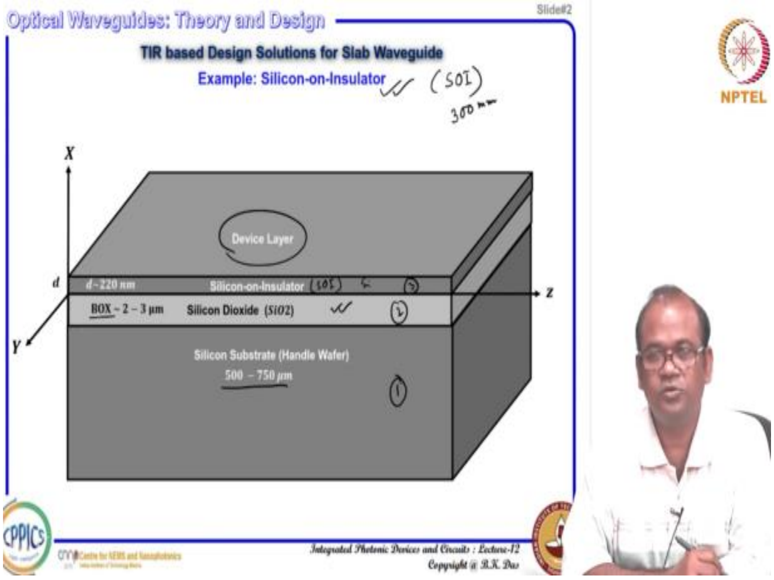
Integrated Photonics Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology - Madras

Lecture - 12
Optical Waveguides: Theory and Design: TIR Based Design Solutions for Slab Waveguides

Hello everyone, today's lecture I will continue optical waveguides theory and design specifically I will be considering slab waveguides design solution. Of course, I will be discussing at this moment based on total internal reflection principle.

(Refer Slide Time: 00:39)



Optical Waveguides: Theory and Design

Slide#2

TIR based Design Solutions for Slab Waveguide

Example: Silicon-on-Insulator (SOI)

300 nm

Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

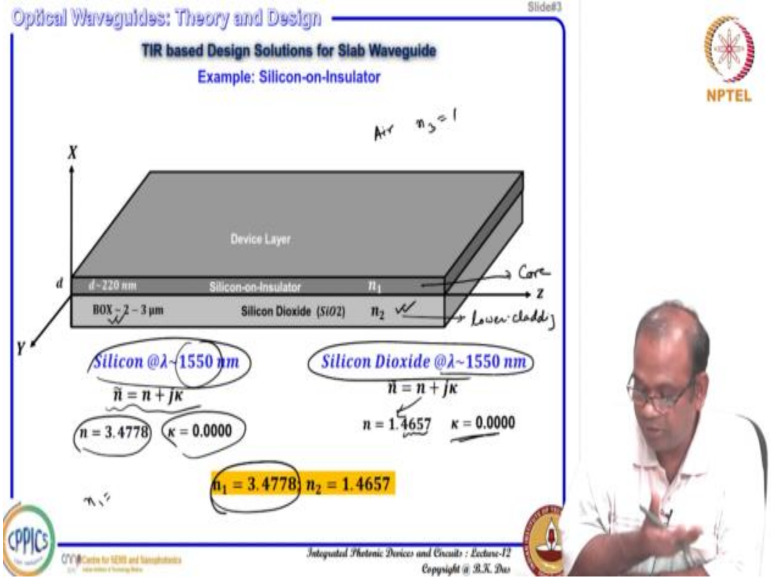
And for that I take 1 example of silicon on insulator in the previous lecture I have shown that different device layer different type of wafers we can use for different application purpose, but, for today's discussion, I consider silicon on insulator typically it is called in short you can call it as a SOI silicon on insulator. So, as I already told you before that this silicon on insulator wafer now it is available up to good quality optical grade silicon on insulator wafer can be available up to 3 millimetre diameter.

But here I have shown a piece of SOI sample it can be diced from a full wafer and if you diced that you can see clearly 3 layers. So, layer 1, layer 2 and then layer 3. The first layer is a thicker one that is actually silicon substrate called handle wafer it actually it has no role in optical waveguide design, but it gives mechanical strength to the wafer. If you are fabricating a chip out of SOI you can get a good strength if you have this much thickness that is the reason this must take less is given as a silicon substrate just handle wafer.

And then you have a buried oxide layer 2 to 3 micrometre typically it is basically silicon dioxide and then in that top that is the actually your device layer called silicon on insulator. This is the layer called SOI this is silicon basically crystalline silicon on insulator. Insulator is silicon dioxide and thickness depending on your requirement you can control the thickness typically it is 220 nanometer is very popular in foundries nowadays for photonic integrated circuits.

But it can be customised to 100 nanometer and can be thicker 1 micron, 2 micrometre device layer. So, this type of SOI wafer I will be considering SOI sample I will be considering while discussing the design aspects of a slab waveguide.

(Refer Slide Time: 03:06)



Optical Waveguides: Theory and Design

Slide#3

TIR based Design Solutions for Slab Waveguide

Example: Silicon-on-Insulator

Silicon @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 3.4778$ $\kappa = 0.0000$

Silicon Dioxide @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 1.4657$ $\kappa = 0.0000$

$n_1 = 3.4778$; $n_2 = 1.4657$

$n_1 =$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

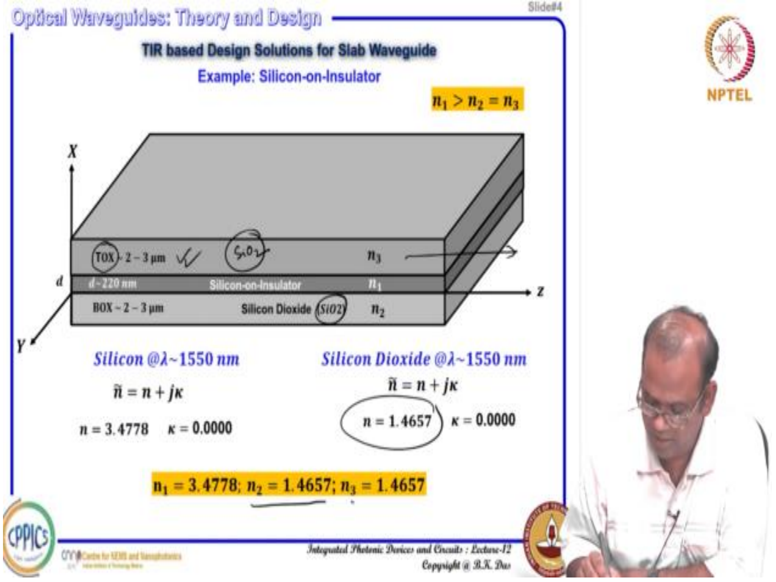
Since the substrate handle wafer that is not useful for guiding purpose. So, I removed in this picture, I just consider only box that is actually silicon dioxide and that actually act as a lower cladding of the slab waveguide and then this device layer actual core where actually light will be confined. And in the top if you do not do anything that means it is air. So, air also you can consider as a refractive index, dielectric material with a refractive index n 3 = 1.

And n 1 here if you just think of silicon crystal and silicon, the refractive index 1550 nanometer wavelength at 1550 nanometer which is the communication band, third generation communication band, optical communication band. And we know that any dielectric material or conductor or whatever, you can actually represent refractive index in complex form, which has the real part is the actual refractive index and the imaginary part is called the extention coefficient of kappa.

And at 1550 nanometers, as you know this is a photon energy sufficiently low and lower than band gap of the silicon crystal that is why it is transparent. That is why you only see the real part of the refractive index so, the imaginary part is 0. So, that is transparent around 1550 nanometer is typically about 1.12 micrometre wavelength it is transparent. So, that is actually shown so, that means, we are now set with n 1 as 3.4778.

Now, silicon dioxide, this is the silicon dioxide box layer you are using box layer also if you consider the same communication wavelength 1550 nanometer and refractive index. So, the real part is about 1.4657 I have just used 4 decimal point, because most of the photonic devices you consider the refractive index change even up to 4th the decimal point that is very important useful. And kappa is also 0 transparent silicon dioxide typically it is almost transparent if it is good quality particularly if the silicon dioxide is native oxide grown on the silicon that is quite good quality. So, there would not be any loss or absorption at 1550 nanometer wavelength.

(Refer Slide Time: 05:53)



Optical Waveguides: Theory and Design

Slide#4

TIR based Design Solutions for Slab Waveguide

Example: Silicon-on-Insulator

$n_1 > n_2 = n_3$

Silicon @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 3.4778 \quad \kappa = 0.0000$

Silicon Dioxide @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 1.4657 \quad \kappa = 0.0000$

$n_1 = 3.4778;\ n_2 = 1.4657;\ n_3 = 1.4657$

Integrated Photonic Devices and Circuits : Lecture-12

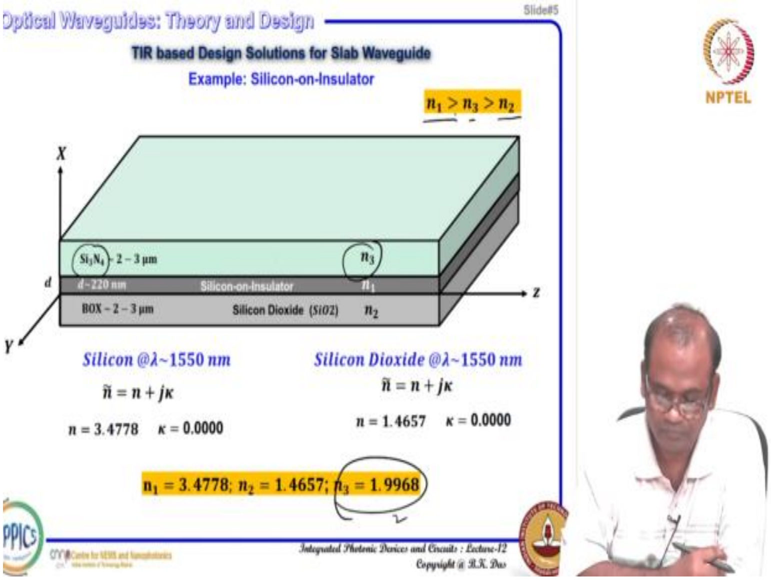
Copyright @ B.K. Das

So, now, you can think of in the top layer you can also have silicon dioxide you can grow silicon dioxide then you can have we can call that is a top oxide layer that can be also 2 to 3 micrometres. So, that will act as a top cladding layer so, instead of air you can have top cladding layer also refractive index of this one around 1550 nanometers. So, in this case TOX and BOX both are silicon dioxide.

This is also silicon dioxide, this is also silicon oxide. So, in this case the core is that is SOI device layer is sandwiched between 2 dielectric material that is silicon dioxide and the same basically that is why this type of slab structure can be considered as a symmetric slab waveguide.

So, because n 3 particularly n 2 = 1.4657, n 3 also 1.4657. So, that is why symmetric planar waveguide structure.

