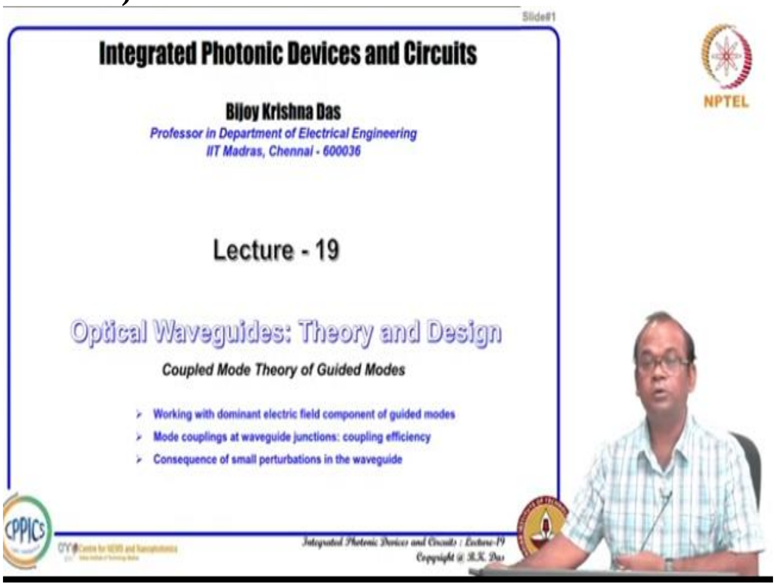
Integrated Photonics Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology – Madras

Lecture – 19
Optical Waveguides: Theory and Design: Coupled Mode Theory of Guided Modes

(Refer Slide Time: 00:17)



Hello, in this lecture today, we are going to start coupled mode theory of guided modes. So, we have already discussed the how to solve guided modes using full vectorial method. And then we also learned that how these guided modes are orthogonal and we have derived also the orthogonality condition. Now, as long as those modes are orthogonal, they cannot exchange energy while propagating co propagating on counter propagating what is I want.

However, if you somehow break the orthogonality condition, then intentionally by your waveguide designers or whatever you can, then there is a chance that these modes start this Eigen modes start interacting each other, they can exchange power among themselves. So, for that purpose, we will be developing a coupled mode theory for a perturbation controlled breaking up orthogonality condition.

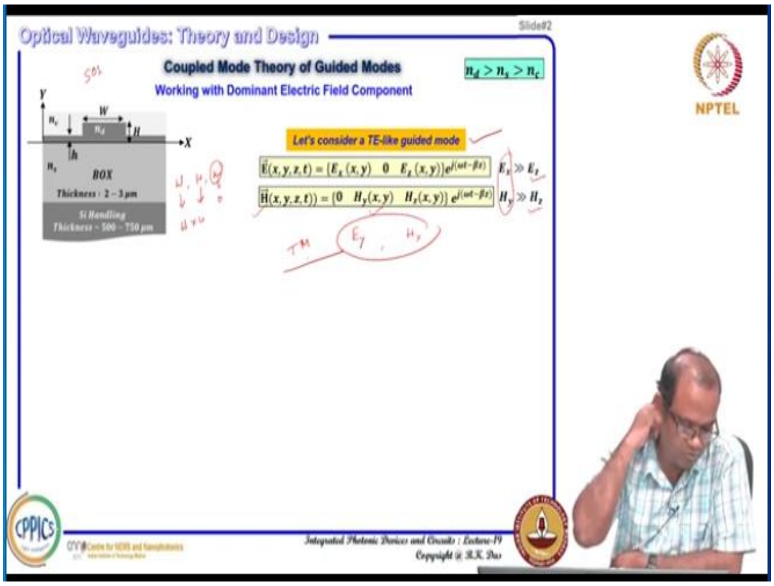
So, towards that in it we would like to discuss now a first thing that we have to modify with our wave equations to convert it into a working model. So, what will be doing, will be working with dominant electric field component. So, you know, there may be in principle hybrid mode, you can have 3 components for the electric field E x, E y, E z. Similarly, 3 components for the H x, H y, H z all these 6 components can be there and TE like mode, TM like mode all those types of things are there.

So, instead of handling all those components, will be just concentrating more on the dominant electric field of the mode. And now we would like to see how that dominant component of the electric field evolved as there are some perturbations or waveguide structure changes, etcetera. For that purpose, we would like to learn first, how to handle only the dominant electric field of the guided modes, how we can approximate that one.

Then next thing is that we will just try to discuss mode couplings at waveguide junctions. That means, if you have 2 different types of wave guides, you can think of one waveguide with a certain cross section and other guide another cross section. So, if you are launching light in one waveguide then what happens in the junction or if you have a waveguide you are launching light from a fiber from the external source and fiber you were just hitting exactly, but coupled to new optical waveguide in a chip. So, in that case what happens?

So, those things we will be learning first that means coupling efficiency that means, we would like to discuss about the coupling of one waveguide coupling up the modes in one waveguide to the another waveguide having its own Eigen mode solutions, that thing we will be learning second thing and then the finally, we will be just discussing in this lecture, the consequence of small perturbations in the waveguide if there is no strong perturbation, then you can somehow a little bit approximate your equations and you can try to see what is the outcome whether you can use it in a effective way or not that I will be discussing.

(Refer Slide Time: 04:21)



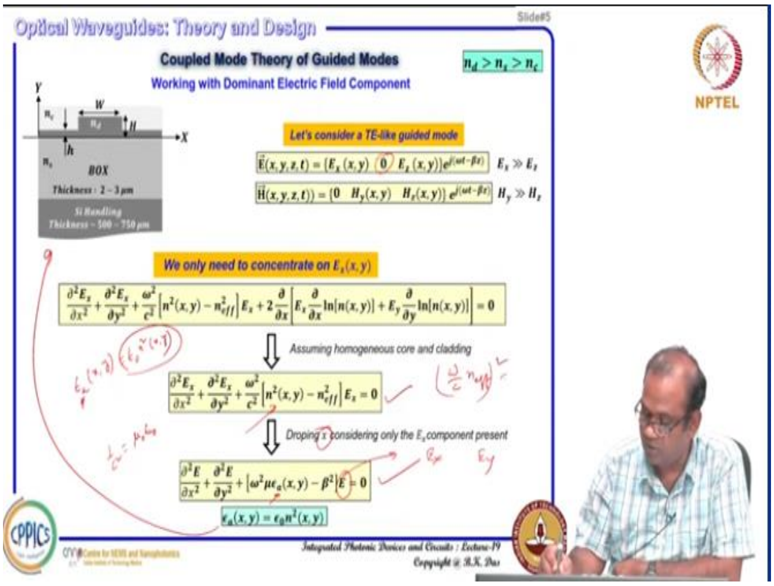
So, next thing so, you know as usual this is our waveguide cross section, we are taking an example of SOI silicon on insulator platform. In this case particularly that is a silicon re waveguide structure where and waveguide parameters are W, H, h and h can be 0 and this one actually fixed by the wave for you are buying. And you can control W value waveguide width. So, if h = 0 then W and H waveguide diamonds and will be W / H.

So that will like for photonic waveguide structure and if you analyse the modes, then what do you see you can have most of the time you will be getting either TE like mode or TM like mode, let us consider only TE like mode in that case you will have electric field in the xz plane in the plane of the substrate and magnetic field will be in the vertical plane that is in the yz plane.

