

Optical Fiber Sensors
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Lecture 29
Phase modulated sensors - 9

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* How to build an optical gyroscope?

- Sagnac effect

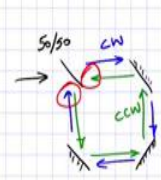


Diagram showing a square fiber loop with a Sagnac loop. Light propagates clockwise (CW) and counter-clockwise (CCW). The Sagnac loop is indicated by a red circle.

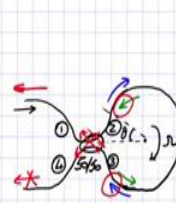




Diagram showing a circular fiber loop with rotation rate Ω and round-trip time T . The Sagnac loop is indicated by a red circle.

$\theta = \Omega T$
 $\Delta\phi = \frac{2\pi}{\lambda} n \Delta L$
 $= \omega \Delta T$

$\phi_{1234} = \phi_{CW}$
 $\phi_{1231} = \phi_{CW} + \pi/2$
 $\phi_{1321} = \pi/2 + \phi_{CCW}$
 $\phi_{1324} = \pi/2 + \phi_{CCW} + \pi/2$

$\phi_{CW} = \phi_{CCW}$
 $\Delta\phi = \pi$ destructive interference
 Zero $\Delta\phi$ constructive interference

During our last lecture we were talking about fiber optic gyroscope and we were looking at the principle of behind the fiber optic gyroscope, which is, what we identified as a Sagnac effect, so essentially we are looking at phase changes between clockwise propagating waves and counterclockwise propagating waves and there, these are the counter clock wave, counterclockwise propagating waves.

And then we said there will be a phase difference, whenever there is a rotation which is what we are actually interested in measuring through this Sagnac interferometer.

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Design of an Interferometric FOG

* Find a relation between rotation rate (deg/hr) and phase change due to rotation / physical parameters (velocity of light, radius of coil etc.)

Under rotation, $\Delta \tau = T_{CW} - T_{CCW} = \frac{(2\pi + \Omega t)R}{v_g} - \frac{(2\pi - \Omega t)R}{v_g}$

$= \frac{2 \cdot \Omega t R}{v_g}$ where $T = \frac{2\pi R}{v_g}$ for N turns.

$\Delta \phi = \Delta \omega_D T = N \frac{8\pi^2 R^2}{v_g \lambda} \Omega$

$\Delta \phi = \frac{4\pi R^2}{v_g^2} \Omega \Rightarrow \Delta \phi = N \frac{8\pi^2 R^2}{v_g \lambda} \Omega$

So, let us go ahead and try to design what is called an interferometric fiber optic gyroscope, FOG in short, it is called interferometric because obviously it depends on the interference between the clockwise and the counterclockwise waves. So, what exactly would we like to achieve. We like to find a relation between, let us say the rotation rate that is what we want to measure, which is typically expressed in terms of degree per hour.

So, we want to find a relation between the rotation rate and the phase change, the phase change due to rotation and we also want to look at the physical parameters. How the physical parameters affect the relationship between the rotation rate and the phase change that we obtained due to rotation. So, this is basically physical parameters such this is corresponding to the velocity of light in the medium of light.

The radius of this coil here and so on, radius of coil, et cetera. So, let us go ahead and try to get a relationship between all of this. So, to do that let us actually redraw this fiber optic gyroscope, which is the sagnac interferometer configuration. So, let us say we are coming off into a loop and this is basically your 50-50 splitter. So, this is your incoming radiation and that sets off, what we used for the clockwise is blue, this is your clockwise propagating wave.

And then it also sets off counterclockwise propagating wave which comes around and interferes with the clockwise propagating wave, and what we looked at was actually this, let us say the radius of the coil is R and because of this rotation, which we are calling as omega, we are having change in the reference point as far as the interference is concerned and that we identified corresponds to omega times t.

What is T ? T is the round trip time for any one of these two waves, and of course, we are gonna try to say that the round trip time actually changes for, between the clockwise and the counterclockwise waves. So, let us actually say this is what we were at before, so we had an expression for $\Delta\phi$ as $\omega \Delta t$, where Δt is the change in the round trip time for the clockwise and counterclockwise wave.

So, let us try to quantify that, so Δt is going to be T_{cw} minus T_{ccw} and when you look at the round trip time for the cw wave, without any rotation that is going to correspond to the circumference of this coil here divided by the velocity with which the wave is actually going to propagate. So, that is $2\pi r$ over V_g , but now under the influence of rotation, so under rotation what we find is this is 2π , you have 2π radians and for the clockwise propagating wave that is actually that angle is increased by ωT multiplied by R .

