**Integrated Photonics Devices and Circuits**
**Prof. Bijoy Krishna Das**
**Department of Electrical Engineering**
**Indian Institute Technology - Madras**

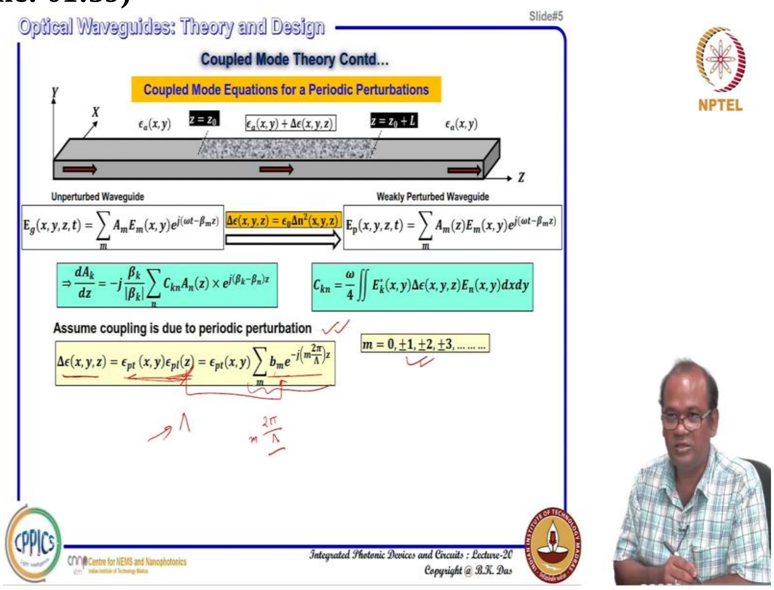
**Lecture - 21**
**Optical Waveguides: Theory and Design: Coupled Mode Theory Contd...**

Hello everyone so in this lecture we will continue coupled mode theory. And today we are going to learn about coupled mode equations for a periodic perturbation all the basic thing we have discussed in the previous lectures. Now we will be developing a working model of coupled mode equations with periodic perturbations. And then we will be discussing using that periodic perturbation in the waveguide.

We will be discussing how 2 modes can be coupled between themselves when they are propagating in the co-direction. Co-direction that means they will be traveling in the same direction and they can be also coupled they can interrupt each other while propagating in opposite direction that is what we call contra directional coupling. So, here also we will try to see how the model of the couple mode equations can be modified in these 2 cases.

And then very special case we will be discussing a rectangular grating coupler that is actually the periodic perturbation that perturbation will be of course periodic but it is rectangular in shape. So that particular shapes are getting is heavily used for light coupling into the chip and out of the chip we will discuss that also today.

**(Refer Slide Time: 01:59)**



So, let us move on, so here we start with a coupled mode equations for general periodic perturbations. Here we have shown some perturbation we have shown that perturbation when it is actually uniform waveguide which is defined by dielectric constant means under epsilon a, x, y that means epsilon a, x, y earlier we have defined something like this epsilon 0 n square a waveguide cross section refractive index profile square.

And perturbation region we have defined from z = z 0 this is z axis here and z 0 to z = z 0 + L we have the perturbation delta epsilon and that delta epsilon can be x, y, z function dependent. And we can say that when waveguide was unperturbed that means uniform waveguide then if there are guided modes and that electric field associated to guided mode obviously you should remember that.

We approximated this electric field dominant electric field and treated as a scalar field and that field can be decomposed into various modes. And here we have shown m is the mode index and sum over all modes this is a field distribution for individual modes mth mode and propagation constant for the mth mode and then amplitude of the mth mode. And of course this amplitude of the mth mode will remain constant as it propagates if it is unperturbed of course lossless waveguide.

Now when we consider this type of perturbation where delta epsilon is defined again epsilon 0 delta n square x, y, z some kind of small perturbation obviously we are considering very small perturbation which eventually give convert this equation we can model like that actually that electric field in the perturb waveguide we can assume that all the modes will travel with keeping it is mode field distribution nearly same that means E m x, y, E m x, y same we have written.

They are propagation constant phase constant also same only thing is that we assumed that because with the perturbation as a function z amplitude will be varying and we have discussed that this amplitude will be varying such that all the modes this thing that you can consider that or maybe you can consider individual modes. When it is not perturb we consider that A m square = 1 watt that is watt we discussed earlier.

And then we have also shown what is that orthogonal condition how it works for that and if individual modes were 1 watt with when it was not perturb then if it is entering into the part of the region we will see because of the presence of this term the energy or power they can exchange each other depending on various factors we will be discussing we have to quantify how much it will be exchanged between 2 modes or how much 1 particular mode will be evolved.

Because of this perturbation that is what we need to quantify if we of course know the type of perturbation if we could define well the perturbation which could be actually fabricated as a device then we can actually in a controlled wave we can actually exchange power between modes among modes. And we have shown that this generic equation we have discussed we have developed in the previous lecture that kth mode how it will evolve?

Because of the scattering or perturbation of all other modes nth mode that is sum over this one beta k is the propagation constant for the kth mode. And the C kn has been defined like this which is nothing but that coupling strength between k and n or you can say that the fraction of power or some parameter which represent the fraction of power exchanged from nth mode to kth mode that is what we can measure we can quantify with this expression.

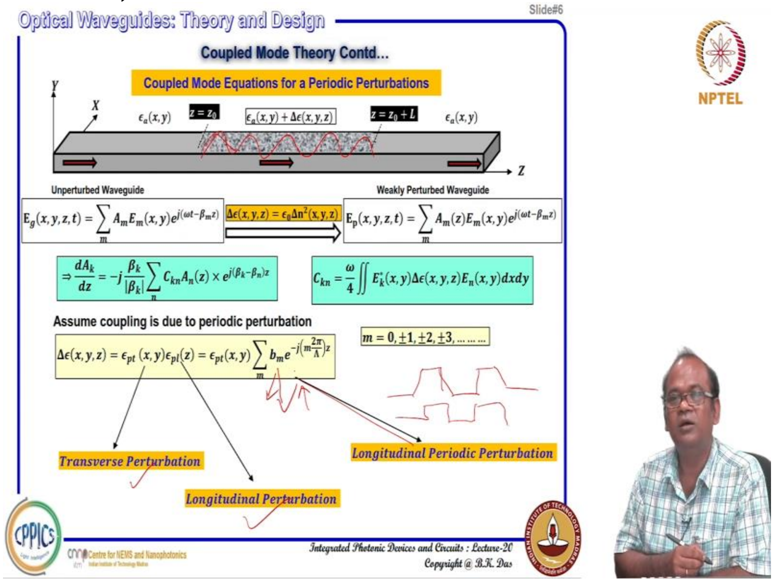
Where E k x, y is the kth mode field distribution complex conjugate delta epsilon is the same thing we mentioned here this is the perturbation and E n is the what is the how much it is affected because of the perturbation nth mode how much it is affected to contribute power exchange towards kth mode. So, this thing we have discussed earlier nothing new so what is the next then we will consider now coupling is due to periodic perturbation this is something we want to discuss here.

If we consider this perturbation obviously you see these delta epsilon x, y, z we can decompose into 2 term 2 factor 1 term x, y dependent perturbation we call it epsilon pt meaning perturbation in the transverse direction x, y is the cross section of the wave guide if you have any perturbation x, y dependent function is there and that function if it is periodic over longitudinal direction so z dependent function we can write perturbation longitudinal direction.

And as we mentioned that this longitudinal direction perturbation can be represented with a periodic function and you know that any periodic function you can actually decompose into Fourier harmonics if the periodicity is lambda then Fourier harmonic that means in Fourier space the harmonics will be 2pi / lambda this is the real space if it is periodicity lambda in Fourier space or k space fondly it is called the harmonics will be integer multiple of m is the integer 2pi / lambda capital lambda is the period.

And you see these are the Fourier coefficient beta b m and this is the harmonics so superposition of all harmonics supposed to give the periodic perturbation along longitudinal direction that is what we mean where this m is nothing but 0 + - 1 + - 2 + - 3 all this integer values we need to consider.

