



**Integrated Photonic Devices and Circuits**  
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**Lecture – 17**

**Optical Waveguides: Theory and Design Dispersion and Polarization of Guided Modes**

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Slide#1



**Integrated Photonic Devices and Circuits**



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**Lecture - 17**

**Optical Waveguides: Theory and Design**

**Dispersion and Polarization of Guided Modes**

- Step-wise full vectorial eigen mode solution
- Guided mode solutions and dispersion relations
- Identifying the polarization of guided modes



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Today lecture I will continue to discuss the guided mode and their dispersion and polarization. So, before going into that details I would like to discuss whatever we have derived the Eigen mode full vectorial method stepwise I will be just giving you some outline. And then I will continue to discuss about the guided mode solutions and their dispersion relations. And finally identifying the polarization of guided modes, whenever you get a solution using full vectorial method and then you need to understand what type of polarization it is there are some classifications I will be discussing on that.

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Optical Waveguides: Theory and Design Slide#2

**Guided Modes and Orthogonality Condition**  
Step-wise full vectorial eigen mode solutions

Thickness : 2 – 3  $\mu\text{m}$   
Si Handling  
Thickness - 500 – 750  $\mu\text{m}$

$h = 0$

Thickness : 2 – 3  $\mu\text{m}$   
Si Handling  
Thickness - 500 – 750  $\mu\text{m}$

photonic wire waveguide

**Design parameters of silicon photonics optical waveguide**

$\lambda \rightarrow$ Operating Wavelength	
$n_d \rightarrow$ Device Layer Refractive Index	$H \rightarrow$ Device Layer Thickness
$n_s \rightarrow$ BOX Layer Refractive Index	$h \rightarrow$ Slab Layer Thickness
$n_c \rightarrow$ Cover Layer Refractive Index	$W \rightarrow$ Waveguide Width

Step-0 Mapping of dielectric constant or refractive index profile

$\epsilon_r(x, y) \rightarrow n^2(x, y)$

*using an analytical equation may not be possible in certain*

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So, I continue that as a reference I take help of silicon waveguide this is a steep structure parameters are  $W$ ,  $H$  and  $h$  known as device layer thickness and  $h$  is the slab layer thickness and  $W$  is the waveguide width and the slab layer  $h$  can be 0 when it is 0 that is called photonic layer waveguide. So, sometimes steep waveguide and sometimes people call also photonic layer waveguide.

So, for more solution we need to know the refractive index of the core and also the cladding materials and the operating wavelength which wavelength you want to operate or you want to know the guiding property that operating wavelength is needed because this full vectorial method is a frequency domain it is Fourier domain solutions. So, first step is that we call it as step 0, step 0 is mapping dielectric constant or refractive index profile.

So, we have shown here the example of silicon waveguide but it can be any waveguide, any cross section any profile refractive index profile that can be represented as a  $n^2(x, y)$  and in the  $x, y$  plane you should have a map every coordinate what is the refractive index. So, it may not be possible to express using an analytical equation for example here if you see this refractive index for a certain coordinates only in the refractive index.

And when we were considering  $y$  less than 0 entire box region refractive index is  $n_s$ . So, it can happens that it is a diffuse channel waveguide it can be some kind of doped waveguide those

type of waveguides you can represent with some analytical formula if not then you need a matrix form you have every x, y coordinates you should have a particular dielectric constant or refractive index. So that mapping x y mapping of refractive index profile you need to go for a solution full vectorial method using full vectorial method.

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Optical Waveguides: Theory and Design Slide#5

**Guided Modes and Orthogonality Condition**  $n_d > n_c > n_s$

Step-wise full vectorial eigen mode solutions

$\vec{E}(x, y, z, t) = \vec{E}(x, y)e^{j(\omega t - \beta z)}$   $\vec{H}(x, y, z, t) = \vec{H}(x, y)e^{j(\omega t - \beta z)}$

$\beta = \frac{2\pi}{\lambda} n_{eff} = \frac{\omega}{c} n_{eff}$

$\vec{E}(x, y) = \hat{a}_x E_x(x, y) + \hat{a}_y E_y(x, y) + \hat{a}_z E_z(x, y)$

$\vec{H}(x, y) = \hat{a}_x H_x(x, y) + \hat{a}_y H_y(x, y) + \hat{a}_z H_z(x, y)$

$\epsilon_r(x, y) \rightarrow n^2(x, y)$  **Need to Solve Vector Wave Equations (Full Vectorial Method)**

$\nabla^2 \vec{E} + \frac{\omega^2}{c^2} \epsilon_r \vec{E} - \nabla(\nabla \cdot \frac{\vec{E}}{\epsilon_r}) = 0$   $\nabla^2 \vec{H} + \frac{\omega^2}{c^2} \epsilon_r \vec{H} + (\nabla \epsilon_r) \times (\nabla \times \vec{H}) = 0$

**Step-1**

1.  $\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\omega^2}{c^2} [n^2(x, y) - n_{eff}^2] E_x + 2 \frac{\partial}{\partial x} [E_x \frac{\partial}{\partial x} \ln[n(x, y)] + E_y \frac{\partial}{\partial y} \ln[n(x, y)]] = 0$

2.  $\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\omega^2}{c^2} [n^2(x, y) - n_{eff}^2] E_y + 2 \frac{\partial}{\partial x} [E_x \frac{\partial}{\partial x} \ln[n(x, y)] + E_y \frac{\partial}{\partial y} \ln[n(x, y)]] = 0$

For a given  $\epsilon_r(x, y)$  or  $n^2(x, y)$ ,  $\{E_x^m, E_y^m\}$  and  $\beta_m$  are solved numerically as eigen vectors and eigen values, respectively

1. For the first three terms in equation 2, E x should be replaced by E y

2. In equation 2, the partial differentiation outside the square bracket is with respect to y

So, now as we have already discussed that a dielectric constant profile for the waveguide if it is given that means the refractive index profile has a core and also surrounding it over refractive index region will be there. So, if that is the case and that is 2 dimensional then your waveguided mode can be represented with an electric field and magnetic field. Electric field you will have an x y dependent some confined well defined mode solution.

And then it should be traveling with omega t - beta z. Similarly magnetic field you will have a well defined vector field in the x, y plane and then it will be traveling with the same omega t - beta z. So that means this electric field and that magnetic field they are coupled to each other and that is how it will be guiding along z direction where this beta is defined by so 2 pi lambda n effective and w / c n effective.

So, our first intention is that to find out this n effective, effective index of a guided mode and then next intention will be the electric field that is the x, y dependent profile of the mode that is a vector field and that vector field can have x component, y component, z component likewise

magnetic field will have x component, y component, z component. So, we have all the 6 components are there and we need to solve for a given refractive index profile.

So, this is for example our vector wave equation we have derived earlier for electric field and magnetic field starting from Maxwell's equation. So, Maxwell's curl equation we try to decouple electric field and magnetic field considering that epsilon r is not a homogeneous medium because it is a core cladding everything will be there. So, you should have divergence D equal to 0 but you cannot write divergence E not equal to 0

Because D involves epsilon 0 epsilon r E, so if epsilon r is x, y dependent then divergence E cannot be equal to 0. So, just using that thing we have just written down the vector wave equations for electric field and magnetic field that what we have already derived earlier in the previous lectures. Now step one what is the step one? Step one from this vector equation we can decompose into 3 scalar equations.