(Refer Slide Time: 06:57)



Optical Waveguides: Theory and Design

Slide#5

TIR based Design Solutions for Slab Waveguide

Example: Silicon-on-Insulator

$n_1 > n_3 > n_2$

Silicon @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 3.4778$ $\kappa = 0.0000$

Silicon Dioxide @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 1.4657$ $\kappa = 0.0000$

$n_1 = 3.4778;\ n_2 = 1.4657;\ n_3 = 1.9968$

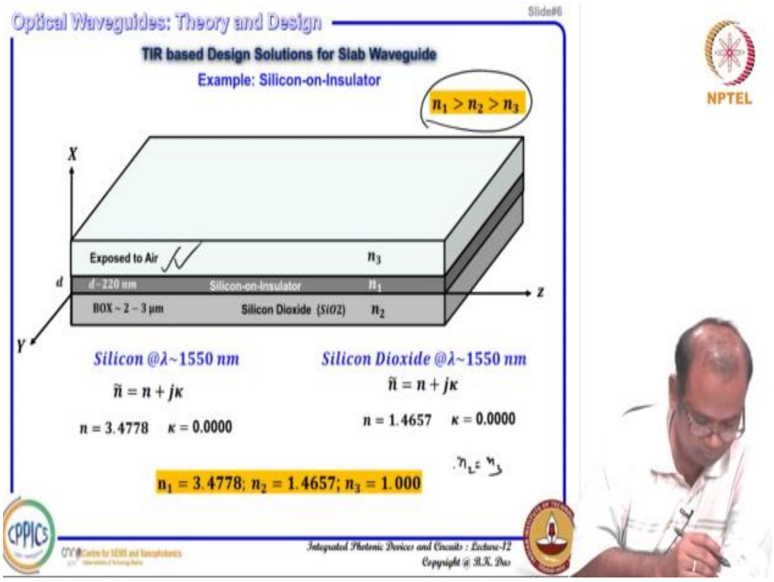
Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

And then sometimes it is useful for a special application purpose instead of silicon dioxide and the top as a TOX you use silicon nitride also that is also CMOS compatible, you can deposit silicon nitride and silicon nitride refractive index is higher than your silicon dioxide n 3, n 3 = 1.9968 close to 2. So, that means your n 2 is silicon dioxide and n 3 is your silicon nitride. So, top cladding layer refractive index is more.

So, you can consider that n 1 greater than n 3 greater than n 2, n 3 is greater than n 2 that is reverse when you are using air that is n 1 greater than n 2 will be greater than n 3 normally top if it is air that is n 3 that refractive index will be lower, but in this case it is upper cladding is higher refractive index compared to lower cladding. So, this can be considered as a planar waveguide structure.

(Refer Slide Time: 08:05)



Optical Waveguides: Theory and Design

Slide#6

TIR based Design Solutions for Slab Waveguide

Example: Silicon-on-Insulator

$n_1 > n_2 > n_3$

Silicon @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 3.4778$ $\kappa = 0.0000$

Silicon Dioxide @λ~1550 nm

$\tilde{n} = n + j\kappa$

$n = 1.4657$ $\kappa = 0.0000$

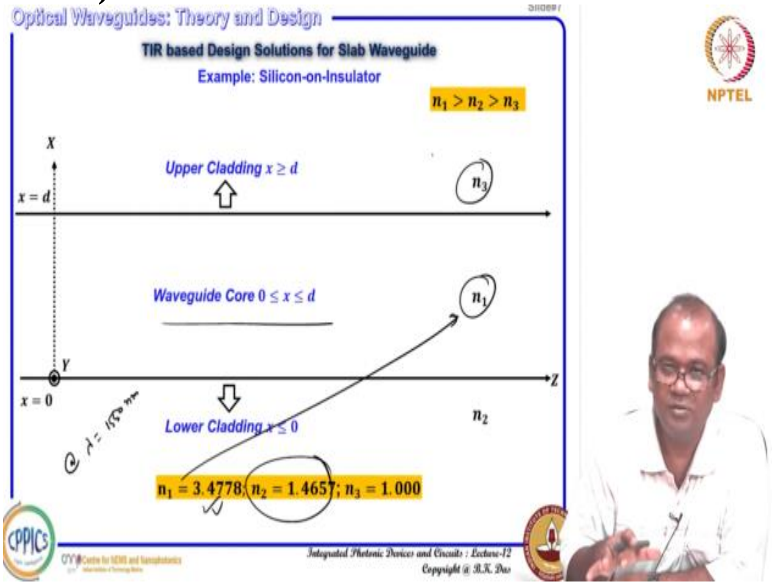
$n_2 = n_3$

$n_1 = 3.4778; n_2 = 1.4657; n_3 = 1.000$

Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

Now, so, that is I just come back to the if top layer is exposed to air completely then we just as I mentioned n 1 greater than n 2 and greater than n 3. For some calculation purpose will be considering this type of asymmetric planar waveguide structure of anywhere necessary for symmetric most of the time whenever you are fabricating photonic circuit, top layer will be oxide coated. So, in that case, we will be considering n 2 = n 3, but occasionally, if you want to use the top layer as the air, you can just remove oxide layer in that particular area so that you can design your waveguide structure with asymmetric SOI structure.

(Refer Slide Time: 08:49)



Optical Waveguides: Theory and Design

Slide#7

TIR based Design Solutions for Slab Waveguide

Example: Silicon-on-Insulator

$n_1 > n_2 > n_3$

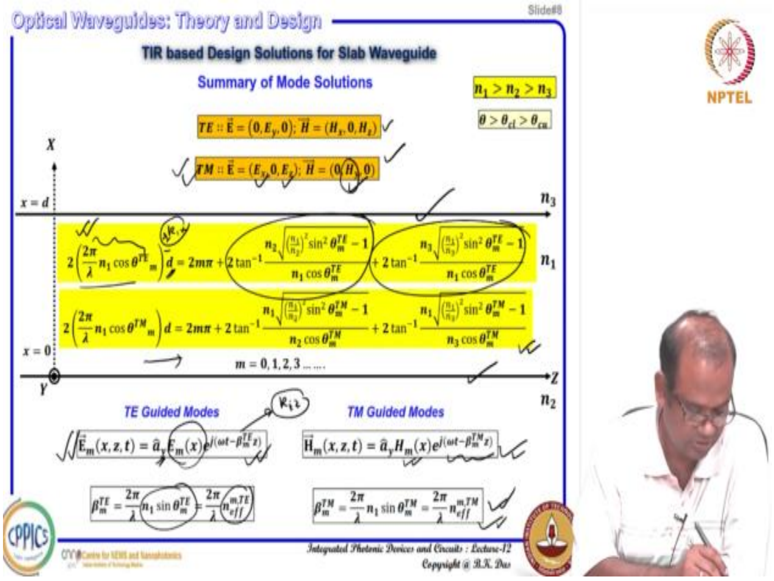
$n_1 = 3.4778; n_2 = 1.4657; n_3 = 1.000$

Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

So, now let us move on so, we have the just instead of that 3D view, I just take the vertical direction. So, along X direction intersections are given because YZ plane is infinitely extended you can consider with respect to your guiding light in the code and you can consider that along the X direction you have your lower cladding, lower cladding will have refractive index of n 2 that is x less than or equal to 0.

You can consider and core regions should be stretched between x = 0 to d with a refractive index n 1, n 1 means this one refractive index 3.4778 all the refractive index value I have just given here they are at lambda = 1550 nanometer third generation communication window and n 3 we can consider here for discussion purpose.

(Refer Slide Time: 09:51)



Optical Waveguides: Theory and Design

Slide#8

TIR based Design Solutions for Slab Waveguide

Summary of Mode Solutions

$n_1 > n_2 > n_3$

$\theta > \theta_{cl} > \theta_{cu}$

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$TM :: \vec{E} = (E_x, 0, E_z); \vec{H} = (0, H_y, 0)$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta^{TE}{}_m\right) d = 2m\pi + 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m^{TE} - 1}}{n_1\cos\theta_m^{TE}} + 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m^{TE} - 1}}{n_1\cos\theta_m^{TE}}$$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta^{TM}{}_m\right) d = 2m\pi + 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m^{TM} - 1}}{n_2\cos\theta_m^{TM}} + 2\tan^{-1}\frac{n_1\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m^{TM} - 1}}{n_3\cos\theta_m^{TM}}$$

$m = 0, 1, 2, 3 \ldots\ldots$

TE Guided Modes

$\vec{E}_m(x,z,t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m^{TE} z)}$

$\beta_m^{TE} = \frac{2\pi}{\lambda} n_1 \sin\theta_m^{TE} = \frac{2\pi}{\lambda} n_{eff}^{m,TE}$

TM Guided Modes

$\vec{H}_m(x,z,t) = \hat{a}_y H_m(x) e^{j(\omega t - \beta_m^{TM} z)}$

$\beta_m^{TM} = \frac{2\pi}{\lambda} n_1 \sin\theta_m^{TM} = \frac{2\pi}{\lambda} n_{eff}^{m,TM}$

Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

Now, we just already whatever we have already discussed earlier for a slab waveguide how total internal reflection helps to confine a light in the core region and these are the governing equation we have discussed in the previous lecture, I have just pulled them here just to continue our discussion and analysis. You know, we can consider one of the 2 polarizations TE polarisation where electric field will be tangential to the interface.

That means along Y direction oscillating and magnetic field will be in the Z plane that is called TE polarisation and alternatively, you can consider TM polarisation or that type of electrical components were E field is in the Z plane x component, z component will be there magnetic field will be the along Y direction perpendicular to the field. So, one of these 2 or both you can consider if any electric field direction if it is oscillating you can decompose into this 2 different type of polarisation you can treat them differently.

Because this will help us to exploit the boundary conditions or the interfaces upper interface and lower interface. This is the upper interface, this is a lower interface and to get the final reflection and the phase change at total internal reflection that expression we have derived using these boundary conditions. So, if this is the condition then we know that TE guided modes can be expressed electric field can be expressed only Y component will be there.

And X dependent confinement will be there and e to the power j omega t - beta m TE m z that is a field. And beta is the something basically longitudinal component of the wave vector which is nothing but 2pi by lambda n 1 sin theta TE and 2pi by lambda this n 1 sin theta TE it is called n effective for TE polarisation similarly, we can consider for TM polarisation and in case of TM polarisation we are considering only H y component.

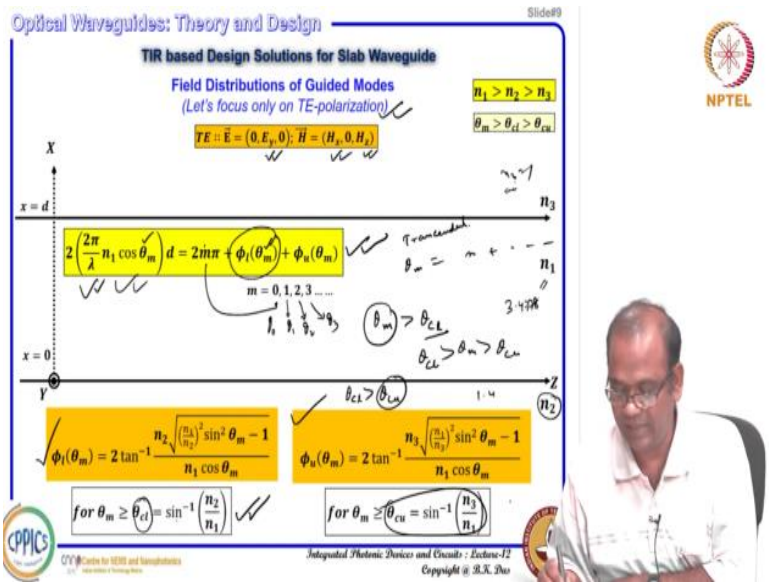
So, I have just same way I put for TM polarisation this y vector y unit vector is there A y unit vector along y direction and magnetic field will be x dependent because light wave will be confined along the X direction and phase also instead of beta TE it will be considered as a beta TM and accordingly n effective also for TM polarisation M is actually called so, called your modal index.

So, order of the mode guiding mode how that is decided that is decided by the Eigen solution Eigen equation for top one is for TE polarisation, this is basically your transverse component. So, K 1 x and this beta z actually defined by K 1 z. So, if a vector is there going back and forth with total internal reflection, so, z components longitudinal component along that direction that is actually known as beta and transverse component you will have plus and minus that is why standing wave will be there, so, round the phase 2 times d.

So, K 1 x multiplied by 2 times d that is the total phase and this total phase and you have to also add the phase change of total internal reflection in the so, called lower interface and phase change corresponding to the upper interface and this 2 should be minus sign here I have taken this side. So, this is one Eigen solution likewise you can go for TM polarisation. So, they are different only this n 2 and n 1 will be interchange the numerator.

