Integrated Photonics Devices and Circuits
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Lecture - 16
Optical Waveguides: Theory and Design: Optical Waveguide with 2D Confinement

Today in this lecture we will continue waveguide theory and design. So, far we have already discussed the details about 1 dimensional optical waveguide, slab waveguide and starting from modal analysis to dispersion curve all those things we have discussed in the previous lecture and today we just take from there to understand how actual practical waveguide can be implemented or designed.

I will say 2D waveguide, 2D confinement that means, a guided mode will be confined in 2 direction and it will be allowed to propagate in the third direction. So, that is the practical waveguide, we can use for photonic integrated circuits.

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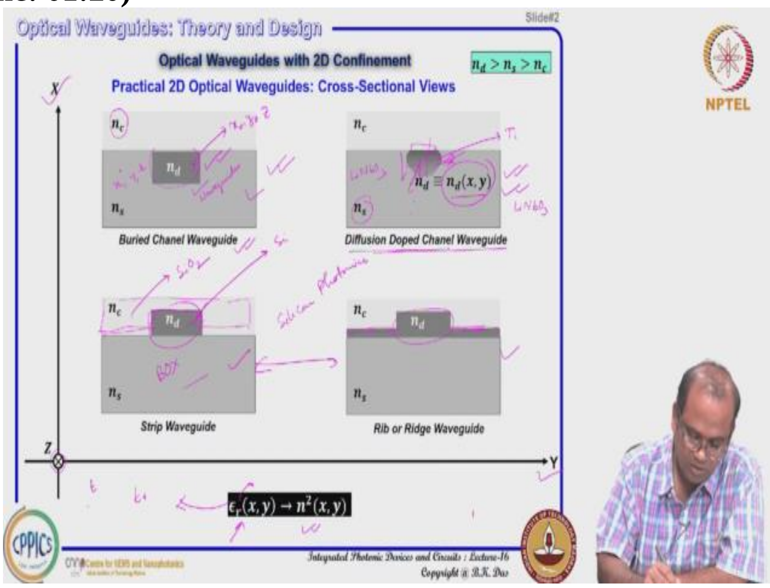

Let us move on and let us see what all different types of waveguide is popular in photonic integrated circuit development? So, first thing is that first type of waveguide is this one is a cross sectional view it is shown. So, if you consider this is X axis this is Y axis and this is your Y axis X axis and then Z axis into the screen now, and then if you are considering a Z propagating waveguide then this type of cross section is continuous invariant as a function Z.

And you can see that this bottom one this is so called n s refractive index is n s, s stands for probably substrate and normally, you know our silicon dioxide silicon on insulator we will consider that is actually buried outside. And then actual device layer will be device waveguide is having a little bit higher refractive index and refractive index is n d as usual we have considered there for the device layer and cover refractive index n c.

So, this can be oxides, air, nitrite anything polymer even all those types of things you can consider and if you see your waveguide core, this is the waveguide core. Waveguide core is buried into the substrate with a rectangular cross section, it can be any cross section, but rectangular cross section is practically easier to implement fabricate and that is why we consider here this rectangular or square whatever you can design you can do that you are free to do.

So, but this type of waveguide it is called buried channel waveguide that means, the core region is buried into the substrate that is why buried channel waveguide we call it buried channel waveguide. And another type of waveguide is normally popular in certain technology platform in which you will see that the substrate it is doped with some other material. So, that in a specific selected region; certain atoms will be in diffused and doped.

And that will enhance some refractive index that can be graded that means from the top to bottom if you see that means that the concentration of the doped material will be decreasing and the refractive index change will be just proportional to the doped atoms or doped material and that is how you get a some kind of so, called graded index waveguide core such a device such a waveguide is known as the diffused diffusion doped channel waveguide.

So, normally you know we have discussed about modulator lithium niobate modulator also we discussed and as an example I can say that if you just dope titanium, diffusion doped titanium atoms the surface of lithium niobate substrate suppose if it is a lithium niobate for example, then titanium doping, titanium doping can be done, you can just deposit a small layer of titanium thin film.

And then you put it in a burners, then titanium will be diffused inside the lithium niobate substrate and that concentration of the titanium actually to be bearing laterally as well as particularly, and that is how you can see the graded concentration of the titanium and refractive index also will be proportional to the titanium atom concentration that is how you can make a lithium niobate waveguide.

And that type of waveguide is very popular for electro optic function electro optic operations particularly high speed modulator purpose that type of waveguide it is used. So, they are called diffusion doped channel waveguide and refractive index obviously it is a some function that function depends on what is the diffusion parameter, diffusion temperature what is the width etcetera?

It can be a function of X, Y we call it n d device refractive index is a function of X, Y in this case you see the core is actually step type refractive index. So, cladding to core refractive index change is abrupt, but in this case refractive index is graded to refractive index change is graded and ultimately towards the deep inside this refractive index will be change of the refractive index will be reducing, reducing and it will be merging to n s.

Finally, substitute refractive index another type of waveguide called strip waveguide that is actually will have substrate and then you have a device layer that is pattern like this rectangular strip and then it will be core and this core is now can be considered is in the top of the substrate and it is you can have some kind of cladding material on the top. So, you can say that this core is now buried inside the upper cladding.

So, this type of waveguide since it is not buried inside the substrate we call it strip waveguide, strip type waveguide that can be pattern and the fourth type of waveguide popular waveguide did the same thing same n d material is their core and it is on the top of the substrate but both side for certain application purpose you leave some here for example, you leave some material here and the pattern like this.

So, that is called the rib or ridge waveguide structure. These 2 types of waveguide structure cross section normally people use for silicon photonics application where this photonics application where this one will be just a box and this one will be your device layer silicon that top can be again SiO 2, air or silicon nitride whatever and you can leave some material silicon for certain kind of again electro optic and electronic integration, electronic circuit integration etcetera.

You can use this device layer and actual guided mode will be confined here surrounding region refractive index will be relatively lower. So, in this type of cross section normally if you see this type of refractive index distinction either it can be strip type, it can be graded or whatever it is only in the XY plane what about Z direction? Z direction it is invariant. So, that is why along the Z direction you do not see any refractive index you can say that if you just consider a particular core you know.

Suppose you just consider this coordinate here suppose this coordinate is x 0, y 0 and now, you vary your Z along Z you will always see refractive index n d if you consider something x 1, y 1 coordinate somewhere here. So, when you vary Z that means Z direction refractive index will be always n s. So, that is why it is actually called Z propagating waveguide since, in the Z along the Z direction the index is not changing only XY cross section.

