**Integrated Photonics Devices and Circuits**
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**Lecture – 22**
# Integrated Optical Components: Y- Junction Power Splitter / Combiner and Mach-Zehnder Interferometer

Today in this lecture we are going to start a new chapter integrated optical components. So, the first simple components will be discussing today that is actually based on Y - Junction Power Splitter / Combiner and also Mach-Zehnder Interferometer. These are the actually some of the important passive component I would say and you can easily understand if you have good knowledge in waveguide theory and coupled mode theory.

Coupled mode theory just we have discussed we will continue that whenever necessary we will just use those coupled equations. And we will be explaining and today we will be just using a little bit idea a couple mode theory to understand the working principle of so called Y-Junction Power Splitter / Combiner and Mach-Zehnder Interferometer, it is one of the most important components among many few components are important for integrated optical applications, integrated optical circuits, so Mach-Zehnder interferometer each one of them.

To continue that first what we will do? We will try to picturize visualize a 2D waveguide mathematical model we want to explain a mathematical model for 2D waveguide. And in terms of 1D picture, so I mean to say you can say that as I mentioned here, it is a 1D mathematical model of 2D waveguide. And then based on that, I will try to explain the working principle of Y- Junction based on single mode waveguides. And then finally I will be discussing your Mach-Zehnder Interferometer.

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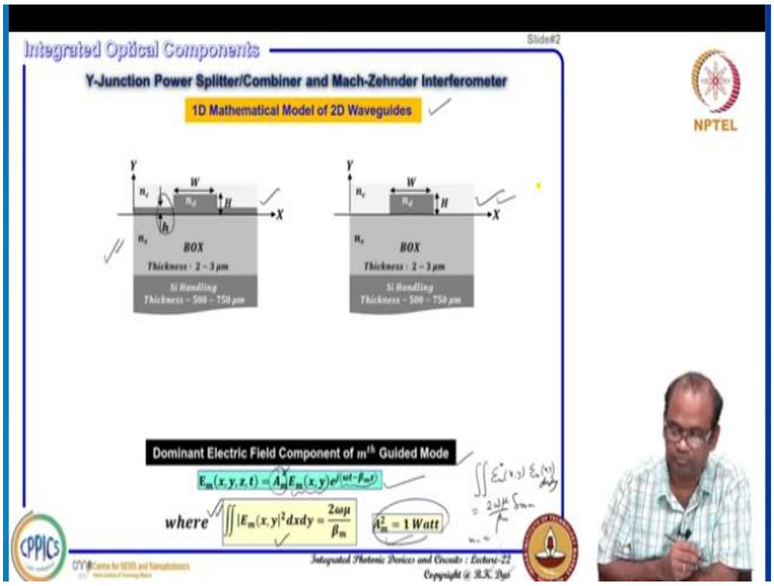


So, now let us try to discuss first how we can develop a 1D mathematical working model of a 2D waveguide. Here one example, as we always consider for SOI silicon on insulator waveguide it is a waveguide structure and this photonic wire waveguide structure, when this h the slab height is 0 then it is called sometimes photonic wire waveguide. And we know that the dominant electric field component for the; mth mode, if it is a multimode waveguide numbers of modes are there.

So, mth mode we can just express in this way E m x y z t, if it is a monochromatic wave electromagnetic wave propagating along the waveguide accesses along z direction, then you have amplitude A m and field distribution, dominant electric field distribution it may be either in the X direction or Y direction will be considering that depending on the TE like mode or TM like mode and then your phase term will be there omega t – beta m z, Z direction phase will be changing.

And we have normalized such that A m square is 1 watt and E m so basically we have learned earlier that E m Star x, y E n x, y dx if you integrate you are supposed to get 2 omega mu / beta m delta m n that is our normality condition according to that we have derived this expression that if it is electric field, if m = n. So, we write this one as 2 omega mu / beta m and if them mode is normalized to 1 watt, we can say that amplitude square in 1 watt. So, simply all the power you need I can impose on A m square. This is what we learned before we started the couple mode theory.

