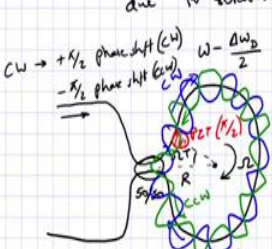


**Optical Fiber Sensors**  
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**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**  
**Lecture 30**  
**Phase modulated sensors - 10**

(Refer Slide Time: 0:15)

Design of an Interferometric FOG

Find a relation between rotation rate (deg/hr) and phase change due to rotation / physical parameters (velocity of light, radius of coil etc.)





Under rotation,  $\Delta T = T_{CW} - T_{CCW} = \frac{(2\pi + \Omega R)R}{v_g} - \frac{(2\pi - \Omega R)R}{v_g}$

$= \frac{2 \cdot \Omega R^2}{v_g} \Omega$  where  $T = \frac{2\pi R}{v_g}$  for N turns.

$\Delta T = \frac{4\pi R^2}{v_g^2} \Omega \Rightarrow \Delta\phi = N \frac{8\pi^2 R^2}{v_g \lambda} \Omega$

$\Delta\phi = \Delta\omega \cdot T = N \frac{8\pi^2 R^2}{v_g \lambda} \Omega$

$$\Delta\phi = \frac{4\pi R L}{v_g \lambda} \Omega$$

Suppose  $\Omega = 10^{-3}$  deg/hr

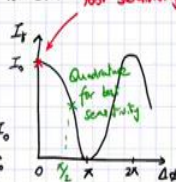


Let  $\lambda = 1.5 \mu\text{m}$ ,  $v_g = 2 \times 10^8 \text{ m/s}$ ,  $R = 10 \text{ cm}$ ,  $L = 200 \text{ m}$

$$\Delta\phi = \frac{4 \times 3.14 \times 0.1 \times 200}{2 \times 10^8 \times 1.5 \times 10^{-6}} \times 10^{-3} \times \frac{\pi}{180 \times 3600}$$

Poor sensitivity

$$\approx 4 \times 10^{-9} \text{ rad}$$

Interferometer output,  $I_T = I_0 [1 + \cos(\Delta\phi)] \rightarrow 4 \times 10^{-9} I_0$

$$= I_0 [1 + \cos(\pi/2 + \Delta\phi)] \rightarrow 4 \times 10^{-9} I_0$$




Hello, the last lecture we started talking about design of an interferometric fiber optic gyroscope and we got to the point where we defined the change in phase as a function of rotation rate and we looked at some typical numbers and we realized that if we want to achieve rotation rate sensitivity in the order of  $10^{-3}$  degree per hour, then you need to be able to pick up fairly small changes in phase.

So,  $\Delta\phi$  in the order of  $4 \times 10^9$  radians and in this context we also realize that it is actually if you put this as part of an interferometer, which is a Sagnac interferometer, you typically are operating at this point if you do not apply any phase bias, but if you apply phase bias we said you could possibly move to about  $\pi/2$  phase shift that is actually the quadrature point.


If you operate around that quadrature point then you can actually improve the sensitivity, but then it is actually a little tricky to operate at the quadrature point because of the fact that both the clockwise as well as the counterclockwise beams are going through the same path, which is very different from the previous case that we looked at, which is for a Mach-Zehnder interferometer we had a reference arm and we had a measurement arm.

So, we could do certain, we could apply a certain phase bias in the reference arm which would not actually affect the beam that is going through the measurement arm, but in this case, since it is actually a common path interferometer, both the clockwise and counterclockwise waves will see the same phase shift in which case they cancel each other and you are still back at  $\Delta\phi = 0$ .

So, when you apply this phase bias, now you have to be a little more intelligent about it and you could possibly exploit the fact that the phase bias could be applied on for the count, for the clockwise beam at a certain level and then by the time the counterclockwise wave comes to the same point, you would have actually switched the phase. So, in that case you could still maintain a quadrature of, the quadrature point operation. So, that is what we are going to look at in little more detail today.