You see the refractive index change we can say that this refractive index change that is coming out of different dielectric constant. So, dielectric constant of silicon, silicon dioxide etcetera different why we are writing like that, because in the Maxwell's equation n square comes where epsilon r is their original Maxwell equation it was this epsilon r is used basically epsilon that is epsilon = epsilon 0 epsilon r as we have discussed earlier.

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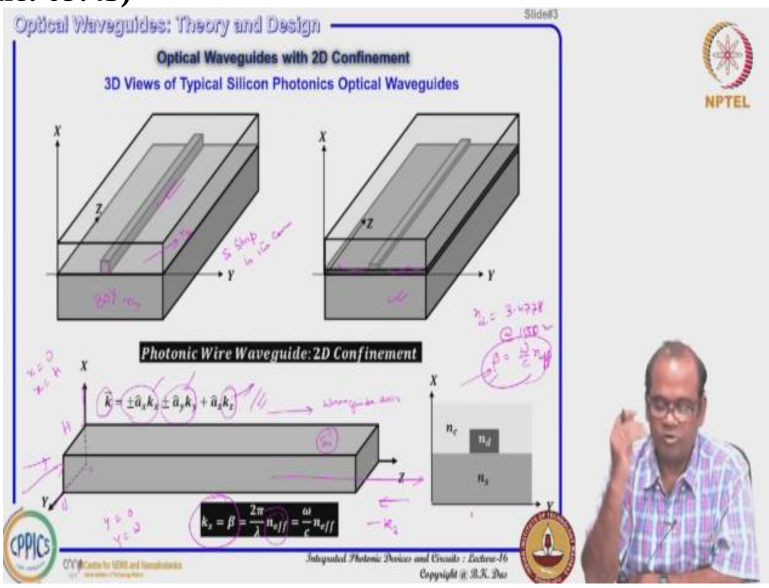

Now, we say that the different types of cross section and different type of platform waveguide cross section is used for different types of technology platform. And as I mentioned that the rectangular type this strip waveguide or this type of strip waveguide and rib waveguide structure or rib waveguide structure is very popular for silicon photonics. So, here I tried to give you a view graph view 3D view of a typical silicon photonics optical waveguides.

One type of waveguide where actually you see this is the core region it is shown here this is your core region and this is your box layer you can consider this is your box and this is your top cladding box will be n s this will be n c and the pattern is the rectangular pattern or square pattern you can see that and that is actually varied uniformly along Z direction you see it is aligned so, that such type such a waveguide is called actually photonic wire waveguide.

And this is actually photonic wire where you are getting actually the strip silicon strip is there silicon strip is the core in this case and here same thing you have against that like rip type of structure you have both sides some device layer is there the side and this side some device layer is there silicon is left and the height of the waveguide the same but both side it is reduced that is rib type of waveguide structure.

So, these 2 type of waveguide structure are popular for photonic integrated circuit basically CMOS compatible silicon photonics integrated circuit we can see this heavily with these 2 types of waveguides are heavily used. Now, how this photonic wire waveguide sometimes they are called photonic wire waveguide as I mentioned, how in this photonic wire waveguide or photonic rib waveguide structure in silicon on insulator platform or this type of structure can be also fabricated using CMOS technology.

As I mentioned earlier in bulk silicon as well in that case, the core material instead of crystalline silicon that will be amorphous silicon and here will be this is grown oxide will be there. So, basically on the top of grown oxide poly silicon is grown, deposited and polysilicon again pattern and that poly silicon core or amorphous silicon can be patterned and that can be used as a waveguide core loss will be a little bit higher, but that is also used in bulk.

So, what I meant to say that this type of structure you can pattern both in silicon on insulator platform as well as bulk silicon platform. So, when you can fabricate that using CMOS technology, so, we need to understand that, we understand qualitatively that if you have a core with higher refractive index and the surrounding region is lower diffractive index region that can be acted as a 2D waveguide that means, 2 dimensional confinement will be there of electromagnetic wave.

So, light wave can be propagated in the Z propagating direction waveguide axis, but, in such a waveguide how to deal with that type of guide what how to find this solution for such waveguides. So, normally we have to develop our theory, which will be compatible to a device which can be fabricated, which can be realised in practical life. So, that way we in the following few today's lecture, I will be discussing mostly how this mode can be calculated how this light can be confined, and how they can be useful for further applications.

So, if you just take the core region only have this photonic wire waveguide structure, so, you can see that this is X axis this is Y axis. So, I just saw on that X axis here and this is the Z axis this is the propagation direction so, waveguide axis or propagation direction waveguide axis. So, we know that any light you are launching here from this side in the XY plane from the left you are launching. So, inside in this silicon it is the silicon core.

For example, it can be any other material surrounding with other lower refractive index material suppose this is silicon, we will normally silicon n d you know that is actually 3.4778 at 1550 nanometer wavelength. So, in that inside that material, if you just consider that wave vector any direction you consider the plane waves entering inside then because of the containment total internal reflection in the XZ surface.

So, you have XZ surface in the surface and at X equal to at Y = 0 and Y equal to say you can consider W some W in this 2 interface at Y = 0 and Y = W in this 2 interface. You can see total internal reflection and that is why you can get forward propagating and backward propagating wave vector wave in the Y direction. Similarly, along the X direction also you can consider the bottom is 0 X = 0 this thing can be considered as H, X = H.

So, you can have X = 0 and X = H. So, you will get some kind of total internal reflection. So, that means total internal reflection also you can see in the interface in the plane in that YZ plane at X = 0 and X = H. So, in that way you can get a kind of standing wave pattern standing wave propagating backward propagating plus minus k X will be there what we have discussed in the waveguide similar thing we can see, but along Z direction, you do not have any discontinuity do not have any interface along the direction.

So, it is continuous you are considering as long as you can maintain that it is continuous that is why along Z direction you have already k Z component although not positive direction. So, if you are launching this side you can have allocated director if you are launching from this site, so, you can have minus k Z that is a minus k Z you will be writing that the only vector v negative that is it.

So, we know that if total internal reflection happening along the X direction as well as Y direction, you can have some kind of standing wave pattern along X direction as well as along Y direction. So, that standing wave pattern can create some kind of field distribution inside the photonic wire waveguide. So, that field distribution a particular set of k X and k Y you can have a particular solution for k Z and that particular solution of k Z we can call it as a this one beta.

So, that pattern will be propagating along Z direction that maintaining that pattern that is called mode that is what we have learned so, far. So, that beta will be calling as a 2pi by lambda n effective, n effective is known to effective index and that can be written as omega by c n effective. So, we will be able to get certain kinds of solution omega by c n effective that will be propagating along Z direction.

So, again same way like whatever you have learned in 1D wave guide here also our challenge to find out this beta value and certain pattern whatever field distribution you will be getting in the XY plane in the XY plane you will have a certain field distribution and corresponding that particular field distribution you will be getting a specific beta value. So, those set of field distribution field means I meant to say electromagnetic field electric fields and magnetic fields associated with a particular solution for beta that will be a mode.