One corresponding to x component and y component and z component. 3 scalar equations you can get and here also you can get 3 scalar equation and if you can solve those 3 scalar equations second order differential equation then you are done instead of solving all those 6 differential equation we have shown that you can concentrate on 2 second order differential equation involving from here.

You can find x component and you can find y component x component equal to 0 y component equal to 0 these 2 equation if you analyze clearly you can see that you have E x and you have E y and E x also x y dependent and these also x y dependent this is there. So, it is a second order derivative partial derivative of this one that is coming from del square you know what is the value of del square, del square is nothing but  $\text{del}^2 \text{del} x^2 + \text{del}^2 \text{del} y^2 + \text{del}^2 \text{del} z^2$ .

And then you are getting like this where you have clearly used this epsilon r instead of epsilon r you have just written n square x, y and you have considered this  $\text{del} \text{del} z = -j \beta$  you have consider that means  $\text{del}^2 \text{del} z^2 = -\beta^2$  and minus beta square instead of beta square you can write  $\omega / c n \text{ effective}$ . So, using that thing you are getting this equation.

And because of the presence of  $\Delta \epsilon_r$  we have this additional coupled term where  $E_x$  and  $E_y$  both terms are there that is directly related to  $n_x, y$ . If this  $n_x, y$  is having some kind of inhomogeneous medium that partial derivative about  $\Delta \epsilon_r$  this is learn and that will be surviving that thing you have to count. So that is why we have just kept as long as we are considering in general solution for any general waveguide.

So, you can this is the perfect equation so we can use this 2 second order differential equation  $x$  component and  $y$  component then what we can get for a given  $\epsilon_r, n_x, y$  and or  $n^2$ ,  $y$  we can solve  $E_x, E_y$  and  $\beta$  you can these are if you see these equations are starting actually derived from Maxwell's equations and Maxwell's equations we have used for linear it is the Maxwell's equation in normal case it is linear.

So, you can linearize this thing this equation and this equation you can it a second order differential equation, you can actually represent them in a matrix form in the matrix form you can represent with something matrix of dielectric constant and then you can write something  $E_x, E_y$  and then you can have some value  $E_x$  and  $E_y$  and  $\beta$ . So, this is a matrix  $E_x, E_y$  and this is something  $\beta$  means  $n^2$  effective square term will be there instead of an  $\omega^2 / c^2 n^2$  effective we can write  $\beta$ .

So, if you just represent this in a matrix form this matrix can be created because of the  $\epsilon_r$  or  $n^2$   $x, y$  this thing in a different form you can write down I think there are various techniques how to solve the second order differential equation in  $x, y$  plane. So, we can represent like this and if you solve this one you can find Eigen vector a series of Eigen vectors and corresponding  $\beta$  if you write their solutions are  $m$ th one and then we will be writing  $\beta_m$ .

So, for a set of Eigen vector you get a Eigen value  $\beta_m$  so after solving these 2 equations you are getting the Eigen vectors with component  $E_x, E_y$   $\beta_m$  can run from just an index to say basically indexing your Eigen values you can start from 0, 1, 2, 3, 4. So, if it is 100s of Eigen values you are solving that depends on how much, what is the order of matrix you are forming using your dielectric mapping, dielectric constant mapping or refractive index mapping. So that

can be done numerically I am not discussing this that is out of the scope of this course. So, just how it is being it is calculated that is what my intention is to explain that.

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**Optical Waveguides: Theory and Design** Slide#6

**Guided Modes and Orthogonality Condition**  $n_d > n_s > n_c$

Step-wise full vectorial eigen mode solutions

$\vec{E}(x, y, z, t) = \vec{E}(x, y)e^{j(\omega t - \beta z)}$   $\vec{H}(x, y, z, t) = \vec{H}(x, y)e^{j(\omega t - \beta z)}$

$\beta = \frac{2\pi}{\lambda} n_{eff} = \frac{\omega}{c} n_{eff}$

$\vec{E}(x, y) = \hat{a}_x E_x(x, y) + \hat{a}_y E_y(x, y) + \hat{a}_z E_z(x, y)$

$\vec{H}(x, y) = \hat{a}_x H_x(x, y) + \hat{a}_y H_y(x, y) + \hat{a}_z H_z(x, y)$

$\epsilon_r(x, y) \rightarrow n^2(x, y)$

**Need to Solve Vector Wave Equations (Full Vectorial Method)**

$\vec{\nabla}^2 \vec{E} + \frac{\omega^2}{c^2} \epsilon_r \vec{E} - \vec{\nabla} \left( \vec{\nabla} \cdot \frac{\vec{E}}{\epsilon_r} \right) = 0$   $\vec{\nabla}^2 \vec{H} + \frac{\omega^2}{c^2} \epsilon_r \vec{H} - \left( \vec{\nabla} \epsilon_r \right) \times \left( \vec{\nabla} \times \vec{H} \right) = 0$

**Step-2**

$\vec{\nabla} \cdot \vec{D} = 0 \Rightarrow \vec{\nabla} \cdot [n^2(x, y) \vec{E}(x, y)] = 0$

$E_z(x, y) = \frac{1}{j\beta n^2(x, y)} \left[ \frac{\partial}{\partial x} [n^2(x, y) E_x(x, y)] + \frac{\partial}{\partial y} [n^2(x, y) E_y(x, y)] \right]$

$E_z^m(x, y)$  are evaluated using  $E_x^m(x, y)$ ,  $E_y^m(x, y)$  and  $\beta_m$

So, once you know  $E_x$  and  $E_y$  you go for step 2 you have solved 2 second order differential equation to find out  $E_x$  and  $E_y$  for different  $m$  can run from 0 to 1 so many Eigen vectors Eigen values you are getting. For every given Eigen vectors so  $E_x$  and  $E_y$  you can calculate actually  $E_z$  also  $E_z$  x y m how that is possible you just start with the Gauss divergence theorem divergence  $D = 0$ .

And you know this divergence  $D$  means as I mentioned earlier that is actually  $\epsilon_r \nabla \cdot \vec{E} = 0$ ,  $\epsilon_r = 0$  can be out in here  $\epsilon_r = 0$  that is constant you can take out. And this  $\epsilon_r$  instead of  $\epsilon_r$  you can write  $n^2(x, y)$  you know your refractive index  $\epsilon_r$  is  $n^2(x, y)$  that is what you will be getting and then  $\nabla \cdot \vec{E}$  vector x y vector will be there you know this is  $n^2(x, y) \nabla \cdot \vec{E}$  and  $\nabla \cdot \vec{E}$  divergence. So, you have 3 terms.

So, you will have say  $\nabla_x \cdot \nabla_x E_x$  plus  $\nabla_y \cdot \nabla_y E_y$  plus  $\nabla_z \cdot \nabla_z E_z$  along with that you have to multiply  $n^2(x, y)$  every time you have  $n^2(x, y)$  where that means you have to this thing will be the first derivative and this thing will be the second derivative and second term and this one will also will be  $n^2(x, y)$  and then again you will do a little bit simplification  $\nabla_z \cdot \nabla_z n^2(x, y)$  is not a function of  $z$ .