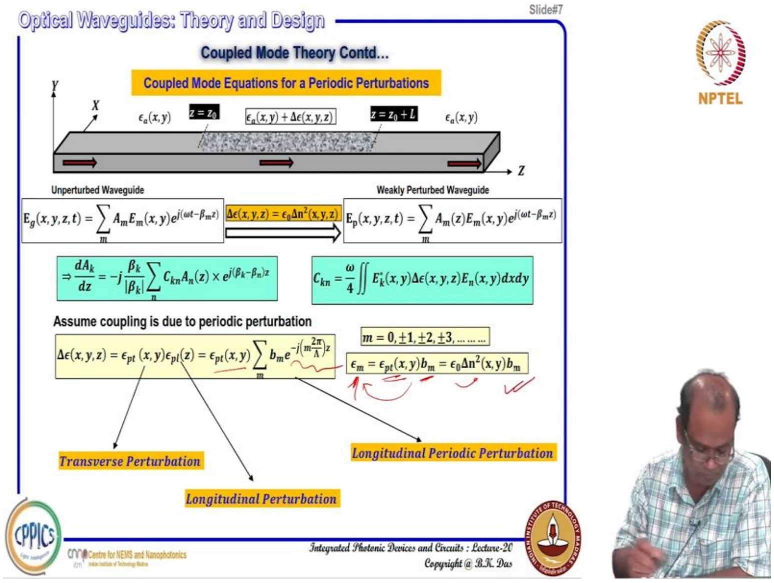
**(Refer Slide Time: 09:27)**




So that is what we mention so this is the transverse perturbation this term is the longitudinal perturbation and obviously this term is representation of longitudinal periodic perturbation if perturbation is periodic any perturbation any periodicity for example I can consider that perturbation is just sinusoidal perturbation here sinusoidal perturbation means you will have only one harmonic in the periodicity.

But if it is something triangular or maybe trapezoidal something like this trapezoidal perturbation, something like this or maybe rectangular perturbation periodic rectangular perturbation or maybe some other way triangular maybe it can be triangular also perturbation. So, all these perturbation periodicity periodic perturbation you can decompose into Fourier harmonics depending on the period. And you can obviously you can find out what is the Fourier coefficient by standard method we know.

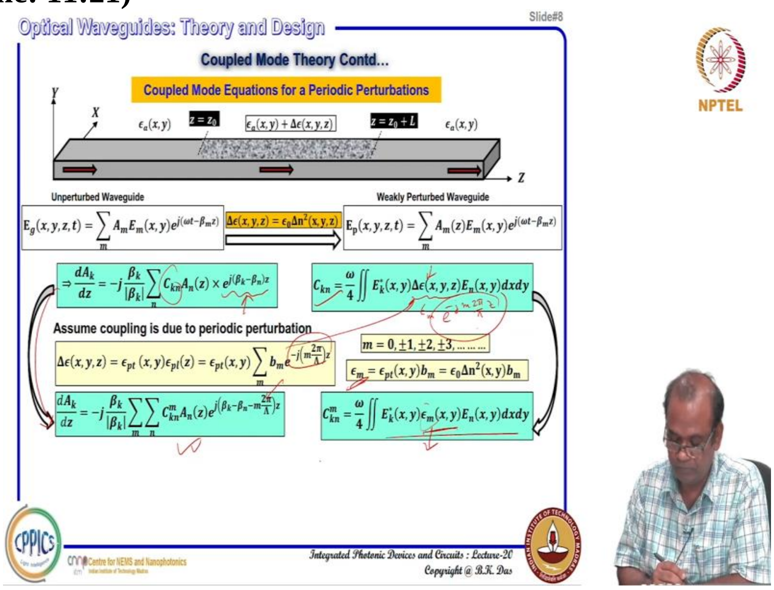
**(Refer Slide Time: 10:29)**




What is that? Before going into that details we define this E pt x, y multiplied by b m that means Fourier coefficient E pt multiplied by Fourier coefficient we can say that transverse perturbation epsilon m is equal to transverse cross sectional perturbation multiplied by mth Fourier coefficient that is what we will be writing epsilon n something like that just to decompose just to dissociate longitudinal perturbation from this periodicity we say that.

We just introduced one more term perturbation term epsilon m which is cross sectional term multiplied by Fourier coefficient which is coming out of longitudinal direction that will be helpful for developing our mathematical model this expression.

**(Refer Slide Time: 11:21)**

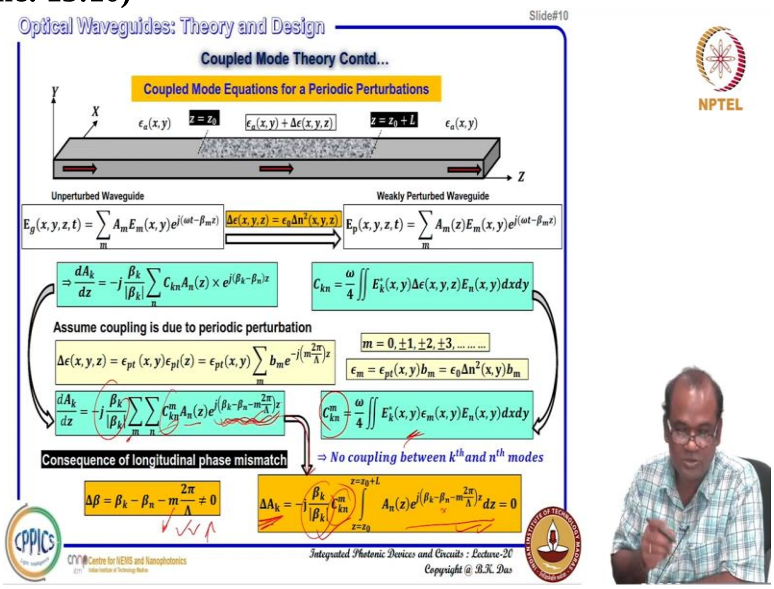



Let us see so with this consideration if we try to see this equation so you know now here in this expression in this coupled equation we have discussed earlier generic coupled equation we have one z dependent phase factor exponential function is there and C kn if you just introduce here then you will be getting instead of delta epsilon you will be writing epsilon m and then this periodic function epsilon m then e to the power - j m 2pi / lambda z.

So, since already in the C kn you are bringing one more z dependent phase factor. So, what we could do? We can redefine the C kn by dissociating the longitudinal term associated with a perturbation that means the z dependent part if we remove them in the C kn term it will be only E m x, y but that z dependent part where I should that we cannot just completely forget. So that thing where I will be including I will be just adding here.

If I add here there my coupled equation will be looking like that C kn because of the mth perturbation. So, this term is added with the exponential and C kn m can be expressed in this form. So, now you see this epsilon m we have defined earlier here it is basically Fourier coefficient mth Fourier coefficient. So, C kn we will be defining like that the coupling strength or coupling coefficient between k and n because of the mth harmonic in the perturbation if it is periodic of course that is what we are discussing so far.

**(Refer Slide Time: 13:10)**




Now consequence of you say, this is we call it as a longitudinal phase. So, when the kth mode dA k / dz we see that, that is the evolution of the kth mode and evolution of the kth mode it depends on how this term is happening up evolving as a function of z if you see this is actually nothing but if you just take a real part it is a exponential term we consider because of the mathematical easiness but ultimately you have to consider the real part.

And maybe you can dissociate the phase real part associated with the phase now we can say that the exponential term it is a sin, cosine, cosine + j sin of this phase factor if this is not equal to 0 highly possible depending on the beta k beta n value that kth mode propagation constant nth mode propagation constant and grating vector and also choice of m integer. So, depending on that it is highly possible that for any lambda frequency the kth mode and n th mode.

And for a given m that may not be equal to 0. So, in such situation what happens? You see from here I tried to see say evolution of kth mode I just defined delta A k that means I have to take I have to integrate right hand side instead of summing over all the modes. Let us concentrate only the evolution of kth mode because of the nth mode. Then we will be taking only 1 mode no summation I will be using.