So, that is our challenge to find now. So, once we know that we can use them and we can design particularly waveguide depending on the design structure parameter sets that I will be able to predict that what will be the mode set, what will be the beta value, what will be the phase velocity, what will be the group velocity, what is the dispersion etcetera? All sorts of things we will be able to know and those are very important to design a waveguide devices using silicon photonics technology platform.

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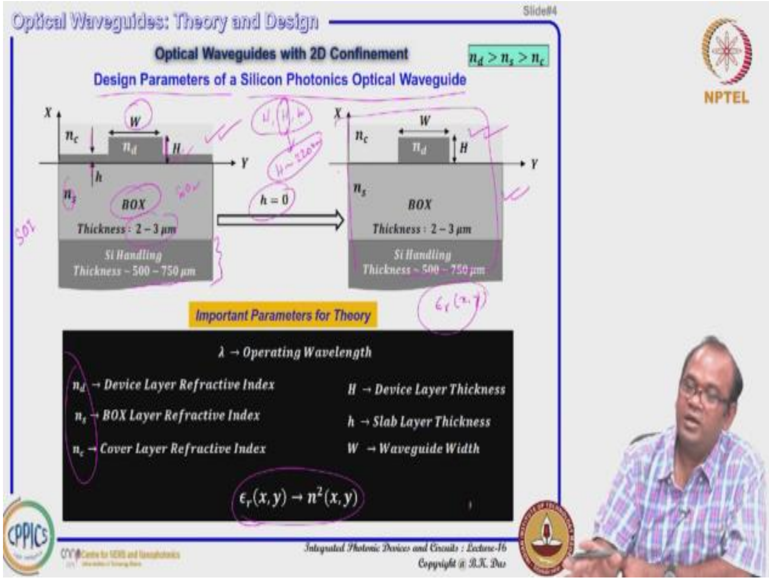

So, now, let us see what are the parameter governing parameter design parameters for silicon photonics waveguide? So, here I mention that design parameters of a silicon photonics optical waveguide. So, 2 types of waveguide I have shown this and this one. So, basically they are identical you see if I just try to see what are the parameters involves first thing is that you have substrate silicon handling of a thickness about 500 to 750 nanometer.

This is SOI substrate silicon on insulator we were considering and then you have a box layer 2 to 3 micron typically needed basically more is better, but again I think 2 to 3 micron since it is insufficient try to go for more because it has to be actually grown. So, this type of the technology part silicon on insulator platform is also very expensive. So, that wafer you need to customise and industry has customised with 2 to 3 micron in that range somewhere.

And that is basically box layer that is silicon dioxide and you call that is the refractive index, as I mentioned here and device layer and in the device layer if you see that we can have parameters like width of the rib and then device layer height H. And then another thing this small h these 3 parameters can be considered as the waveguide parameter, geometrical parameter. In fact, you can derive this structure photonic wire structure.

Once you put h this small h = 0 then you get the structure. So, any way you can think of that, this waveguide these are generic waveguide structure. So, here you can vary W value, h value, small h value and you can design different types of waveguide cross section and different type of solutions for that also, obviously, whenever you are buying a silicon on insulator.

This part is normally constant that is actually whatever available in the market you have to use it or you are very much interested to do something novel something different types of devices you can customise that small capitalised device layer thickness or you can have your own laboratory that you can grow that, that way you can control that, but normally in industry today mostly most of the silicon photonics, silicon on insulator substrate they use that device layer thickness h equal to in the order of 220 nanometer.

But that is not that also foundry to foundry that is actually reading. Now, we have this parameter W h small h and other parameters as you I mentioned that they are refractive indices and to solve the mode normally ray optics picture we have already mentioned that, that is not sufficient for a 1D waveguide as well for this 2D waveguide. It is good to have directly to solve the electric field mode solution from the Maxwell's equation itself using the dielectric constant matrix like this.

You can define epsilon r x y matrix in the x, y plane you can have every coordinate you know you just map this one because these type of cross section you are getting from the technology point of view. So, you know that this is the cross section and you know this region I want to solve what is the mode and what is the propagation constant or phase velocity etcetera? So, you can map this 1 x y coordinate wise and then you use them in Maxwell's equation to solve the rest electric fields propagation constant that is it all.

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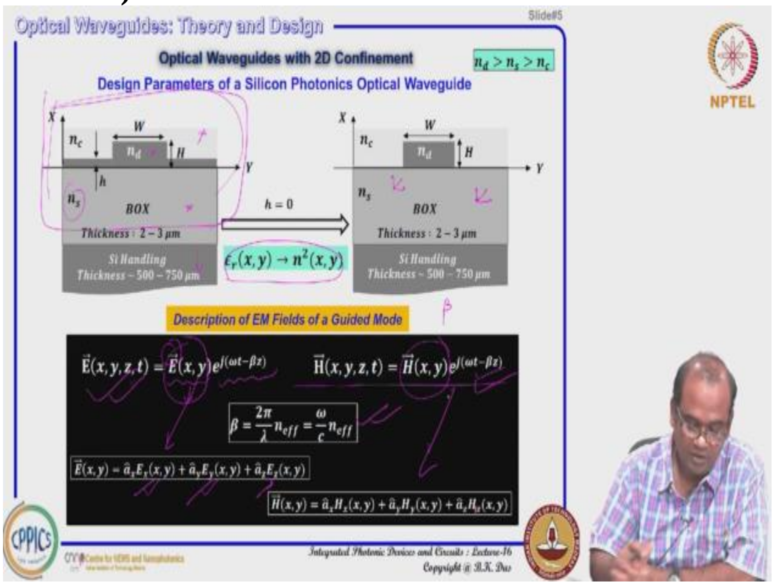

So, now, if you just think about that, this epsilon r that is actually heading in the x, y plane for example, you take a coordinate here what is the refractive index n s and you take a coordinate here what is the refractive index that is n d you take a coordinate here refractive index n c you do not need to bother about what is there in the bottom, because that is just for handling purpose.

So, your area where you want to solve the dielectric constant that is this area. So, box layer, you can consider box layer that is considered as your substrate so called for waveguide analysis and then cladding you can take some cladding region and this cross section you can consider that cross section is invariant along Z axis. So, you can imagine that this is your electric field associated with a mode what is that?

This electric field associated with a mode having a propagation constant for example beta of that you know that this field is x y z dependent because all this dielectric constant is different in this cross section and along Z direction it is not changing, but Z direction it is propagating will be propagating we can say that this pattern X direction Y direction confinement that will give you certain kinds of pattern that pattern function is this one along with that I put a vector because the electric field will have position to position.