So, this you are operating on a Eigen vector  $E_y$   $\nabla^2 z$  can give you minus  $j\beta E_y$ . So, using that thing actually you can actually find what is the value of  $E_z$ . So, this is actually  $E_z$ , this will be  $E_z$ . So,  $\nabla^2 z E_z$  this is  $E_y$  this is  $E_x$ . So,  $E_z$  you can be find  $z$  component and if you start with  $m$ th Eigen mode then whatever you are getting, you will be getting  $E_z^m$ . So, for set up Eigen vectors solved using first 2 differential equations and using  $\beta_m$  you can find  $E_z^m$ . So that means all the 3 components  $E_x$ ,  $E_y$ ,  $E_z$  is solved now that is in step 2.

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**Optical Waveguides: Theory and Design** Slide#7

**Guided Modes and Orthogonality Condition**  $n_d > n_c > n_s$

Step-wise full vectorial eigen mode solutions

$\vec{E}(x, y, z, t) = \vec{E}(x, y)e^{j(\omega t - \beta z)}$   $\vec{H}(x, y, z, t) = \vec{H}(x, y)e^{j(\omega t - \beta z)}$

$\beta = \frac{2\pi}{\lambda} n_{eff} = \frac{\omega}{c} n_{eff}$

$\vec{E}(x, y) = \hat{a}_x E_x(x, y) + \hat{a}_y E_y(x, y) + \hat{a}_z E_z(x, y)$

$\vec{H}(x, y) = \hat{a}_x H_x(x, y) + \hat{a}_y H_y(x, y) + \hat{a}_z H_z(x, y)$

$\epsilon_r(x, y) \rightarrow n^2(x, y)$  **Need to Solve Vector Wave Equations (Full Vectorial Method)**

$\nabla^2 \vec{E} + \frac{\omega^2}{c^2} \epsilon_r \vec{E} - \nabla(\vec{E} \cdot \frac{\nabla \epsilon_r}{\epsilon_r}) = 0$   $\nabla^2 \vec{H} + \frac{\omega^2}{c^2} \epsilon_r \vec{H} + (\frac{\nabla \epsilon_r}{\epsilon_r}) \times (\vec{\nabla} \times \vec{H}) = 0$

**Step-3**  $\vec{\nabla} \times \vec{E} = -j\omega\mu_0 \vec{H}$

$\vec{H}_x = \frac{j}{\omega\mu_0} \left[ \frac{\partial E_z}{\partial y} - (-j\beta) E_y \right]$   $\vec{H}_y = \frac{j}{\omega\mu_0} \left[ (-j\beta) E_x - \frac{\partial E_z}{\partial x} \right]$   $\vec{H}_z = \frac{j}{\omega\mu_0} \left[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right]$

$H_x^m(x, y), H_y^m(x, y), H_z^m(x, y)$  are evaluated using  $E_x^m(x, y)$ ,  $E_y^m(x, y)$ ,  $E_z^m(x, y)$  and  $\beta_m$

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Now for step 3 what we do? We take help of its curl equation curl of  $E = -j\omega B$  it is curl  $E$  we know that expression curl  $E = -\nabla B / \nabla t$ ,  $\nabla B / \nabla t$  again  $-j\omega B$  and  $B$  can be written as  $\mu_0 H$ . You can write you have all the components for  $E_x$ ,  $E_y$ ,  $E_z$  you have solved already? So, use them here then you can find  $x$  component left hand side  $x$  component right hand side,  $y$  component left hand side  $y$  component right hand side,  $z$  component left hand side  $z$  component right hand side you can equate.

And then you can get the expression for  $H_x$ ,  $H_y$ ,  $H_z$  obviously wherever  $\nabla^2 z$  terms comes you are using minus  $j\beta$ . By using this minus  $j\beta$  minus  $j\beta$ . So, if you see you can represent  $H_x$  if you know  $E_z$  and  $E_y$  you can find  $H_x$ , if you know  $E_z$  than  $E_x$  you can find out  $H_y$  if you know your  $y$   $x$  then you can find  $H_z$ . So, all these 3 components for the magnetic field also known now just solving first order differential equation here.

Just you have to make a derivative of electric field components  $E_z$ ,  $E_y$ ,  $E_x$  if it is known then all the 3 components are known along with the  $\beta$ . So that means for a given  $n^2$   $x$ ,  $y$  I can get a series of solutions Eigen vectors Eigen values as a  $\beta$  propagation constant and then we can just inspect those solutions some of these solutions will see that they are well confined within the core they are guided mode.

And they have a Eigen corresponding Eigen value they will be travelling along  $z$  direction with a phase velocity this one  $\omega / \beta$  and so on that is traveling, but you can get also many more solutions which may not be giving you well confined solutions. And that may be it is like a leaky solutions that is in Ray optics picture we have seen that sometimes when effective index solution or maybe angle coming less than critical angle.

So that means it will be leaking to the substrate or air that is radiating mode. So, all the radiating mode solutions also you will be getting a Eigen value solution but you have to after solving you have to see the field distribution, electric field distribution and magnetic field distribution. If you see that you get a very well defined mode shape and outside at  $x$   $y$  if you are increasing outside the core and slowly, slowly decaying like even it is an field then that is a guided mode, otherwise they are leaky mode that is the way normally we used to do.

Because you know normally when you are using certain technology and you have to rely on that technology and you are getting a structure which is convenient or which is comfortable to fabricate using a standard technique for example see mass fabrication technique you have a certain process line where you can fabricate waveguide structure like this  $H$  can be 0 all those types of things.

So that particular structure to solve you needs really numerical method like full vectorial method. So, there are commercial software I have already explained you can use them to solve this thing you can develop by your own code also computer program to solve otherwise there are many commercial software's are available you can use that.

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Optical Waveguides: Theory and Design Slide#8

**BOX**  
Thickness - 2 - 3  $\mu\text{m}$

**Si Handling**  
Thickness - 500 - 750  $\mu\text{m}$

**Guided Modes and Orthogonality Condition**  $n_1 > n_2 > n_3$

Step-wise full vectorial eigen mode solutions

$$\vec{E}(x, y, z, t) = \vec{E}(x, y)e^{j(\omega t - \beta z)} \quad \vec{H}(x, y, z, t) = \vec{H}(x, y)e^{j(\omega t - \beta z)}$$

$$\beta = \frac{2\pi}{\lambda} n_{eff} = \frac{\omega}{c} n_{eff}$$

$$\vec{E}(x, y) = \hat{a}_x E_x(x, y) + \hat{a}_y E_y(x, y) + \hat{a}_z E_z(x, y)$$

$$\vec{H}(x, y) = \hat{a}_x H_x(x, y) + \hat{a}_y H_y(x, y) + \hat{a}_z H_z(x, y)$$

EM Fields of a guided mode indexed by an integer number  $m = 0, 1, 2, 3, \dots$

$$\vec{E}_m(x, y, z, t) = \{E_x^m(x, y) \hat{a}_x + E_y^m(x, y) \hat{a}_y + E_z^m(x, y) \hat{a}_z\} e^{j(\omega t - \beta_m z)}$$

$$\beta_m(\omega) = \frac{\omega}{c} n_{eff}^m(\omega)$$

$$\vec{H}_m(x, y, z, t) = \{H_x^m(x, y) \hat{a}_x + H_y^m(x, y) \hat{a}_y + H_z^m(x, y) \hat{a}_z\} e^{j(\omega t - \beta_m z)}$$

$$v_p^m(\omega) = \frac{\omega}{\beta_m} \quad v_g^m = \frac{d\omega}{d\beta_m}$$

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So, now electromagnetic fields of guided mode and that should be indexed by an integer number  $m$  can be expressed like this  $x$  component,  $y$  component,  $z$  component and all of them will have a phase factor it will be traveling with it to the  $j\omega t - \beta m z$  and associated magnetic field also will have  $x$  component,  $y$  component,  $z$  component all these component when I am writing within this curly bracket.