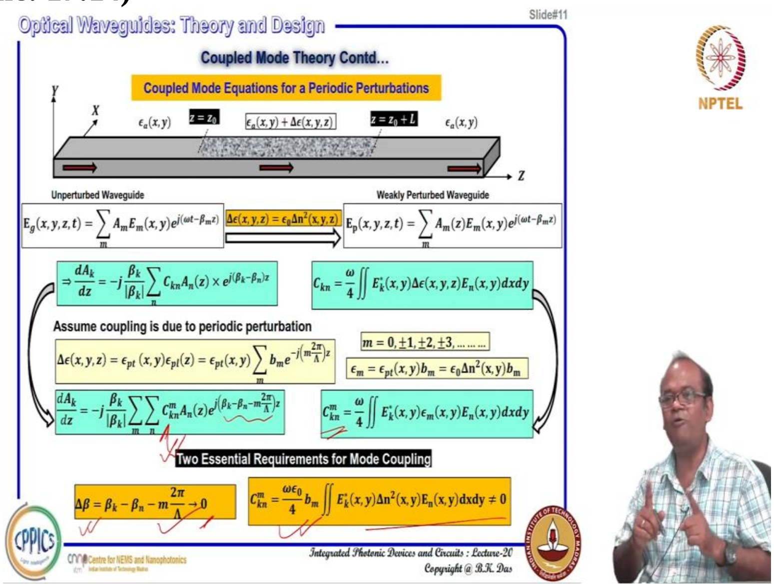
And I will be considering only 1 harmonic m that is why the song also I am not taking here. So, without considering just 1 time 1 mode nth mode and 1 Fourier harmonics because of the 1 Fourier harmonics how much evolution of the kth mode if I try to see it will be looking like this that means delta k equal to this term I am retaining here and C kn m I am retaining here that is what this value basically.

And then you have to integrate for example if your perturbation is z = z 0 to z = z 0 + L and just integrating now you know this is something not equal to 0 exponential time. So that means if I take a real part in principle if you see if you are integrating over z over length sinusoidal function over length the integration will be 0. So, what we conclude then in case the phase factor because of the beta k beta n.

And whatever coming because of the grading periodicity if that is not equal to 0 for n th mode and kth mode and because of the mth harmonic then that particular k and n and m will not contribute coupling or will not contribute coupling between kth mode and nth mode that will be giving 0 as it propagates it will not exchange any power k and n will remain immune to the perturbation if this condition is occurring.

So, this is called longitudinal phase mismatch if it is 0 we will be calling that has a longitudinal phase matching because that only gives you this exponential term perfectly 1 and in that case you will be able to get some kind of evolution of the kth mode of course this C kn must not be equal to 0 that should have some value then we can say that kth mode will be evolving and these value is 0 that means it will be calling as a phase matching condition.

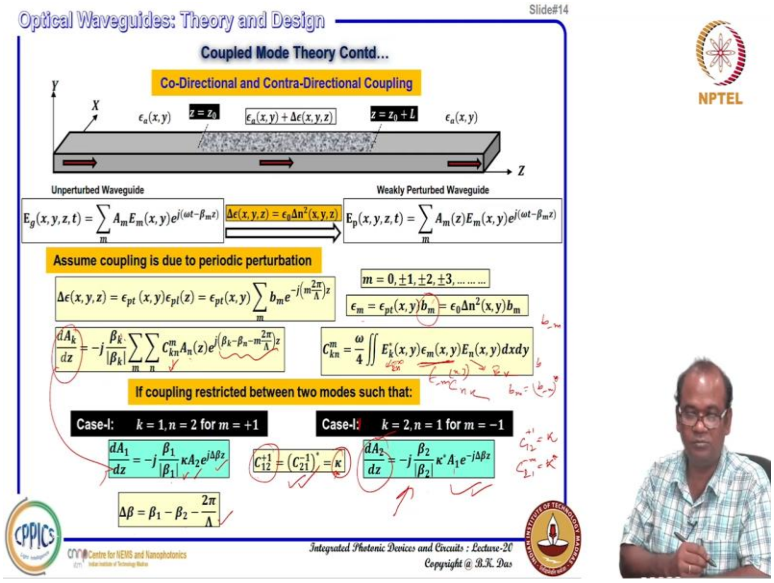
**(Refer Slide Time: 17:14)**




So, in this way we can conclude that 2 essential requirements for coupling between 2 modes. So, kth mode and nth mode what are those 2? One thing is that this longitudinal phase must be equal to 0 or close to 0 at least so that at least something will be building up if it is tends to 0 this periodicity phase mismatch periodicity will be long enough but you can get in the meantime some coupling.

And another thing is that this coupling strength this value which is coming here there should not be 0 that should have some value. So, this should be 0 so one thing should not be 0 another thing should be 0. So, these 2 conditions need to be fulfilled then only you can ensure that coupling between kth mode and nth mode because of the mth Fourier harmonics.

**(Refer Slide Time: 18:13)**




Now I just repeated whatever we have discussed so far. So, we have the periodic perturbation here and we have the evolution term of kth mode and we have the coupling strength between kth mode and nth mode because of the transverse perturbation and of course because of the mth Fourier coefficient is included in epsilon m that is what we just put down to proceed forward. So, what is next we will be just now restricting ourselves.

We will be slowly, slowly trying to develop a working model for a particular function to get certain functions. So, most of the time we will be dealing with either single mode or few mode waveguides there will be supporting either single mode or maybe 2 more 3 modes and so on not like 1000s of modes in a waveguide you can handle if there are 1000 modes and there is a perturbations then you have to have 1000s of equation to be solved.

So that is what it is really not useful so if you want to use this perturbation in an integrated photonic circuit your waveguide should be supporting only 1 or 2 modes. And out of these 2 modes it can happen that 1 mode is propagating in the forward direction another mode is propagating the backward direction that means single mode waveguide forward direction backward direction those 2 modes are orthogonal also we have shown earlier.

And they can be coupled also or we can say that 2 modes propagating in the forward direction and if a perturbation is there whether they can be coupled because of the perturbation these 2 modes. So, we will consider 2 situations we are considering kth mode nth mode and mth harmonic of the Fourier coefficient. So, let us consider we want to concentrate here let us consider indicate the k means mode number 1 or first mode whatever things,

And n th mode we are considering here we are considering suppose that is the second mode we are indexing like 2 now if we just define this power evolution or amplitude evolution of the kth mode happening because of the k = 1 because of the n = 2th mode and because of the Fourier harmonics m + m = +1 we know that m can be 0 + - 1 + - 2 + - 3. Obviously when m = 0 if you see this one will become just not periodic at all when m = 0 that is a DC part of the Fourier transform.

So that will not contribute anything for this longitudinal phase matching and so on. So, we will be considering 2 cases case 1 and this is would be case 2 case 1 and case 2. So, case 1 is coupling between 2 modes when we are indexing kth mode 1 nth mode 2. And that is happening because m = + 1 so, what we can write you see here we are putting k = 1 that means dA 1 / dz this one coming k will be 1 beta 1 k will be 1 beta 1.

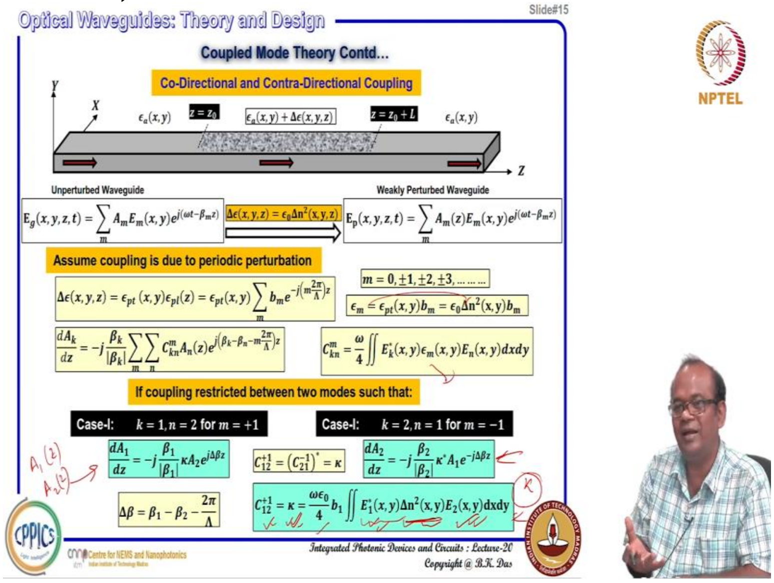
And this C kn means C 1, 2, m C, 1, 2 m + 1 we are representing like this that we are representing as a kappa. So, when we are restricting ourselves coupling between 2 modes then instead of just considering C 1, 2, + 1 this type of complicated symbol we just simplify it as a kappa. So, I am writing C kn as a kappa and a n = 2 and this one this beta 1 - beta 2 m = 1 + 1 2pi / lambda that we are just if there is some delta beta is there not equal to 0 we are writing delta beta and this is a first equation.