(Refer Slide Time: 3:22)

How to maintain quadrature point operation in I-FOG?



$$\Delta\phi = \phi_s + \phi'_{cw} - \phi'_{ccw}$$

$$= \phi_s + \phi\left(t - \frac{T}{2}\right) - \phi\left(t + \frac{T}{2}\right) \quad \text{where } T = \frac{L}{v_g}$$

Suppose  $\phi(t) = \phi_m \cos(2\pi f_m t)$



$$\Delta\phi = \phi_s + 2\phi_m \sin\left(\frac{\omega_m T}{2}\right) \sin(\omega_m t)$$

$$= \phi_s + \phi_m \sin(\omega_m t) \quad \text{when } \phi_m = 2\phi_m \sin\left(\frac{\omega_m T}{2}\right)$$

Max occurs when  $\omega_m T = \pi$

$$f_m = \frac{\pi}{2\pi T} = \frac{1}{2T}$$

For  $L = 200\text{m}$

Suppose  $\phi(t) = \phi_m \cos(2\pi f_m t)$


$$\Delta\phi = \phi_s + 2\phi_m \sin\left(\frac{\omega_m T}{2}\right) \sin(\omega_m t)$$

$$= \phi_s + \phi_m \sin(\omega_m t) \quad \text{when } \phi_m = 2\phi_m \sin\left(\frac{\omega_m T}{2}\right)$$

Max occurs when  $\omega_m T = \pi$

$$f_m = \frac{\pi}{2\pi T} = \frac{1}{2T}$$

For  $L = 200\text{m}$ ,  $v_g = 2 \times 10^8 \text{ m/s} \Rightarrow f_m = \frac{v_g}{2L} =$



So the question is how to maintain quadrature point operation in I-FOG? So, what do you have to do to maintain the quadrature point operation? And essentially we can draw the I-FOG you have this loop which might actually consist of multiple turns, so this is your 50-50 splitter and what we are talking about is applying a PZT element, so you wind certain part of the loop over a PZT cylinder, a piezoelectric transducer cylinder and if you apply a certain voltage to that then you could.

So, you if you apply some voltage to that then you could possibly maintain this quadrature point, but the question is what is this waveform that you would apply over here, what is the frequency, what is the amplitude of that waveform that you would apply to this PZT so that

you can maintain quadrature point operation. So, that is what we want to look into a little more detail.

And key thing is that we are trying to make sure that there is a difference in the phase between the clockwise and counterclockwise waves, so if you are looking at  $\Delta\phi$ , which of course, has this signal phase which is corresponding to the rotation rate that you want to measure, signal phase corresponding to this rotation rate, plus now you have  $\phi_{cw}$  minus  $\phi_{ccw}$  due to your PZT, so maybe you can just say  $\phi'$  or something like that.

So, what should this be? Now, of course, what we say is that if these are like static values they cancel each other because both of them are going through the same path, but now you have to clearly make it varying with respect to time. So, how do you get it to vary? So that is the question. So, what we may want to do is actually change the  $\phi$ , so you basically say this corresponds to  $\phi(t) = \phi_0 + \frac{\omega T}{2} t$ , where  $T$  is your round trip time plus  $\phi$  of  $t$  plus  $T$  by 2.

So, essentially, so where  $T$  correspond to the length of the loop, length of the fiber in the loop divided by the group velocity of light in the fiber, so what we are essentially saying is that within a round trip you are actually changing the phase that is applied and so that corresponds to phase difference between the two waves and you could potentially operate in the quadrature point in that sense.

So, if we say that suppose you take  $\phi(t) = \phi_0 \cos(2\pi f t)$ , if  $\phi(t)$  is that function, then what you have here is something like  $a \cos(\omega t - \frac{\omega T}{2}) + b \cos(\omega t + \frac{\omega T}{2})$ , if you do that, that is essentially going to look like you have  $a \cos(\omega t - \frac{\omega T}{2}) + b \cos(\omega t + \frac{\omega T}{2})$ , so minus  $a$  plus  $b$ .

So, you have some something of that form and that we know corresponds to  $2 \sin a \sin b$ . So, you can essentially look at now  $\Delta\phi$  will be  $\phi_0 \sin(\omega t + \frac{\omega T}{2}) + 2 \phi_0 \sin(\frac{\omega T}{2}) \cos(\omega t)$ , so where  $a$  could be  $\sin(\frac{\omega T}{2})$  and  $b$  is simply  $\sin(\omega t)$ , you get as  $2 \sin a \sin b$  term and this of course, you can write it as  $\phi_0 \sin(\omega t + \frac{\omega T}{2}) + 2 \phi_0 \sin(\frac{\omega T}{2}) \cos(\omega t)$  where  $\phi_0$  corresponds to  $2 \phi_0 \sin(\frac{\omega T}{2})$ .

So, the question is where does this become maximum? So, now you can, these are free parameters, so  $T$  is given for you,  $T$  is the round trip time, so we already saw in our

previous example that l, we took as 200 meters and Vg as 2 into 10 power 8 meter per second, so T is fixed, but now omega m is actually a free parameter, so you can choose your omega m, so that you can maximize this.