That means this one multiplied by this one that is the  $x$  component, this one multiplied by this one that is the  $y$  component and this one multiplied by this one  $z$  component. Similarly for electric field also that means all the field components electric with all these 6 components together will give you a certain mode this is all solutions will give a mode that mode will have a phase velocity actually  $\omega / \beta m$ .

That is actually traveling wave it is along  $z$  direction that is what how we can represent. And not only that once we know phase velocity if we find some  $\omega$   $\beta$  curve is not linear  $\omega$   $\beta$  relationship then at any particular frequency or  $\beta$  value you can get a slope of that curve  $d\omega / d\beta$  then you can find that  $z$  group velocity, normally you know group velocity is very important.

Whenever you are sending any information modulating signal and frequency components are there as a data to transmit one place to another place. So, this is what we know this mode

solutions are often. Now onwards we will be thinking that we know how to solve the different modes and also beta m these are now known. And we can solve any given structure will be able to solve that can be numerically that can be analytically if certain functions are favourable. Otherwise we will be considering that these modes can be calculated or evaluated very easily, that is pursued we are taking granted now onwards.

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**Optical Waveguides: Theory and Design** Slide#9

**Guided Modes and Orthogonality Condition**  $n_d > n_s > n_c$

Step-wise full vectorial eigen mode solutions

$\vec{E}(x, y, z, t) = \vec{E}(x, y)e^{j(\omega t - \beta z)}$      $\vec{H}(x, y, z, t) = \vec{H}(x, y)e^{j(\omega t - \beta z)}$   
 $\beta = \frac{2\pi}{\lambda} n_{eff} = \frac{\omega}{c} n_{eff}$   
 $\vec{E}(x, y) = \hat{a}_x E_x(x, y) + \hat{a}_y E_y(x, y) + \hat{a}_z E_z(x, y)$   
 $\vec{H}(x, y) = \hat{a}_x H_x(x, y) + \hat{a}_y H_y(x, y) + \hat{a}_z H_z(x, y)$

EM Fields of a guided mode indexed by an integer number  $m = 0, 1, 2, 3, \dots$

$\vec{E}_m(x, y, z, t) = \{E_x^m(x, y) \hat{a}_x + E_y^m(x, y) \hat{a}_y + E_z^m(x, y) \hat{a}_z\} e^{j(\omega t - \beta_m z)}$      $\beta_m(\omega) = \frac{\omega}{c} n_{eff}^m(\omega)$   
 $\vec{H}_m(x, y, z, t) = \{H_x^m(x, y) \hat{a}_x + H_y^m(x, y) \hat{a}_y + H_z^m(x, y) \hat{a}_z\} e^{j(\omega t - \beta_m z)}$      $v_g^m(\omega) = \frac{\omega}{\beta_m}$      $v_g^m = \frac{d\omega}{d\beta_m}$

Power carried by the  $m^{th}$  guided mode

$P_z^m = \frac{1}{2} \text{Re} \int \int (\vec{E}_m \times \vec{H}_m) \cdot \hat{a}_z dx dy = \frac{1}{2} \text{Re} \int \int [E_x^m(x, y) H_y^{m*}(x, y) - E_y^m(x, y) H_x^{m*}(x, y)] dx dy$

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Now what about the power carried by a mode say mth mode, you know the power carried by a mth mode is you can use your pointing vector theorem that means energy flow per unit area per unit time and here we have to consider along z direction. So, along z direction E m electric field and magnetic fields complex conjugate and you have to take real part and then half that will give you the time average power flow.

And if it is a mth mode we are writing mth mode and if it is waveguide cross section though waveguide core is well defined but you know mode field it can be extended outside the core as even it is tell that is why we can say that entirely if you integrate even up to the infinity if you integrate x equal to minus infinity to plus infinity, y equal to minus infinity to plus infinity that is fine because for a guided mode at infinity.

If you are integrating from minus infinity to plus infinity you will be getting only the field around the core region core and surrounding region. So, in fact if you just extend your

integration minus infinity to plus infinity for convenience that is fine if you can put down that limit but you have to take along z direction you have to take a z component and then you have to integrate this pointing vector along z direction and then you have to integrate.

So that is why we are taking a dot z that vector whatever vector you are getting that you have to take projection along the z axis and that will be the energy flow along z direction and if you have all these 6 components you use then this is x a y and you have to write E cross H right E cross H and z component, z component E cross H if you just try to find out that means this is something like this matrix form you can write.

So, then you can write E x multiplied by H y E x multiplied by H y y star of course you are taking a cross b E cross H and then minus E y H x that is actually the z component if you just take E cross H considering all the 6 component presents in a mode. So that mode we carry energy along z direction if you solve this equation if you can integrate this equation if you know the x y dependent profile for the electric field of the x component and magnetic field y component and vice versa. So that means only x and y component of the electric field and magnetic field that actually contributes energy pro along z direction.

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Optical Waveguides: Theory and Design
Slide#11

**Guided Modes and Orthogonality Condition**  $n_d > n_c > n_s$

Guided modes and dispersion relations

Thickness: 2 - 3  $\mu\text{m}$   
Si Handling Thickness: 500 - 750  $\mu\text{m}$

$$\vec{E}_m(x, y, z, t) = \vec{E}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y, z, t) = \vec{H}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{E}_m(x, y) = [\hat{a}_x E_x^m(x, y) + \hat{a}_y E_y^m(x, y) + E_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y) = [\hat{a}_x H_x^m(x, y) + \hat{a}_y H_y^m(x, y) + H_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

$H = 220 \text{ nm}, W = 1000 \text{ nm}, h = 0$   
 $n_d = 3.4778, n_c = 1.4657, n_s = 1.0000$

$H = 220 \text{ nm}, W = 1 \mu\text{m}, h = 0$

$\beta = \frac{\omega}{c} n_{eff}$   
 $\beta_0 = \frac{\omega}{c} n_d$   
 $\beta_0 = \frac{\omega}{c} n_c^2$   
 $= \frac{2\pi}{\lambda} n_{eff}(\lambda)$

So, next thing what should we do? Let us try to find that dispersion relation we know that how much power it will be flowing energy it can carry by guided mode we can calculate once we

know the mode field distribution and propagation constant. Now we know that energy flowing is not enough this type of waveguide or optical interconnect, fiber optics communication etcetera we want to transmit data.