Now you think about reverse situation I can put k = 2, n = 1 highly possible. So that is what case 2 this is mistakenly typed here one that should be 2. So, k = 2 and n = 1 that means d A 2 / dz first term is dA 2 / dz - j beta 2 / beta 2 so k will be 2 and C kn will be C 21 - 1 m = - 1. So, we can show later that C 12 plus will be just complex conjugate of C 21 - 1 that can be shown very quickly that is actually we are writing kappa.

So, if I just interchange for example instead of this one it is written at C nk. So that means this will be E n star and this will be E k. So, instead of E k star E n star is there E k star and this one if it is complex it should be written as minus m epsilon - m x, y and we know the epsilon - m x, y only thing Fourier coefficient coming b m means b - m will be coming. So, we can say that this b m should be equal to b - m star.

If that is happening then obviously this is also is correct. So, in that case when we will be writing this one C 12 + 1 is kappa then C 21 - m should be kappa star just complex conjugate. So that is why this equation we could write like this. So, now you see first equation it is a complex equation between A 1 and A 2 but it is showing how A 1 will be evolving because of the A 2 and equation this one it is showing how A 2 will be evolving that means mode 2 how it will be evolving because of A 1.

**(Refer Slide Time: 24:13)**



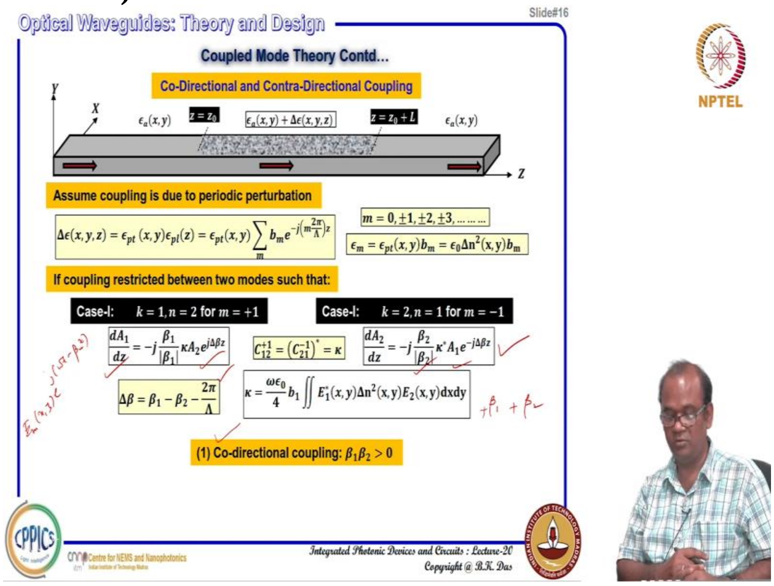

Now you see that is what we have written if I just try to define C 12 as a kappa. Kappa can be written as omega / 4 is there and epsilon m we are just returning epsilon m value epsilon m is the m = 1 obviously so epsilon 0 delta and square and b 1 since b 1 is Fourier coefficient that is a constant that can come outside the integration and here you will be writing E 1 star x, y delta n square.

That means what is the perturbation modulation periodic perturbation delta n square x, y; x, y dependent lateral refractive index profile perturbation and this is a second mode and if you take a integration where both E 1 star E 1, E 2 and delta n square non 0 perturbations would be there that particular region you have integrate to all 3 should be non 0 then only that value will be there. So, wherever they are non 0 you have to sum up them and that value actually is the major of kappa. So that kappa is coming in your coupled mode equations.

So, assume that this kappa you can calculate because of the some software mode solver you can calculate the electrical distribution and if you can define what is the transverse refractive index perturbation introduced that can be introduced by user can define that and they can fabricate that also. So, if you know all these then you will be able to find kappa so once you know kappa and once you solve the mode profiles then you will know the propagation constants also.

Then you can solve these 2 equations and then you can find how A 1 z and A 2 z 2 equations 2 unknowns can be solved analytically. So, this is not completely analytical this type of working model is called semi analytical because you need help of numerical techniques to solve mode profiles mode profiles full vectorial method it is not so easy to solve analytically because of the nature of the refractive index profile of the waveguide cross section.

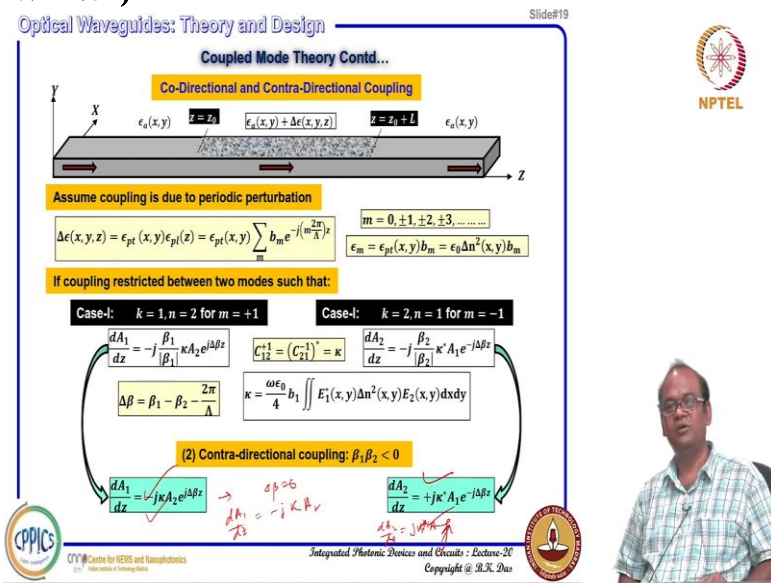
**(Refer Slide Time: 26:29)**




Then what is next I just repeated same thing here and here. And we just consider co directional coupling case. So, for co directional coupling case if it is one situation you consider you have 2 modes and 2 modes associated propagation constants are beta 1 and beta 2 if you have both the modes in the waveguide they can be guided mode they are propagating in the same direction either in the positive direction both or negative direction both.

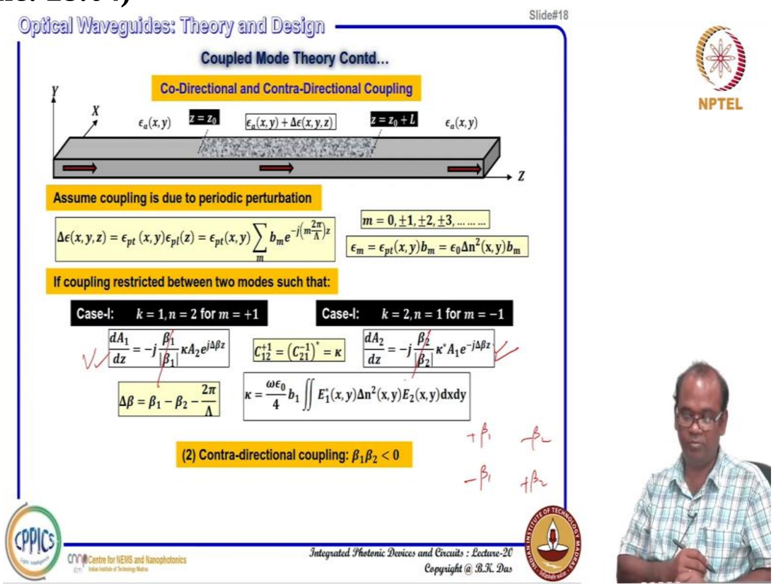
So, if it is positive z direction both then beta one will be just here whatever they are I will be using beta 1 and beta 2 plus according to these because original equations of mth mode we have just consider something like this E m x, y e to the power j omega t - beta m z that is actually forward propagating wave. So, accordingly we have derived these 2 equations that means we have considered forward propagating wave for both beta 1 and beta 2.So, in that case we do not need to change any sign here for beta 1 and beta 2.