And typically most of the time you will find that this E x component is much higher than E x component and here our magnetic field H y component will be also higher than the longitudinal component, nevertheless will be this transverse component E x and H y normally they contribute power propagation in their direction that is why we will be it is good enough to concentrate only one transverse components and in case of TM, this will be changed this electric field will be E y and magnetic field will be H x.

So, these are the 2 components for the TM polarization you need to consider so let us start with a TE like mode, so we can develop our working theory so that you can use them effectively.

(Refer Slide Time: 06:17)



So, now so as I mentioned that we need to only concentrate only one x component of the electric field for TE like mode in that case, we know that the scalar wave equation associated with the vector wave equation only E x component if you use you know that x component this is x component E x and then you are using this refractive index profile, this refractive index profile is nothing but epsilon r = epsilon 0 we can write this one as something like this n square xy.

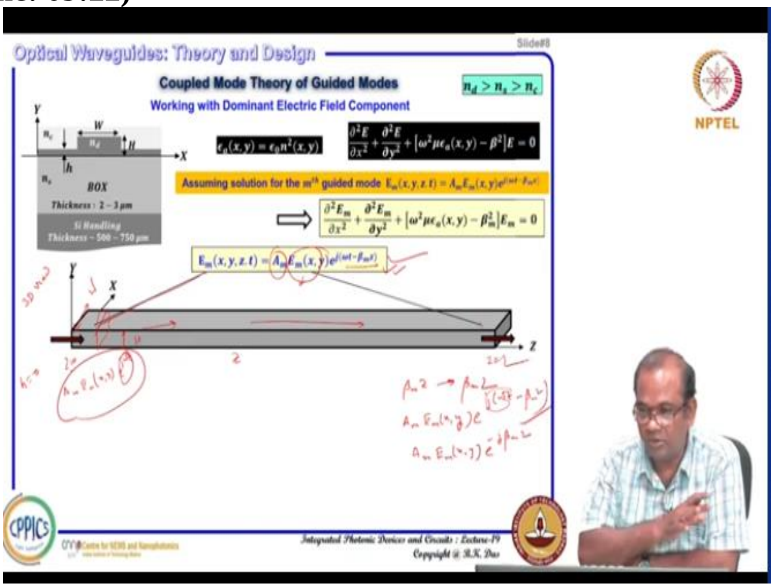
So, that way if we write that is the refractive index profile we can write and effective index of the guided mode and then you have coupling equation if this refractive index has a gradation in the xy plane, then we should have this term will be there otherwise you can ignore that assuming homogeneous core and cladding. So, this part I am just cancelling and in that case, I have very simple equation they we can consider this a 2D waveguide for a homogeneous refractive index core and cladding this is the equation this equal to 0.

So, only E x is involved E x means that must be obviously, that can be a function of xy there it is mentioned. So, now, next thing is that since we are only concentrating only one x component E x we do not need to bother about what is it what did you what were you basically in this case we are considering much less 0 approximately 0 here we are considering electric field.

So, we can keep in our mind that this is the x component I am considering let us drop this x subscript only keep in mind that is the x component and it is a scalar equation, this is also our scalar wave equation only whenever I am considering E now on ward we will be considering that it is a E x for TE polarization and if it is for TM polarization, it should be considered as a E y nothing else so, just to TE to TM if I am dealing with the electric field then I will be writing E x for a TE like mode and it will be E y like a TM like mode.

Nevertheless, let us only concentrate E whatever the dominant electrical component and it just scalar equations and of course waveguide cross section we can just represent the waveguide cross section the dielectric constant can be represented something like this a it is just symbolized as a waveguide cross section somewhere core and cladding is there and that is obviously what I mentioned earlier that should be epsilon 0 n square xy that is what we mentioned same thing we are just repeating here so far so good.

(Refer Slide Time: 09:22)



So, let us move on so, I have just written down this thing for using in following discussions. So, this is our scalar wave equations, where we have just multiplied omega square in this equation I forgot to mention that in this equation this omega / c square I just multiplied here, and that will be c square you can write 1 / c square is nothing but mu 0 epsilon 0. So, mu 0 epsilon 0 we can write here in that case, omega square mu 0 and I can write epsilon 0 times n square xy that means you are epsilon a.

So, you are just writing dielectric constant directly and omega square / c square and you know omega / c if you multiply here, omega / c n effective square which is nothing but your beta squared that is what it is written here. So, that is what the scalar wave equation and electric field is the dominant electric field component as I put in the heading and this is the dielectric constant profile of the waveguide which can be represented again in terms of the refractive index profile.

Now, if you have somehow mth guided mode, you know if it is a waveguide that can actually support a number of modes. So, you can represent mth mode m can be you can index like m = 0, 1, 2, 3 and so, on. So, any arbitrary mode you are considering E m say x, y, z t again we are just we can consider this as a scalar that is the space dependent scalar only will be concentrating only the dominant field component .

Then this is it beta x E m x y that means, I am just trying to express the mode fill distribution in xy plane that is with represent by this one and phase part is this one. So, if it is m = 0 I will be writing A 0, E 0 and beta 0 that is it difference. So, if it is mth waveguide mth guided mode if we are considering and if you can replace this E as E m then I get this equation. So, this E m means basically it is nothing but E m x y.

E m x y that means field distribution of the, that is your dominant electrical field component of the mth guided mode that is what E m stands for here. Now, next thing is that if it is a waveguide now, I am just trying to show you a 3D view of a photonic waveguide so, this is your x direction x direction this is you can say this is a W and this is a vertical direction this is your H photonic waveguide I have just assumed H = 0 this is just for discussion purpose and you are launching light from here.

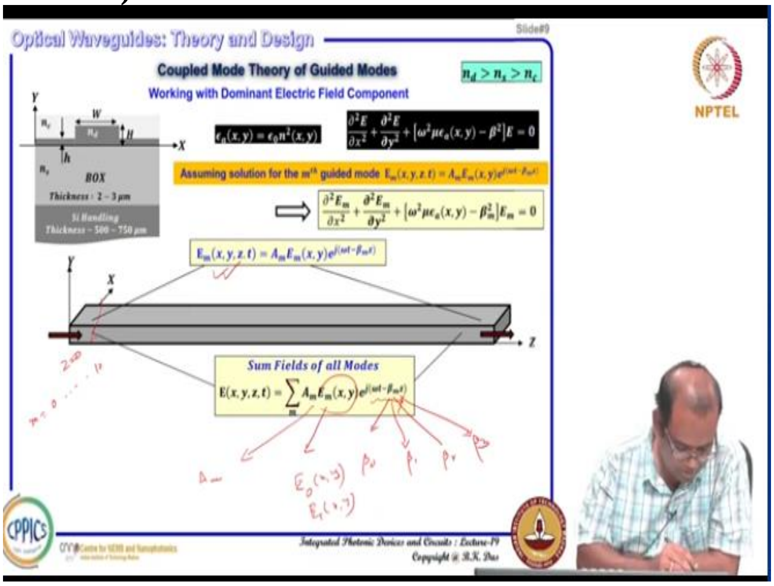
And you just want to see at a distance this may be z = 0 this is z = L and so on. And you are launching light if you are launching a mth mode at the input mth mode is expressed like this and obviously we can consider E m as just kind of amplitude and this is just electric field distribution profile in x, y pipe line, and that will be propagating along the z direction. So, that means the mode if I just see what is the mode here in this cross section, as it propagates.