So, when you try to look at that max for the above thing occurs, when omega m T by 2 or let us just say omega m T is equal to, you have, so that is pi, so when omega m T equals to pi sin, it becomes sin of pi by 2 which is equal to 1 or you can just say fm is given by pi over 2 pi multiplied by T, so which is basically 1 over 2t, so if you choose fm such that, it is 1 over 2 times T, where t is the round trip time.

You can possibly just extend this a little bit, so what that says is fm is equal to t is given by 1 over Vg, so you are talking about Vg over 2l, so let us actually just look at some example number for l equals to 200 meters, Vg equals to 2 into 10 power 8 meters per second. If you substitute that that essentially says that this is 2 into 10 power 8 divided by 4 into 10 power 2, so that is about megahertz, so it is about 0.5 megahertz or rather 500 kilohertz.

So, what it says is that if you use fm equals to 500 kilohertz, then you could achieve this operation in at the quadrature point, you can maintain quadrature point operation as far as the I-FOG is concerned. So, that is one of the key, so you essentially drive this at fm equals to, just erase this now 1 over 2t, so that is the frequency at which you need to drive this PZT, so that you maintain that quadrature point.

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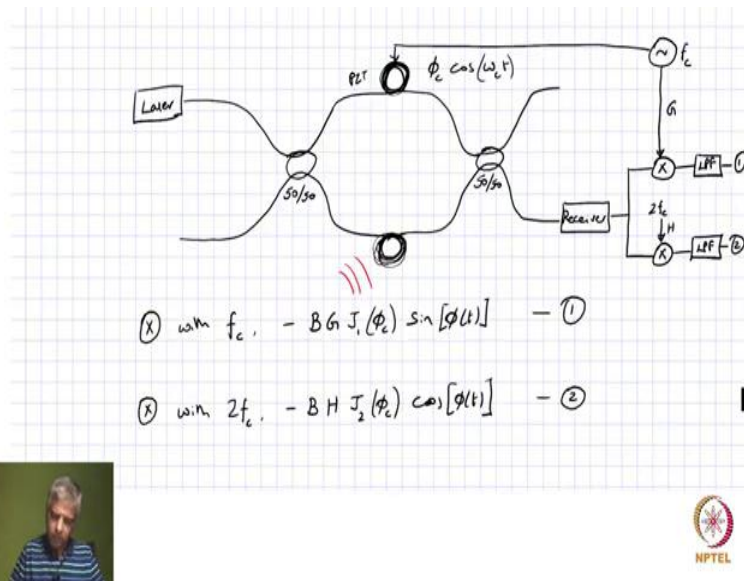
$$I_f = I_0 [1 + \cos(\Delta\phi)]$$

$$= I_0 [1 + \cos(\phi_s + \phi_m \sin \omega_m t)]$$

$$\frac{I_f}{I_0} = 1 + \cos \phi_s \cos(\phi_m \sin \omega_m t) + \sin \phi_s \sin(\phi_m \sin \omega_m t)$$

$$= 1 + \cos \phi_s \left[ J_0(\phi_m) + \sum_{k=1}^{\infty} 2 J_{2k}(\phi_m) \cos(2k \omega_m t) \right]$$

$$+ \sin \phi_s \left[ \sum_{k=0}^{\infty} 2 J_{2k+1}(\phi_m) \cos((2k+1) \omega_m t) \right]$$



So, that is one part, but then now what we see is as far as the interferometer output is concerned, if you look at the interferometer output we just wrote it as simply It equals to  $I_{naught} + 1$  multiplied by  $1 + \cos$  of  $\delta\phi$  and this we can write it out as  $I_{naught} + 1 + \cos$  of, now  $\phi_s + \phi_m \sin$  of  $\omega_m T$  and that again is of the form that it is a  $\cos$  of a plus  $b$ , so then you say it is equal to  $\cos a \cos b + \sin a \sin b$ .

So, I can just write this simply as let us say I take the  $I_{naught}$  over the other side, so I just normalize the intensity at the output with  $I_{naught}$ , then you get  $1 + \cos$  of  $\phi_s$ ,  $\cos$  of  $\phi_m \sin \omega_m T$  that is one term plus  $\sin$  of  $\phi_s \sin$  of  $\phi_m \sin \omega_m T$ . Does that look familiar? Well, that is pretty much what we handled when we were talking about phase generated carriers.

So, when you have a  $\cos$  of  $\sin$  you essentially have all these even terms, even Bessel function terms, so you have component 0,  $2\omega_m$ ,  $4\omega_m$ ,  $6\omega_m$  and all that, that is what this term is giving you and this term is going to now give you all the odd Bessel terms, so we can write it out like we did previously, so you have  $1 + J_{naught}$ , so  $1 + \cos \phi_s$  and multiplied by  $J_{naught}$  of  $\phi_m$  plus, you have summation of  $K$  going from 1 to infinity of  $2 \times J_{2K}$  of  $\phi_m$  and  $\cos$  of  $2K \omega_m t$ .