And denominator and tan inverse function that will be interchange n 1 by n 2 here n 1 by n 3 in the top interface. So, this equation we have derived from Fernyl equation and we have discussed a little bit detailed in previous lecture I just summarised in this slide to progress to continue our discussion for today's lecture.

(Refer Slide Time: 14:23)



Optical Waveguides: Theory and Design

Slide#9

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$$

$m = 0, 1, 2, 3 \ldots\ldots$

$$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}}{n_1 \cos\theta_m}$$

$$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1}}{n_1 \cos\theta_m}$$

$for\ \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$for\ \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

Now, let us focus only on TE polarisation. So, because you know the equations are almost identical if you know how to solve in one polarisation you can easily solve for the other polarisation. I will discuss only for TE polarisation where electric field component is your concern that is actually easy to handle. And other components we can easily derive from your Maxwell's curl equations wants to know electric field and magnetic fields they are related through Maxwell's equation and frequency domain.

So in that case, I just repeat that equation for TE polarisation, instead of entire phi expression, I just put that this phi lower that means phase change happening at the lower intern interface due to total internal reflection that must be theta dependent and upper also theta dependent. So, if you solve this equation for a given value of n, you can get a solution for theta that is theta 0 if m = 0, you will get solution for theta 0, m = 1 you will get theta = theta 1 and m = 2 you get theta 2 and theta 3.

So, all these discrete theta solutions you can get if you can actually solve this equation, it is not so, easy to solve this equation, because it is a transcendental equation. So, you cannot really derive expressions for example, you cannot write something like that theta m equal to something m plus something right hand side without theta. So, then it would have been easier you could calculate easily.

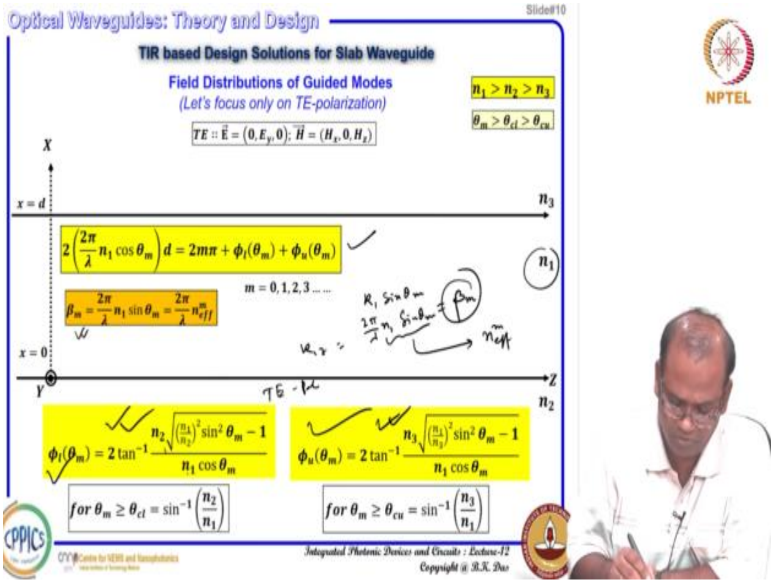
But this is transcendental equation we have to go a little bit different way I will discuss that today also the topic and I have just inserted a phi l writing here I just put down here for your memory purpose if the equation is seen again and again by you then it can be memorised so, that you can utilise anytime for any problem solution. So, couple of more things you should get a knowledge here.

If you try to see what is the critical angle at the lower interface that should be lower interface n 2, n 1. So, sin inverse n 2 by n 1 so, we can calculate critical angle for the lower interface I know what is the value of n 1 in silicon on insulator platform. So, it would be SOI silicon 3.4778 you know that you can easily calculate and this one 1.46 something like that and then it will be here so, n 3 = 1.

So, n 1 you will know and n 2 you can find out so, you can get critical angle n 2 by n 1, n 2 by n 1 that is 1.4 by n 1 that is correct. And similarly, I can find also theta cu for the upper interface critical angle for the upper interface. And one thing you must be knowing that total internal reflection ensure that if the angle theta whatever solutions you will be getting that must be greater than the critical angle of the board interface in this case, you know that theta this is n 2 is greater than n 3, that is why I can easily say that theta cl is greater than theta cu.

So, that means theta cl greater than theta cu that means, your theta m solutions must be greater than theta cl at least if it is greater than theta cl that means that it will be obviously greater than theta cu that means, if you get a solution which is greater than theta cl that ensures the total internal reflection is both the interface. If you are getting a solution, which is actually showing something theta m maybe it is greater than theta cu. But less than theta cl then theta m 3 since it is less than theta cl in the lower interface total internal reflection will not take place. So, it will be leaked out so, it will not be guiding so, that information is should be remembering all the time.

(Refer Slide Time: 18:40)



Optical Waveguides: Theory and Design

Slide#10

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$TE :: \vec{E} = (0, E_y, 0);\ \vec{H} = (H_x, 0, H_z)$

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$$

$m = 0, 1, 2, 3 \ldots\ldots$

$$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$$

$K_1 \sin\theta_m$

$\frac{2\pi}{\lambda} n_1 \sin\theta_m = \beta_m$

$K_{1z} =$

n_{eff}^m

TE - TM

$$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}}{n_1 \cos\theta_m}$$

$for\ \theta_m \ge \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1}}{n_1 \cos\theta_m}$$

$for\ \theta_m \ge \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$

Integrated Photonic Devices and Circuits : Lecture-12

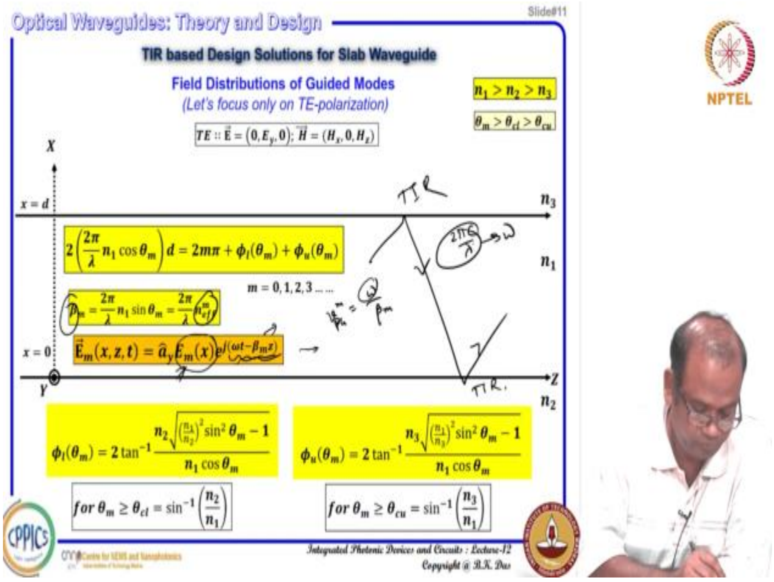
Copyright @ B.K. Das

Now, you see now, as I said that we can actually find the propagation constant and longitudinal direction typically K 1 and sin theta m and K 1 is nothing but the wave vector in homogeneous medium n 1 that is 2pi by lambda n 1 that is K 1 and sin theta m that is actually K 1 z and that we are defining as a beta and if I put in theta m then I will write beta m because theta m is discretized.

So beta m will also will be discretized and this n 1 sin theta m we separately call each other n effective m that means effective index of the mth guided mode and beta m is the propagation constant of the mth guided mode, this is the expression we can write and what is new here? So, this equation all the theta m I have not used anymore this TE, TM something like that we just keep in mind that we will be using this phi expression, this expression this is only for TE polarisation we are using.

Because TE to TM polarisation the difference only in this phase term phase term depolarization will be one expression and TE polarisation will be other expression and simply it will be instead of n 2 by n 1, n 1 by n 2 and n 1 by n 3 that is for TM otherwise the same.

(Refer Slide Time: 20:15)



Optical Waveguides: Theory and Design

Slide#11

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$$

$m = 0, 1, 2, 3 \dots\dots$

$$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$$

$$\vec{E}_m(x, z, t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m z)}$$

$$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$for\ \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$for\ \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$

TIR

$\frac{2\pi c}{\lambda} \to \omega$

$v_p^m = \frac{\omega}{\beta_m}$

TIR.

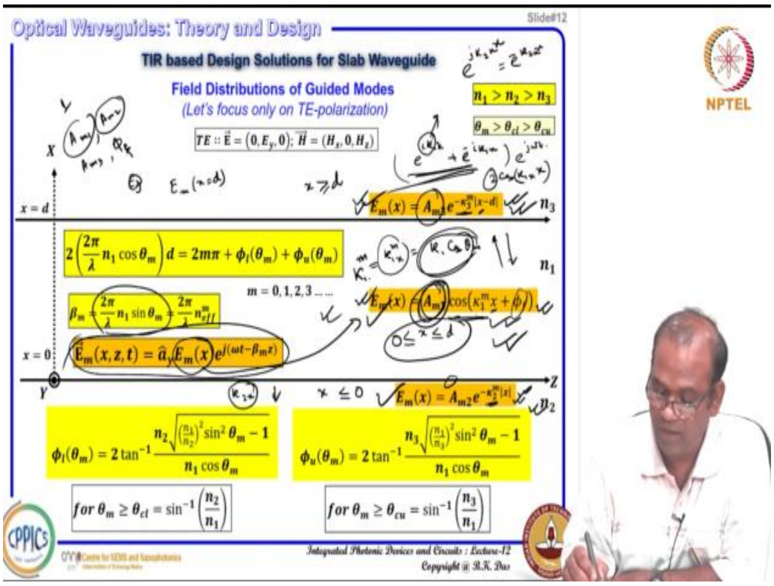
Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

Next, if I try to see what is the solution for the guided mode? So, I will see that because it is a mode, so, I can expect that there will be a good confined well defined distribution along X direction, because along the X direction, you get total internal reflection here and you get the total reflexion here also, TIR happening here and TIR happening here. So, that means your electromagnetic wave is confined within the core region.

So, you can get a just a good well defined x distribution and that particular distribution depends on m I will discuss that and the phase that particular distribution will have a particular solution for beta m and that will propagate along Z direction with this phase parameter omega t - beta m obviously, if this is a phase for a travelling wave the velocity phase velocity you can write for a mth mode you will be omega by beta m.

So, once you know n effective m you beta m and you will know the frequency 2pi c by lambda that is actually your omega then you can find the phase velocity for that particular mode once you get a beta m you can find what is the phase velocity it will be.

(Refer Slide Time: 21:36)



Optical Waveguides: Theory and Design

Slide#12

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$E_m(x \geq d)$

$x \geq d$

$E_m(x) = A_m e^{-\kappa_3^m |x-d|}$

n_3

$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$

$m = 0, 1, 2, 3 \ldots\ldots$

$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$

$\vec{E}_m(x, z, t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m z)}$

$\kappa_{1x}^m = k_1 \cos\theta_m$

n_1

$E_m(x) = A_m \cos(\kappa_1^m x + \phi)$

$0 \leq x \leq d$

$x \leq 0$

$E_m(x) = A_{m2} e^{-\kappa_2^m |x|}$

n_2

$x = d$

$x = 0$

X

Y

Z

$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}}{n_1 \cos\theta_m}$

$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1}}{n_1 \cos\theta_m}$

$for\ \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$for\ \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right)$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

Now, you see I want to see what would be this distribution in x less than d greater than or equal to 0 in this region and when x greater than or equal to d and x less than or equal to 0 as I know that this function in this region it is a standing wave type things. So, standing wave we can represent in terms of cosine or sin function. So, cosine and sin function we can write like this.