It can have different electric field direction. So, every point I just mentioned, how electric field is there at that particular point and that pattern will be invariant along z axis only difference is that it will have some kind of phase as a function of time and space as it propagates as time passes at same coordinate you can see how that will vary. And for same instant you will see what is the along Z direction what is there total particular snapshot you can get.

And in that particular mode you know electromagnetic wave only electric field is no meaning you have to consider magnetic field also for that particular mode you will have also magnetic field and magnetic field also will have a x, y profile vector and also the same phase velocity because they are electromagnetic waves belong to a particular mode or having a propagation constant beta.

So, propagation constant will be shared by both the electric field and magnetic field distribution because that is a pattern it is called mode. So beta eventually will be getting like this, it can happen that like whatever you saw in your 1D wave guide, you can have a multiple solution for n effective beta and multiple set of this vector and they are actually the guided mode or confined mode.

We can say that those types of solutions we have to accept and as usual we can say that this electric field x, y dependence is there that means, this transverse electric field transverse direction whatever the electrical distribution whatever X, Y direction your X, Y, but it can have x component it can have E y component it can have a E z component at any point. Suppose you consider this point at this point electric field can have y component can have x component and can have also z component any direction.

It can orient you change x different coordinate point here they are also you can consider some direction will be there you will be getting the E x component you will be getting certain other E y component and some other E z components will be getting. So, point to point I can have different field strength along with that direction as well. So, I can consider that there are 3 components you can imagine in this Cartesian coordinate system for electric field.

Similarly, same for magnetic field you can have x component, y component, z component that will be varying as a function of x y because that is a mode and that will remain invariant along Z direction which will be propagating which will be carrying energy with this phase information omega t – beta z so, far, so, good so, we have the everything is set.

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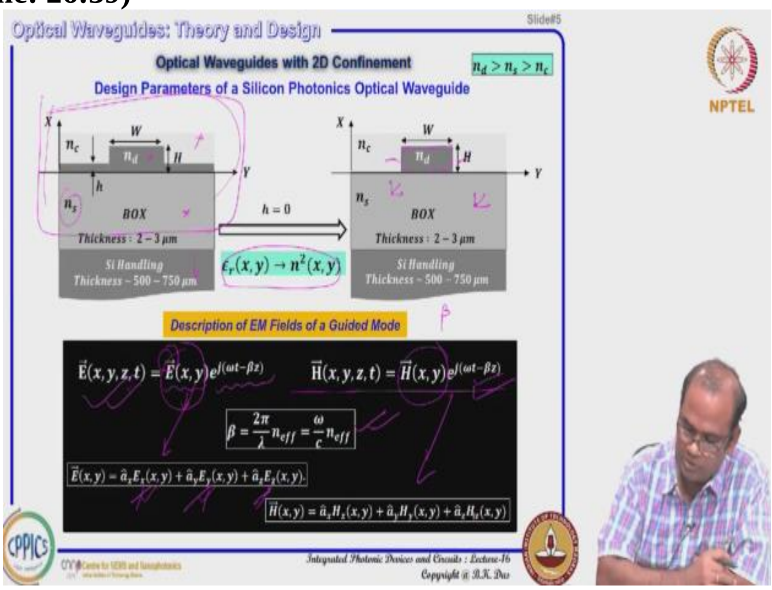

Now, what we have to do? We have to now solve these all the 6 components if you see all these E x component E y component E z component H x component H y component H z component. So, normally all the 6 components can have in a particular mode, when we discussed about 1D waveguide. For example, in 1D waveguide we know that 2 different types of polarisation will consider T mode and T m mode.

And in case of T mode we have considered only E y component will be there for electric field and in case of magnetic field T m polarisation T m mode, we have considered only H y component y component of the magnetic field will be there and that is how T, T m we could decompose easily and we analyse them independently individually, but when you say that is actually consistent with the Maxwell's wave.

Maxwell's equations for propagating energy or carrying energy in 1 waveguide that was consistent, but when you see that light it is confined in both X and Y direction and you have a particular shape of the core. In that case, you can imagine that in a mode all sorts of a component is required for guiding purpose for a guided mode. So, you can have all the components and sometimes you can have a mode maybe some of the components may be negligibly small.

So, out of these 6 components maybe 2 or 3 components you may not need to consider so, small after solution you may find. But at the beginning it all depends on what is the waveguide structure what is the refractive index distribution? What is the refractive index of the core to cladding refractive index? Contrast save corners all those staple thing that involves the field pattern. But that field pattern actually this is this would be a specific type of pattern for a certain mode solutions.

So, at the beginning, we will be considering as if all the 6 components are there in the mode, and they later on we will see that for any solutions can we need to consider all those modes or some of the modes, some of all those components or some of the components can be ignored? That we will see later, for the moment, let us consider that we have to consider we have to give equal importance to all the components, all the 6 components, E x, E y, E z and H x, H y, H z. So, all these 6 components, we have to consider good it is not like 1D waveguide.

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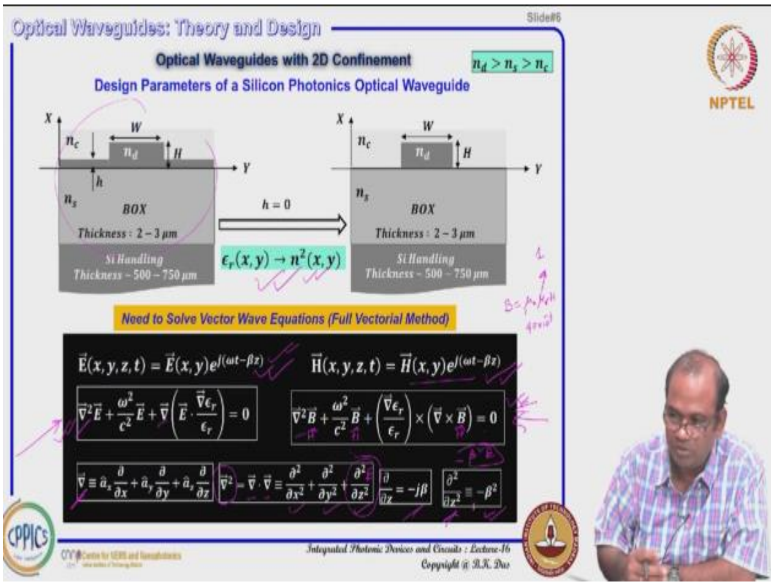

So, now, as we also mentioned that for an inhomogeneous medium inhomogeneous structure, normally we have to solve vector equation inhomogeneous in the sense that you have a path and you have a structure for example, in this cross section, everyone not same refractive index coordinate to coordinate you vary your refractive index is different, you have this type of function.