It will propagate with keeping this type of chip paste and this type of paste as you are traveling way it will be traveling all the way that is how I can I can so there whatever the mth mode at present here same mth mode will be represented here also here only thing is that instead of beta m z it will be just you will be writing beta m L and in this case beta m z if z = 0 that part will be 0. So, it will be just A m is E m x y e to the power j omega d will be there.

So, this one will be there and here it will be A m E m x y the same will be same and phase will be now, at same instant of time if I am considering this will be beta m L that is the difference this will be input and that will be your output if you are considering at time t instant what is the situation then this will be 0 this will be 1 and this will be 1 then you will be getting A m E m = 0 instant I am talking x y then you will be getting minus t beta L.

So, if you are taking a snapshot at t = 0, I can actually define what is the field strength at every point of z I can define using this equation, this is the Eigen mode solution that will travel carrying its own energy and power along the waveguide direction we know how to calculate the power of mth guided mode.

(Refer Slide Time: 14:45)

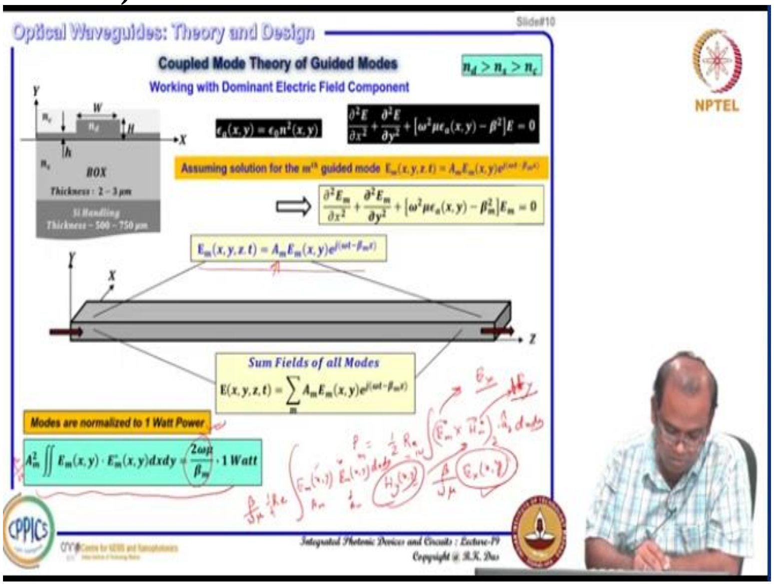


So, now if you have instead of only one mth mode, if you have all modes, you can think of that m runs from 0 to if it is supporting return modes 0 to 10 modes then what you have to do at any point at z = 0. For example, we are considering what is the total field, you have to just consider sum up all fields at z = 0 you put z = 0 beta m. So, this is the sum all mode times as it is an electric field.

So, anywhere if you have a multiple mode are propagating so at any point you want to see what is the total electric field, you have to see the superposition of all electrical along with its phase that is why all individual mode they will have pitch pattern like this beta m z it if it is first mode, it will beta 0 second mode, it will be beta 1, third mode beta 2, fourth mode beta 3 and so on all these mode obtain modes will be there.

So, you just see even profile will be E 0 x y and then E 1 x y that profile will be different also mode set will be different and they are amplitudes also can be different, we will discuss that. So, individual modes they can have different amplitude also, but at any point if you want to see what is the total field you have to see the sum of all electric fields there along with its phase because one field if it is just phase 0 another mode it is coming phase with opposite that means exactly minus then total field will be to be subtracted. So that is how you have to just consider superposition of all field along with its sign or phase.

(Refer Slide Time: 16:29)



So, now the modes are normalized to 1 watt that that is what we have discussed earlier you know the power in a mode we can calculate like this half real part and then E m if it is a mth mode for example, E m cross H m star and you have to see all the z component that means a z you have to see and dx, dy. Now you know normally this electric field magnetic field electric field is in the x component and magnetic field will have only y component assuming longitudinal component not contributing to the power propagate flow in the z direction.

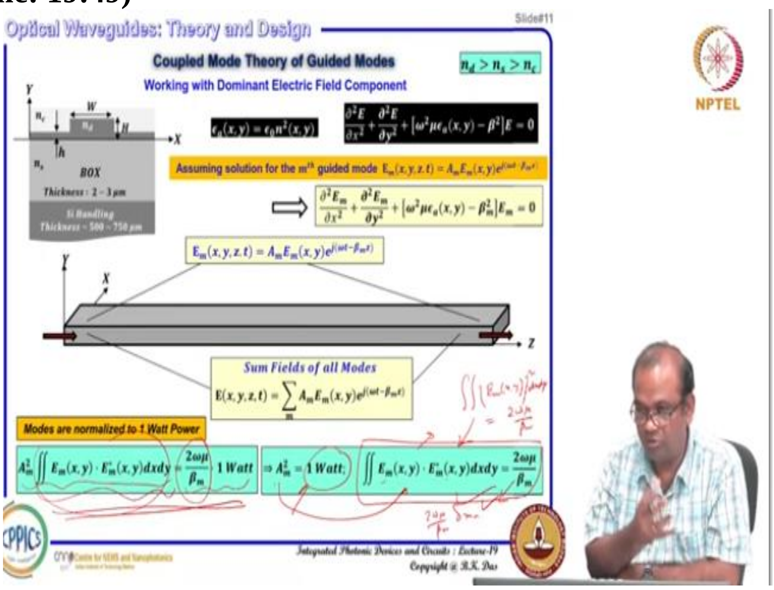
So, in that case I can express that this is your H y is this one magnetic field is a H y and electric field along the x direction magnetic field along y direction. So, we can see from the call equation we have shown in the previous lecture also that this H y so, that will be beta y omega mu times E x so it is just simple relationship. If you know the electric field distribution along the x direction then you can find magnetic field distribution along y direction.

So, if you just substitute this one here, then what will be getting ultimately will be getting E m x y and E m x y star integration dx dy and then half real part and then obviously, you have this beta / omega mu beta / omega mu will be there this beta / omega mu that is the total power. So, if you are taking cross product that cross product will give you that only E x component and H y component and H y component will be this one if you multiply and we can find out that if this E m x y actually associated with amplitude A m.

Then I can write one amplitude will come here and other amplitude will come here that is what A m square so we can write this way. So, beta / 2 omega mu that can be taken to the right hand side if this total power is 1 watt that means I can write this one. So, if you are just using that the assumption that every mode if it is mth mode mth mode, if it is mth mode mth mode, mth mode if it is carrying 1 watt power, then I can write this is the mth mode is represented like that.

Then this 1 watt expression can be written like this 2 omega mu / beta m times 1watt because beta m/ 2 omega mu should be this side when you are calculating your total power that is actually 1 watt. So, you take that this side then it is coming like that.