**(Refer Slide Time: 27:37)**




So, in that case what we will be getting? We will be getting this one same equation will be written here beta 1 and beta 1 because sign already consider. So, they will be canceled they will be canceled and you will be getting minus j kappa star and here j kappa A 2 this is delta beta z this is delta beta z. So, your life is more simpler so you can solve these 2 equations we will see how they can be solved in later lectures.

**(Refer Slide Time: 28:04)**



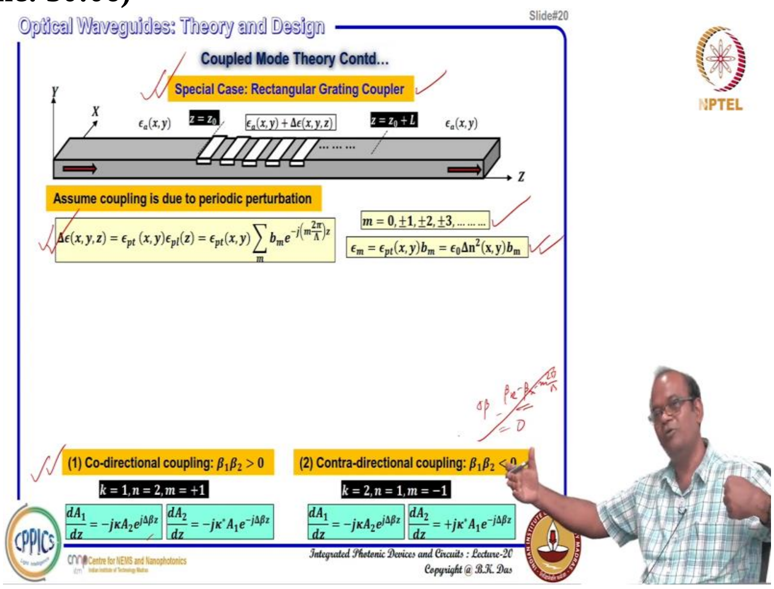

Now we will consider if these 2 modes mode 1 and mode 2 if they are counter propagating one is positive direction another is negative direction. So, when I can consider beta 1 positive and beta 2 negative if it is negative direction or beta 1 I can consider negative and beta 2 positive. So, in that case I can if I considered beta 1 is negative beta 2 positive then negative sign here negative, negative sign positive but here no sign change will be there.

So, in that case these 2 signs will be opposite. So, for contra directional coupling case these 2 equations will be the same but only sign has to be changed when we will be removing this one and this one. So that is what we have written you see one is plus another is minus. So, co directional coupling and contra directional coupling you can use same equations only by changing the sign one of them you will be just changing the sign then good enough.

So, when we will be dealing with that type of device when coupling is happening because of the perturbation when 2 modes are propagating in opposite direction then we have to start dealing this equation we will see that later on we will be following these so you have to remember these equations but counter propagating. So, you know counter propagating means opposite proposition so opposite sign will be there.

If they are propagating in the same direction they will be same thing minus, minus. So that is just easy to remember and kappa is the strength and you know this is very easy to find. If delta beta is 0 then you can get dA 1 dz = - j kappa A 2 standard couple mode equations. And here you will be delta beta = 0 means this will be opposite dA 2 / dz = j kappa star A 1 so something like this. So, it is straight forward we will show that how to deal with these types of equations later.

**(Refer Slide Time: 30:06)**

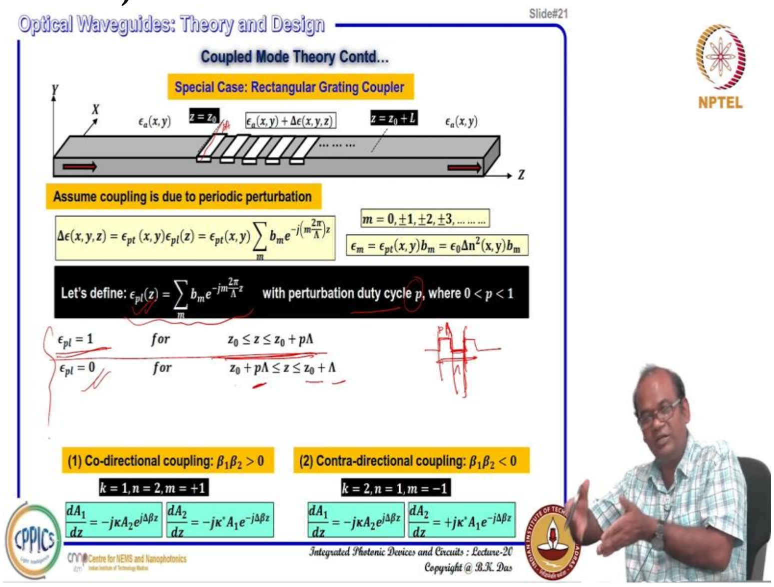



Before that we want to discuss important thing the special case as I mentioned the last one. So, we have discussed the genetic coupled equation coupling between 2 modes because the periodic perturbations and then we want to discuss this rectangular getting coupler what happens? I have just written down whatever discussed so far here that is a perturbation periodic perturbation and Fourier coefficient and b m and m can run from 0 + - 1 + - 2 + - 3.

Obviously we have only considered m = + 1 and minus 1 you can always consider m = + 2 and minus 2 but you have to check that it is close to delta beta = 0 or not if it is coming or not that we have to check because always you have to consider beta k - beta n - m 2pi / lambda. So, whatever for a given beta k and beta n can you can use m for different value so that this delta beta can be close to 0 phase matching condition we have to match for different m.

So, if it is phase matching condition between 2 modes it is close to m = 0 or m =1. So, you can only consider m = 1 you do only to consider all other harmonics all other harmonics will be happening for different beta meaning different frequency omega beta value change means independent frequency mean if that frequency or omega or lambda it is beyond your operating window you do not need to consider them that is a practical situation. So, we have just considered this 2 thing and this thing and we will be now considering this.

**(Refer Slide Time: 31:40)**



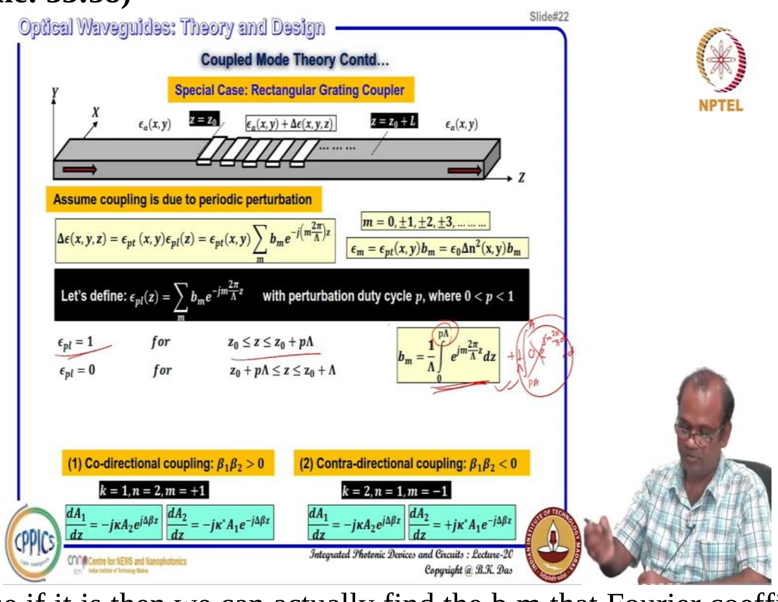

Rectangular perturbations this is the longitudinal direction perturbation Fourier transform typical expression we are writing that all the harmonics superposition that gives you longitudinal direction perturbation z dependent perturbation and this perturbation we are considering rectangular that means this thing this type of perturbation so this is your basically your lambda and you can consider if this is lambda out of lambda p factor lambda it is considered a duty cycle p is the duty cycle.

So, p lambda is the you have perturbation means dielectric constant is more and rest of the region up to here sorry period is up from here to here this is the period. So, this region; it will be different perturbation so perturbation is just a step type rectangular step type. So, we are considering p the fraction of period is actually having your perturbation some additional dielectric constant and obviously p can be that type of duty cycle can be between greater than 0 and less than 1.