So to deal with this type of function, we have developed from a Maxwell's equation 2 vectorial equation. Starting from Carl equation, one when I eliminated magnetic field I get 1 vectorial equation as involving electric field alone. And another equation we got that involving magnetic field alone here B we have placed just for used just for a very concise type of equation.

We get normally you should be finding that B = mu 0 mu r H normally mu r you are considering 1 non magnetic material and mu 0is constant 4pi into 10 to the power - 7 Henry per metre. So, basically B and H they are related to the mu 0 only. So, we can just use H here, H here, H here and mu naught can be factored out. So, instead of B I can easily write H vector here, H vector here, H vector here that does not matter.

So, it is traditionally vector equation is written in terms of E and B that I have written here. So, we have seen how to derive these 2 equations earlier I am not going into the detail again here. So, all only thing is that you should keep in mind we are using this nebula square nebula all those types of things are there we know what is that in Cartesian coordinates and nebula is a vector we consider differential vector that is actually x component will be del del x, y component will be del del y, z component del del z.

And then in that case del square means del dot del nebula dot lambda. So, that will be not productive to take that is actually a square, square, square and once you know you are getting this one you have the modal solutions like this. So, z dependent function the basically when something mode is propagating along Z direction, so, z dependent dependency is known to you, e to the power - beta z for a given mode that is why del del z means if you are just making a derivative of any magnetic field components or electrical component.

If you are just making a partial derivative with respect to z then you are going to get minus z beta and if you do once more derivative with respect to z then you are getting minus beta square. So, this information we know easily so, anywhere we are a facing this type of thing suppose multiplied by you want to operate del del square with E. So, I can write instead of that I can write minus beta square E.

So, del del square you do not need to consider because along Z direction what is the variation we know and what is the derivative second order derivative what is the partial second order derivative what was the value we know that, that is why you can just simply put to reduce the mathematical load.

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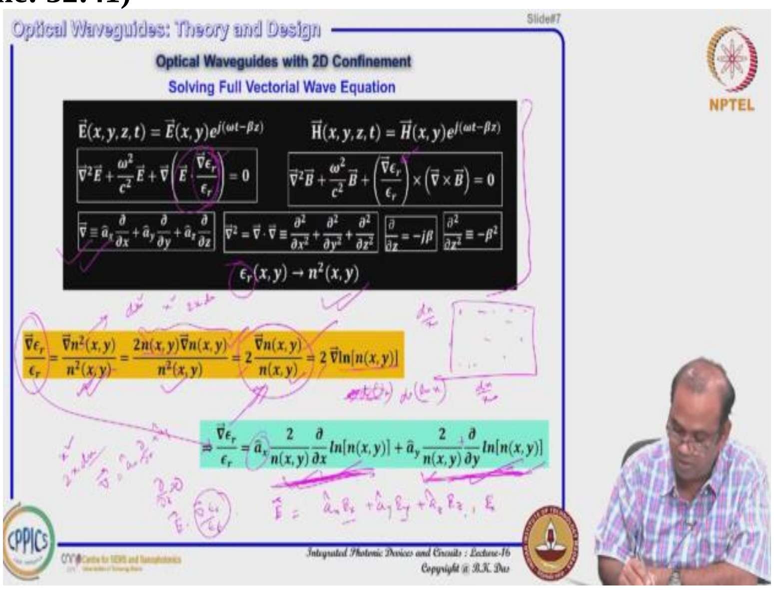

Now, I just again repeated here to proceed further. So, you know that we have the epsilon r known to you it can be matrix form or whatever it is given to you n square x it is you can actually extract you can generate that matrix if you have a cross section of the waveguide cross section and you are considering every point you know what is the refractive index that matrix is known to you if it is not well defined function.

Normally it cannot be a well defined function when you were thinking about the practical device whenever something coming practical reality you cannot get ideal type of mathematical function type refractive index profile. So, you have delta epsilon r will be there and you see in the vectorial equation for electric field vectorial equation for magnetic field delta epsilon r by epsilon this term is there, we just try to see how to deal with that fast then the rest of the things will take care.

Suppose we are writing this, epsilon r can be written as n square x, y just we are writing like this, that is a straight forward. Now, you know, this is n square numerator here my ultimate goal is that this n square x, y is in the denominator and this is derivative. So, if you can just try to find out that here refractive index here is also reflective index, if we can find some operator in which a single refractive index function you can operate.

So, numerator let us consider this is as if it is considered dx square type things dx square you can write you are making derivative of x with respect to x for example, so, if you are doing derivative of x square that you will get 2x dx that type of thing you will be getting. So, you are writing the same square this del, del means derivative basically only the vector here. So, we can write n x, y delta n x, y.

So, n square can be written as 2 times this one. So, it is something like that x square you make a derivative you get 2x dx. Similarly, it is a 2x n is the x and dx is that del n x we are writing like this and denominator as it is we are n square x, y writing and then you see this n square this n; n, n cancel 1 n will be cancelled then we can write 2 times delta n here and n x, y d of del n x, d of ln x derivative of ln x is actually you can write dx by x.

So, that is what we have written to is there 2 is there and this is we can write like this. So, that is a one way of representation, but, we will concentrate only this one this is what we know that, what is the refractive index profile of the waveguide cross section? You know that. So, what we can write this is actually you see refractive index is the x, y is a function of x, y, z invariant.

So, and whenever you are taking a gradient on that nebula operator on that you are getting a vector and that vector you just try to write epsilon r this one we can write it will have an x component and it will have a y component. So, you get a x component with derivative this one and y component with derivative of this one refractive index z component also will be there because you know this is nebula is equal to a x del del x + a y here it is written.

So, you can just follow this one del del z if you operate del del z on x, y because this is a function of x, y only so, del del z will be 0. So, we can write in that respect del del z = 0 that is after putting that you are getting 2 terms one is x component term and another is y component term. So, delta epsilon r we have written this one that will be equal to this one this is this one and this one, we have only x component and y component will be this one, I can insert this one directly here.

So, I have x component y component and then I have E is for example, I have E x + a y E y + a z E z. So, if I try to get a dot product of this one E dot delta epsilon r by epsilon r that is what here we see this dot product if you see these has all the x component y component these are the x component, y component, z component. So, ultimately you will be getting x component that means, E x you will be getting E x times say this component is this one multiple component dot product. So, accordingly you can insert there and you can try to find out your solution.