(Refer Slide Time: 19:49)

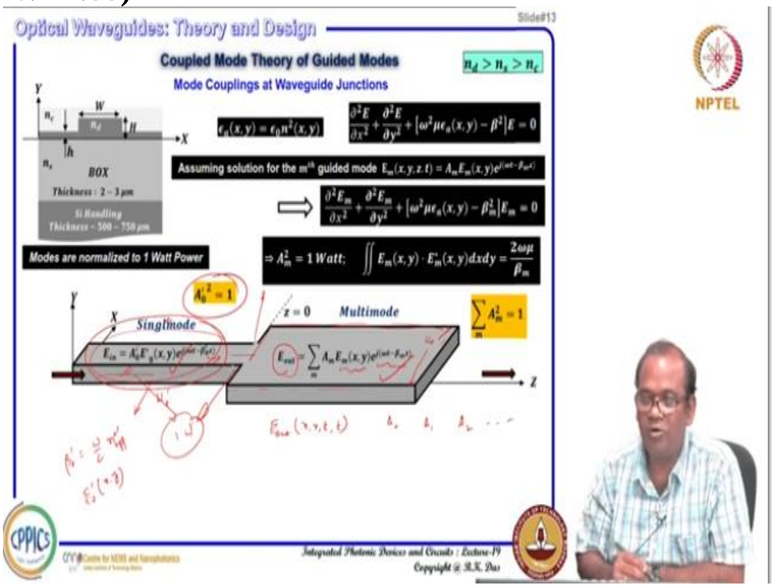


So, now what do you do? You have this expression equal to this expression. So, I can consider that this A m square actually equal to 1 watt and this one is equal to 2 omega mu / beta m. So, from this consideration that if every mode is carrying mode is 1 watt. Then the amplitude I can impose 1 watt amplitude square can be imposed as a 1 watt and whatever this expression is their integration E m, that means this one should be written like this E m x y square dx dy that should be equal to mth mode 2 omega mu / beta m so that will be the expression.

So, if you just multiply this one and this one A m square this one then you will be getting 2 omega mu / beta m 1 watt that is the expression you will be getting. So, I have now another thing so all these we are following from the orthogonality condition, orthogonality condition what is that, if these 2 modes are different, if it is m and this is n, then we can write here 2 omega mu / beta m times delta mn.

So, if m not equal to n that should be equal to 0 according to the orthogonality condition, If m = n then this is the expression will be using this expression again and again when we will be developing our coupled mode theory. So, you should keep in mind so you should not only keep in mind this expression, you should know where from there originated. So, this is the explanation.

(Refer Slide Time: 21:33)



So, far so good we just repeated all the things so, far we discussed waveguide profile, dielectric constant profile governing equation which dominant electric field component it can be E x it can be E y whichever is suitable if it is dE / dx for TM it is E y component that is the dominant component and if you are defining a mth mode guided mode out of many modes are there you are just considering nth guided mode.

So, this thing if you just put here substitute here the equation it will be beta square will come at beta m square and whatever profile will be there and you have to solve this one to find out

E m x y. So, you have to solve this equation that is known as long as epsilon a x y this value is given you know how to calculate numerically that same methods you can use and obviously, we have used that if the guided mode is defined by this one A m E m x y that is the profile and this is a phase part.

Then A m square we have considered 1 watt and this one we have considered 2 omega mu / beta m and that is in the previous slide where from there coming just assumption is that you should keep in mind that if I am putting A m square = 1 watt that means mth mode is getting 1 watt. If you are putting m = 0, then fundamental mode is carrying 1 watt m = 1 that means fast m mode carrying 1 watt and so on.

Now, let us consider this thing what do we want to discuss? We wanted to discuss mode coupling at waveguide junction now. Suppose you have 2 waveguides one waveguide here, let us consider this a single mode waveguide this obviously is missing here single mode waveguide. And in a single mode guide, you have only one mode is launched this is A naught square prime E naught x y and propagation constant of these waveguide.

So, this waveguide is characterised by these one and one fundamental mode is supported here beta naught you can like beta naught prime equal to omega / c n effective naught c prime. So, effective index are the fundamental mode if you solve this part waveguide you can get this one and you know what x y it can be x component or y component depending on the TE and TM and polarization that also you know suppose this is known and obviously you can consider this thing A naught square = 1 watt normalization you can consider.

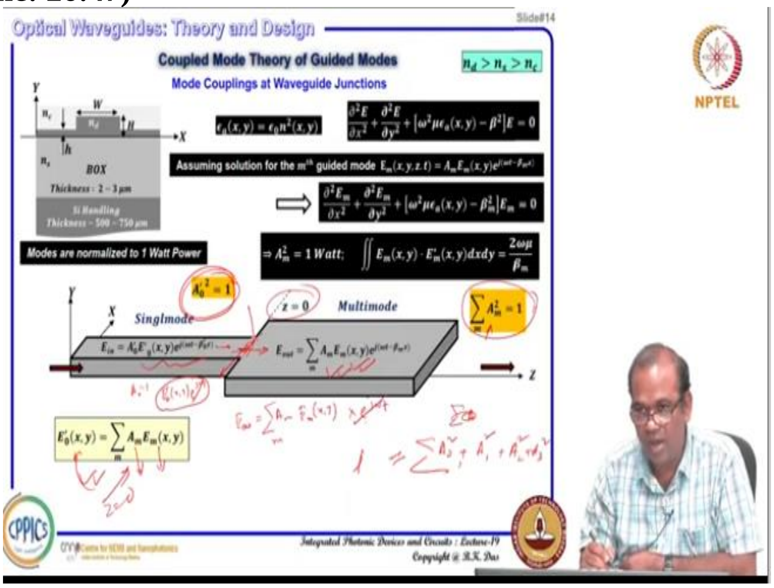
Now, this single mode waveguide is connected to with a little bit wider waveguide structure. So, suppose this is W naught here and this is just modified the width only just wave carried vertical direction you have not changed. So, what happens here you can have many modes it is multimode waveguide. So, if you just think of that, what is the total field E out if you are considering x, y, z and t at any point if you want to know what is the electric field output in this output waveguide this is a multimode waveguide junction here.

Then I can say that all the modes are there they are A m A 0 A 1 A 2 and so on and the field distribution is this one and corresponding phase this one superposition of them at any coordinate at any instant of time you can find now, thing is that these are the solutions here and these are the solution here. Now, all these solutions there does not mean all of them are active, you can actually selectively choose to excite a certain mode.

Suppose if it is A m = 0 to 10 modes are there, there you can you should have in your own choice to accept maybe fifth mode or fourth mode or maybe all the modes or none. So, how that is being decided, what happens suppose, I have a input waveguide where I have only fundamental mode we are launching here at the junction it is open wide, it is a wider waveguide and I have all possible solutions A m = 0 to 10 solutions are there guided mode solutions and I am considering that no other losses are happening here.

For example, in the junction, so, whatever power carried by this mode that is should be transferred to the multimode waveguide region, but I would like to know which mode will accept how much power suppose it is carrying 1 watt does it mean that this 1 watt power will be distributed equally to all modes, if it is coming here at the junction or it will be completely coupled only to fundamental mode other modes will not be excited. So, that is the question now we have to discuss. So, let us proceed how we can deal that.