We will be considering this one it is rectangular. So, rectangular function we can define like this suppose you have z = z 0 you are starting from z = z 0 between z = z 0 2 z 0 + p lambda. So, this is the p lambda region you have this perturbation is 1 that perturbation can be maybe remove all of your silicon core a little bit during that for that length duration. If that is we are writing 1 that means perturbation is there.

And rest of the part of this period that means z = z 0 + p lambda to z 0 + lambda that means this region to this region that is actually we are considering 0. So, this type of definition we are just introducing the duty cycle it is not 50% duty cycle for example in general case you can engineer that duty cycle when you are fabricating your device.

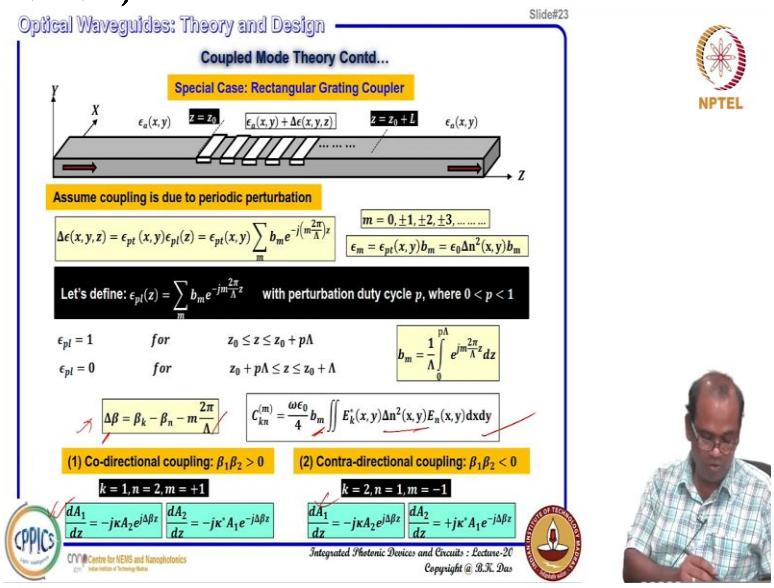
**(Refer Slide Time: 33:58)**




So that is the case if it is then we can actually find the b m that Fourier coefficient and in this form you are taking average over lambda and you are integrating 0 to p lambda because 0 to p lambda is your longitudinal perturbation 1 and rest of the things if you just try to get another integration p lambda to lambda 0 times e to the power j m 2pi lambda z dz so 0 that means it will go 1 / lambda should be there also.

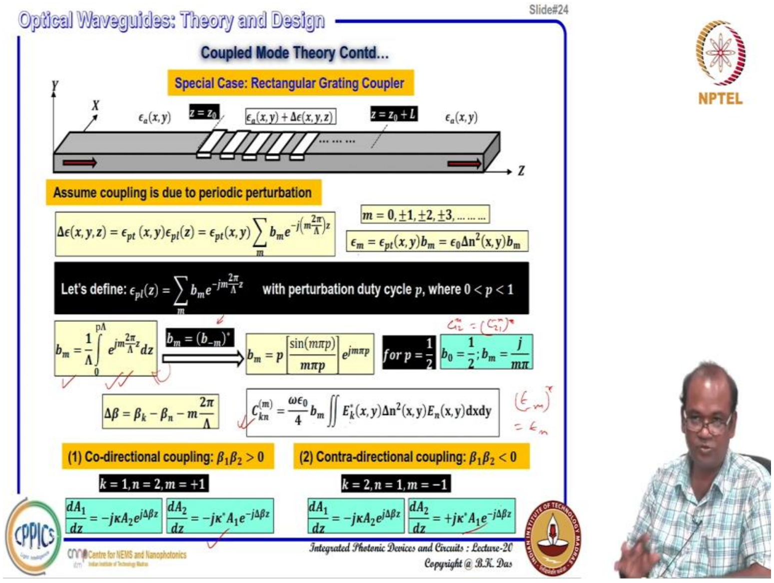
So, we are just decomposing 2 parts where it is adjusting the perturbation and where it is not where it is not existing perturbation that is obviously will come in 0. So, we have to integrate where the perturbation is there that region that length and but you have to average over the entire period. So that is how your duty cycle comes into the picture of finding the Fourier coefficient. So, it is straightforward so, we know how to find the Fourier coefficient just you have to integrate this one let us see.

**(Refer Slide Time: 34:59)**




So, one more thing we just added that beta k - beta - m 2pi / lambda that is actually delta beta we suppose considered and then C kn we just repeated here omega epsilon 0 b m will be there and refractive index that definition we bring back so that our everything in the same slide clear picture now periodic perturbation and then Fourier coefficient Fourier expansion of the periodic perturbation. And you have the coupled mode equations 2 coupled mode equation that is fine.

**(Refer Slide Time: 35:33)**

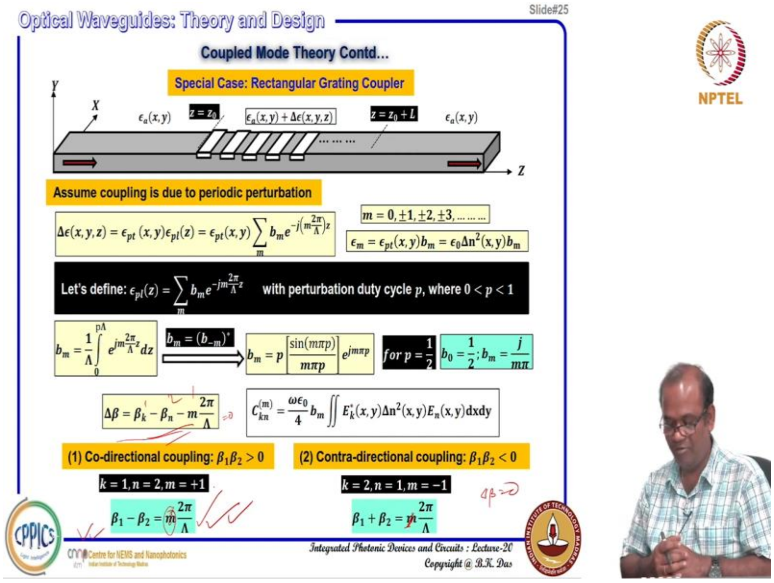



So, one requirement is that so for example if we just do that one here I have just tried to I promised you that I will be showing that the coupling coefficient and complex conjugate that means earlier we have shown that we will be able to show, C 12 m should be equal to C 21 - m star that complex conjugate should be same when you are coupling in the opposite reverse direction.

So, here if you see if b m is expressed like this you can easily show if you are just putting m = - m then you have to take complex conjugate then you will be getting same expression. So, it is proven so if b m is complex conjugate then obviously you are this one epsilon m and epsilon - m that should be equal to epsilon m and you take a complex conjugate so that means kappa coupling where you will be getting epsilon m or b b - m.

So, you have to take complex conjugate because k will become n m will become k. So that means the complex conjugate if you take when coupling strength in the reverse direction they must be equal. So, complex conjugate terms that is how it is introduced in your coupled equations this is for co directional coupling minus sign minus sign contradiction coupling minus sign plus sign that is what we have discussed next thing.

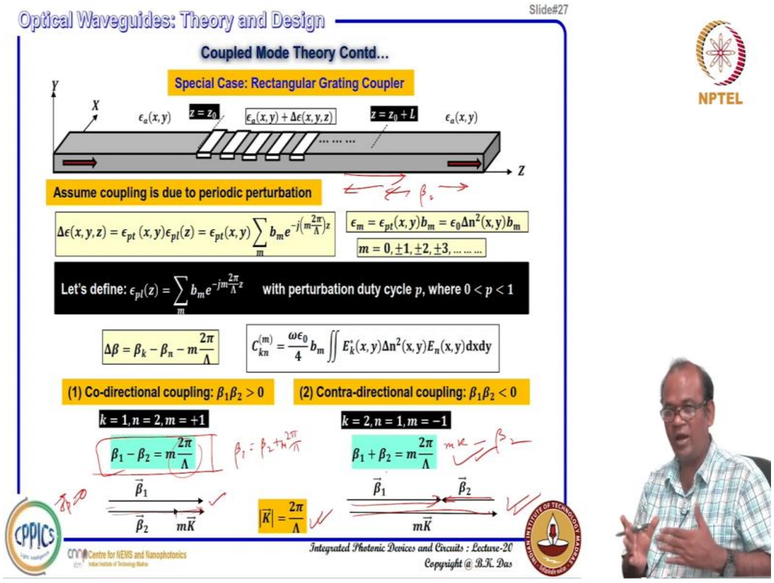
**(Refer Slide Time: 36:58)**




Now you will know that the another requirement is there couple mode equation is not enough but to get a meaningful coupling between 2 modes we have to have this propagation phase constant this longitudinal phase matching should happen for k = 1, n = 2, m = 1 we know beta 1 - beta 2 - m / 2 pi lambda that should be equal to 0. 1, 2 and m = 1 that should be equal to 0 that gives you beta 1 - beta 2 = m over 2pi lambda.