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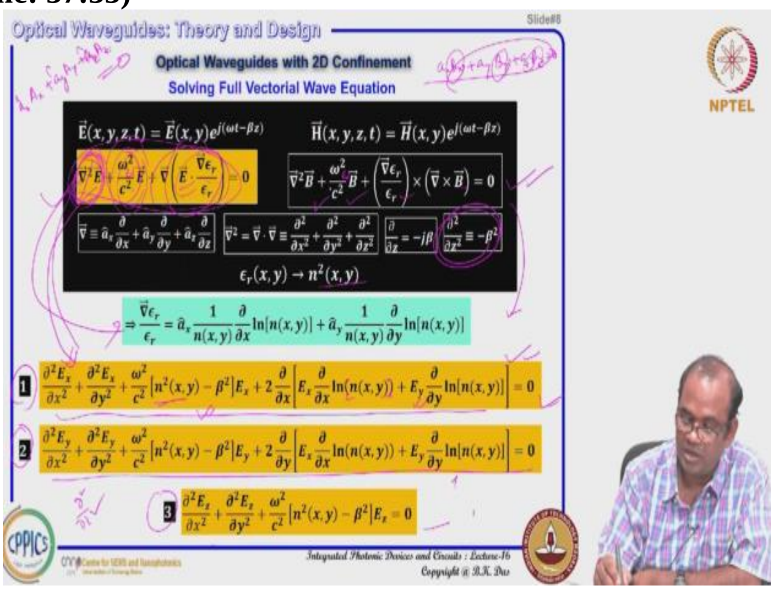

So, I have repeated here I concentrate this equation only electric field vectorial equation for the electric field this one del square E this one and this one here for magnetic field I will consider that one later, if we consider this one and insert this one there, delta epsilon r and what we will do you know this is a vector left hand side if you consider this will become a vector second term is also vector.

And here E dot delta that will be dot product of 2 vector that will be a scalar and when scalar again you are taking a gradient that will be a vector again. So, that means, this vector I can consider something like this a x A x a y A y A z sorry a z A z that is actually equal to 0. So, when a vector is equal to 0 what does it mean left hand side together it will be a vector that vector is ultimately 0.

So, you are considering this vector and adding this vector and adding this vector you will get a closed loop for example, so, total resultant value is 0 when a vector is 0 that means, individual component must be 0 x component will be 0 y component also will be 0 z component will be also 0. So, what I will be writing for a if I just try to find out x component here x component will be looking like this simply here del square E this one I am writing and del 2 z del z to you know that is minus beta squared we have written here.

So, I have x 2 del del x 2 second derivative with respect to y and with respect to z beta square is there and in this expression one thing is missing in this there will be epsilon r here, epsilon r here I will be correcting in the slide, this epsilon r epsilon r will be there here that epsilon r is written n square x y that is missing in this you should keep in mind. So, nevertheless this equation is correct omega square by c square n square x, y.

And then you have also other term this term will be coming ln n x y ln x y you will be getting like this that is your x component. So, that means x component or equal to 0 that is a scalar equation you can consider only x and E y component also of course involve and if you are just considering y component all the y component to act together they must be equal to 0 y component.

You take from here y component x or y component tech from here they are adding together to be 0 you can write down this you can step forward we just try to find out what is the x component if you know the vector algebra, it would not be a difficult task and similarly, you can consider z component of this thing and z component this thing z component this thing add them together 0 we just named equation number 1, 2 and 3.

So, from here I could decompose this one into 3 scalar equation involving E x, E y, E z so, identically I can again use this equation vectorial equation for magnetic field and again I will just writing so, for example, a x B x + a y B y + a z B z = 0 so, that means, the vector as a whole is becoming 0 all the x component to consider from here, here, here and then add them together B x that must be equal to 0, B y must be equal to 0, B z must be equal to 0. So, that means, any vector we are considering as a B not like magnetic field here. So, all things again identical you try to write down for the magnetic field here.

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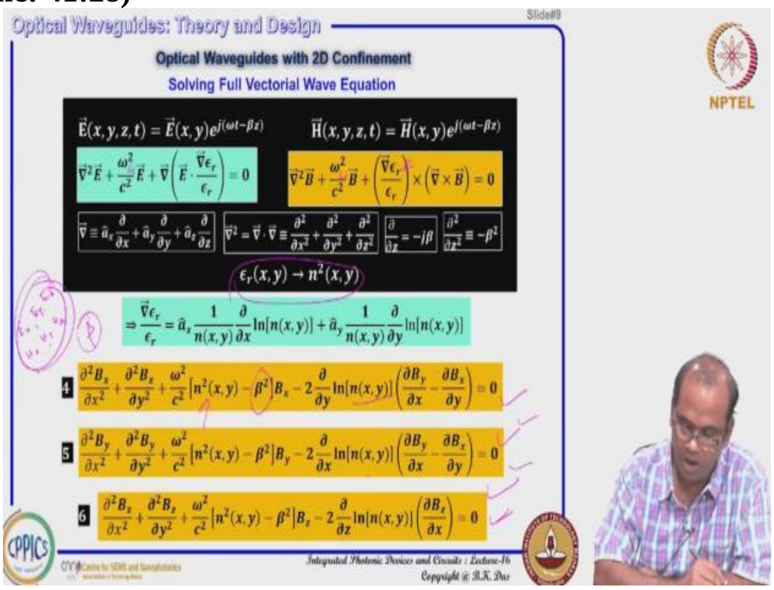

So, we can just get this thing B x x component B y, B z similar thing it is as simple algebra slightly different here, because you have this equation and this equation you see again missing epsilon r so, consider that thing. So, epsilon r epsilon r you considering because it is curl of r a vector here that is cross again curl of B you are considering so, slightly different. So, if you see the pre scalar equation coming out of elliptical vector equation is something slightly different compared to whatever you are getting from the magnetic field.

Vectorial equations so, these are the things you are getting so, far, so, good. And then once you get these 6 components, so, basically you can say that I want to solve E x, E y, E z and H x, H y, H z for a given dielectric distribution across the waveguide cross section. So, that is actually possible only if you concentrate all this you have to solve all these 6 scalar equation by inserting whatever n x, y if n x, y no just insert n x, y.

And you can solve and normally it has to be solved numerically and numerically whenever you are solving the set all our second order differential equations you can you have to solve them simultaneously, then only these 6 components will be getting along with that given beta value also as the Eigen value you will be getting these will be coming out as a Eigen vector and this will be coming out as a Eigen value.

So, you have to solve them numerically, because it is a complicated structure of the waveguide cross section and particularly this one you are getting right and second order derivative. So, normally Maxwell's equation if you solve you can just solve using finite difference technique in a Fourier domain for a particular frequency operation, then you get all these 6 equations you will be able to solve.

But normally you know these you can develop your own code to solve mathematics you can use your python or C Programming etcetera. So, that you can solve them, but I would just explain here that you do not need to solve all these six equations second order differential equation only these 2 equation if you solve if you get E x and E y from these 2 equations, equation number 1 and equation number 2 simultaneously solved.