So, data transmission means we have to follow the group velocity and dispersion relation to all those are very important. So, we need to study now dispersal notion for the individual guided modes you consider a shortened waveguide dimension for example here standard thickness device layer thickness  $H = 220$  nanometer we have considered assume waveguide width is 1 micrometer and slab height is 0 if we consider and standard  $n_d = 3.4778$ ,  $n_s$  equal to that substrate box reproductive index 1.4657.

And cladding refractive index we consider that is the air cladding that means 1 it can be oxide also as I mentioned again and again. And we know that the waveguide solutions comes that means every mode Eigen mode solutions comes with  $n$  effective  $m$  series of effective indices and corresponding  $\beta_m$  you will be getting and we represent the  $m$ th mode associated electric field is expressed like this.

This is a vector and this is also a vector  $x, y$  dependencies will we have written, now if we solve if we just plot we just remember that these refractive index when I am saying this is actually for  $\lambda = 1550$  nanometer. And that is a bulk material bulk silicon refractive index is this one at  $\lambda = 1550$  and bulk silicon dioxide this is the refractive index at 1550 nanometer here is the all the wavelength is normally 1 refractive index.

But when I try to solve the dispersion relation we have to sweep the  $\lambda$  value, we want to see the  $\beta$  versus  $\omega$ , so  $\beta = \omega / c n_{\text{effective}}$  that is the things I want to calculate effective index as a function of  $\omega$  or  $\lambda$   $\omega = 2\pi c / \lambda$ . So, I have to sweep the  $\lambda$  so if you sweep the  $\lambda$  to see what all the solutions what do you have to do we need to know  $\lambda$  dependent refractive index or the material dispersion we need to know.

But here in this case we have considered that the material dispersion is almost 0 there are material dispersion for the discussion purpose for present discussion we have considered this material dispersion is 0 that means because you will consider 1500 nanometer to 1600 nanometer very short range if you consider within this range if you are considering within that range you can see that these refractive indexes change many minimum.

But whatever that thing is there you must consider of course but what I want to discuss here that when a electromagnetic wave is propagating as a guided mode confined within a certain area within 1 micrometer vertically 220 nanometer you know 220 nanometer vertical depth and you are considering  $\lambda = 1500$  nanometer. So, it is much, much lower than the wavelength, the thickness of the waveguide dimension is lower than the wavelength width is in the order of wavelength.

So, in that range if your wavelength is varying a bit then your mode solutions will vary rapidly, that is qualitatively you can understand. So, whenever you are talking about waveguide like a silicon waveguide which is very small cross section 220 nanometer width maybe also 400, 500 nanometer here we are considering 1 micrometer. So, in that case your operating wavelength is 1550 nanometers.

So, more than the dimension and because silicon has a very high refractive index that is why the wavelength you can pack your electromagnetic wave as a mode within a area which is much, much lower than the wavelength of the electromagnetic wave in the free space. So that is the reason but at the same time you should keep in mind that if you change the  $\lambda$  slightly from 1500 to certain other wavelength.

Then you will see that there your mod will see different type of mode field distributions as well as a different type of effective index solution and we try to solve with this parameter and to solve it found that we actually tuned the wavelength in our solution from 1.3 micrometer to 2 micrometer that means 1.3 micrometer we have consider because you know  $\lambda = 1310$  nanometer that is the second generation communication window fiber optic communication window.

And that is also transparent in silicon and we have just understanding purpose we can calculate it also 1.55 as well as extended up to 2 micrometer just for understanding purpose. And the x axis if you see this is the wavelength 1550 nanometers it is marked here and y axis we are considering effective index and if we see we concentrate on mode 1 that means  $\beta_0$  we are considering equal to  $\omega c / n_{\text{effective } 0}$ .

And you can consider  $\beta_0 = \omega / c n_{\text{effective } 0}$  or you can consider  $2\pi / \lambda n_{\text{effective } 0}$ . So, if you vary  $\lambda$  then you calculate  $n_{\text{effective}}$  then that  $n_{\text{effective}}$  versus  $\lambda$  curve we are plotting here effective versus if we were tuning the  $\lambda$  value and you are calculating the effective, if you are tuning the  $\lambda$  you are solving your full vectorial equations all the equations you are solving.

And you are getting and since you are getting Eigen mode  $m = 0$  all the Eigen values corresponding to  $m = 0, 1, 2, 3, 4$  you just plot it. So, this is actually founded mode 1, this is mode 2, mode 3, mode 4, mode 5. So,  $m = 4$  means mode 5 because we are counting 0. So, they take a message here is as you increase the wavelength if you are just restricting yourself with one mode, you see effective indexes reducing almost like a linear but it is exactly not linear if you just to amplify.

And check and next mode if you see effective index reduced but same term almost but it is a longer wavelength it is actually the separation is mode and next mode if you see  $n_{\text{effective}}$  we calculate and see effective index as a function of  $\lambda$  it is coming like that and this way and this way and then it is like this. And here this dashed line if you see this is marked at  $n_s$ ,  $n_s$  means substrate refractive index.

So that means silicon dioxide refractive index you are getting a solution sometimes for example if you just cross above 1550 nanometer, your solutions you are getting effective index which is lower than the box with lower cladding refractive index that means that is obviously not a guided mode but sure looking into this you can just conclude that. So, beyond this one  $m = 4$  will not be guiding.

So that means beyond 1550 nanometer wavelength you may have only this one solution and second solution, third solution, fourth solution 4 guided modes will be there and corresponding effective indices you will be getting also. So, if you just consider if you are designing one wavelength one particular waveguide parameters 221 micrometer everything and suppose you are operating at 1310 nanometer here 1.3 micrometer.

Then definitely you are getting 1, 2, 3, 4, 5 different guided modes multimode waveguides. However if you just operate same waveguide dimension if you go further say maybe 3 micron, 4 micron of course beyond 4 micron you cannot use that because beyond 4 micron it will be absorbed by the lower cladding. So, if you see that this term will come like this, this will be coming like this. So, after certain value you see here maybe 3.3 micrometer.

So, here you will see it will become single mode, so if you consider these waveguide parameters and if you operate beyond 3 micrometer for example then it can be single mode waveguide but as you reduce your wavelength more number of modes will be coming, these modes will be coming. So, it will become more and more multimode so choosing the correct parameters waveguide parameters very important to have to restrict to control the number of maximum guided modes in the waveguide 5.