Here I have written kappa 1 n because this is related to mth mode, mth mode how it will be direct you have some kind of amplitude A m when we were writing and then cosine kappa 1 m x phi x some phase I have just added what is that kappa 1 m x? That is nothing but it is K 1 x component for the mth mode which is nothing but K 1 cosine theta m. So, K 1 sin theta m is the beta contributing beta K 1 sin theta m is contributing beta and K 1 cos theta m that is actually this direction standing wave is creating.

So, standing wave you can have e to the power iK 1 x + e to the power - iK 1 x if you are just considering this thing one forward propagating wave and another backward propagating wave obviously, they can be multiplied by e to the power j omega t and so, on. If you just add them together what you will get you will be getting plus plus so you will be getting cosine 2 cosine K 1 x times x.

So, this will be the value so, these 2 can be absorbed in the amplitude. So, it is basically a cosine function so, this kappa 1 is nothing but this one kappa 1 m x and we can add phi a little bit of phase can be there so, that I can find out what would be the value because when you put x = 0 you can get some value what is the value here, we can find we can add some phase if it is there, you have to solve that value by utilising the boundary condition I will be discussing later.

So, this is your solution standing wave solution along the x direction within this region. But what happens in this region or in this region x less than 0. So, you know, x less than 0 the x component x direction phase should be decaying because I know that this K to x is imaginary and since it is imaginary, and you do not have any standing wave type of things only get to x only this direction.

It will be there in core and you have K 1 x plus and K 1 x minus standing wave in because of the total internal reflection but when x less than 0 you have only K 2 x in this direction. So, it is like travelling with but once it is imaginary, you will see some kind of attenuation exponential decaying, but here K 1 x is the real value you will be getting as long as theta m is real K 1 is real K 1 x is real.

So you get this thing but once it is imaginary if you put this imaginary value here one more K 1 x is imaginary K 2 x for example e to the power if you just write a j K 2 x and if K 2 is your imaginary then you can write minus K 2 x x. So, this is nothing, so, it will be as a function of x it will be decaying that is what we have written similarly, it is in the this direction so, that means, this is your solution this is your entire mode solution.

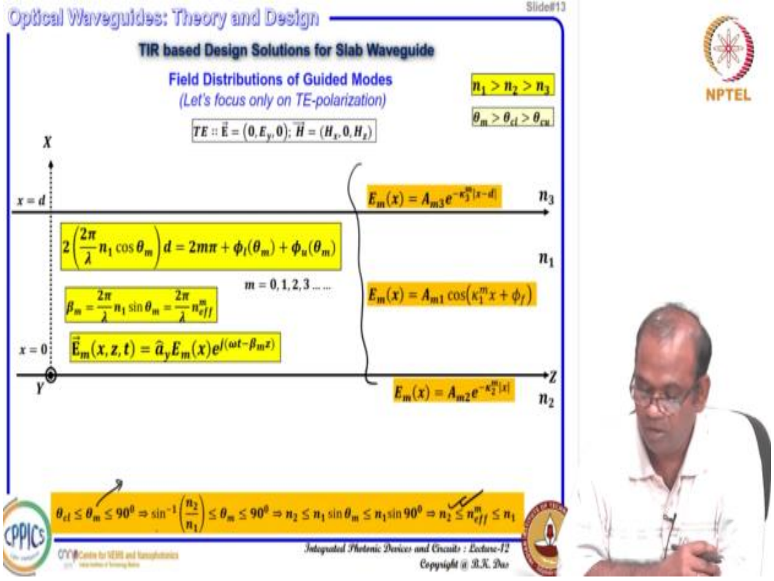
But this entire mode solutions you will know what is the nature in the core region in the upper cladding region and lower cladding region and since it is only y component is there in the electric field, so, tangential. So, I can write I can find out these constants by utilising boundary condition because I can say that e m x = d this is when I am using this one at x = d and this one also x = d both are valid at x = d these 2 equations just x = d I will be putting here x = d.

I will be putting here then I can get one equation. Similarly, I can say that this equation and this equation they are same this would be satisfying because of the boundary condition at x equal to 0. So, if I put x = 0 here x = 0 here you will be getting another equation in this way using those equation I can find what is the relationship between A m 1, A m 2 and A m 3 and also I can find the phase value also has a phi m.

So, once I solve all these I can consider the A m 1 this one this value whatever you are getting that can be just normalised to 1 then I with respect to that I can find out using boundary condition what is the A m 2 value? And what is the A m 3 value? So, once we get that then we can also find phi m. So, in this way I can actually get what is the phase distribution along the x direction that is the mode solution.

And that mode solutions you are getting for a particular theta m once you get a particular theta m particular K 1 x and particular K 1 x will define K 1 kappa 1 here and here we are considering kappa 3 like this and here we are considering kappa 2. So, we need to know kappa 1 x here we can solve easily and we can find out what is the kappa 2 value kappa 3 value I discussed earlier I just repeat here once more.

(Refer Slide Time: 27:34)



Optical Waveguides: Theory and Design

Slide#13

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$

$m = 0, 1, 2, 3 \ldots \ldots$

$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$

$\vec{E}_m(x, z, t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m z)}$

$E_m(x) = A_{m3} e^{-\kappa_3^m |x-d|}$

$E_m(x) = A_{m1} \cos(\kappa_1^m x + \phi_f)$

$E_m(x) = A_{m2} e^{-\kappa_2^m |x|}$

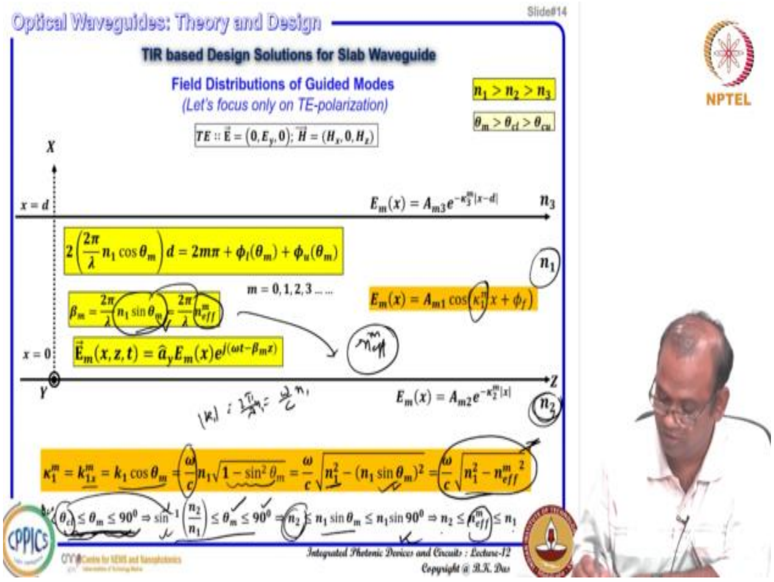
$\theta_{cl} \leq \theta_m \leq 90^0 \Rightarrow \sin^{-1}\left(\frac{n_2}{n_1}\right) \leq \theta_m \leq 90^0 \Rightarrow n_2 \leq n_1 \sin\theta_m \leq n_1 \sin 90^0 \Rightarrow n_2 \leq n_{eff}^m \leq n_1$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

So, this is the solution so, I repeat here all these things, but one things you should keep in mind that your this theta m whatever you solutions you get that should be between theta cl less than theta m, theta m should be greater than critical angle until is the 90 degree that I will be discussing a little while later for the moment you do not need to consider that here so far so, good.

(Refer Slide Time: 28:02)



Optical Waveguides: Theory and Design

Slide#14

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$E_m(x) = A_{m3} e^{-\kappa_3^m |x-d|}$ n_3

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$$

$m = 0, 1, 2, 3 \ldots\ldots$

$$\beta_m = \frac{2\pi}{\lambda}(n_1 \sin\theta_m) = \frac{2\pi}{\lambda} n_{eff}^m$$

n_1

$E_m(x) = A_{m1} \cos(\kappa_1^m x + \phi_f)$

n_{eff}^m

$$\vec{E}_m(x, z, t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m z)}$$

$E_m(x) = A_{m2} e^{-\kappa_2^m |x|}$ n_2

$|k_1| = \frac{2\pi}{\lambda} n_1 = \frac{\omega}{c} n_1$

$$\kappa_1^m = k_{1x}^m = k_1 \cos\theta_m = \frac{\omega}{c} n_1 \sqrt{1 - \sin^2\theta_m} = \frac{\omega}{c}\sqrt{n_1^2 - (n_1 \sin\theta_m)^2} = \frac{\omega}{c}\sqrt{n_1^2 - {n_{eff}^m}^2}$$

$$\theta_{cl} \le \theta_m \le 90^0 \Rightarrow \sin^{-1}\left(\frac{n_2}{n_1}\right) \le \theta_m \le 90^0 \Rightarrow n_2 \le n_1 \sin\theta_m \le n_1 \sin 90^0 \Rightarrow n_2 \le n_{eff}^m \le n_1$$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

Now, let us see what is K 1 m? Here we have used K 1 m, K 1 m is nothing but K 1 x m, K 1 cos theta m I discussed and cos theta m I can write 1 - sin square theta m and K 1 can be K 1 you know K 1 mod = 2pi by lambda and can be written as omega by C. So, we are writing omega by C times n 1, that is K 1 and then sin theta m 1 - sin square theta m and then these n 1 I take inside n 1 square inside the square root n 1 square and then n 1 sin theta m square.

And we know n 1 sin theta m, n 1 sin theta m that is actually nothing but n effective m. So, I have written n 1 sin theta m just n effective m. Now, if you see n effective m which one is higher n 1 or n effective m so, since we can so, easily here that n effective m must be less than or equal to n 1. So, this one always will be positive how that is you just consider this one I know this is the condition for total internal reflection in both interface.

So, it should be theta cl is again greater than theta cu, because lower cladding n 2 is higher than n 3. So theta cu is less than theta cl. So, if you are satisfied solution theta m greater than theta cl that is sufficient. So, you just consented this one now theta cl that means critical angle at the lower interface. Critical angle at the lower interface means n 2, n 1 that means theta cl sin inverse n 2 by n 1 that should be less than or equal to theta m 90 degree. So, this one I just replaced this way.

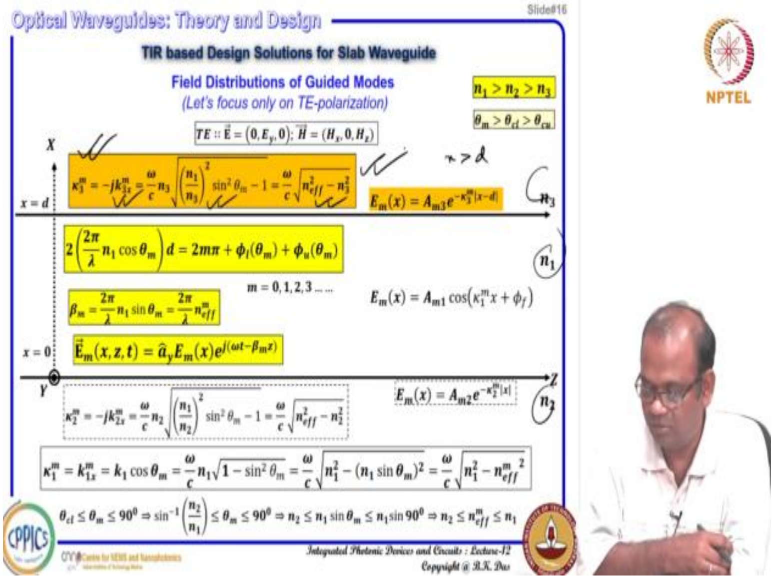
Now, this is the relationship if I just remove the sin inverse then it will remain n 2 and sin inverse here it is coming you are dividing that means it will be sin theta m and then it will be sin 90 degree and n 1 also multiplying both sides up to and removing inverse I just multiplied n 1, n

1 multiplication n 1 goes. So, this side will remain n 2 and this side will remain n 1 sin theta m and this side remain n 1 sin 90 degree so, this is equation.