So, that is all the even terms, plus you have  $\sin$  of  $\phi_s$  multiplied by summation of  $K$  equal to 0 to infinity, you have  $2 \times J_{2K+1}$  of  $\phi_m$  and then  $\cos$  of  $2K+1 \omega_m T$ . So, that is actually going to give you all the even terms. So, we know how this works, so like we looked at previously, you look at in terms of  $\omega_m$ , if you see the even and odd terms, so I can just use say green color for representing all the even terms.

So, you have basically something at 0, some component at  $2f_m$  and some component at  $4f_m$ , and all that and if I want to represent this I would say it start from 0 here and then you have basically some component at  $f_m$ , some component at  $3f_m$  and some component at  $\phi f_m$ , and all that, so you have both the odd and even Bessel components, which is actually carrying the phase information that you want to pick up.


So, that is carrying the information about  $\phi$ , so essentially you have converted the change in, whatever, so this rotation rate that is actually showing up in the phase difference which is corresponding to  $\phi$  and that you have now encoded into the Bessel component. So, now you can actually pick up, say either you can take the ratio of this component and this component or like what we did previously was taking the ratio of this component versus this component.

So that can actually give you the phase that you want, that can essentially give you the  $\phi$ . And the significant difference here is the fact that we are trying to maintain a quadrature operation previously, when we were looking at phase generated carrier, so let me just go all the way back in the, when we are applying phase generated carrier  $\omega_c$  was totally a free parameter, you can choose whatever value of carrier frequency you want.

So, there was no strings attached, only thing you were looking for was you go to a frequency which is, well, outside of your  $1/f$  noise, the pink noise or the flicker noise and so you typically went to 10 kilohertz or something like that, that frequency. But here and that is because of the fact that two waves are going through completely different paths.

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How to maintain quadrature point operation in IFOG?



$$\Delta\phi = \phi_s + \phi'_{cw} - \phi'_{ccw}$$

$$= \phi_s + \phi(t - \frac{T}{2}) - \phi(t + \frac{T}{2}) \quad \text{where } T = \frac{L}{v_g}$$

Suppose  $\phi(t) = \phi_{m0} \cos(2\pi f_m t)$


$$\Delta\phi = \phi_s + 2\phi_{m0} \sin\left(\frac{\omega_m T}{2}\right) \sin(\omega_m t)$$

$$= \phi_s + \phi_m \sin(\omega_m t) \quad \text{where } \phi_m = 2\phi_{m0} \sin\left(\frac{\omega_m T}{2}\right)$$

Max occurs when  $\omega_m T = \pi$

$$f_m = \frac{\pi}{2\pi T} = \frac{1}{2T}$$

for  $L = 200\text{m}$ ,  $v_g = \dots$




But here when we talk about what we have as far as the I-FOG is concerned, since they go through the same path, both the clockwise and the anticlockwise components, they go through the same path. Now you need to make sure that you are switching it at a frequency while keeping the such that within that round trip it goes from one phase value to another phase value and you can maintain that phase so that you can achieve maximum phase operation.

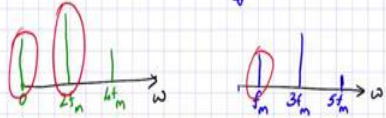

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$$I_f = I_0 [1 + \cos(\Delta\phi)]$$

$$= I_0 [1 + \cos(\phi_s + \phi_m \sin \omega_m t)]$$

$$\frac{I_f}{I_0} = 1 + \cos \phi_s \cos(\phi_m \sin \omega_m t) + \sin \phi_s \sin(\phi_m \sin \omega_m t)$$

$$= 1 + \cos \phi_s \left[ J_0(\phi_m) + \sum_{k=1}^{\infty} 2J_{2k}(\phi_m) \cos(2k\omega_m t) \right]$$

$$+ \sin \phi_s \left[ \sum_{k=0}^{\infty} 2J_{2k+1}(\phi_m) \cos((2k+1)\omega_m t) \right]$$




So, that is the key difference as far as I-FOG is concerned, but once again it is at the end of the day when you look at picking up phase you are picking up at a very specific frequencies,



so all the advantages of a phase generated carrier in terms of noise suppression, because you are interested in only this component, you can essentially just put a, just beat it with those frequencies and put a low pass filter, so you can reduce the external noise, whatever external perturbations there are to a large extent.