And what is the velocity of light in this medium? Let us say this we are considering a fiber, so we are going to be looking at the group velocity of light and the fiber which is given by V_g and V_g of course, can be written in terms of velocity of light in free space with c over n_{eff} , is the effective refractive index that the wave sees within this fiber.

So, that is for the cw and similarly for the ccw you have 2π and for the ccw case it is actually minus ωT multiplied by R divided by V_g and so this is going to be 2 times R , which corresponds to $2\pi r$ over V_g , well maybe we can just write it out in a minute. So, this is 2 times r times R divided by V_g multiplied by this ω , which is the rotation rate.

And of course, we know that, so we can write T is $2\pi r$ divided by V_g , so we can actually bring that into this expression and so we can write this as ΔT equals $4\pi r^2$ divided by V_g^2 multiplied by ω . So that is actually the ΔT and this implies that $\Delta\phi$, if you write it that is going to be $2\pi f$ multiplied by this quantity and we know that V_g over f is λ , so you have a 2π multiplied by 4π .

So, you have $4\pi^2$, R^2 divided by V_g multiplied by λ , λ is nothing but V_g divided by f , this if you just expand that is $2\pi f$, so V_g over f is actually what is giving you this λ multiplied by this ω . So, this now is the phase change that you incur because of rotation and interesting thing to note here is this is a phase change that we incur for a single loop, a single fiber loop.

But you could possibly increase this $\Delta\phi$ by going for multiple loops, basically you can take the fiber and you can actually make a coil, let us say with n turns and if you do that

then this entire thing, this $\Delta\phi$ that we have is going to be scaled by that factor, so you can just, sorry, you can just say that corresponds to n times for or N turns, we get $\Delta\phi$ equals to N times $R^2 \frac{8\pi}{Vg}$.

So, that is actually a major advantage of using a fiber because you can just coil the fiber and you do not have to worry about alignment and anything like that, so that is one of the key reasons why these gyroscopes people prefer a fiber optic gyroscope because you can just coil the fiber relatively easily and you can get a scaling in terms of the sensitivity with a factor of N . There are some downsides of that and we will come back to that in a few minutes.

Now, this is what we actually achieved by looking at just a change in the transit time that is the round trip time for the clockwise and counterclockwise waves, but we talked about potentially looking at this from a perspective of a doppler shift, so essentially you can think of this to be similar to a case where, let us say you have a sound wave propagating and it is getting reflected, so the wave is propagating, it is getting reflected off the mirror.

Let us say just just you are looking at the echo from from an object and when the object is stationary, whatever wave is going in, whatever frequency you have that is the same frequency you are going to actually get back, but suppose your object is moving away, if the object is moving away, this sinusoidal wave, this acoustic wave is now going to get stretched and so, whenever, whatever is actually reflected back from this object is going to have a downshift in terms of the frequency.

And that downshift is what we call as a doppler shift. So, it is downshift if the object is moving away, but if the object is moving towards the wave, then that ends up actually compressing the wave somewhat and that actually means that the reflected wave is going to have an upshift in terms of the frequency. So, you have a similar situation here in which we see that the count the clockwise wave is seeing a reference that is moving away.

So, it is going to see a downshift, so the clockwise will have basically, you have ω minus, say $\Delta\omega$ D's shift divided by 2, whereas for the counterclockwise wave the reference is coming towards the wave, so in that case you have ω plus $\Delta\omega$ D divided by 2. So, it is going to have a relative shift and that the total shift is actually what we are calling as $\Delta\omega$ D.

But you just attribute to the clockwise and the counterclockwise that is why we have $\Delta\omega$ D divided by 2, so essentially $\Delta\phi$ if you look at that doppler picture, that would

corresponds to, that would correspond to $\Delta\omega d$ multiplied by the round trip time, so that will give you the $\Delta\phi$ and of course, in that case also you can actually find out this $\Delta\phi$ difference and there is your $\Delta\omega D$ can be written in terms of $\Delta\beta$.

And once you work that out you find that you have a similar sort of expression that you get. So, you can either go for this time picture or the frequency picture, both are equivalent, you should get the same result in both cases. So, now the question is how do we increase $\Delta\phi$? Obviously, this $\Delta\phi$ is going to be converted to a change in intensity because of your, because this is an interferometer.