So, once you get that this n 1 sin theta m is nothing but n effective m that is what we have written here n 1 sin theta m. So, that means, n effective m must be less than or equal to n 1 because it is coming back from here and it must be greater than n 2. So, any solutions you are getting for n effective m, n effective m that solution must be greater than n 2 but less than n 1 if it is greater than n 2 it should be greater than n 3 as well.

So, that ensures the confinement so, whatever solution whatever wave is propagating in this slab waveguide that should have a effective index in between it will be less than or equal to n 1 but should not be lower than n 2, n 2 is higher than this is the n 1 greater than n 2 so, this thing many solutions you get n effective m or any theta m you should get these conditions will be satisfied. So, this is the condition must be satisfied.

(Refer Slide Time: 31:36)



Optical Waveguides: Theory and Design

Slide#16

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\kappa_3^m = -jk_{3x}^m = \frac{\omega}{c} n_3 \sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1} = \frac{\omega}{c}\sqrt{n_{eff}^2 - n_3^2}$

$E_m(x) = A_{m3} e^{-\kappa_3^m |x-d|}$

$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$

$m = 0, 1, 2, 3 \ldots\ldots$

$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$

$E_m(x) = A_{m1} \cos(\kappa_1^m x + \phi_f)$

$\vec{E}_m(x, z, t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m z)}$

$E_m(x) = A_{m2} e^{-\kappa_2^m |x|}$

$\kappa_2^m = -jk_{2x}^m = \frac{\omega}{c} n_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1} = \frac{\omega}{c}\sqrt{n_{eff}^2 - n_2^2}$

$\kappa_1^m = k_{1x}^m = k_1 \cos\theta_m = \frac{\omega}{c} n_1 \sqrt{1 - \sin^2\theta_m} = \frac{\omega}{c}\sqrt{n_1^2 - (n_1 \sin\theta_m)^2} = \frac{\omega}{c}\sqrt{n_1^2 - {n_{eff}^m}^2}$

$\theta_{cl} \le \theta_m \le 90^0 \Rightarrow \sin^{-1}\left(\frac{n_2}{n_1}\right) \le \theta_m \le 90^0 \Rightarrow n_2 \le n_1 \sin\theta_m \le n_1 \sin 90^0 \Rightarrow n_2 \le n_{eff}^m \le n_1$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

So, I have the K 1 value I can write this n effective m is smaller. So, this is a positive value so, standing with I can get easily. Now, what is K 2 x, K 2 kappa 2 I would say kappa 2. So, kappa 2 we you know that is actually the imaginary because transverse component of the x component in the medium 2 that is imaginary we write like that this type of solution and kappa 2 by the way we can solve you remember that we could consider that.

Since there is a tangential component we can consider some fixes as imaginary theta t and that theta t can be converted into using Snail’s law theta m. So, we have already discussed this that this imaginary part since it is an imaginary it is an imaginary field in the cladding region. So, that imaginary part will be coming 1 minus this one because this will be always greater than 1 when theta m is greater than critical angle.

So, that is why it is also you just multiply into inside you are getting like this expression this is your kappa 2. This is now you see n effective square must be greater than n 2 that is what we have shown that n effective m I would write here n effective because the m is either this would be m, m should be written here typo mistake here. So, that means, this is n effective square is greater than n 2.

So, this is a positive value and positive value that means, I take x mod because this side is x negative. So, I can express that this side once you know the kappa 2 from this formula, if you know lambda, then you can calculate omega and you can actually if you know n effective then you can calculate your kappa 2 then you will know how much it will be stressed. So, if you know kappa 2 m then you can say that that particular mth mod how much it will be penetrating 1 by eth it will be delta m penetration depth will be delta m = kappa 2 m.

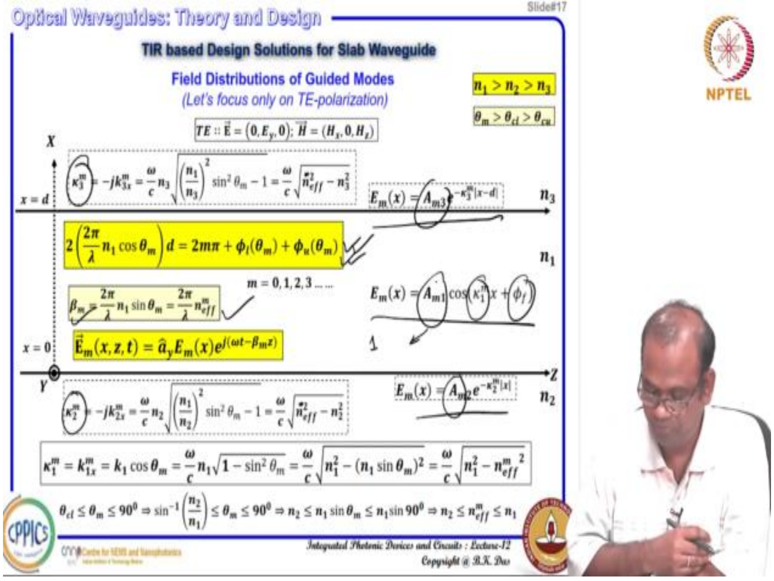
So, for mth mode if I can calculate kappa 2 I can find what is the penetration depth of the mth mode exponentially to be decaying and here you can see some kind of sinusoidal wave and they are also exponential decay. So, you can get these types of solutions or you can get this type of solution like this these are the guided mode when it is exponential decaying but in between this will be oscillating like cosine function.

So, they will be actually guided mode and each of them depending on their value they will be propagating with a phase velocity so, this is the idea whole idea. So, we get kappa 2 similarly, we can find kappa 3 and we can say that beyond when x greater than d there is entailed. So, I write K kappa 3 m x - d kappa 3 also same way I can write here, I can express this way I can express this way so, this is also positive value.

So, once we know lambda I can find how much it will be penetrating. So, thing is that you can easily find out which particular mode if you know the effective index of a particular specific mode you can say and if you know of course n 1 and n 3 value and if you know lambda then you can find how much it will be penetrating in the cladding region that is very important cladding region how much it is penetrating that actually is the important part of integrated optics that is reminiscent tail reminiscent field strength.

That actually helps how much it is insulated to the neighbouring waveguide or how much it can be coupled to the neighbouring waveguide that is very important that is why I would emphasise more on this. This expression you should keep in mind all the time and you know, it is very easy to derive as I have mentioned earlier discussed earlier.

(Refer Slide Time: 35:34)



Optical Waveguides: Theory and Design

Slide#17

TIR based Design Solutions for Slab Waveguide

Field Distributions of Guided Modes

(Let's focus only on TE-polarization)

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$TE :: \vec{E} = (0, E_y, 0); \vec{H} = (H_x, 0, H_z)$

$\kappa_3^m = -jk_{3x}^m = \frac{\omega}{c} n_3 \sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1} = \frac{\omega}{c}\sqrt{n_{eff}^{m\,2} - n_3^2}$

$E_m(x) = A_{m3} e^{-\kappa_3^m |x-d|}$

$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$

$m = 0, 1, 2, 3 \ldots\ldots$

$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$

$E_m(x) = A_{m1} \cos(\kappa_1^m x + \phi_f)$

1

$\vec{E}_m(x, z, t) = \hat{a}_y E_m(x) e^{j(\omega t - \beta_m z)}$

$\kappa_2^m = -jk_{2x}^m = \frac{\omega}{c} n_2 \sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1} = \frac{\omega}{c}\sqrt{n_{eff}^{m\,2} - n_2^2}$

$E_m(x) = A_{m2} e^{-\kappa_2^m |x|}$

$\kappa_1^m = k_{1x}^m = k_1 \cos\theta_m = \frac{\omega}{c} n_1 \sqrt{1 - \sin^2\theta_m} = \frac{\omega}{c}\sqrt{n_1^2 - (n_1 \sin\theta_m)^2} = \frac{\omega}{c}\sqrt{n_1^2 - n_{eff}^{m\,2}}$

$\theta_{cl} \le \theta_m \le 90^0 \Rightarrow \sin^{-1}\left(\frac{n_2}{n_1}\right) \le \theta_m \le 90^0 \Rightarrow n_2 \le n_1 \sin\theta_m \le n_1 \sin 90^0 \Rightarrow n_2 \le n_{eff}^m \le n_1$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

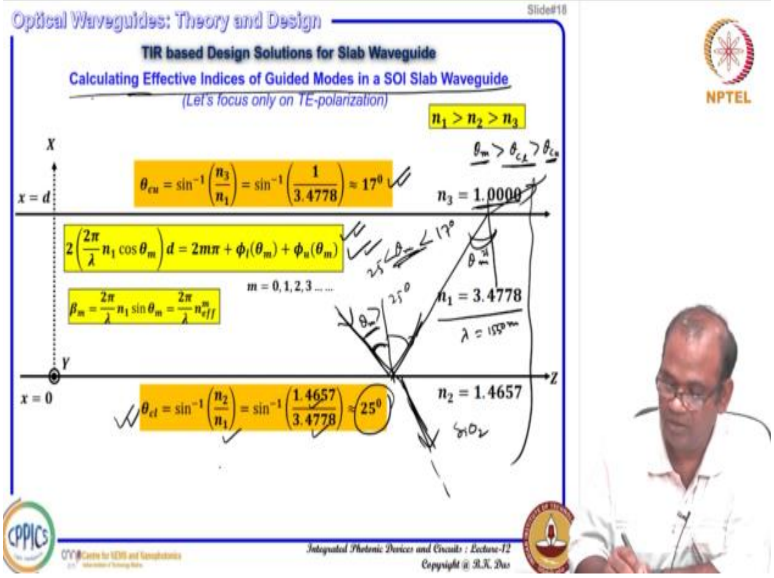
Now, all the equations relevant equation for field distribution in 3 different region corresponding constants how to solve this thing, this thing and this phi f and this thing they are related through boundary condition because tangential component this is U i component. So, continuous are the boundary I can put down I can find out easily I leave this as your homework assignment to find out what is the relationship between these things.

And once you find that relationship, you can just set this one as your 1 then you can easily you can plot using some MATLAB programme or Python programmer C Programming etcetera what would be the mode looking like but 1 important parameter you need to solve first to see different modes because you need to solve this theta m, but theta m solution is this one and you can solve theta m from here.

And then once you solve theta m then you can find the theta m and n effective m once you know n effective m you can find out this is a m is missing here this is also m this a mth mode I would say missing here. So, you can find what is the kappa 2 m for mth mode what is kappa 2 value?

For mth mode what is kappa 3 m? And for mth mode what is kappa 1? Then you can just plot and you can find out how is the field distribution can be in the slab waveguide. So, only thing is that you need to know n effective m then you can find what is the nature is really straightforward.

(Refer Slide Time: 37:08)



Optical Waveguides: Theory and Design

Slide#18

TIR based Design Solutions for Slab Waveguide

Calculating Effective Indices of Guided Modes in a SOI Slab Waveguide

(Let's focus only on TE-polarization)

$n_1 > n_2 > n_3$

$\theta_m > \theta_{cl} > \theta_{cu}$

$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) = \sin^{-1}\left(\frac{1}{3.4778}\right) \approx 17^0$

$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$

$m = 0, 1, 2, 3 \ldots\ldots$

$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$

$\theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.4657}{3.4778}\right) \approx 25^0$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

So, I move on to that how to get that. So, I just consider SOI silicon on insulator, I just want to calculate calculating effective indices of guided modes in SOI, you get that is what our next target once we solve that one effective index then we know almost all properties have a slab waveguide needed to understand the mode concepts everything we will know. So, I know that that my theta m should be whatever solutions you get from here that should be greater than theta cl and since theta cl is greater than theta cu.