(Refer Slide Time: 26:47)



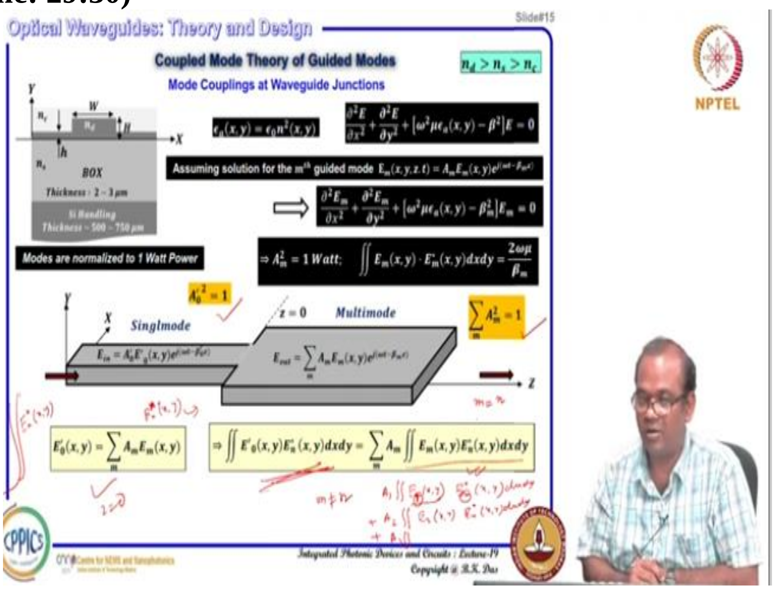
Now you see if I just put z = 0 this is z = 0 I am considering this position z = 0 and z = 0 that means I can get what is z = 0 here this is actually A naught is not equal to 1 you can consider A naught = 1 prime and E naught x y e to the j omega t, this is here what is the actual you will be getting here E naught prime x y e to the omega t and from this one what you will be getting here, E output at z = 0, I will be getting E out, E output will be equal to A m all the things and E m x y times e to the power j omega t.

So, e to the power j omega t, e to the power j omega t will cancel at z = 0, I am just considering like this. So, I am assuming that all the power is somewhat is going into the secondary again that means whatever amplitude here whatever the field here that can be considered superposition of all these modes at z = 0 whatever the modes are there, you can consider superposition of all modes will produce this mode that is what mathematically we can express this one.

So, this is the mth mode field distribution in mth mode amplitude and if you are adding all these mth mode field distribution and they are amplitude that should produce the input mode profile if you are considering this A naught = 1 that means 1 watt power is launched in the input waveguides. So, I can easily write at z = 0 this is the expression. And obviously, since I have assumed that no other losses is there, all the power coming in that is going into the multimode I can consider individual mode.

Suppose, how much power will be there in the first mode this will be A naught square and what will be in the second mode that is A 1 square third mode, A 2 square fourth mode, A 3 square. So, all these if you sum of all the power you sum of that should be equal to 1 watt because you are launching at the input one watt that is why we are writing A m square all the coefficient square if you add this is we can consider coefficient of the mth mode and if you are taking square then you can get that to be equal to 1.

(Refer Slide Time: 29:30)

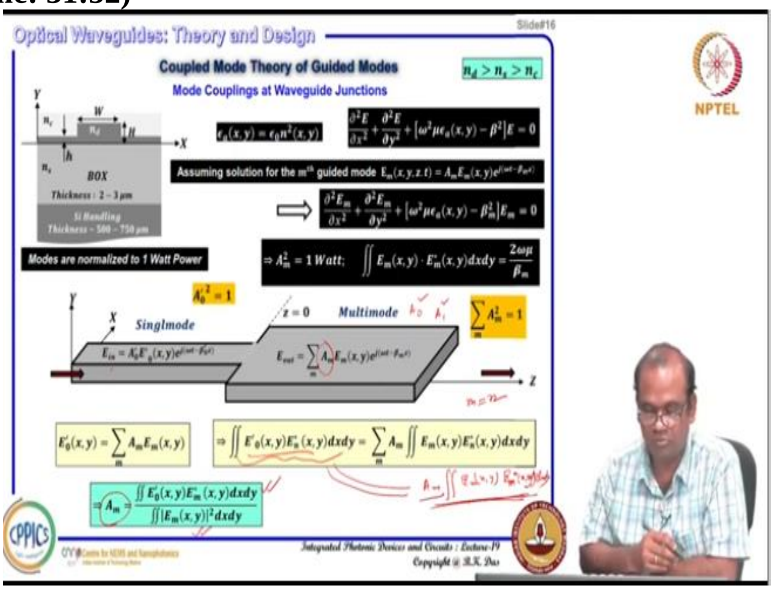


So, now, with this I have just written this one here, this one here and this is that z = 0 now, what we do with this, what I will do? Let me try, I just multiply E n star x y this side E n star x y this side both side I multiplying and after multiplication, I just integrate work extra cross section that is the left hand side, I multiply A m star x y and I am integrating over dx dy. Just simply I am just E n is the E n star x y means that is the field distribution of the nth mode of the multimode cross section that is what I am multiplying in both sides.

So, if you multiply this side E n star right hand side also I multiplied E n star E m this one so simply multiplication and integration both side is there. Now, let us concentrate this one it is actually first thing is that I will be considering E m x y or E 1 and E n star x y dx dy that is the first one I consider A 1 and second one it will be A 2 and E 2 that is plus x y E n star x y dx dy and third term A 3 and so on. So, all these terms you have will be adding, but what is the An E n star that is the inner complex conjugate of the mth mode and this is a first mode.

So, we know that according to the orthogonality condition, if m not equal to n that means, this value and this value are equal then they are overlap integral that should be equal to 0 that is what we learn from orthogonality condition. So, these all these terms some terms on they will be vanishing, but, when m = n that particular term will be surviving in the right hand side.

(Refer Slide Time: 31:52)



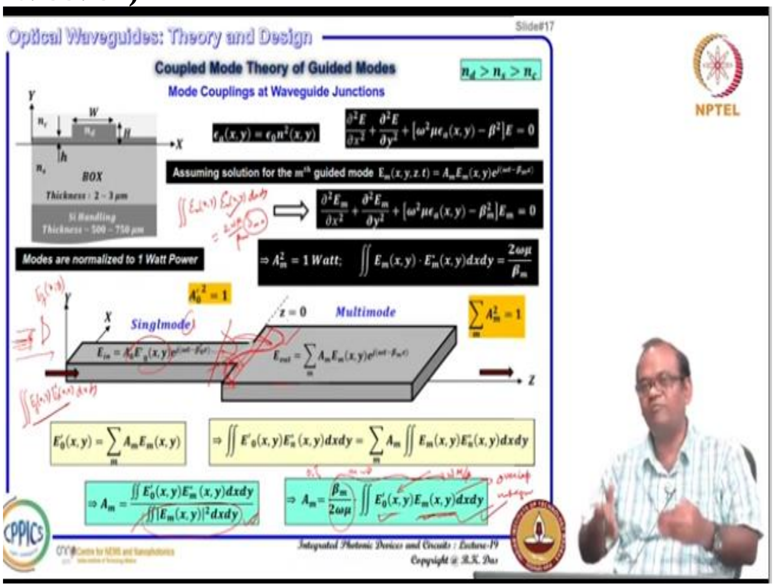
So, we simplify that one that is so, only that part will be surviving the right hand side will be A m E m x y and E n star x y n will be become E m dx x y dx dy. So, right hand side will be reduced to only one term when A m equal to be applying just orthogonality condition when m

= n that term will be surviving all other terms will be 0 because of the orthogonality condition and then if it is so, then this is A m that will be equal to your left hand side is this one.

So, if I just equate this to what will happen I can find out what is the value of A m, so A m equal to this one divided by this one I have written here this one means E n x y star, so square it is written so, I can find out A m that means, if I know how much power is coming these expression, I know the field profile filed profile I can calculate easily and of the multimode region as well as single mode region. And if I know if I am launching fundamental mode, then how much fraction of power will be coupled to the nth mode that is decided by the A m.