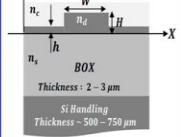
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Slide#12

**Optical Waveguides: Theory and Design**

**Guided Modes and Orthogonality Condition**

Guided modes and dispersion relations  $n_d > n_s > n_c$



$$\vec{E}_m(x, y, z, t) = \vec{E}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y, z, t) = \vec{H}_m(x, y) e^{j(\omega t - \beta_m z)}$$


$$\vec{E}_m(x, y) = [\hat{a}_x E_x^m(x, y) + \hat{a}_y E_y^m(x, y) + E_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

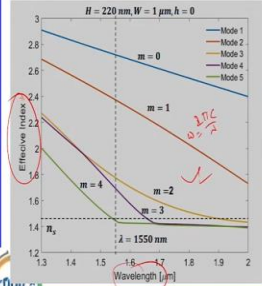
$$\vec{H}_m(x, y) = [\hat{a}_x H_x^m(x, y) + \hat{a}_y H_y^m(x, y) + H_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

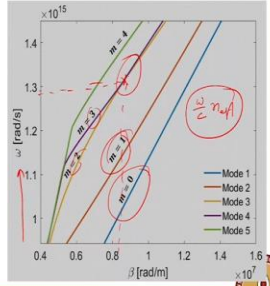
$$H = 220 \text{ nm}, W = 1000 \text{ nm}, h = 0$$


$$n_s = 3.4778, n_c = 1.4657, n_c = 1.0000$$


$$\beta_m = \frac{2\pi}{\lambda} n_{eff}^m = \frac{\omega}{c} n_{eff}^m$$












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Now next thing is that this is actually we can call as a one way one calling it as a dispersion relation effective index as a function of lambda, same thing I because we are interested to know omega beta curve omega versus beta omega beta relationship because from there we can directly extract what is the phase velocity at a particular frequency and what is the group velocity at a carrier frequency omega c for example.

So, what we could do? We can actually this wavelength we can convert into  $2\pi c / \lambda = \omega$ . So, lambda can be converted into omega and that omega we put in the x axis and effective index whatever we are getting we will be multiplying that with omega / c n effective if we multiply then it will be beta. So, if I just multiply omega / c corresponding omega / c and n effectively multiply then you will be getting beta.

So, if I just plot same thing we are plotting in omega beta is just a little bit scaled omega beta it directly then you can get this mode one how it will be looking like mode  $m = 0$ ,  $m = 1$ ,  $m = 2$  the nature will you see for example here  $m = 3$  and  $m = 2$  the crossing at this particular point because you know sometimes you can have a degenerate solution also. So, for example at this frequency you get it per  $m = 2$  and  $m = 3$  you are getting the same beta value that means effective index.

So that type of solutions also can have what is the reason for that? That is because you are getting different Eigen modes some of the Eigen modes can have Eigen values same. So, how that is possible one particular mode and another mode they are same because that actually depends on how the electric field and magnetic field distributed around the cladding depending on that actually that particular mode how it will be traveling with a certain phase velocity that decided.

Suppose some field you will see in the vertical direction it is extended more for a certain face component another field maybe in the horizontal direction it is extended more such that total impedance or total effective index you are seeing in both cases they can be identical also. So, this type of identical solution does not mean they are same mode but they are different mode field



distribution can be different but Eigen value can be same, Eigen vector can be different but Eigen value can be same these types of things we can call degenerate solutions.

(Refer Slide Time: 35:17)

The slide, titled "Optical Waveguides: Theory and Design" (Slide#14), discusses "Guided Modes and Orthogonality Condition" and "Polarization and Field Distribution of Guided Modes". It shows a cross-section of a waveguide with core thickness  $h$ , cladding thickness  $H$ , and width  $W$ . The core has refractive index  $n_1$  and the cladding has  $n_2$ . The substrate has refractive index  $n_s$ . The waveguide is labeled as a "BOX" with a thickness of 2-3  $\mu\text{m}$  and an "SI Handling" thickness of 500-750  $\mu\text{m}$ . The condition  $n_1 > n_2 > n_s$  is noted.

The field distributions are given by:
 
$$\vec{E}_m(x, y, z, t) = \vec{E}_m(x, y) e^{i(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y, z, t) = \vec{H}_m(x, y) e^{i(\omega t - \beta_m z)}$$
 The electric field components are:
 
$$\vec{E}_m(x, y) = [\hat{a}_x E_x^m(x, y) + \hat{a}_y E_y^m(x, y) + E_z^m(x, y)] \times e^{i(\omega t - \beta_m z)}$$
 The magnetic field components are:
 
$$\vec{H}_m(x, y) = [\hat{a}_x H_x^m(x, y) + \hat{a}_y H_y^m(x, y) + H_z^m(x, y)] \times e^{i(\omega t - \beta_m z)}$$
 The propagation constant is:
 
$$\beta_m = \frac{2\pi}{\lambda} n_{eff}^m = \frac{\omega}{c} n_{eff}^m$$

Parameters for the  $m=0$  mode are:  $W = 500 \text{ nm}, H = 220 \text{ nm}, h = 0$ ;  $\lambda = 1550 \text{ nm}$ ;  $m = 0, n_{eff}^0 = 2.3828$ .

Three plots show the field distributions for  $m=0$ :  $H_x^0(x, y)$ ,  $H_y^0(x, y)$ , and  $H_z^0(x, y)$ . The  $H_y^0(x, y)$  plot shows a central peak with a red circle highlighting it.

Logos for NPTEL, CPPICs, and IIT Madras are visible. The slide is part of "Integrated Photonic Devices and Circuits - Lecture-17" by Dr. S. Das.

So, next we move on to look into the different components for example we take another waveguide parameters silicon waveguide parameters  $W = 500$  nanometer,  $H = 220$  nanometers,  $h = 0$  that means a completely put on a wave guide and 500 nanometer sub micron. And of course you should remember that all the solutions you can solve by using any mode solver Eigen mode solver commercially available or you can develop yourself the method I have shown how to develop.

So, now if you consider these waveguide dimension and then you use your mode solver and find different solutions, when you get the first Eigen value  $m$  equal to 0 that means first Eigen vector that means  $E_x, E_y, E_z$  component you will be getting and corresponding beta value you will be getting that beta value corresponding  $n_{eff}$  you are getting like this 2.3828 which is higher than the slab effective index that means  $n_s$ ,  $n_s$  is how much,  $n_s$  is not giving here

$n_s$  is 1.46 something like that so  $n_{eff}^0$  is more than that when you are getting. So, once you get that is the guided that means that is your confined mode then we look at individual field components first we looked at  $E_x$  component  $x$  component of the electric field,  $y$  component of

the electric field, z component of the electric field all of them are present and we will see the scale this color bar here you see the red is the maximum.

That means or the center it is normalized to one and then as you core this is your way guide core it is shown here rectangular box and at the center the  $E_x$  component is maximum there and then as you go away it will be reducing but at the boundary you see boundary there is a discontinuities there because you have to see the continuity equation. So, normally  $E_x$  is the normal component in the vertical boundary here.

So, normal component it is continuous only  $D$  displacement vector is continuous dielectric constant difference in both sides that is why electric field we will see some kind of so called discontinuity but the electric field you can see here. In that scale if you see y component, y component if you see is very weak you see this somewhere here 0.18 or something like that but only visible in the corners a most of the places it is actually almost 0.

That means  $E_y$  component almost it is almost nothing with respect to  $E_x$  but there some field component is there and z component it is a relatively stronger so you have  $E_x$  component  $E_z$  component but  $E_y$  component is not there almost 0 but you cannot rule out completely some value is there. Now part the same mode I have all the 3 components how it will be looking we have seen now what is magnetic field for the same mode.