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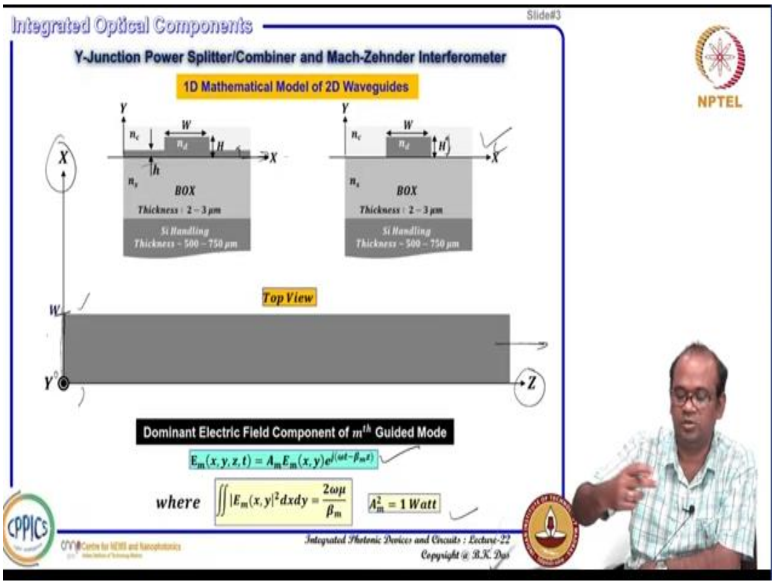


Now, we can just see W h H, we can just try to picturised for understanding purpose explanation purpose, we can just only picturise top view, that means this is your X axis, this is your Z propagation direction, and then we are looking from the top surface region of the waveguide then you can see along Y direction, along the X direction, this is your X direction width is W so along the X direction, we have this is the width.

So, from X = 0 to W that is width, Y is now perpendicular to the screen and we can have this is the waveguide is propagating direction this one. We can just picturised this, but we should keep in mind that the mode filed distributions will be looking like that and they are normalized like that. So, this is some kind of model we already we would not sketch 2 dimensional picture of cross sectional view, because we would like to explain because all these waveguides we will be using for planar circuit, a circuit in a plane.

So, in plane whatever changes is happening that matters and also we should keep in mind that this device layer thickness is a fixed parameter that depends on your silicon on insulator wafer you are buying typically 220 nanometre and you can leave some h slab height and that we are not picturised in here we are just considering only W with top view. When I see the top view that means I am just talking about top pictorial scheme.

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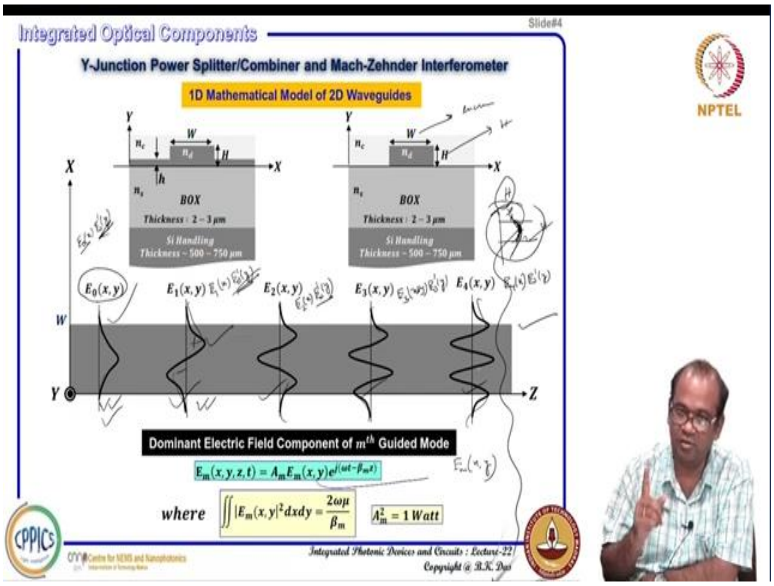


Then if i just try to see that there are multiple modes will be there, so for example if I write this is the mode field distribution E m x y this is the mode field distribution of the mth mode. So, that will be actually x y field profile you will be able to see you can evaluate that and we have discussed that earlier also when we solve the full vectorial equation for a waveguide mode. However, I know that my height is at just is such that vertical direction. Vertical direction you will be observing in the Y direction only one.

If you said just see this is Y direction because this H Y direction you will see only one standing mode standing that means round trips would be such that one peak will be there and we can always consider that even though you are increasing your width if this width is increased, but vertical distribution field distribution profile will not change. Along H direction and along Y direction field distribution will not change because as long as H fixed.