So, that means if I ensure that it is the solutions is greater than theta cl then it is obviously greater than theta cu critical angle in the upper interface critical angle the lower interface then I can say that that solutions not only ensuring total internal reflection also ensuring the page condition to form a mode. So, I tried to find out the critical angle for silicon on insulator substrate where your n 1 is refractive index this one lambda obviously, we are considering around 1550 nanometer wavelength and at the same wavelength.

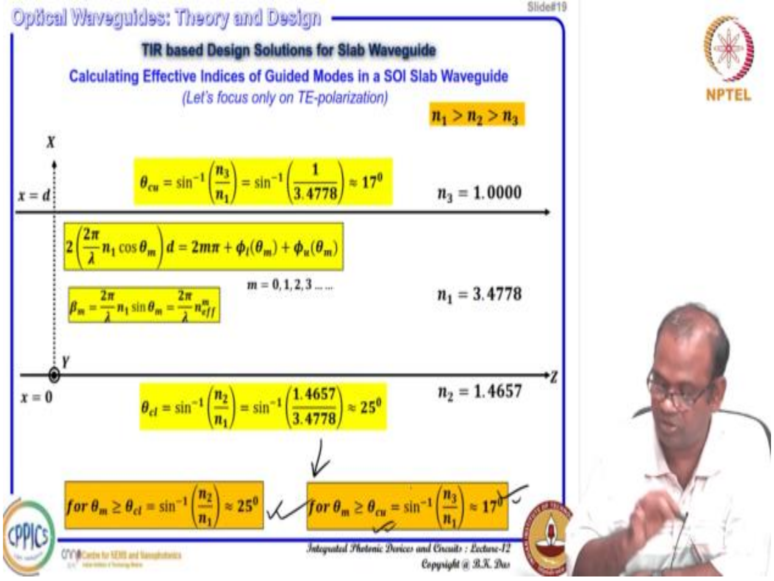
If top layer is air, that is actually 1 and bottom layer is silicon dioxide. So 1.4657 utilising this I calculated what is the critical angle critical angle follower interface sin inverse n 2 by n 1, n 2 I put here n 1 I put here then I get 25 degree. So, that means anything if it is coming this angle theta m that theta m should be greater than 25 degrees this should be greater than 25 degrees should be greater than theta cl.

So, whatever angle it is coming to go for total internal reflection that angle solutions should be greater than 25 degree in the upper interface which is actually 17 degree same thing critical angle you calculate a 17 degree but silicon on insulator. So, that means if your this angle theta m is greater than 25 degree obviously, when it is it is hitting here, there is also the time that is also greater than 25 degree because this one and this one should be same.

So, if you are inserting 25 degree here, that 25 degree also coming here also that means which is obviously greater than 17 degree, so that means total internal reflection will happen. So anything this theta m less than 25 degree that means it will be leaking something here total internal reflection will not take place. So, if it is something time you are getting it is greater than 25 degree but less than 17 degree.

That means in this range, it can get total internal reflection here, but since it is less than 17 degree it will be leaking here upper interface. So, that time here it will not leak to the substrate lower cladding, but it will be leaking to the here. So, you have to make sure that this theta m should be greater than theta cl and theta cu and that calculation you can do this thing this is fine.

(Refer Slide Time: 40:26)



Optical Waveguides: Theory and Design

Slide#19

TIR based Design Solutions for Slab Waveguide

Calculating Effective Indices of Guided Modes in a SOI Slab Waveguide

(Let's focus only on TE-polarization)

$n_1 > n_2 > n_3$

$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) = \sin^{-1}\left(\frac{1}{3.4778}\right) \approx 17^0$

$n_3 = 1.0000$

$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$

$m = 0, 1, 2, 3 \ldots\ldots$

$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$

$n_1 = 3.4778$

$\theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.4657}{3.4778}\right) \approx 25^0$

$n_2 = 1.4657$

$for\ \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \approx 25^0$ ✓

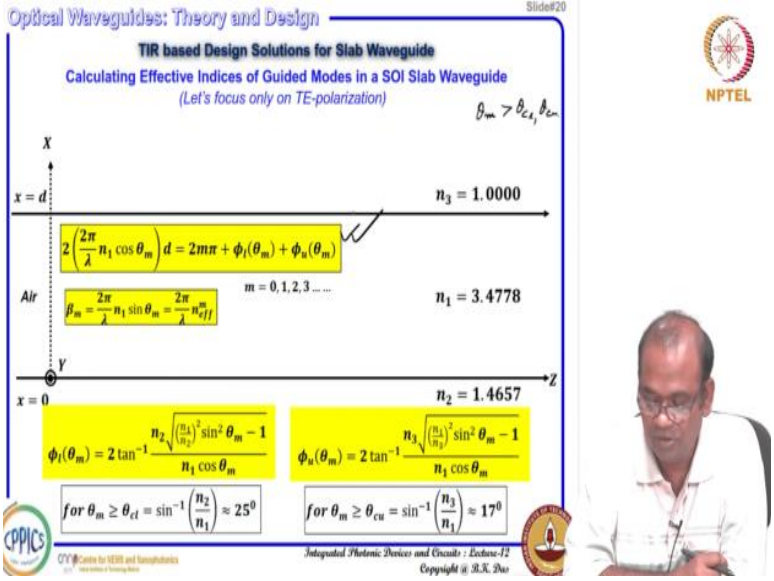
$for\ \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) \approx 17^0$ ✓

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

Next thing is that this is what I have just written down here same thing theta m would be greater than theta cl and theta m should be greater than theta cl. So, whatever solutions you get, but this condition I have calculated 70 degree and 25 degree.

(Refer Slide Time: 40:40)



Optical Waveguides: Theory and Design

TIR based Design Solutions for Slab Waveguide

Calculating Effective Indices of Guided Modes in a SOI Slab Waveguide

(Let's focus only on TE-polarization)

$\theta_m > \theta_{cl}, \theta_{cu}$

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$$

$m = 0, 1, 2, 3 \dots\dots$

$$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$$

$n_3 = 1.0000$

$n_1 = 3.4778$

$n_2 = 1.4657$

$$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$\text{for } \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \approx 25^0$$

$$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

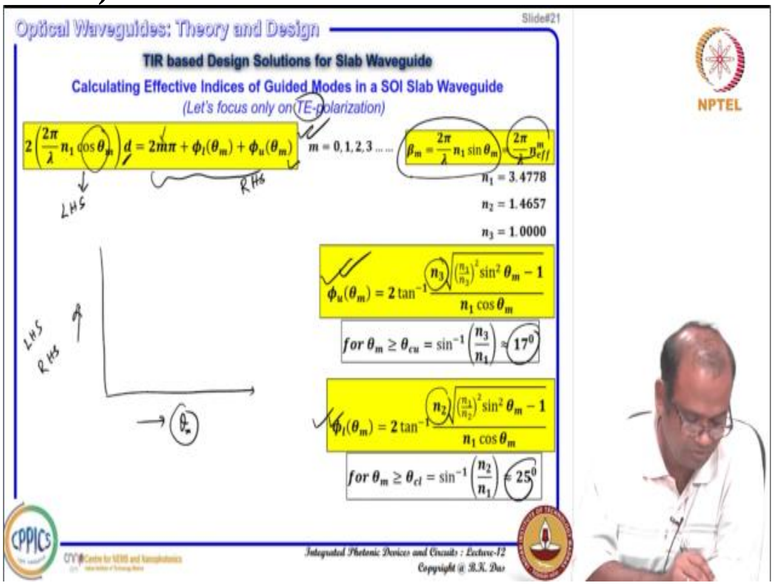
$$\text{for } \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) \approx 17^0$$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

Now, my intention is that how to find out this theta m solutions which satisfy at least it should be greater than theta cl obviously theta cl theta cu should be there how to solve because so, far we have not discussed how to solve this one I said that this is transcendental equation which is not so, easy to solve I will try to show how we could solve in a graphical way.

(Refer Slide Time: 41:10)



Optical Waveguides: Theory and Design

TIR based Design Solutions for Slab Waveguide

Calculating Effective Indices of Guided Modes in a SOI Slab Waveguide

(Let's focus only on TE-polarization)

$$2\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m)$$

$m = 0, 1, 2, 3 \dots\dots$

$$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$$

$n_1 = 3.4778$

$n_2 = 1.4657$

$n_3 = 1.0000$

$$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2 \sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$\text{for } \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) \approx 17^0$$

$$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2 \sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$\text{for } \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \approx 25^0$$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

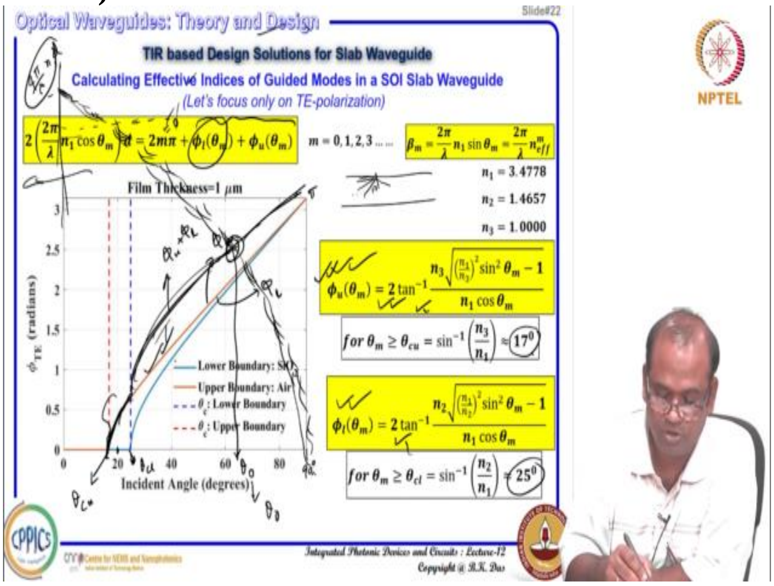
Let us see I have written this one this equation I have written here that thing simply rewritten there this one I just 2mpi phi l theta m and phi u theta m, phi u theta m is this expression phi l theta m is this one that means lower interface whatever phase because of the total internal reflection and upper interface what is the phase happening because of the total internal reflection because upper interfaces n 3 lower interface n 2.

That is why this is the there obviously, this is for TE polarisation you should keep in mind and beta m whatever solution this thing I have just written for your understanding purpose and I have also shown the calculated value here to keep in mind while solving this thing. Now, you see I just tried to you say what we could do I can plot a figure this axis is theta and these I can call these at the left hand side and this entire thing I can call it right hand side right hand side I have different values m = 0.

I can put one right hand side m = 1 I will get another right side m = 3 another right side 4 any number you put right hand side is changing, but left hand side what will be changing because for a given thickness of the waveguide n 1 it is fixed lambda is fixed. So, only thing is that theta m will be changing. So, I can plot as a function of theta what is your left hand side and I can plot as a function of theta what is your right hand side, left hand side and right hand side.

I be plotting then I can find that that 2 curve will be intersecting in this plot and that intersecting is the solution for theta m. That I will be getting a one particular theta m solutions. I will just for that first I will so that if we just plot only this right hand side for m = 0 how it will be looking like.

(Refer Slide Time: 43:25)



Optical Waveguides: Theory and Design

Slide#22

TIR based Design Solutions for Slab Waveguide

Calculating Effective Indices of Guided Modes in a SOI Slab Waveguide

(Let's focus only on TE-polarization)

$$2\left(\frac{2\pi}{\lambda}n_1\cos\theta_m\right)d = 2m\pi + \phi_l(\theta_m) + \phi_u(\theta_m) \quad m = 0, 1, 2, 3 \ldots$$

$$\beta_m = \frac{2\pi}{\lambda}n_1\sin\theta_m = \frac{2\pi}{\lambda}n_{eff}^m$$

$n_1 = 3.4778$

$n_2 = 1.4657$

$n_3 = 1.0000$

$$\phi_u(\theta_m) = 2\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2\sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$for\ \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) \approx 17^0$$

$$\phi_l(\theta_m) = 2\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$for\ \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \approx 25^0$$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

It is shown here this is shown here that for m = 0 you are considering if you just put m = 0 then you have phi u expression this one and phi l expression this one and you will know this thing these 2 equation is valid only for theta m greater than upper critical angle this is for upper interface and this one also true only if it is greater than critical angle less than critical angle these values are 0 no phase change happens and exactly critical angle this thing will be also 0 just above the critical angle.

