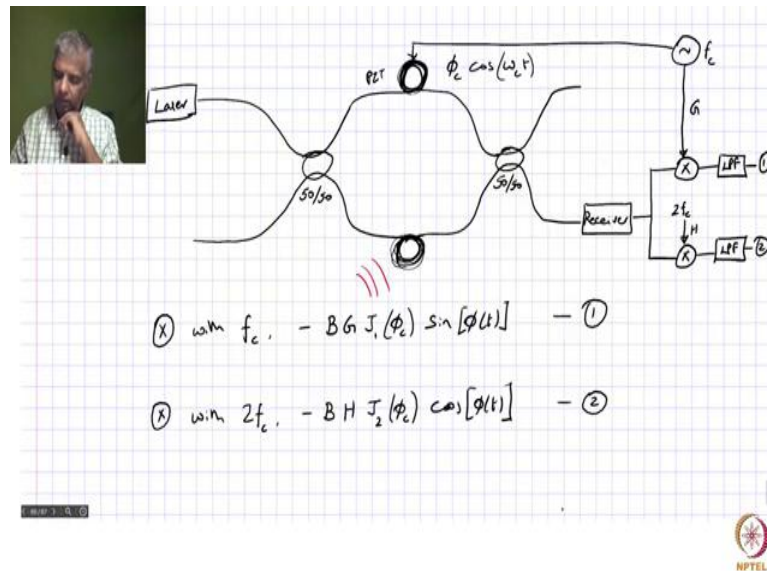
And what they mentioned there is they are able to get to about 10^{-6} radians to about 1 radian in their work, which is of course, has been this was almost 40 years ago. So, it has been improved since then. But but they also make another important point which is - what is the minimum detectable limit as far as this phase generated carrier technique is concerned, what is it really limited by.

What they observe is limited by the fluctuation of the emission frequency of the source, if you have a single frequency source, then you have a single phase, very definite phase associated with it, but if your source is actually having multiple frequencies, then the phase actually does not become deterministic.

There is some uncertainty in the phase itself and that uncertainty is going to constitute the minimum detectable limit as far as the phase detection is concerned. So not so surprisingly that detectable limit in terms of phase is proportional to $\Delta \nu$ where $\Delta \nu$ correspond to

the spread in the frequencies of your source. So, we will look into this aspect a little more detail possibly in the next lecture.

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So that is what we have a phase generated carrier technique to pick up phase changes in the presence of environmentally induced phase noise.