If you see  $H_x$  component here you can see some value here. But you see scale 10 to the power minus 3 that means  $H_x$  component is there but very weak but what about  $H_y$  component you see  $H_y$  component is also there 10 to the power -3 some component is there and  $H_z$  components also power -3 some is there. So, overall I think here this scale must be a little more it will be I think here somewhere some mistake is there  $H_y$  components should be higher.

Now of course it is magnetic field strength you have to relate with the electric field strength that is why it is showing but this  $H_y$  component is the dominating factor here it is scaled with the  $E_x$  value that is why it is lower, here also it is lower but in the corner and  $H_z$  component is also similar things.

(Refer Slide Time: 39:18)

Optical Waveguides: Theory and Design Slide#15

**Guided Modes and Orthogonality Condition**  $n_d > n_s > n_c$

Polarization and Field Distribution of Guided Modes

BOX  
Thickness: 2 - 3  $\mu\text{m}$   
Si Handling  
Thickness: 500 - 750  $\mu\text{m}$

$$\vec{E}_m(x, y, z, t) = \vec{E}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y, z, t) = \vec{H}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{E}_m(x, y) = [\hat{a}_x E_x^m(x, y) + \hat{a}_y E_y^m(x, y) + E_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y) = [\hat{a}_x H_x^m(x, y) + \hat{a}_y H_y^m(x, y) + H_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

$$\beta_m = \frac{2\pi}{\lambda} n_{eff}^m = \frac{\omega}{c} n_{eff}^m$$

$W = 500 \text{ nm}, H = 220 \text{ nm}, h = 0$        $\lambda = 1550 \text{ nm}$        $m = 1, n_{eff}^1 = 1.5838$

$m = 1$

$E_z^1(x, y)$

$E_y^1(x, y)$

$E_x^1(x, y)$

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Now go to the  $m = 1$  solution next Eigen value Eigen vector you see  $E_x$  1 is almost nothing is 8.7 this scale is given and  $E_y$  1 that will be stronger is 2.9 you are saying that this thing field distribution is different now the it is higher order mode field distribution is different and then you see this one z component is like this. So, all these 6 components are present some of them are stronger some of them are not stronger.

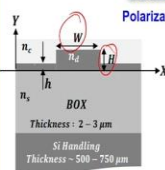
So, we know how it will be looking like all different field distribution but particularly silicon waveguide we have confirmed that whatever we predicted in full vectorial solutions that all the 6 components will be there they are coming. Now series this is for magnetic field.

(Refer Slide Time: 40:14)

Optical Waveguides: Theory and Design Slide#19

**Guided Modes and Orthogonality Condition**  $n_d > n_s > n_c$

Polarization and Field Distribution of Guided Modes



$$\vec{E}_m(x, y, z, t) = \vec{E}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{H}_m(x, y, z, t) = \vec{H}_m(x, y) e^{j(\omega t - \beta_m z)}$$

$$\vec{E}_m(x, y) = [\hat{a}_x E_x^m(x, y) + \hat{a}_y E_y^m(x, y) + E_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$



$$\vec{H}_m(x, y) = [\hat{a}_x H_x^m(x, y) + \hat{a}_y H_y^m(x, y) + H_z^m(x, y)] \times e^{j(\omega t - \beta_m z)}$$

$$\beta_m = \frac{2\pi}{\lambda} n_d^m f = \frac{\omega}{c} n_d^m f$$

**Case-I: TE-Like Mode**  
 Electric Field Vector is in X-Z Plane and Magnetic Field Vector in Y-Z plane  
 $\vec{E}_m(x, y) = [E_x^m(x, y) \quad 0 \quad E_z^m(x, y)]$   $\vec{H}_m(x, y) = [0 \quad H_y^m(x, y) \quad H_z^m(x, y)]$

**Case-II: TM-Like Mode**  
 Electric Field Vector is in Y-Z Plane and Magnetic Field Vector in X-Z plane  
 $\vec{E}_m(x, y) = [0 \quad E_y^m(x, y) \quad E_z^m(x, y)]$   $\vec{H}_m(x, y) = [H_x^m(x, y) \quad 0 \quad H_z^m(x, y)]$

**Case-III: Hybrid Mode**  
 The Electric Field and Magnetic Field Vectors are not restricted in X-Z or in Y-Z planes  
 $\vec{E}_m(x, y) = [E_x^m(x, y) \quad E_y^m(x, y) \quad E_z^m(x, y)]$   $\vec{H}_m(x, y) = [H_x^m(x, y) \quad H_y^m(x, y) \quad H_z^m(x, y)]$



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Now we based on the fraction of the components normally people used to categorize classify polarization of the guided mode. So that is why now we want to discuss polarization and field distribution of the guided modes, polarization what different type of polarization set we can have. So, sometimes some solutions you may get say in this configuration this is the x axis, this is y axis x component will be there, z component will be there in the electric field.

So that means electric field in the xz plane, xz plane means it is parallel to the tangential to the substrate. So, electric field will be oscillating only in the plane of the substrate, plane of the waveguide surface and magnetic field will be in the yz plane x component is not there. So, magnetic field will be in the vertical plane yz plane so if electric field is oscillating in the xz plane z is the propagation direction.

And magnetic field is in the yz plane it has a y component and z component electric field x component and z component then this type of mode will call as a TE like mode. So, in 1D waveguide we know that the x component will be the electric field and y component will be there in the magnetic field that is actually so called TE polarization. But in this case you have electric field in the plane because 2D confinement.

So, you cannot just say that only one direction, so it will be in that one plane and magnetic field in the vertical plane that will be called TE like this is just a nomenclature just for understanding

purpose it is all whatever you are getting as a solutions that is that absolutely have to consider but sometimes some solutions you will be getting in this nature if in this nature we will be calling that TE like mode because this TE like mode we can easily refer for certain application.

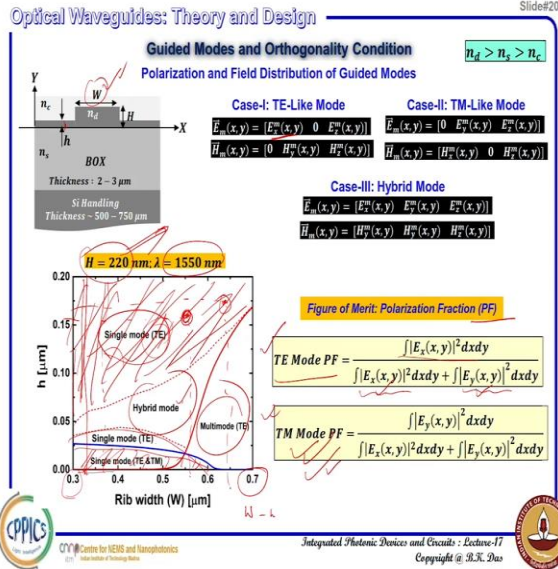
If it is TE this type of it is sensitive to this type of application if it is some other orientation then it will be sensitive to other types of solutions. So, likewise we can have another option that your electric field can be in the yz plane vertical plane and magnetic field in the horizontal plane. So, TE like mode electric field in the horizontal plane magnetic field vertical plane and now magnetic field in the horizontal plane and electric field in the vertical plane.

So that type of field will be called mode will called TM mode. So, transverse magnetic transverse electric, transverse electric means your that means a transverse in the plane of the waveguide electric field and transverse magnetic, magnetic field in the plane of the waveguide. So that will be the TM mode and if all the components are there per example like this all the component then we call that the hybrid mode.