Because is A naught square is a 1 watt here and A 1 square for example, A 0 square we will consider that is a fraction of power it will be coupled to the fundamental mode when A 1 square that is the fraction of the power will be coupled to the first order mode and so on. So, we know that this expression if we can evaluate mathematically numerically, then we know how much power will be coupled to the different modes depending on this m value according to the profile you have to write.

(Refer Slide Time: 33: 54)



So, that is know you this thing again we have derived that this one is nothing but normally we have discussed earlier that is actually if you are integrating E m x y E n star x y dx dy applying the orthogonality condition and 1 watt power this is actually 2 omega mu / beta m. When m = n that will be surviving otherwise, it is delta mn m not equal to n that will become 0 m = n that will become one that is the chronical delta function.

So, if I use this denominator this concept that m multiplied by m star that means, it will be 2 omega mu / beta m that means A m can be written as beta m / 2 omega mu so, this is the thing. So, only thing is that, I need to know what is this this was the field distribution of the input mode here in our prime and this is what this is the field distribution of the mth mode. And we are integrating over x y cross section. So, that means if you know, what is the overlap of the field distribution, this is actually we call it a overlap integral.

So, if we know the overlap integral, how much overlapping you have to integrate, you have to multiply every coordinate in the cross section and then you have to integrate, you have to add all of those and then if you multiply be down 2 omega / mu, then whatever value you will be getting, if you just square it, that is the fraction of power that is the amount of power you can get suppose 1 watt you are getting here.

If this value is coming like that says 0.5 that means for m = 0 that means, I can say that 500 milli watt power is coupled to the fundamental mode and I can individually I can calculate, so, this is what very important information about how modes can be coupled, how you can excite a particular mode for example it can happen that. So, for example, a light coming from here from a fiber for example fiber is a core like this and here fibre is there so, core and cladding is there. So, light is coming here along the fiber.

So, you know, what is the field distribution of the fiber so, you can just express that if you E f x y that is the field distribution of the fiber mode. Then if I want to calculate how much power we coupled to this fundamental mode, then I can easily say that that is actually proportional to E f fiber mode field distribution times E naught prime x y dx dy. So, better overlap between these 2 modes you can have better coupling if these modes are actually exactly matching then your overlap will be 1.

Because this field if this field is matching exactly, then these value will become 2 omega mu / beta something like that same if they are identical then 2 omega mu / beta and this beta m / 2 omega mu they will cancel then you will be getting one. So, if you want to coupled light from one waveguide to another waveguide first thing you should ensure that they are mode should be matched their overlaps should be equal unity.

So, if they are mode field distributions are different then you will not get overlap integral unity and your coupling efficiency will be less. So, any 2 fiber they appear you have to see what is their field distribution waveguide can be different configuration, core can be different cladding can be different for the input waveguide core of the output waveguide and cladding of the output can be different.

But you have to see if they are modes are field distributions are equal and their overlap is coming almost in unity then you can say that maximum power transfer can take place you are actually ignoring if there is any impedance mismatch are happening or not mode matching is happening impedance matching means effective index of the input waveguide and effective index of the output wave guide this would also match then only you can say that there is no final reflection back to the input waveguide or input site.

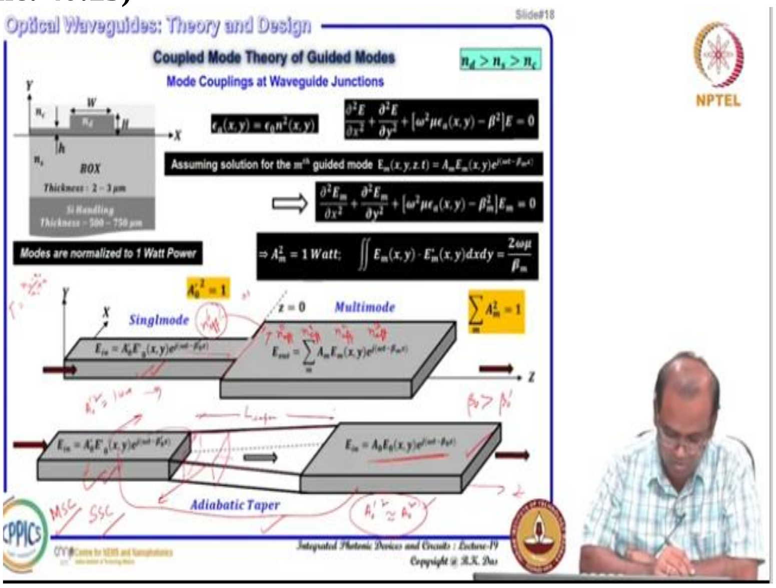
So, 2 things you have to concentrate in coupling from one waveguide to another waveguide or maybe input fiber to output waveguide. So, one thing is that effective index should be matched that is the first condition and then mode field distribution field, if an electric filed distribution should be exactly equal to the electric field distribution of the output guide, then maximum power transfer will take place as it go away if they are mismatching then your coupling efficiency will be dropping according to the how much fraction it is overlap you are getting.

So, here also you can think of suppose you can consider here in this waveguide suppose, you are considering a first order mode like this field distribution and fundamental mode will be like these in the centre. So, if you take the overlap first half will be getting positive and second half if you integrate it will be negative. So, it is assumed that if your input waveguide is exactly positioned in the centre and your output waveguide is just going out here like that.

Then you can say that this first order mode where you are one half is positive and other half is negative overlap will be 0. So, that mode will not be excited because the overlap integral will be 0. So, in that case what you have to do you have to position these waveguides a little bit often straight, then you can get certain overlap also then there is a chance that the first order mode will be excited.

So, it depends on input waveguide positioning and output waveguide positioning with respect to that depending on that you can actually control which mode you want to excite if you want to excite all the modes accordingly you have to just set that overlap integral for the all modes, so, be nonzero and you can find what the coupling coefficients this is very important for integrator portent applications.

(Refer Slide Time: 40:23)



Now, I know that effective index here n effective not one and effective index here is n effective not for example and n effective 1, n effective 2, n effective 3 so on I know that how much power will be coupling here by calculating overlap integral, but if I know n effective is this one how much power will be coupled to the first mode and how much it will be reflected back to the first mode there. Then you have to see the effective index mismatch at the fundamental mode.

Fundamental mode how much that you have to see and you have to use your final reflection condition like reflection coefficient you know that reflection coefficient is n 2 – n 1 / n 2 + n 1 for normal incidence theta 1 = 0 we have learned when we are discussing about plane waves and so on. So, that thing will happen that some kind of reflection will be there. And after reflection whatever coming in suppose 1 watt coming and then maybe 0.1 milli watt is reflected totally because of the finally reflection.

Then 0.9 watt will be distributed among the output waveguide modes and among 0.9 watt they will be again can be calculated depending on their overlap integral so that is the idea, but can we do something so, that I want to I have a single mode theorem I am launching here and here I have many modes I do not want to lose any power but I want to coupled all the power from the input to the output waveguide but not to the all modes I want to excite only one fundamental mode how that is possible. So that is actually very much possible.