And it is kind of whenever you are solving you are solving like a you are just making some problem like x something a psi = lambda psi something like that you will be getting equation you are solving like this type of case and you have an operator and then you are when operating and then you will be getting Eigen value and then this psi is an Eigen vector. So, all these components E x and E y components you will be getting as your Eigen vector.

And corresponding beta value you will be getting as your Eigen value, but you have to solve them numerically that type of numerical programming can develop and now it is commercial subtasks are available you can solve directly you just use their software and you can get to solve E x and E y using this thing this request and if you solve because these 2 equations if you see.

If you ignore this part sometimes what happens if this diffractive index you take a lon and then again you take a derivative sometimes lon if you are taking the value will be reduced and the gradient will be reduced and then if you take a derivative with respect to x that is actually compared to all other terms that can be ignored, this time can be ignored. So, in that case you have only this equation.

So, only in this equation, it would be easier to solve your electric field x component directly from equation 1, but since equation 1 and 2. It is important this coupled region is the E x, E y is coupled a couple term is there. So, you will not be able to use only 1 equation independently to solve E x. So, you have to simultaneously solve equation 1 and 2 to get E x and E y and corresponding certain value of beta.

So, using this equation you are set to solve E x and E y and beta value these things are solved you may get a set of solution E x and E y as a Eigen vector and every Eigen vector you can get 1 solution for beta those are correspondingly particular mode for the system or structure.

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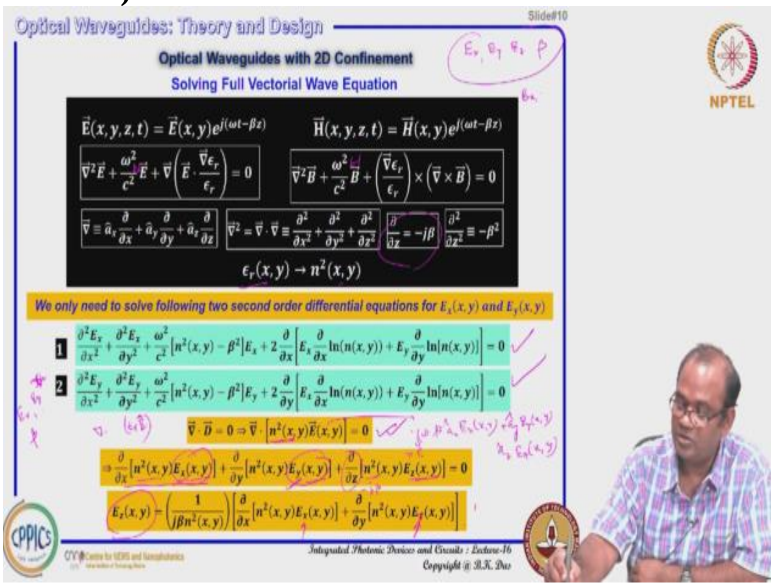

So, now, after solving this we are considering concentrating these 2 equations again you just do not forget to include epsilon r here everywhere. So, both cases set is missing and then once I know E x, E y, E z no E z I have not solved yet and beta I have solved now, we turn back to our Maxwell's equation, GOS equation divergence D = 0. What is that, if you just put divergence D that means the divergence basically, epsilon r E = 0 epsilon r is n square x, y.

So, divergence of this one, so, if you just expand a bit here, so, you can write del del x n square E x del del y dot product you are taking divergence you are taking. So, you take del del x n square E x because this is also vector E x, A x, x, y if you are just considering A y, E y, x y and A z, E z, x, y and you are taking divergence of that one divergence of a vector. So, it is a scalar so, you can write del del x n square times E x, del del y n square time E y, del del z n square time E z both n square n square is a function of x, y.

And we know that a solution coming out of E x and E y they are also a function of x, y. So, what do we know that from here if you are taking a the del del z we know that what is the value minus beta. So, instead of del del z because everywhere all the components you have this term is involved omega t -beta yet, so, del del z - z beta. So, here if you are considering minus z beta times n square and z x y that means, I can find E z x, y in terms of E y and E x n square x, y know.

So, every point if I have E x this value known this value known and of course beta known minus z beta. So, E z x, y you also will be known from this expression. So, every point every code in it, you just insert what is the value of E x what is the value of E y that should be y x component is a component. There is the x component y component that will be there. So, you get to know what is the E z? So, now, at this point of time, you already know E x, E y is it as well as beta value what is leftover, you need to know B x, B y and B z.

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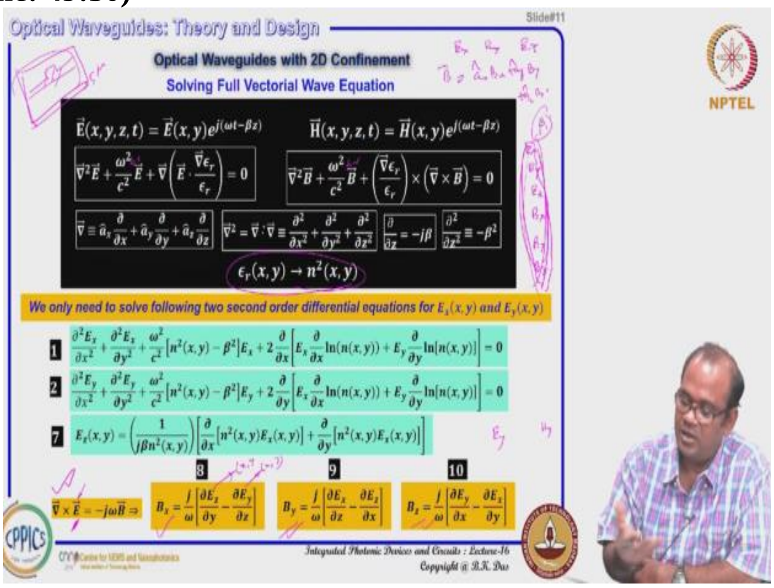

So, how will you know that you go to next thing you turn to here also you just consider epsilon r is there already and you turn to another curl equation that is called equal to minus j omega B. So, all the components E vector is known, I have E x, E y, E z they are known. And B means B x B u + B y + B z I can write B vector means a x b x + A y B y + a z B z. So, I just insert E and then every component B x I can derive B x, B y, B z. So, E z is also known E y known they are a function of x y.

So, you just make a derivative and that is the rotation B x I can get B y I can get B z come back and so, in this way I know now all E x, E y, E z and B x, B y, B z and that is known for a given cross sectional dielectric constant distribution or refractive index distribution and correspondingly you get a beta a series of values you will be getting a particular set of all the 6 components correspondingly we will be getting one beta another set of all this you will be getting another beta these are these can be considered your Eigen vector.