Of course, we did start by saying that in the FOG the external perturbations are not as high compared to what you would normally have with Michelson or a Mach-Zehnder interferometer, to start with you have relatively no noise, but as you do this measurement, as you have increase in the rotation rate, some of those noise components start showing up and in that case this scheme actually helps to suppress all the external perturbations.

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I FOG Limitations

- ① Polarization → Environmentally-induced polarization changes will affect CW/CCW phase ⇒ FM fiber
- ② Coherent Rayleigh Backscattering → ARB along the length of fiber
- ③ Faraday effect → due to external magnetic field, polarization changes (non-reciprocal)
- ④ Shot Noise → due to random arrival of photons @ receiver



How to maintain quadrature point operation in IFOG?

$$\Delta\phi = \phi_s + \phi'_{CW} - \phi'_{CCW}$$

$$= \phi_s + \phi(t - \frac{T}{2}) - \phi(t + \frac{T}{2}) \quad \text{where } T = \frac{L}{v}$$

Suppose  $\phi(t) = \phi_m \cos(2\pi f_m t)$

$$\Delta\phi = \phi_s + 2\phi_m \sin\left(\frac{\omega_m T}{2}\right) \sin(\omega_m t)$$

$$= \phi_s + \phi_m \sin(\omega_m t) \quad \text{where } \phi_m = 2\phi_m \sin\left(\frac{\omega_m T}{2}\right)$$

Max occurs when  $\omega_m T = \pi$

$$f_m = \frac{\pi}{2\pi T} = \frac{1}{2T}$$

For  $L = 200m, \dots, v_s$



$$\Delta\phi = \frac{4\pi R L}{v_g \lambda} \cdot \Omega$$

Suppose  $\Omega = 10^{-3}$  deg/hr

Let  $\lambda = 1.5 \mu\text{m}$ ,  $v_g = 2 \times 10^8$  m/s,  $R = 10$  cm,  $L = 200$  m

$$\Delta\phi = \frac{4 \times 3.14 \times 0.1 \times 200}{2 \times 10^8 \times 1.5 \times 10^{-6}} \times 10^{-3} \times \frac{\pi}{180 \times 3600}$$

$$\approx 4 \times 10^{-9} \text{ rad}$$

Interferometer output,  $I_T = I_0 [1 + \cos(\Delta\phi)] \rightarrow 4 \times 10^{-9} I_0$   
 $= I_0 [1 + \cos(\theta_L + \Delta\phi)] \rightarrow 4 \times 10^{-9} I_0$



So, now let us look at all the possible limitations as far as a fiber optic gyroscope is concerned. So, let us, specifically we are looking at the I-FOG, so let us look at the overall limitations as far as I-FOG is concerned. So, the first limitation that would come around is similar to what we have been seeing in other cases also, so the first one that we should list out is polarization.

So, we know that environmentally induced polarization changes, if there are any environmentally induced polarization changes, can actually give, so whenever we talk about polarization change we are talking about a relative phase change between two orthogonal polarization components, so that does mean that there is actually a change in the phase for any of these two waves cw or counter clockwise waves.

So, these environmentally induced polarization changes will affect cw or ccw phase, so you will have a change in the phase for the cw and ccw and in reality, since they are traveling separately they might actually go through different phase changes at any particular location corresponding to the environmental condition when the wave comes around that location. So that could be a limitation.

Of course, we would say that how do we counter that? We counter that by choosing a polarization maintaining fiber. So, by using a polarization maintaining fiber we could potentially reduce these effect of environmental changes and that is what we saw previously for our other cases also, for the Michaelson as well as the Mach-Zehnder based interferometers.

So, you use a pm fiber, so we are not going to discuss that in much detail now, because we already spent a lot of time discussing that previously. In fact, it is fair to say that the science and technology related to polarization maintaining fibers was actually really motivated by a lot of requirements for a gyroscope application back in the late 70s and the early 80s.

So, we could possibly use a panda type of fiber like we talked about previously to limit that and of course, there is this other type of fiber called an elliptical core fiber, which can possibly give you even better sort of protection, better protection from environmental changes, so those are things that are well documented in the literature.

So, there is this one company called KVH Corporation in the USA, which actually assured in this elliptical core fibers and that was actually a big hit for them. So, the early fiber optic gyroscopes were all made by KVH using elliptical core fibers rather than panda fibers. We do not want to go into those details right now.

But the other thing that can actually limit the performance of an I-FOG is what is called coherent Rayleigh backscattering and this is actually a very critical type of effect that we have to be very vary about. So, what is coherent Rayleigh by scattering? Well, if we go back and look at this picture of a FOG, we are looking at the phase difference between clockwise and the counter clockwise propagating beams.