So, how that is possible you have an input waveguide here and input waveguide the field distribution you can express like this, this is the beta naught prime is the propagation constant that is us solving for the input waveguide and this is the field distribution and this is A naught square is not prime you can always consider that 1 watt and all the orthogonality condition you can think of then you have this is a waveguide where you have multiple modes are they are excited, but this is the fundamental mode.

What I want to do? I want to convert this entire power into this one so, that approximately A naught times square approximately should be equal to A naught square how that is possible. So, you do one thing you instead of just abrupt junction, waveguide junction you do slowly tapering the waveguide we will do very slowly you allow some length L taper link some length you allow and slowly you increase and margin to this waveguide.

So, that what happens every point is every cross section if you see here and every cross section if you see here. So, nearby cross section, if you see waveguide diamonds and is not suddenly changing slightly delta among these increased delta amount to effective index is increased. So, in that case, you can consider this mode when it is going from one position to another position along z axis for example, then it does not see abrupt change of effective index.

So, in that case you will see loss final reflection loss will be at minimum from you can just consider local normal modes you can solve and you can check the defective index differences not that great. So, the reflection on will not be that kept that high. So, you can actually slowly you can see this E naught prime also will be evolved a little bit bigger and the effective index also will be a little bit increased slowly. And if you just keep on doing going further as it propagates you will see that they are actually changing.

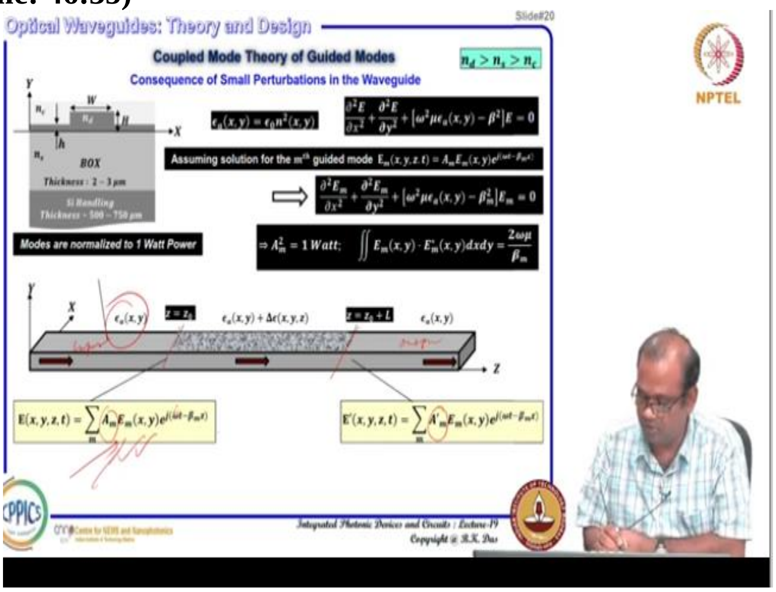
They are increasing the field the diameter the distribution area that will be increasing also slowly and effectively index also will be increasing slowly obviously, you know this beta not because wider waveguide beta naught is higher than beta naught prime. So, slowly it will be increasing so as it propagates if you just allow them to slowly evolve. Then it is possible to achieve this type of power coupling and you can actually only 2 coupled to the fundamental model even though this waveguide can support many modes.

So, that is called adiabatic tapering and practically people actually demonstrated such a adiabatic taper in silicon platform and insertion loss can be much lower than 0.20 almost no loss, but the staple length actually that depends how fast you are tapering, if it is faster tapering if this length is small then it will be loosely, you should allow sufficient length so that it can evolve slowly and to take a shape of the fundamental mode of the multimode waveguide.

So, that is one useful technique, when you are making different types of waveguide and you want to transfer power from one particular device to another particular device have different diamonds and for example, waveguide diamonds and can be different you can couple the intel light with less loss in less or loss that is called that is why since it is no loss no energy is being lost ideally I would say then it will be calling and that is why it is called an adiabatic taper, adiabatically it is mode is converting from smaller size to a bigger size.

So that is sometimes it is adiabatic taper and sometimes it is also called as a spot size converter SSC sometimes it is called spot size converter and sometimes it is called mode size converter. So, in literature textbook they define differently, but it is basically the same mode size converter or spot size converter.

(Refer Slide Time: 46:35)



Next thing is that suppose, you have a Benson sunny here this is a nice thing that we found that if you are slowly increasing your waveguide width it that means, your dielectric constant or waveguide diamonds are effective index is slowly increasing impact your mode shape can remain same and your fundamental mode is evolved into fundamental mode in the multimode waveguide even the higher order modes are there they will not be excited, that is a beautiful thing.

So, let us see, if somewhere in your waveguide you have your some kind of perturbation you know this input waveguide is defined by epsilon a x y like this and output side also it is defined like that a certain region from z = z naught, z = z naught + L this region we have some kind of small perturbation what is the consequence of small perturbation in the waveguide. So, if you have a very small perturbation maybe this width may be a little bit high low, high low or something like that happen.

Your height of the waveguide may be changing high low high low or maybe some kind of roughness is there some kind of edge roughness is there in the waveguide that can be considered as a perturbation that means the dielectric constant is delta epsilon is changing or you can think that you know any waveguide if you have a waveguide mode is like that if you have some another waveguide also coming nearby suppose another waveguide also coming nearby like this suppose this is another waveguide coming side by side.

So, what that means, this waveguide if there is a evolution field this mode solutions you get here. This mode will see outside instead of here after a certain time it sees the tail to secondary light it can see this secondary guide can be considered as a delta epsilon for the faster waveguide and vice versa. If you are considering this waveguide presence of the other waveguide you can consider this guiding mode here.

It will see some kind of disturbance, some kind of fringing effect for the neighbouring waveguide that type of perturbation if it is there, if these 2 waveguides are far away then no problem because the venison field to be quickly to be reduced to almost 0 before the second waveguide is comes in. But if this waveguide comes closure, you can see that they are actually disturbing each other that disturbance I can consider this waveguide with perturbation of this one.

So, in general I want to discuss first if this delta epsilon is there some kind of perturbation this can be present sub secondary neighbouring secondary guide, this can be some kind of intentional waveguide with variations or maybe it can be intentional periodic perturbation, some kind of grading structure you are making or it can be that grading structure you can make some kind of corrugation surface corrugation or etching some kind of periodic etching etcetera you can do so, those type of perturbation if it is there.

What could be the consequences, what would be the outcome of this ledger you know for example it is just perfect waveguide homogeneous waveguide you can see number of modes can be guiding and superposition of all those modes I can write like this. We have so far we are discussing it in the Fourier domain frequency domain only one frequency monochromatic to serve one frequency we are considering another frequency you can just treat them with another frequency.

Otherwise sometimes you can directly solve your Maxwell's equation using your so called FDTD technique finite difference time domain technique. We just considering frequency domain it is easier to understand all these coupled mode theory etcetera. So, here in the input side I know that this is the all the modes and field at any point that can be superposition over all the modes and in the output side also I can consider the mode because there is no perturbation here it is as good as input side.