So changes in phase is is going to correspond to change in intensity and you want as large a change in intensity as possible, so the question is can we actually increase that $\Delta\phi$. How do you increase that? Well, so $\Delta\phi$ is proportional to rotation rate, so higher the rotation rate, then you can also increase $\Delta\phi$, so that is obvious. But you can also increase this R , that is the radius of the loop, so you can essentially go for a larger radius and you could increase $\Delta\phi$.

But of course, you may be limited in terms of the size of the gyroscope, you cannot just make a very large gyroscope, you need to keep it relatively small. So, that might limit it, but you do have a relatively what looks like a relatively free parameter in N . So, let us actually rewrite this, now you do find that there is a $2\pi R$ component, which is corresponding to the circumference of this coil over here and N times $2\pi R$. What does that represent? So, that is basically N times of the circumference, which corresponds to the length of the loop, length of the fiber in the loop.

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$$\Delta\phi = \frac{4\pi R L}{v_g \lambda} \Omega$$

Suppose $\Omega = 10^{-3}$ deg/hr

Let $\lambda = 1.5 \mu\text{m}$, $v_g = 2 \times 10^8 \text{ m/s}$, $R = 10 \text{ cm}$, $L = 200 \text{ m}$

$$\Delta\phi = \frac{4 \times 3.14 \times 0.1 \times 200}{2 \times 10^8 \times 1.5 \times 10^{-6}} \times 10^{-3} \times \frac{\pi}{180 \times 3600}$$

$$\approx 4 \times 10^{-9} \text{ rad}$$

Interferometer output, $I_f = I_0 [1 + \cos(\Delta\phi)]$
 $= I_0 [1 + \cos(\pi/2 + \Delta\phi)]$

NPTEL



Design of an Interferometric FOG

Find a relation between rotation rate (deg/hr) and phase change due to rotation / physical parameters (velocity of light, radius of coil etc.)

Under rotation, $\Delta T = T_{\text{CW}} - T_{\text{CCW}} = \frac{(2\pi + \Omega R)R}{v_g} - \frac{(2\pi - \Omega R)R}{v_g}$

$$= \frac{2 \cdot R T}{v_g} \Omega \quad \text{where } T = \frac{2\pi R}{v_g}$$

for N turns, $\Delta\phi = N \frac{8\pi^2 R^2}{v_g \lambda} \Omega$

$\Delta\phi = \Delta\omega_D T = N \frac{8\pi^2 R^2}{v_g \lambda} \Omega$

NPTEL



So, I can actually now write this as delta phi, if I take that circumference as $2\pi R$, if I take that out I have a $4\pi R$ multiplied by the length of the loop divided by v_g times lambda multiplied by omega, so that is a easier expression to deal with. So, now we can actually look at what are the relative quantities here? So, what exactly, so let us just get a feel for what are the relative values that we are dealing with?

So, to do that let us actually start putting some numbers. Suppose, we want to build a navigational grade gyroscope, suppose you want to build something with which is capable of picking up omega of 10^{-3} degree per hour, if you want to pick that small rotation rate, let us just see what does that entail in terms of delta phi? To determine that we need to, of course, substitute values for some of these other things.

So, let us choose λ to be 1.5 microns, so 1.5 happens to be the optical communication wavelength, so the components are available relatively easily at 1.5 micron and then the group velocity is, can say it is about the effective index about 1.46, but let us just approximate that to be about 1.5, so the group velocity you can approximate it as 2×10^8 meters per second.

Because it is just trying to get some order of magnitude numbers here and then let us choose R , so r should not be too large because it might make the entire gyroscope very bulky. Let us just choose a R of like 10 centimeter, this is already on the higher side, but let us just take something like that something that basically it is the radius is corresponding to 4 inches, which is the size of my palm here.

So you can imagine what the size of that loop is going to be and let us say we choose l , which is at this point a free parameter, but let me just say l is about 200 meters, so if I substitute all of this I will find $\Delta\phi$ is equal to $4 \times 3.14 \times R \times l$ is 200, so R is given in 10 centimeters that but we are converting everything into meters, so that is 0.1 meters divided by V_g , which is 2×10^8 lambda, which is 1.5 micron.

So, 1.5×10^{-6} and then multiplied by 10^{-3} , but this is given in degree per hour, so typically we want to find $\Delta\phi$ in terms of radians, so what do you do, so you have to multiply this by π by 180 and you want to convert this to seconds, so that is basically 3600 seconds. So, if you crunch all these numbers, you will find that the final value is in the order of about 4×10^{-9} radians.