That is what we are trying to pick up. What if there is actually some scattering from this point, so that the clockwise propagating wave gets scattered and and propagates back, it goes back so that is actually, it is getting mixed with this counterclockwise beam and that is actually introducing an error and that could be a fairly huge error. We will go into the details of that in a minute.

And this is happening all along the length of the fiber because we know that in an optical fiber you have density variations at a scale that is much smaller than the wavelength of light that is propagating, so you have this Rayleigh scattering happening all along the length of fiber and that can pose a huge limitation in terms of the rotation rate that you are going to be able to pick up. We will come back and see this in more detail.

And the other thing that can pose a limitation is what is called the Faraday Effect. So, what is the Faraday Effect? Well, whenever you have light propagating through a medium and if that medium is subjected to a magnetic field, then you will have a rotation of polarization within

that medium, so we are talking about say a clockwise propagating beam that is going through this fiber even something as simple as the Earth's magnetic field, that is enough to cause a small rotation in the polarization.

And even that small rotation in the polarization is enough to cause changes in the phase, like we said any polarization change also involves a phase change and that phase change is enough to basically increase the noise floor of your measurement. So, that is something that we will have to be very vary about. So, this is due to external magnetic field like we talked about, it could be the Earth's magnetic field or in a environment where you have some other magnetic flux inducing elements, then that will also play a role in this.

Because due to external magnetic field polarization changes and the key point is this is non-reciprocal in nature. Anything that your clockwise wave is experiencing it is the same thing the counterclockwise wave is experiencing as well, then if it is common for both of them then they cancel each other, so you do not really have to worry about it.


But in this case you have one direction that the clockwise wave is propagating and so in that direction let us say it is coming towards you, it is actually, polarization is changing this way because of the external magnetic field. For the other polarization that is going away from you, that is the counterclockwise polarization, the polarization changes happen in the other direction, because it is a Faraday Effect is a non-reciprocal effect and because of that you will have some phase changes.

So, we will come back and look at how to deal with it. And probably one of the more fundamental things that we started talking about previously is a shot noise at the receiver and this we know is due to large or rather random arrival of photons at the receiver, so that can actually also introduce some fluctuations and because finally we are looking at changes in intensity and like we saw previously to increase your sensitivity you need to actually have, you need to increase  $I_{\text{avg}}$  the intensity for each of those waves.

And if you increase  $I_{\text{avg}}$ , then you have more light falling on the photodiode, incident on the photodiode and that actually causes shot noise. So, we need to be very about that as well. So, let us actually look into a little more detail and like I said coherent Rayleigh backscattering is this one of the major limitations. So, let us look into that little more detail.

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Coherent Rayleigh Backscattering (CRB)



Function of light collected  $\approx \beta^2/4$

Ratio of scattered power w.r.t incident power  $= \frac{P_s}{P_i} = \frac{\beta^2}{4} \alpha_j L$        $\beta \rightarrow 0.1 \text{ rad}$   
 $\alpha_j \rightarrow 0.5 \text{ dB/km}$   
 $L \rightarrow 200 \text{ m}$

Max. phase noise  $\Delta\phi_{\text{max}}^{\text{CRB}} = 2 \sqrt{\frac{P_s}{P_i}} = \beta (\alpha_j L)^{1/2} = 0.035 \text{ rad}$





Function of light collected  $\approx \beta^2/4$

Ratio of scattered power w.r.t incident power  $= \frac{P_s}{P_i} = \frac{\beta^2}{4} \alpha_j L$        $\beta \rightarrow 0.1 \text{ rad}$   
 $\alpha_j \rightarrow 0.5 \text{ dB/km}$   
 $L \rightarrow 200 \text{ m}$

Max. phase noise  $\Delta\phi_{\text{max}}^{\text{CRB}} = 2 \sqrt{\frac{P_s}{P_i}} = \beta (\alpha_j L)^{1/2} = 0.035 \text{ rad}$

CRB limited rotation rate  $\Omega_{\text{CRB}} = \frac{v_g \lambda}{4\pi R L} \beta (\alpha_j L)^{1/2} = 341 \text{ deg/hr}$




$$\Delta\phi = \frac{4\pi R L}{v_g \lambda} \Omega$$

Suppose  $\Omega = 10^{-3} \text{ deg/hr}$

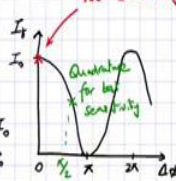

Let  $\lambda = 1.5 \mu\text{m}$ ,  $v_g = 2 \times 10^8 \text{ m/s}$ ,  $R = 10 \text{ cm}$ ,  $L = 200 \text{ m}$

$$\Delta\phi = \frac{4 \times 3.14 \times 0.1 \times 200}{2 \times 10^8 \times 1.5 \times 10^{-6}} \times 10^{-3} \times \frac{\pi}{180 \times 3600}$$