You can always consider m =1 also plus 1 it is a plus 1 this will be 1 we can write and this also 1 you do not need to write m that is fine and contradiction coupling case it is it will be because it is sign will be differently it will be k = 2, n = 1, m - 1. So, in that case the phase matching condition if you write delta beta = 0 this will be beta 1 + beta 2 = 2pi / lambda. So, here it is co directional case propagation constant difference in propagation constants should be adjusted by the 2pi / lambda that is called grading constant grading wave vector we would say I will discuss that why it is called wave vector and for contradiction the expression will be looking like this so fine.

**(Refer Slide Time: 38:21)**




Now next thing is that I just kept this 2 intact once more repeated here all this thing I kept so that entire picture is in your memory to proceed further what we will be doing now will be just trying to get a little bit different things we will try to represent in a vector notation because we have constant beta 1 and beta 2 they are associated with a guided mode. That means beta 1 will be either in the positive direction or negative direction same cases beta 2 will be positive direction or negative direction.

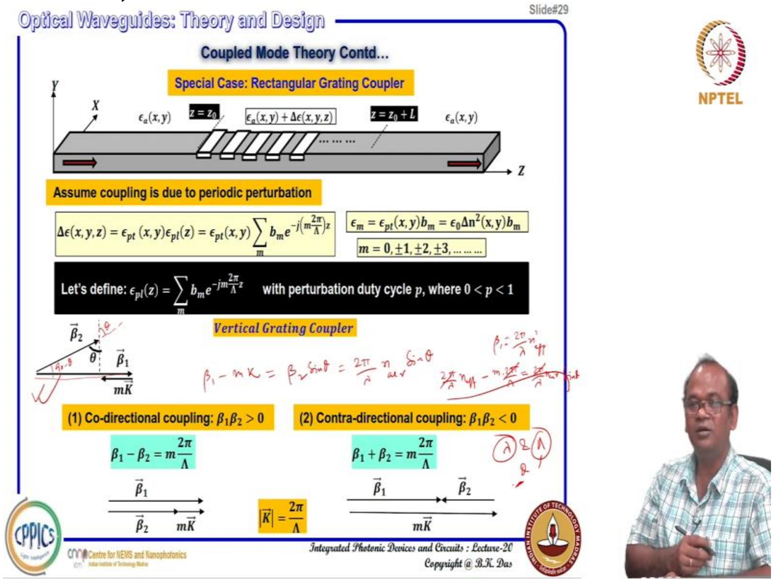
So, we have been using so far as a scalar since the axis is well defined otherwise this is a wave vector. So, wave vector can be a vector basically so in vector notation why we are representing that you will be just you will come to know a little while later with some application. So, we are considering supporting beta 1 is this one this one we are trying to use as a vector sum.

So, beta 1 you say beta 1 should be equal to beta 2 + 2pi lambda if mth is there so you can write m so that is what we written beta 2 up to this point then m k. So, this one we are writing as a wave vector also getting vector beta that is defined like this k is the 2pi / lambda so m k. So, this match vector you have to add and if it is equal to beta 1 then they will be coupling and in that case delta beta will be equal to 0 in vector notation.

So, you are considering delta beta as a vector phase mismatch longitudinal phase mismatch instead of longitudinal phase mismatch will be coming that it may not be only through only for longitudinal phase mismatch some other application will be discussing. So, in case of this phase matching condition we say that this is m k this one is m k. So, m k = beta 1 + beta 2 or we can write m k - beta 2 that will be equal to your beta 1.

So, we can represent in a vector notation like this. So, now this is said as long as you are considering coupling between guided modes this vector notation is perfect. Now we consider a special case that is called vertical grating coupler.

**(Refer Slide Time: 40:53)**




Vertical grating coupler means this type of grating will be using. So that light that grating will help us to couple light from the wave guide to the output region outside region I want to couple light out of the waveguide if that is possible or not you see both beta 1 and beta 2 here in the same direction that means in the waveguide direction can we make this beta 2 because if I am considering light coupling from mode 1 to mode 2 if this beta 2 has a vertical component.

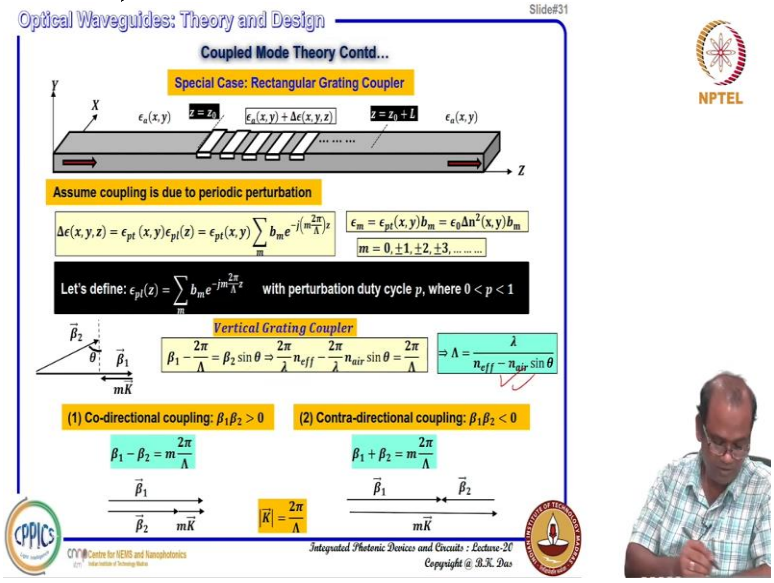
Here longitudinal component somehow it has a beta 2 in this direction this is beta 2 it will have a longitudinal component as well as vertical component if it can have some vertical component then we can say that beta 2 will not be any more along the waveguide direction that will have some kind of angle. So, how that is possible let us take the help of this sketch you see this thing is one as a beta 1 that is the guided mode we are considering.

Along the waveguide meaning beta 1 equal to I can say 2pi / lambda n 1 n effective 1 say for example that is beta 1 that is propagating in this direction. And now let us consider beta 2 which is not in the waveguide plane waveguide axis it is making an angle to the normal say theta for example here this is your beta 2 now to couple light from beta 1 to beta 2 mode, mode 1 to mode 2 having a propagation constant beta 2.

What we can do? I will try to see the longitudinal component of the beta 2 should contributed to the phase matching condition that means I can say that beta 1 - m k beta should be equal to beta 2 this is theta that will be 90 - theta that should be equal to beta 2 sin theta and beta 2 if it is in the air outside the waveguide then beta 2 can be written as 2pi / lambda n air refractive indexes outside is not effective index n air sin theta.

What is theta? Theta is the angle created with respect to the normal drawn from the waveguide surface. And now we can write this equation as beta 1 we can write this one write 2pi / lambda n effective 1 maybe guided mode and minus m 2pi / lambda = 2pi / lambda n air typically that is 1 into sin theta that is what this is the expression then 2pi / 2pi cancel 2pi, 2pi cancel. Then I can get a relationship between lambda and lambda with theta if I control the lambda is a fixed operating wavelength and if I control period then I can define which direction the light will be out coupled.