So, that is actually quite a small value 4×10^{-9} radians and especially, if you think about how we are going to use it, we are going to use it inside an interferometer, so the interferometer output is going to be given by let us say I_0 , which is basically the input intensity multiplied by $1 + \cos(\Delta\phi)$, let us put a square bracket over here.

And so when you are talking about 4×10^{-9} radians that means it is actually a very small intensity and on top of that if you really look at how we are operating this, if I look at I as a function of $\Delta\phi$, it is going to be something like this, so you start with I_0 when $\Delta\phi$ equal to 0 and then it goes to a minimum, it goes to a 0 and then maximum and then a minimum and so on.

Where it goes to a minimum that will be at a value of $\Delta\phi$ equal to π , and then it goes to a maximum at 2π and and so on, but if we look at this operating point over here, we are

starting with a case where your clockwise wave and your counter clockwise waves are essentially going through the same path length, so their phase will cancel each other and the only thing we are looking at is if there is any rotation then the phase might actually show up a little bit around here.

So, your operating point is over here and that means at this point, because the cos function is relatively flat over here, that is actually poor sensitivity. So, where do you get the most sensitivity? Well, that would correspond to somewhere over here in the linear part of this curve and if you pay close attention, you will find that this corresponds to a value of $\pi/2$. So, you need to operate in what is called this quadrature point for best sensitivity.

So, like we did in the previous cases, we can operate in the quadrature point, provided we have, let us say a phase shift that is incorporated inside this loop. So, let us actually first look at what if we operate in the quadrature point, this is basically says $I_{naught} (1 + \cos(\pi/2 + \Delta\phi))$, which corresponds to $\sin(\Delta\phi)$ and for small values of $\sin(\Delta\phi)$ that is equivalent to $\Delta\phi$ by itself.

So, what we are looking at is this component is actually going to, if you call this ΔI that is the change in intensity because of this change in phase, so that actually, for this particular case it is 4×10^{-9} multiplied by I_{naught} , so you are looking at very small changes in intensity, 4×10^{-9} changes in intensity or in other words you can also write it as in terms of power.

So, we are looking at 4×10^{-9} multiplied by the input power. So that almost suggests that as you increase your input power you are going to end up having better sensitivity. So, you have a larger signal that you can pick up. But what is the downside of that? Well, as you keep increasing the power the amount of light that is falling on the photodiode is going to keep increasing and that essentially gives rise to shot noise like what we have seen before.

So, it is not like you can just keep increasing the optical power that is, that you are feeding into the loop, so there is a limit to that, we will come back and take a look at it, but the question is how do we actually bias it for best sensitivity. And for that we need to go back here and then see how to introduce this bias. Now, I can introduce this bias by basically say incorporating a piezoelectric element, PZT element, which we saw previously.

We introduced that in one of the arms of a Mach-Zehnder interferometer previously to realize that hydrophone, to realize that phase generated carrier. So, you can do a similar thing over here except for the fact that if you have a phase change incorporated which is a static phase change, then both the clockwise as well as the counter-clockwise waves are going to experience that and then they will essentially cancel each other.

So, you essentially set this PZT for this quadrature phase point. So, you basically, you introduce a π by 2 phase shift there, but π by 2 will be the phase shift that both the clockwise as well as the counter clockwise waves are going to see, so they will essentially cancel each other. So you will go back to this same point over here, so you are not moving over here. So, to really move over here what you are going to have to do is you apply, you give a phase shift of π by 2, initially for the clockwise wave.

So, initially for the clockwise wave you have let us say plus π by 2 phase shift and by the time the other wave comes around, by the time counterclockwise wave comes around you need to actually shift the phase, so if you keep the same phase, then it is actually going to just cancel between the two, but if you shift the phase, so that the counterclockwise actually, so this is for the clockwise beam.

But before the counterclockwise wave comes around, which corresponds to roughly this round trip time, this T you change the phase shift. So, essentially when you change that phase shift you are going to have a condition where you will keep the interferometer and quadrature position at that point itself.

And of course, you can play this game with doing this whatever switching between the phases, you can do that at a particular frequency and you can lock to that frequency and all of that the same things that we saw as far as the interferometric FOG is concerned. So, previously we saw that for the hydrophone, but you can apply the same principles for the interferometric FOG as well. So, let us look into some of those details in the next lecture.