If theta value then you see the phase change happening. So if you just consider say upper interface, this one upper interface, what is the critical angle 17 degree, so this is 20 degree this x axis this is 20 degree, so this is the 70 degree point. And from 17 degree if you plot if you see that the phase is increasing as a function of theta up to 90 degree, this is your 90 degree and increases up to phi.

So, your theta, suppose you have a waveguide here and core region here. So, that means, your phase change total internal reflection starts happening starting from 17 degree in the upper interface. So, this is your 70 degree and if you just keep on increasing, increasing up to 90 degree then it will be sync total internal reflection and phase also it will be changing why phase.

It will be changing because of your different component the tangential component and normal component of the vector is going to change that is the reason phase will be changing. So, it will be if you just plot with a MATLAB programme you will find this red curve and if you just plot what is happening phase change for the lower it will start from 25 degree this is 20 degree or 25 degree it is starting.

So, for m = 0 I can plot this one this is actually corresponding to phi u that is phi u and this one corresponding to phi l and here this is actually we can call it theta cu and they know the theta cl and this is actually theta cu, theta cu is 17 degrees here you have your theta cl. Now, if you want to add them together this 2 this is phi u this is phi l. So, if you add them together up to this point you have other value phi l is 0 from here.

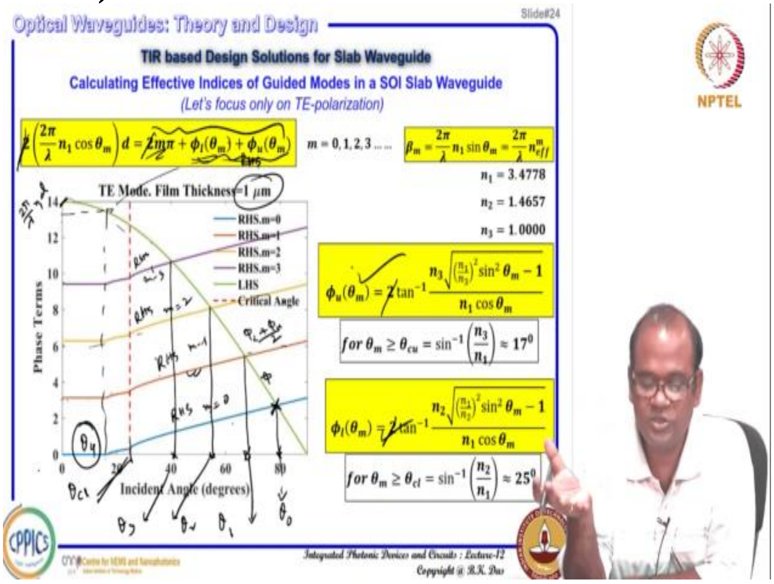
If we add them together you will be getting something like this. So, this one will be added here sorry I can be a little bit cleaned up. So, you can say something like this, this will be like this, because this value will be added again phi. So, these will be your this curve can be considered as a phi u + phi l for m = 0 and m = 2pi means all the data points you are getting you have to add 2pi more and if m = 2 that means, again you have to add 4pi same curve, the curve will be seated upward 2pi upward, 4pi upward, 6pi upward.

So, this curve you can generate depending on the many set of curves you can generate just changing m value. So, but left hand side if I just plot as a function of theta m so, how it will be looking like? So, theta m you are just changing from this one, theta m = 0 this will be 4pi by lambda 4pi by lambda n 1 some value you will be getting some value 4pi this means 6 lambda 1.55, so, 12 so, it is more than 10 somewhere here you will be getting maybe here then you will be getting something like this.

So, it will be your left hand side like this it is a cosine function at theta m 90 degrees it will be 0 and theta m 0 it will be 4pi somewhere this one 4pi by lambda n 1 d. So, that will be the value add here and it will be there and where this left hand side and right hand side crossing for m = 0 you will get that value theta that will be your theta 0. So, you are getting m = 0 value theta 0 solutions you are getting.

Now, this curve if you add 2pi then what you will be getting 2pi maybe you will be getting 6pi somewhere here you will be getting and then after 2pi adding again that curve will be added so, you get another solution there so, that is clearer here.

(Refer Slide Time: 48:25)



Optical Waveguides: Theory and Design

Slide#24

TIR based Design Solutions for Slab Waveguide

Calculating Effective Indices of Guided Modes in a SOI Slab Waveguide

(Let's focus only on TE-polarization)

$$\cancel{2}\left(\frac{2\pi}{\lambda} n_1 \cos\theta_m\right) d = \cancel{2}m\pi + \phi_l(\theta_m) + \phi_u(\theta_m) \qquad m = 0, 1, 2, 3 \ldots$$

$$\beta_m = \frac{2\pi}{\lambda} n_1 \sin\theta_m = \frac{2\pi}{\lambda} n_{eff}^m$$

$n_1 = 3.4778$

$n_2 = 1.4657$

$n_3 = 1.0000$

$$\phi_u(\theta_m) = \cancel{2}\tan^{-1}\frac{n_3\sqrt{\left(\frac{n_1}{n_3}\right)^2\sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$for\ \theta_m \geq \theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) \approx 17^0$$

$$\phi_l(\theta_m) = \cancel{2}\tan^{-1}\frac{n_2\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2\theta_m - 1}}{n_1\cos\theta_m}$$

$$for\ \theta_m \geq \theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \approx 25^0$$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

So, if we just show that maybe it is better to look into it. So, this is solved for the right hand side that means this one right hand side for m = 0 that this curve that means m 0 this one 0 I am adding this to then I get this curve just use this curve and this curve plus and then you just plot and then m = 1 you see you have to I think what happens in this curve it is supposed to be 2pi means 4pi but here you are adding just 3 point something pi you added that is because these 2 term, 2 term is cancelled.

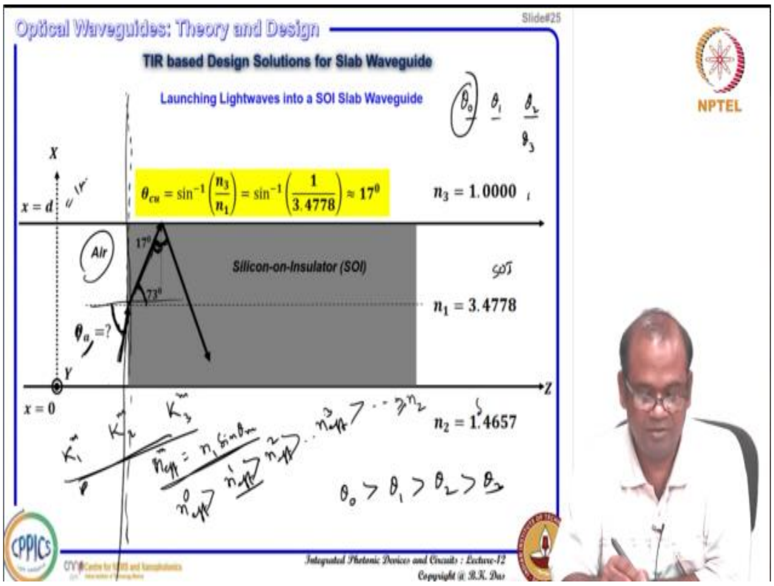
So 2, 2 cancel here and this is 2 also that means this one actually calculated for phi u total divided by 2. So how it is plotted? It is not pi l theta m 2 times, 2 is not multiplied. So, left hand side also these 2 are not multiplied. So, that means this value is nothing but 2pi by lambda n 1 d, d is the thickness of the film that one is this one and this one is basically phi l + phi u by 2 that is whatever value divided by 2 it is plotted.

So, you see, if you are just adding m = 1 then you get this plot and m = 2 you get this plot of the right hand side right hand side for m = 1 right hand side for m = 2 right hand side this one, this one, this one, this one for m = 3 and left hand side you are just putting 2pi lambda n 1 cos theta this thing you are plotting. So, you get a one solution here and here and here this value you are getting one solution you are getting one solution here you are getting on solution here and for this calculation you have considered d = 1 micrometre.

So, if d = 1 micrometre if you are considering these solutions you are getting this is for m = 0. So, whatever solution you will be getting that is actually theta 0 and the solution you will be getting theta 1 the solution theta 2 the solution this is actually critical this is actually what? This is 25 degrees this is actually theta cl if you add one more for m = 4 you get a solution somewhere here another 3 if you add here.

So, it is 9 point something, so, somewhere here you will be getting and that solutions you will be getting, which is actually theta 4 you see this theta 4 is less than theta cl. So, this solution is not acceptable that will not give you guided mode. So, it is actually the very easy way if you just use your MATLAB numerically, you just can solve all these theta 0, theta 1, theta 2, theta 3.

(Refer Slide Time: 51:43)



Optical Waveguides: Theory and Design

Slide#25

TIR based Design Solutions for Slab Waveguide

Launching Lightwaves into a SOI Slab Waveguide

$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) = \sin^{-1}\left(\frac{1}{3.4778}\right) \approx 17^0$

$n_3 = 1.0000$

$n_1 = 3.4778$

$n_2 = 1.4657$

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

And once you get them, so, you can find your so called so, n effective m you will be getting for example, your say n effective n 1 sin theta m so, all the value you can enter then you can get a n effective 0, n effective 1, n effective 2, n effective 3 since theta 0 we have solved the theta 0 is greater than theta 1 greater than theta 2 greater than theta 3 if you just go back to the previous slide.

So, theta 0 greater than theta 1 this one theta 1 greater than theta 2 theta 2 greater than theta 3. So, higher order mode your theta value will be reducing, reducing. So, if that is the case, so, we can say that n effective 0 will be greater than this one because that value is this one theta 0 is higher. So, this one and so on you will be getting. So, in this way you can find a solution when actually n effective is exactly greater than some value exactly it is n 2 or slightly higher that will be the highest order mode that can be supported by the slab waveguide.

Now, so, we know now, how to calculate n effective once we know how to calculate n effective you can find out what is that kappa 1 and m and what is that again kappa 2 m and kappa 3 m because they actually involve in the field distribution we have shown earlier. If you just go back here I just discussed here you see this kappa 3 m, kappa 2 m, kappa 1 m they all are actually n effective dependent, n effective once you solve then you get all the parameters.

And then you can find out how it will be distributed how we exponentially decaying in this top cladding bottom cladding and how would we it is actually sinusoidally so, you can easily plot your electric field. So, you can try that you can solve it give a certain value of d = 1 micrometre it is given you can use 1 micrometre and try you know that for 1 micrometre you get 4 solutions for TE polarisation, you can get also similar type of solution for TM polarisation.

So, if you are 1 micron thickness, you are dealing with 1 micron thickness of the SOI then you can say 4 different modes for TE polarisation, 4 different modes for TM polarisation but normally you know, these are the general discussion, but most of the time you would be interested to have a sorry, you need to have a single mode guide. So, in that case what to do, what do you have do?

You have to do you are just getting this value this curve that is that left hand side, if this left hand side can be reduced downward suppose you are getting something like this something you are getting like this. So, you get only 1 solution sorry 1 solution here below 1 solution below the above the critical angle. So, if I reduce this value, how we can reduce that either I can reduce n 1 or increase lambda or decrease d since material platform n 1 is fixed lambda is fixed what I can do I can reduce the thickness.

So, if I reduce the thickness slab thickness then you can answer that it can be designed for just 1 mode in vertical direction x direction that is why the d value typically it is used 220 nanometers, so, that vertical direction you can at least you can confine only 1 mode not more than that, if it is more than that, then actually it will be multimodal at least vertical direction. So, that is the reason all the foundry and silicon photonics industry photonic circuits everything.