And this can be whatever beta value we will be getting that is coming as the Eigen values it is similar to 1D waveguide or whatever we have discussed, but in this case since, it is a 2 dimensional and dielectric constant is not a particular function to solve your vectorial wave equation, so, you have to solve it numerically. So, numerically you can solve all the things found and these are called actually solving full vectorial equation full vectorial full vector and means, all the components I am solving already are in case of 1D waveguide.

For TE polarisation we concentrate only one E y and for T m polarisation we concentrate on H y because they are the tangential component and they have been solved and that is actually not full vectorial you are just considering only 1 component and you are solving ultimately you are getting other vectors, but here directly you are using vector wave equation and you are imaging numerically.

Of course, you may not get very accurate solutions pop from the numerical solutions because your numerical solution has to be very powerful, because this is all involves differential equation. So, you have to use a difference finite difference technique method sometimes some people also developed they are this full vectorial equation solver using finite element method also.

So, depending on the elements height depending on the element size your accuracy will depend. So, if you have a waveguide cross section you are considering including buried oxide and then your cross section core etcetera together you are considering this area is for example, if it is 5 micrometre by 5 micrometre area you are solving then you can get so, different type of solutions if you are going different type of solution very fast.

But if you are having a very large structures and you are finding element sizes finite, infinite is small then you have to solve you have to handle huge amount of data that may take a longer time to solve nevertheless, nowadays I think plenty of companies they are offering the this type of mode solver you can also if you are very much passionate to develop the software you can use your skill to develop your software to solve electric field components solution mode solutions as well as their propagation constant Eigen values.

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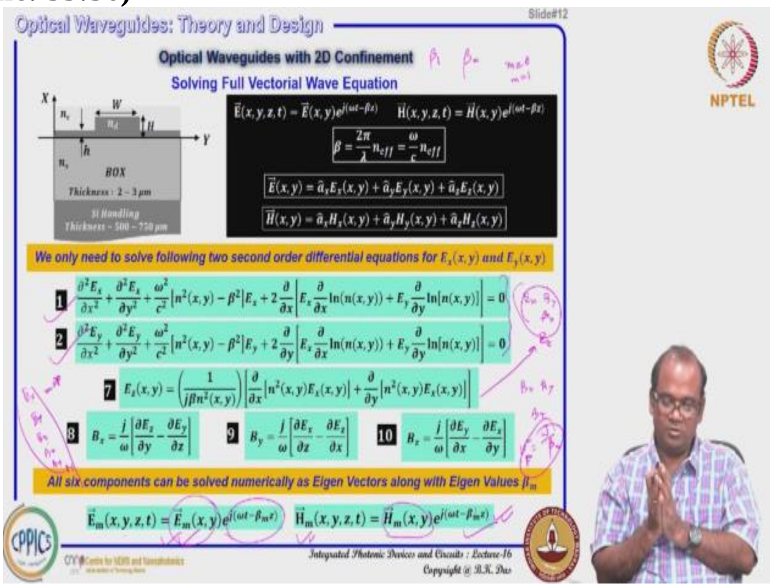

So, far we could actually get all these equations 1, 2, this 2 equations gives me E x and E y as well as beta and then this equation gives me E z and then these 3 equations gives me B x, B y, B z all these you will be getting and you will be getting for a particular solution beta because E x and E y E beta will be up when you solve this one you will be getting a series of values.

So, series of values you can index them as a beta in propagation constant is beta m, m can be first one you can assign as 0 and second one it is like 1 you can assign as 2 etcetera and for m = 0 you get a set of solutions for E x, E y, E z, B x, B y, B z that will be corresponding to m = 0, 1 solution will be getting m = 1 here you will be getting 1 beta 1 here. We will be getting m = 1 another set.

So, field distribution all the field components will be getting as a function of x, y and corresponding beta value you will be getting you will be solving once you know that, then you can find a mode you can define like this. So, a mode mth mode of electric field associated with the mth mode x, y, z, you have a x dependent distribution which gives you x, y, z 3 components, each of these components it will have a different value at different points, but every point it will be oscillating with this phase, E to the power j omega t.

Similarly, for magnetic fields associated to the mth mode, you can have field distribution in the x, y plane x x component, y component, z component each of them will have different value it can be different value, but their identity is the omega t - beta m z. So, identity of all these components carried by the this one they have a particular pattern of distribution in the x, y plane and that pattern will be travelling with a certain phase velocity with that phase velocity will be again b p m will be writing as omega by beta m phase velocity.

So, now in we are set this is a full vectorial method normally photonic waveguide structure or silicon photonic waveguide solution to design a single mode guidance or what how many number of modes are there, what is the field distribution etcetera to solve it is very, very difficult to solve analytically. So, you have to take help up some kind of numerical method numerical analysis, numerical method I would say.

And nowadays, as I just mentioned that there are plenty of softwares available, you can use them to solve field distribution once you know the field distribution and proportion constant you can go for you know everything about the wave guide, and now, use this wave guide for developing your circuit, photonic integrated circuit. So, we are very much close to develop photonic integrated circuit and before going into that detail, we will be learning how this mode actually propagates.

So, suppose you have in a waveguide you can have multiple modes solutions you will be getting when you solve these things, some of them will be very confined well defined within the waveguide and some of them are lossy, it is just leaky modes, we will not concentrate on leaky modes, we will be exciting only the confined guided mode. And we will see that if this confined guided mode, there are 3, 4 guided modes you are exciting.

We would like to know how they propagate, we will show that while propagation inside the waveguide if multiple modes are there, they do not talk to each other, they do not share energy from each other, they do not interact at all they are not coupled they can propagate independently as your Eigen mode. So, suppose you have a 5 modes each mode scattering say for example, 1 watt power every mode you are launching at the input side one more 1 watt power per mode 1, another 1 watt for mode 2, another 1 watt 4 mode 3.

So, as long as your wave guide is maintained uniformly that 1 watt power will be carried by mode 1 another 1 watt will be carried by another they will not exchange their power in between that is the beauty of these are Eigen mode solutions you will be getting and practically that is true. However, if you are waveguides how it is getting disturbed, then this energy exchange can take place between mode 2 mode and that is very important that is very crucial how they are exchanging power, how you can control them.

And by controlling them whether you can achieve some functionality or not do things the subject of photonic integrated circuit integrated optics. So, first in the next lecture, we will be learning about this so called how these? These are Eigen modes, and they have a specific property called orthogonality property. So, that orthogonality property first we will be discussing and following that we will be trying to develop certain kinds of coupled mode equations for understanding photonic integrated components and devices. With this, I just close for this lecture and thank you very much.