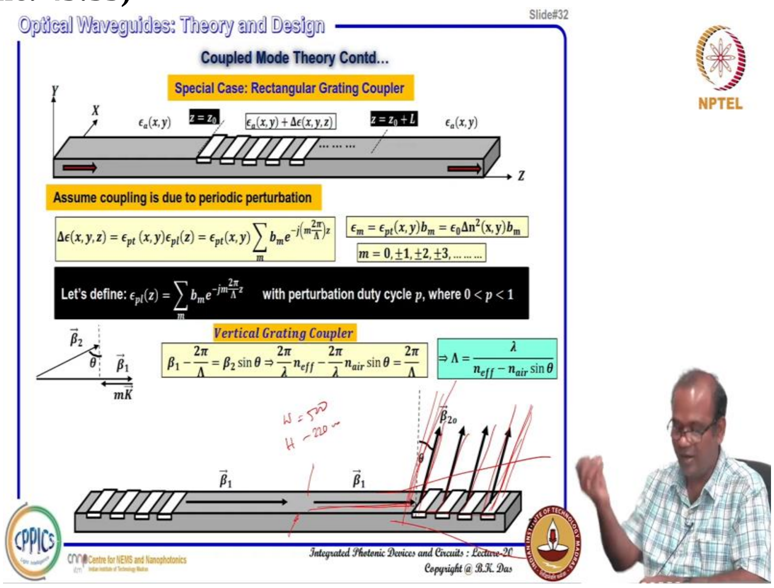
**(Refer Slide Time: 44:31)**




So that is what the expression I have just written down here beta 1 this one whatever I discussed that beta 1 = 2pi / lambda n effective 2pi / lambda I have written and then this one beta 2 sin theta I have taken this side and 2pi / lambda that side in air beta 2 = 2pi / lambda n air and sin theta that should be 2pi / lambda. So, this you just do 2pi, 2pi cancel 2pi 2pi cancel.

And then I can get this period in terms of lambda / n effective - n air sin theta. So, if theta is defined certain angle you want certain direction it is should be coupled then you can define a period your periodic perturbation rectangular perturbation what type of period you need for a given wavelength to have a light out coupling along theta direction and that is actually a very interesting thing we have given this expression

**(Refer Slide Time: 45:33)**



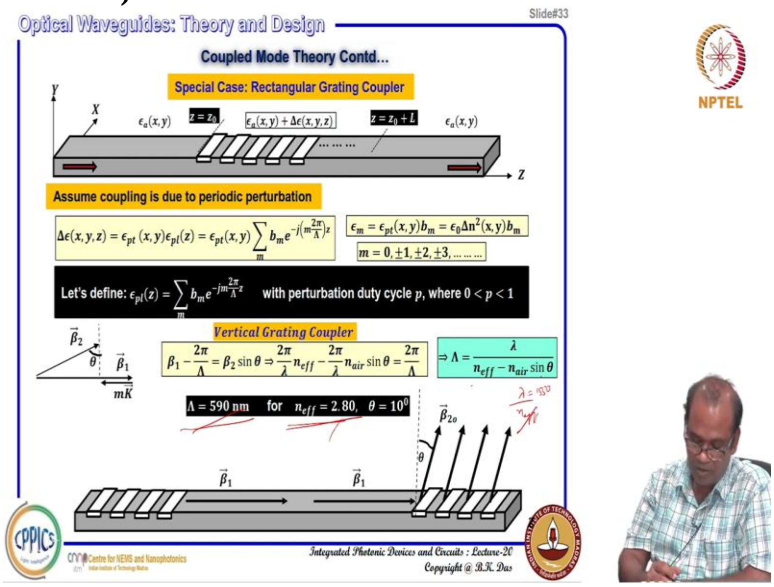

And this is nicely presented here you see this is your beta 1 incoming and beta 2 output. So, output if it is a guided mode if this grading is there it will be coupled. So, fraction all I will be coupling like this direction so what do you if you put a fiber along this angle this theta angle if a certain lambda is calculated then you will be getting theta if you bring it optical fiber and then light can be coupled into the fiber then.

So, you can take light into the fiber for distance remote communication otherwise also you can just say that for free space communication nowadays people are using also for secure communication quantum key distribution etcetera free space satellite communication also people are doing the people are planning that how signal processed on chip light signal processed on chip send to satellite in free space.

So, you can just direct that angle where the satellite according to that you can just position and you can communicate that is actually possible and also you can have a fiber input here then you can couple light output there. So, the fiber of course you know fiber core is typically 10 micrometer but our photonic ware waveguide that is in the order of W = 500 nanometer and H equal to around 220 nanometers very small.

So, 20 times lower cross section or some if you see that dimension is just half a micron and your waveguide core is about fiber core is about 10 micrometer. So, in that case what you could do this thing this where you are coupling with the fiber that region you get a taper idiomatic taper and grating you define taper to 10 micron. So, your fiber can come directly core can be directly sitting on the gratings and you can couple so you are getting length can be 10 micrometer. And what is the periodicity required for a silicon waveguide we have calculated here.

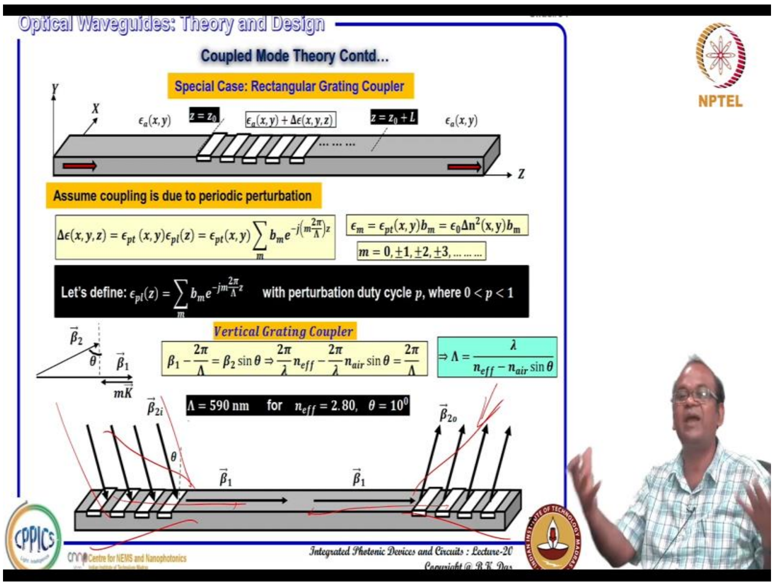
**(Refer Slide Time: 47:37)**




If a guided mode is having effective index of 2.80 we have calculated earlier for a effective index of guided mode that is in the order of 2.8 or something like that and if you want that coupled light into the air that should go in 10 degree then period requires 590 nanometer. So, you can define this period so that you can control light at 10 degree angle you can have completely 90 degree. So, for 90 degree what would be the theta value? Theta value will be 0.

So, if theta is 0 that means lambda / n effective so if you want a 90 degree coupling upward then your period would be lambda / n effective lambda you are putting 1550 nanometer and n effective 2.80 whatever value comes you will be getting that thing that will be a little more. So, in the same process what you could do you could do this thing.

**(Refer Slide Time: 48:37)**




That since light can be coupled this direction. So, in the reverse direction light comes this direction 10 degree angle if it is incident on to the waveguide then light will be coupled to the guided mode also the reverse phase matching condition so that is counter propagating and co directional propagating situation like that. So, you can have in the input side also you can have a grating structure and you can bring your fiber.

And you can input light into the waveguide you can launch light into a photonics chip with a input wave guide getting coupler and you can have a output getting applied like a pad you can just consider like a pad and you can bring your fiber tip as your probe then you can test your photonics chip you know normally nowadays CMOS compatible silicon photonics foundries are available here actually you are fabricating your device circuits in 300 millimeter wide wafer diameter 300 millimeter.

That means it is what we call that 300 millimeter means a 12 inch wafer size it is like a lunch plate. So, there you will have a large dyes large number of circuits large number of devices but how to face to those devices that after fabrication they are working or not. So, people use this type of grating coupler at the input each device or circuit elements you will have an input and output and they are getting coupler will be there like a grating couple of contact pad optical contact pad.

So, you bring your light through fiber and just directly contact at an angle of course depending on your periodicity you have defined your fiber you need to bring at that particular angle then light will be coupled and you can take it out also and you can check whether that device is working or not. So, this is a very important element and discovery which is actually technologically successful.

And people are implementing for chip level testing wafer level testing as well as also sometimes this type of technique grating coupler is used for pilot pickling also for a packaged device with this I stop here. And next we will be continuing discussion for various integrated optical components used for photonic integrated circuits thank you very much.