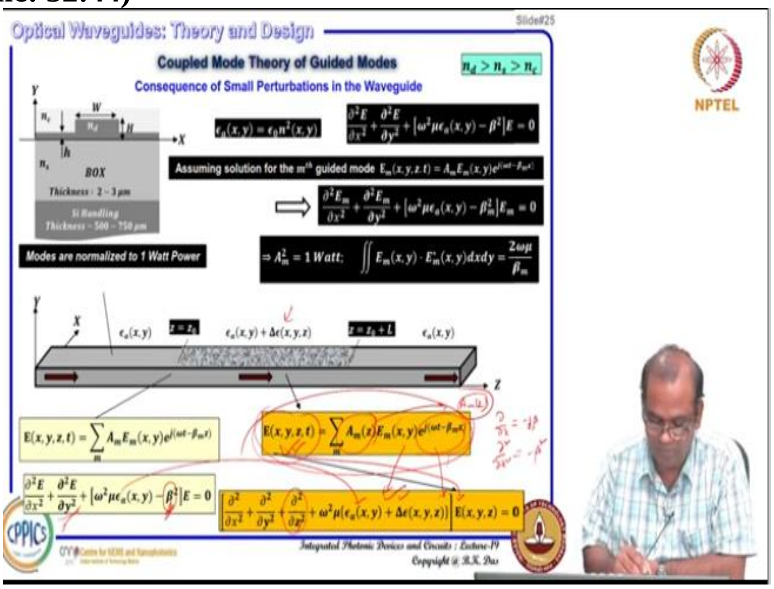
This is your input waveguide side and this is your output waveguide site. So, they are identical, but thing is that in between you had a perturbation something has happened something in between. So, in the output side also you can see similar type of modes at the output they will be propagating but only thing is that because of the perturbation maybe this A m this slightly changed that means, there might be this perturbation region there might have happened that all the modes they extend somehow some power.

Because you have already broken the orthogonality condition there because as long as your waveguide cross section is maintained then your orthogonality condition is not broken and that is why you have all the modes propagating like independent way individually. So, that thing if it is happened what would be the consequence how you could actually change modify your equation that is what we are going to discuss let us see.

So, here you see governing equation I can just simply write the same equation here this equation I have written here that equation here and for this side also I can use this equation here omega square mu a with it is directly coming from this scalar wave equation. Now, how to define here this region if you have a small perturbation you have broken the orthogonality condition, but not abruptly it is a small perturbation.

So, in that small perturbation, then what type of equations would be involved here, can we modify these equations these equations so, that I can get a certain approximate model here so, that I can surely find because of the magnitude of the delta epsilon the randomness of the delta epsilon we can say that how the outcome results that can be calculated.

(Refer Slide Time: 52:44)



So, let us see what we can say that since you are breaking the orthogonality condition here can you write like this. What is that you see, what is the difference between these 2 this here A m is constant as it propagates individual modes will carry same power same energy same amplitude. But here because of this perturbation what I consider that as it propagates mth mode, will see some kind of variation in amplitude along the z direction.

Because amplitude actually because of the perturbation, you see certain kind of power exchange orthogonality condition broken however, we can say that if delta epsilon is very small, if it is small, then we can say that this mode field distribution is not changing much it is called the mode field electric field distribution from the vector wave equation etcetera that is not going to be changed because the delta epsilon this dielectric constant or waveguide or whatever changes that is very small.

So, I can maintain that field distribution exactly, and I can also say that individual mode will not see much change in the propagation constant. So, that after this is approximate and you know just to use Maxwell's equations, all the scalar equation to develop a working model for waveguide and integrated optics you need some kind of approximation, so, that is what engineering is called. So, we have just considering this approximation, this is just approximation.

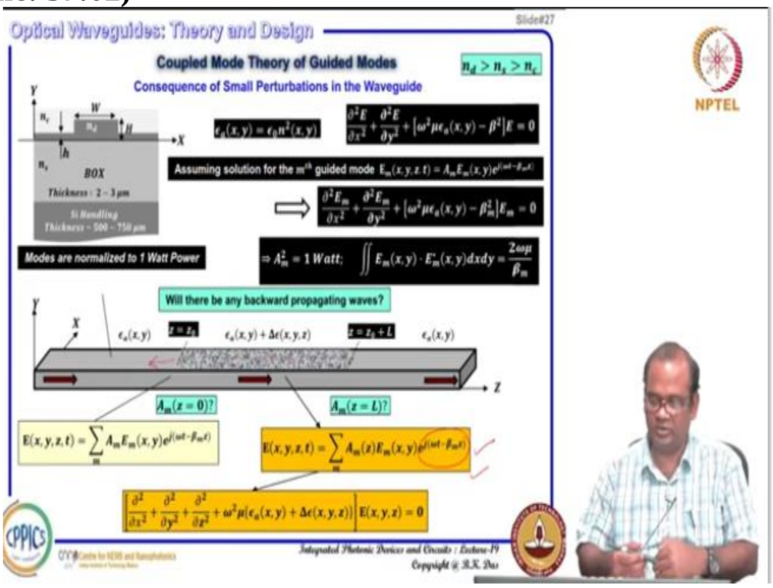
So, as long as it this approximation is good, your devices good and you can use for experimental demonstration or practical device applications. So, we can consider this one then if we just consider this one can we use this equation directly. Obviously, not because you are this epsilon a x y is now converted into this one. So, you need to insert this one here, electric field you can insert here and this one you can insert here. So, once you insert here, that will be the new equation.

So, you are just converting epsilon + epsilon a but by the time what we have done, since I am using this total electric field z dependence things is there, because this beta square comes because of the del del z = - d beta and del 2 del z 2 equal to plus minus beta square. So, if that beta square you are writing here - beta, beta square you are writing that side de del z was not there here, but if I just use z dependent function inside the electric field, then I can just write down del del z 2 this one I have included and beta squared not given there.

So, now what we have to do? We have to solve this equation by substituting this here this electric field I will be substituting here and I will try to solve what is the evolution of this individual mode amplitude. So, if I insert here and try to solve for A m z then I am done I will be able to know as your function of z what is the situation of power of mth mode. So, that is one major thing we can say that just controlling that particular delta epsilon.

We can control this A m z value and that can be very good successful device application we can think we can we will see also in course of time. So, we would like to develop this type of model I will be considering in the part of region the period of individual mode that can be can have some kind of evolution dependent variations will be there that is actually represented by this amplitude. Amplitude it is going to change as function of z because of this delta epsilon as long as this delta epsilon is there so, this A m keep on changing evolving.

(Refer Slide Time: 57:02)



Now next thing so now next question is that in that case I need to know what is the value of A m all the modes are they are A m z = 0 what would the value obviously, whatever you are launching here that would be the z = 0 whatever the value is there you can just consider that one from the input side whatever coming for that particular mode whatever the excitation will be there that we can find out and then what will happen what about A m z = L.

At L whatever the amplitude will be there that means, after that I have again smooth waveguide no perturbation nothing. It will be propagating like that. So, in that case, I would like to find out what is that value if we know that then I can say that because of the perturbation, how much I lost what is the survival of individual modes, how much attraction is surviving. So, next thing is that will there be any backward propagating waves? So, all everywhere if you see we are considering forward propagating wave.

But because of the perturbation you can consider some kind of scattering points, but macroscopically we are trying to get a model it is all our power propagating wave and we have used Maxwell's equations etcetera, but because perturbation means some kind of dialectic perturbation. Dielectric perturbation means, you can think of panel deflection, deflection scattering, everything will be there that scattering can cause anything constructively towards backward direction or not.

Will there be any power reflected backward direction and can take up some mode shape and propagate in the backward direction if so, how you can actually control that, that is the thing we need to discuss in the next lecture. That means will be trying to develop a control coupled equations so that we can use them for various applications. Thank you very much.