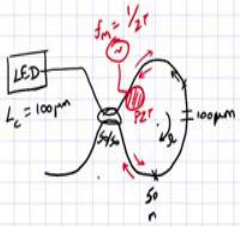
Poor sensitivity

$$\approx 4 \times 10^{-9} \text{ rad}$$

Interferometer output,  $I_f = I_0 [1 + \cos(\Delta\phi)] \rightarrow 4 \times 10^{-9} I_0$   
 $= I_0 [1 + \cos(\pi/2 + \Delta\phi)] \rightarrow 4 \times 10^{-9} I_0$


How to maintain quadrature point operation in IFOG?



$$\Delta\phi = \phi_s + \phi'_{cw} - \phi'_{ccw}$$

$$= \phi_s + \phi\left(t - \frac{T}{2}\right) - \phi\left(t + \frac{T}{2}\right) \quad \text{where } T = \frac{L}{v_g}$$

Suppose  $\phi(t) = \phi_{m0} \cos(2\pi f_m t)$



$$\Delta\phi = \phi_s + 2\phi_{m0} \sin\left(\frac{\omega_m T}{2}\right) \sin(\omega_m t)$$

$$= \phi_s + \phi_m \sin(\omega_m t) \quad \text{where } \phi_m = 2\phi_{m0} \sin\left(\frac{\omega_m T}{2}\right)$$

Max occurs when  $\omega_m T = \pi$

$$f_m = \frac{\pi}{2\pi T} = \frac{1}{2T}$$

for  $L = 200 \mu\text{m}$ ,  $v_g = \dots$

So, what is coherent Rayleigh backscattering? So, which is in short is known as CRB. Now, when you look at what is happening within a fiber, let us just zoom into the core of an optical fiber, we know that we have a wave that is coming in, but it is encountering even as it is propagating in this medium, it is encountering density fluctuations, which are actually much smaller in scale, compared to the wavelength itself.

And in this sort of scenario you have a light that is scattering in all directions, some of which is going back the same path, so anything that goes back is going to cause us problems because this is going to confuse the light that we are getting at the coupler, the clockwise radiation may look like anti-clockwise or vice versa, so any backscattering is going to be very bad as far as this FOG is concerned.

So, if you want to quantify this, let us just try to look at this. Now, it is not like all the light that is coming back is going to be captured, like for example, this one that is going in the transverse direction, just escape out of the fiber, but the numerical aperture which corresponds the cone of angles that is captured by the optical fiber comes into play over here.

So, if you say that numerical aperture corresponds to beta, then angle beta, beta by 2 is the half angle over here and then of course, this is in three dimensions so this is an elevation, so you need to also look at it in the same perspective when you look at a solid angle. So, you have a similar beta by two on the other axis as well.

So, when you look at the area of the solid angle or specifically the fraction of light collected that can be approximated as beta squared over 2 the whole square, because it is happening in both

directions, so that is that can be approximated as  $\beta^2/4$  and where  $\beta$  corresponds to the numerical aperture of the fiber, which once again is determined by the contrast in the refractive index between the core and the cladding.

So, for better confinement you want better contrast, but then if you have higher contrast then you have larger value of  $\beta$  and so the larger value of numerical aperture, larger value of  $\beta$ , so larger will be the amount of light that is collected from the backscattered like. So, that is the fraction of light collected. So, but if you have to quantify the amount of power scattered, so what we are interested is the ratio of scattered power with respect to the incident power, which we can denote as  $P_s/P_i$ .

So, that this is a fraction that is scattered and, or the fraction that is collected, so you have  $\beta^2/4$ , but the scattered, the amount of scattering that is happening is determined by the scattering coefficient  $\alpha_s$  and that is integrated all along the length of the fiber, where the length of the fiber in our case, the loop length is what we have assumed as  $l$ . So, of course, the scattered power as it propagates it is given us  $E$  power minus  $\alpha_s l$ , but  $\alpha$  is actually such a small quantity that you can just approximate it like this.

Now, what we are having is because of the scattered power this essentially is like a noise component that you have, so this component can be looked upon as a noise power or noise variance, so if you are interested in the phase deviation that we are getting of the maximum phase noise noise due to this CRB - Coherent Rayleigh Backscattering. You can just say  $\Delta\phi_{\max}$  due to CRB is going to be given by...

Since, you are interested in the RMS value, you are going to take root of this variance, so this is  $P_s/P_i$ , root of  $P_s/P_i$  and because there is this noise that is independently happening for both your clockwise as well as the counter clockwise waves, you can multiply this by two. So, if you plug in this expression over here, you get a  $\beta/2$  because of this, so you have a  $\beta$  outside and then you have  $\alpha_s L$  to the power of half.