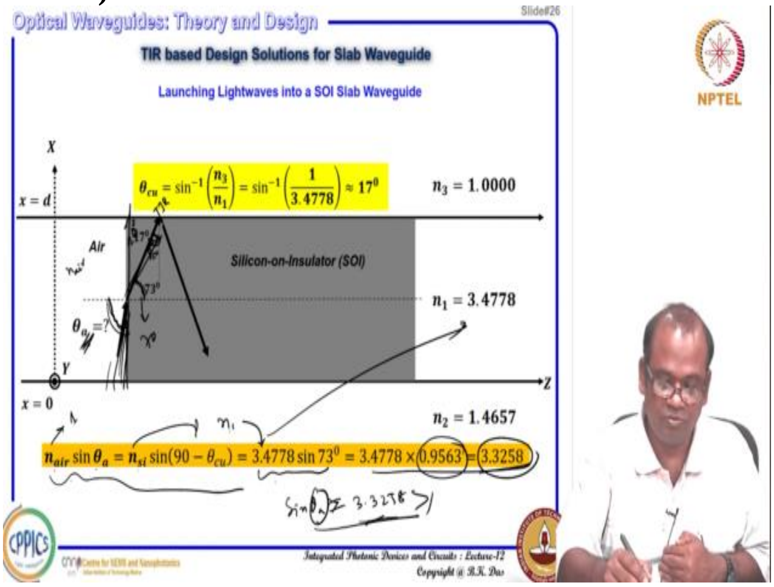
They are fabricating with a SOI device layer thickness of around 220 nanometer or less than that we can so, that even less than 220 nanometer, even 45 nanometer thickness, 30 nanometer thickness people also demonstrated single motor guide in the vertical direction. Now, next thing that important so, I want to learn that we have a slab waveguide and we have a discrete solution for theta 0, theta 1, theta 2 and theta 3 these 4 solutions are there for d = 1 micrometre.

For example and they are basically you can find out the solutions are theta 0 is about 80 degree and theta 1 = 70 degree and so on you are getting so, I would like to know when I am launching light suppose you have somehow diced here vertically placed to the substrate this is your device layer SOI and this is silicon dioxide below will be substrate and air now, you just cut this point and polished and you want to launch a light you have 4 different modes it can support TE polarisation TM polarisation also another 4.

But I want only to excite a theta 0 which actually even though it supports 4 more than 4 modes, but I want to excite only theta 0 then I can ask you a question that what would be the angle of incident here in the outside from here. So, that you get exactly theta 0 with this example, I tried to give you a very good calculation that what is the maximum possible angle you can have.

So, that you are this angle will be exactly critical angle. So, you are making an incident here this is your interface this is your normal and this is incident wave and this is your transmitted wave. So, here angle is theta a and if it is 17 degree critical angle this one sorry this one only this one critical angle, then this one will be it would be 73 degree and here it is n 1 and it is here.

(Refer Slide Time: 58:33)



Optical Waveguides: Theory and Design

Slide#26

TIR based Design Solutions for Slab Waveguide

Launching Lightwaves into a SOI Slab Waveguide

$$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) = \sin^{-1}\left(\frac{1}{3.4778}\right) \approx 17^0$$

$$n_{air} \sin\theta_a = n_{si} \sin(90 - \theta_{cu}) = 3.4778 \sin 73^0 = 3.4778 \times 0.9563 = 3.3258$$

$\sin\theta_a = 3.3258 > 1$

Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

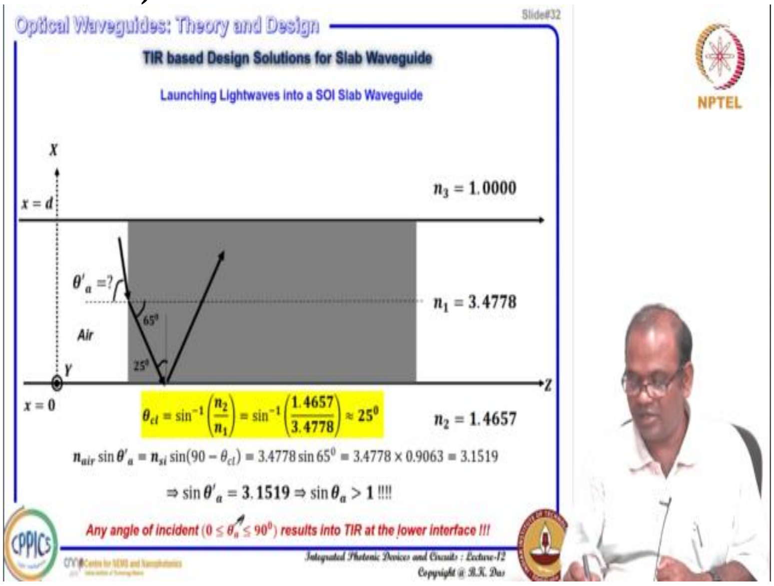
So, what I can do, I can use a Snail’s law. If you use a Snail’s law, use Snail’s law that means n air refractive index here is n air, n air sin theta a = n silicon sin 90 - theta cu this is theta cu 90 - theta su that is 17 degrees. So, here in this case n 1 is n silicon is nothing but in one nothing but this one that is coming here, I put down I get this value, this value I get because science 73 degrees about 0.9563 if you multiply with n silicon, you get 3.32 by the simple calculation, but very odd.

If you see n air is 1 then sin theta a whatever you are getting sin theta a = 3.3258 which is greater than 1 sin theta greater than 1 that means theta a must be imaginary sin theta cannot be greater than 1 must be imaginary. So, that means, this is the angle where I can have critical angle this theta a exactly critical angle which is coming as a imaginary if theta value less than that, then only if it is less than 73 degree will be less this incident of 73 degree to be less so, if it is less than it will be more than 70 degree.

So, suppose it is 70 degree so, that this will be 20 degree then if it is 20 degree that means, more than critical angle, so, here total internal reflection can take place. So, that means, whatever solution I am getting here that is actually that should be something the maximum possible theta a and that maximum well possible theta a is imaginary. So, that is the reason actually at any angle you just go from the incident that actually create insight it is greater than critical angle if you are of course.

If you are not increasing exactly liquid angle if you are giving so, whatever angle it is coming here appearing inside the crystal in SOI device layer that will be greater than 90 degree. So, basically this theta a is you can consider as a 90 degree this exactly 90 degrees theta a that is acceptance angle. So, because of the very high refractive index of the silicon device layer your acceptance angle is very wide. So, anywhere any angle if you just launch light can be coming and total internal reflection can take place.

(Refer Slide Time: 01:01:29)



Optical Waveguides: Theory and Design

Slide#32

TIR based Design Solutions for Slab Waveguide

Launching Lightwaves into a SOI Slab Waveguide

$$\theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.4657}{3.4778}\right) \approx 25^0$$

$$n_{air} \sin\theta'_a = n_{si} \sin(90 - \theta_{cl}) = 3.4778 \sin 65^0 = 3.4778 \times 0.9063 = 3.1519$$

$$\Rightarrow \sin\theta'_a = 3.1519 \Rightarrow \sin\theta_a > 1 \text{ !!!!}$$

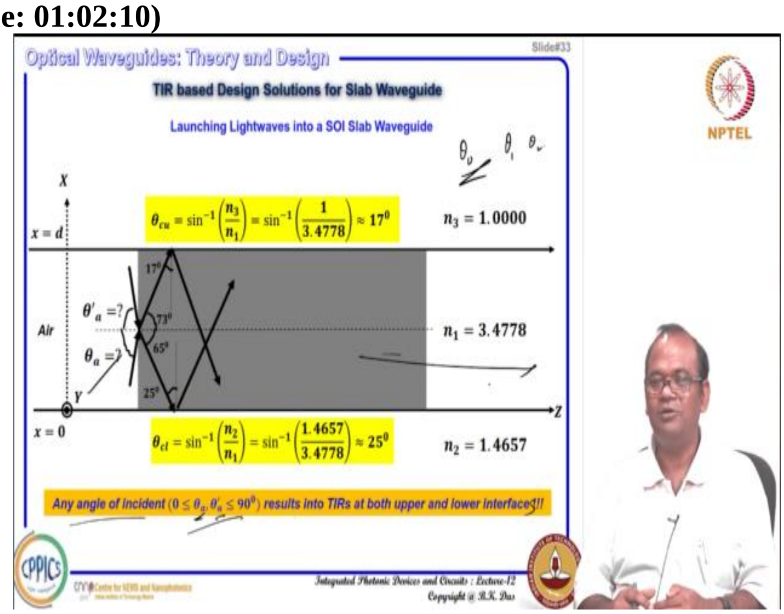
Any angle of incident ($0 \le \theta'_a \le 90^0$) results into TIR at the lower interface !!!

Integrated Photonic Devices and Circuits : Lecture-12

Copyright @ B.K. Das

Same argument is true if you are just considering the other side also. So, here theta a prime also this is coming also here in this calculation if you see it is also coming greater than 1. So, it is something that anywhere this side or this side anywhere any direction you sin, so, that will satisfy your more than critical angle in both interface and that will be guiding. So, that is very interesting easy to couple light inside the silicon device layer. So, that is why we say that any angle of incident that is theta a, that is 0 to 90 degree results into total internal reflection at the lower interface.

(Refer Slide Time: 01:02:10)



Optical Waveguides: Theory and Design

Slide#33

TIR based Design Solutions for Slab Waveguide

Launching Lightwaves into a SOI Slab Waveguide

$\theta_{cu} = \sin^{-1}\left(\frac{n_3}{n_1}\right) = \sin^{-1}\left(\frac{1}{3.4778}\right) \approx 17^0$

$n_3 = 1.0000$

$n_1 = 3.4778$

$\theta_{cl} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.4657}{3.4778}\right) \approx 25^0$

$n_2 = 1.4657$

Any angle of incident ($0 \le \theta_a, \theta'_a \le 90^0$) results into TIRs at both upper and lower interfaces!!

Integrated Photonic Devices and Circuits : Lecture-12
Copyright @ B.K. Das

And so, finally, we can say that any angle of incidence of theta a, theta prime results into total internal reflections at both upper and lower interfaces. So, I can launch any direction it can be satisfying one of the solutions you desire theta 0, theta 1, theta 2. So, you do not need to have that that solution can come to this incident angle that can be anywhere it can be, but total internal reflection can take place that particular angle ensure total internal reflection.

But may not ensure the solution for guided mode, guided mode solutions whatever way we discussed that has to be followed hopefully, you get an idea how a slab waveguide works, I have taken example of silicon on insulator, but any slab waveguide you consider indium phosphide slab waveguide, gallium arsenide slab waveguide, lithium niobate insulator slab waveguide, silicon nitride slab waveguides.

The principle same you can calculate and you can design your how many modes will be there depending on the thickness, how many modes it can be supported, and what would be the minimum thickness required or maximum thickness required so, that you can particle direction only 1 mode it can support if it is 1 mode, it is supporting whatever angle you will sin if at all it is guiding that will be fundamental mode because other mode even if it is excited that will be leaked that will not carry energy propagate as a mode.

So, if it is a single mode, I can ensure that that particular mode has a fixed phase velocity. So, I do not need to bother about that, but if it is a multimode different modes are there in the slab waveguide region. So, all the modes will travel with a different phase velocity so, that is called modal dispersion. So, that is a problem for designing devices, but this is something ground level work for understanding guided modes in slab waveguide.

And then we will be discussing same approach I will be just considering how to design a 2D waveguide to the confinement and what would be the possible solutions and then I will discuss that If there is any limitations or not, and if some limitations are there, the alternative path we have to consider for understanding more rigorous photonic integrated circuits, more complex photonic integrated circuits, with this, I stop here. Thank you very much.