So, in this case your E y component is not there, in this case E x component is not there. And if both E x and E y component is E z component has to be there some way and if E x and E y components are comparable then we can call that as you cannot classify like TE like or TM like mode electric field and magnetic field is no more in a plane. It has all the 3 components x component, y component, z component it is not either xz plane or yz plane.

So, it is actually called hybrid mode so this is 3 different types of classification normal in the textbook and literature people refer normally. So, some solutions you get like TE like mode we can say that TE like mode, some solutions will be these components will be there this TM likes solution. So that depends on all the parameter of course W, H and h this waveguide parameter in n d, n s, n c all those parameters involves.

**(Refer Slide Time: 44:13)**



Now we try to see these 3 different type of solutions for a certain waveguide here with W h we consider  $h = 220$  nanometer which is very popular in CMOS Foundry and to consider lambda equal to only 1550 nanometer wavelength operation and because this is a device layer we are getting from the wave for as we are buying. Now we would like to in our hand if I want to fabricate waveguide or design waveguide in our hand then we have 2 parameters.

I can control W or this h, in this plot what I try to show here that waveguide width varied from 300 nanometer to 700 nanometer and h the slab height that is actually where it from 0 to h can be 0 to 200 nanometer total height it is 220 nanometer that is given fixed, lambda 1550 fixes then for every waveguide width I varied your h and then try to see the solutions every point similarly I change my W value here and I vary again h then try to map a plane.

I tried to find the guided mode solution in Wh plane suppose if I consider this point, this point means I have to consider W value is here and h value is here. And if I use this h value and this W value then what type of solutions I will be getting? I will be getting only one solution and that is TE like mode similarly if I just consider this point that means corresponding W value this one if I solve this one I can get h this one they are also I will be getting TE mode single solution.

So, this region if you see this region for any W and h value you get it will be like a TE like mode and now I say that TE like mode that means the x component, z component will be there and E y

component will be 0 normally  $E_y$  component early will be getting 0 because earlier we have seen that component also has some value but when I say 0 it is negligibly small but it will be certain value maybe 5%, 10% that is also still we can call as a TE like mode.

So, this TE like mode we can quantify the TE like mode or TM like mode then hybrid how to quantify that for that purpose normally in literature people use 2 different figure of merit, what is called TE mode polarization fraction, if you say TE mode normally we say that x component will be dominating x component and intensity square we are taking integrating over into only x component square I am integrating what entire wave guide.

And then I am normalizing with the x component or y component total intensity, intensity contributed by x component, intensity contributed by y component I am adding them and normalizing to the what is the fraction of the  $E_x$  component that it means what is fraction of  $E_x$  component this measures. So, if this fraction is very high close to the 90 more than 90% then we will call it is a TE polarized.

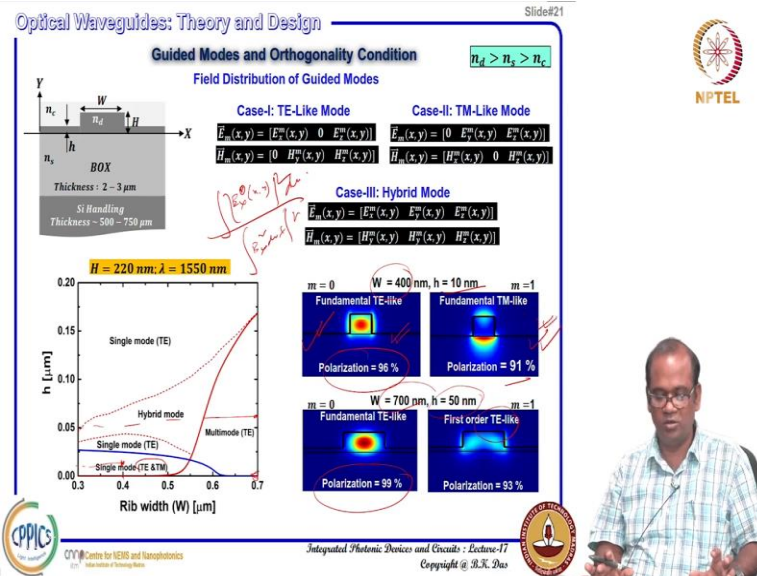
If it is less than 50% maybe only 10% or something like that that means this fraction will be higher TM polarization mode PF polarization will be higher. So, we will be calling that as a TM mode. So, based on that when we find something TE polarization fraction more than 90% then we will be calling TE and if it is less than 90% or if it is something 10% this is more than 90% then it will be called TM otherwise you will be calling a hybrid.

So, in that sense we find different zone in  $Wh$  plane where you will be getting single mode, where you will be getting multimode and where you will be getting hybrid mode and this region you will be getting single mode TE and this region this particular region under the blue curve you will be getting both TE and TM mode. One single mode TE and that is not really a single mode you get 2 modes are there but one will be exactly TE like mode and other will be TM like mode.

So, 2 modes will be there so that way you can easily follow for a silicon waveguide. We have calculated this thing to find out suppose I need a certain type of mode to be guided and this

would be single mode or this would be multimode I need both TE and TM then I can looking into this map I can find out I need this much w value and this much h value.

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So, now this is something we have calculated also that when are you just considered  $W = 400$  nanometer,  $h = 10$  nanometer you try to find out the solution for  $m = 0, m = 0$  you will be getting TE like mode because TE like mode polarization fraction you will be getting 96% that means you are getting  $E_x^2$  divided by this is integration  $dx dy$  and  $E_x^2 dx dy$  plus this one is getting 96% that means the x component is 96% compared to the total value of  $E_x$  and  $y$ .

That is why we were considering this is a TE like mode and if you are solving for the higher order mode fundamental mode that is actually  $m = 1$  you will be getting then you will getting TM like mode that means  $E_y$  component is 91%  $E_y$  component fraction that means electric field will be dominating in the  $y$  direction and in this case electric field will be dominating in the  $x$  direction.

So, this one will be calling like a TM like mode and these will be calling a TE like mode. So, here that is why it is 400 nanometer if you are just considering 400 nanometer and  $h = 10$  nanometer some point you are considering here 10 nanometer here. So, you see one it will be



still some people used to call it as a single mode because one is completely TE like mode and other is completely TM like mode.

So that is why it is single mode but both the polarization can guide that way but in principle in mode solver point of view it is actually still multi mode 2 modes it supports. Similarly for 700 nanometer we have a fundamental mode and 700 nanometer and slab height 50 nanometer. So, 700 nanometers slab height 50 nanometer here, so it is a multimode region. So, fundamental mode you will be getting TE like mode we are TE like 99% polarization.

And next higher order mode also will be TE like no TM like both the mode  $m = 0$  also TE like,  $m = 1$  also TE like and in this case  $m = 0$  is TE like,  $m = 1$  next Eigen value solution like a TM like mode. So that means these TE like, TM like mode it is our own definition but ultimately full vectorial solution this vector wave equation whatever you are solving that is the ultimate things. So, with this I just stop this lecture today and we will continue next about the orthogonality condition in the next lecture.