Now, let us actually start putting some numbers to this, so  $\beta$  is typically about 0.1 radian,  $\alpha$  is let us say that is a scat scattering losses, it can be maximum of up to 0.5 db per kilometre, so to get it in meters you divide this by  $10^3$ , so you have 0.5 to power minus 3 and then converted to nippers, you would multiply it by or divide by 4.3, so if you do that you will come to some number in the order of  $10^{-4}$  per meter.

And then  $l$  we have taken for our case to be 200 meters, so if you plug all these in you get to a value of about 0.035 radians, that is a huge value, if you remember what we were talking about getting is something the order of  $10^9$  radians and here we are talking about something the order of  $10^3$  radians, so which is very-very high.

And if you look at it from a perspective of say the CRB, if CRB was the only noise component that you had, so the CRB limited rotation rate  $\omega_{CRB}$ , to get that you would essentially go back and say look at this expression, so for  $\Delta\phi$  is given by this, so for  $\omega$  you can basically invert that, so you if know  $\Delta\phi$  you can actually get  $\omega$ .

So, you can just invert that and say that CRB is going to be given by  $V_g$  multiplied by  $\lambda$  divided by  $4\pi RL$  and then you have this  $\Delta\phi$  that is coming to the picture, so  $\beta$ ,  $\alpha$  is 1 to the power of half. So, if we do the same calculation that we did before  $\lambda$  is 1.5  $V_g$  is  $2 \times 10^8$  and  $R$  is like corresponding to 10 centimeter, if you plug all those in, you will get some value around 341 degree per hour.

So, that is a huge number, so we are saying that Coherent Rayleigh Backscattering can essentially kill this entire idea and our requirement is in the order of 0.01 degree per hour, so that is 5 orders of magnitude lower, so how can we, so we cannot deal with this at all, so how can we, the question is how can we reduce this? Well, so this is the key point. To reduce it you need to reduce  $L$ .

But you cannot reduce  $l$  directly because you need that  $L$ ,  $L$  is your scale factor,  $L$  actually gives you this sensitivity with respect to small rotation rates. But what you could do is play with the coherence. What exactly are we talking about? If we actually use a source which is relatively incoherent, so if we just go back to this.

So, if we use say an incoherent source like an LED, for example, where you have your LC that is a coherence length is in the order of 100 microns, then what we are doing is we are limiting the amount of backscattering that is actually relevant. So, what does this mean? If there is a path length that is greater than 100 microns, pathline difference that is greater than 100 microns, then you do not have any effect of backscattering.

Or in other words, I will get to only 100 microns worth of backscattering that actually plays a role in confusing my signals, the rest of the backscattering wherever else it happens, you have a path length difference because for the clockwise, suppose it happens over here, so let us say



it happens about at a distance of 50 meters from this point for the counter clockwise wave, it would have happened at 150 meters away for the clockwise wave.

So, then the path difference is 100 meters, so you are not going to actually have any interference because of that. So, essentially the noise or the coherent beating noise that you have with your clockwise signals or the anti-clockwise signals, the signals that we are interested in, is going to be quite low. So, essentially only within 100 microns whatever backscattering is happening, those are the ones that are going to be coherent and that is all that would matter.



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If we use low coherence source

$$\Omega_{CRB} = \frac{V_g \lambda}{4\pi R L} \beta(\theta_s, L_s)^{1/2}$$

low coherence source

$L_c \rightarrow 100 \mu m$   
 $L \rightarrow 200 m$   
 6 orders of magnitude lower

So, when we talk about this rotation rate limited by CRB, we talked about  $V_g \lambda$  over  $4\pi R L \beta \alpha s L$ , but now if we use low coherence source, then the amount of scattering that is relevant is only over  $L_c$ , and since  $L_c$  is actually much smaller, so in this case  $L_c$  is 100 microns, whereas  $L$  is 200 meters. So, previously you are accumulating all this backscattering over 200 meters, now whatever you accumulate within 100 microns is all that matters.

So that is actually 6 orders of magnitude, so this is basically 6 orders of magnitude lower, and because of that if you plug that back into this  $\Omega_{CRB}$ , now you will be able to get to something which is much more comfortable, so it will be in the order of a fraction of a degree per hour. So, the key for this part, where  $L_c$ , it should be as small as possible, so you are essentially saying that you need to have as low coherence source as possible.

That in other words it means that as broadband source as possible, if you use that; then you will be able to reduce  $L_c$  and you will be able to reduce the limitation of due to Coherent Rayleigh Backscattering.