So, as long as H is fixed, we assume that vertical direction field distribution is not changing. So, in this 2D diagram here along propagation direction from the top if we just try to see what we will see that different mode, it will show a different type of distribution, this will be your fundamental mode that is along whatever I am showing here along X direction, Y direction is always looking like that whatever I just shown.

So, all the fields Y direction will be like this, but along the X direction because width is allowing multimode, so more width you can have more number of modes along that direction more number of modes and the modes along the X direction, but Y direction since it is fixed a thickness. So, we can assume that the field distribution is like this. So, schematically though mathematically we will be writing E naught x y that means it has a x y distributions.

So, what we will be writing it can be simply written as E naught x and something E naught prime y, Y direction here and E naught here also I can write E 1 x and then E naught prime y. Here I will be seeing E 2 x and E naught that means Y direction, the field profile same everywhere E naught E naught, E naught, E naught prime I am just writing E 3 x y, E naught prime, E 4 x and then E naught prime y.

So, that means if I just normally when we solve we can just assume that this field distributions are can be actually expressed by separation of variables. So, x dependent variable, y dependent variable is there y dependent variable function same everywhere I imagined assumed then along X direction I can see the mode field distributional will be varying for fundamental mode will be like almost like a Gaussian SOI model.

Then exponential decay will be there and then higher order mode X direction you will be getting one plus another minus field at any point. And then higher order modes you will see more number of nodes and anti-nodes like this. And you must have object that the evanescent tail as you go for higher order mode that is extending towards cladding mode that is actually here soon extending more and more.

So, if you solve for the higher order mode, then you will see that it will be almost leaky. So, you can have certain number of modes will be supported by this waveguide that you can derive. This is how we can just so, the 2 dimensional 2D waveguide but we can just explain in 1 dimension I can sketch what is the field distribution, considering the fact that other direction field distribution does not change, change for any other all the modes.

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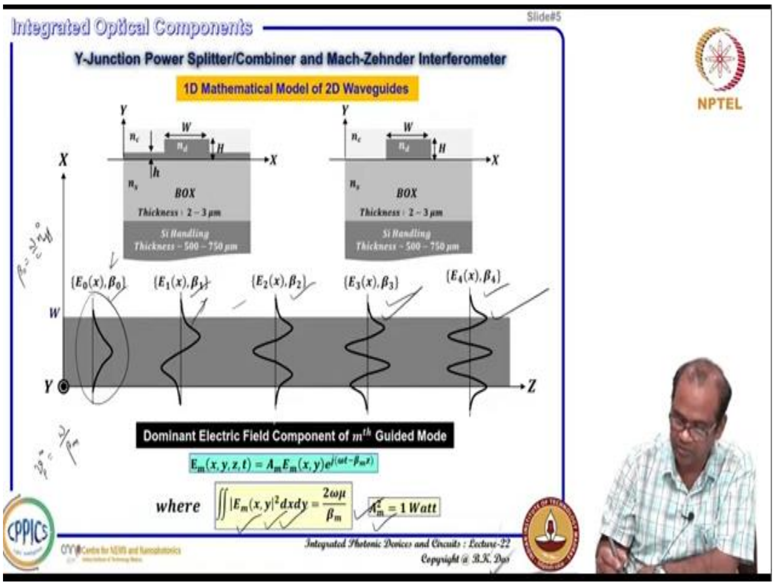


Now, that is why we are updating we can characterize fundamental mode E naught x and along with its beta naught, beta naught means we know we have solved earlier omega / c n effective naught effective index of the fundamental mode. Similarly, I can just represent here this E x beta 1, this one I can represent beta 2, beta 3, beta 4. So, these beta 1, beta 2, beta 3, beta 4 also important besides this field distribution; because they actually decide the phase velocity of the mth mode because of my frequency same.

So, this is your beta m, so different mode they will have different propagation constant, different effective indexes, so as we know that higher the mode effective index drops, effective index drops means higher mode travels faster, phase velocity will be higher. So, that is what it is explained. Now, if we see, but we should keep in mind all the time that this is a 2D waveguide, we are showing in the 2 dimension and we should keep in mind that this condition valid and amplitude is also valid that is every all the modes.

They are normalized to 1 watt, this mode also normalized 1 watt, this mode also normalized 1 watt that means this condition fulfilled for all the modes that is what we assume for mathematical analysis purpose. So, if we have pipe what that means A m square will be pipe that is it nothing else just a scaling factor.

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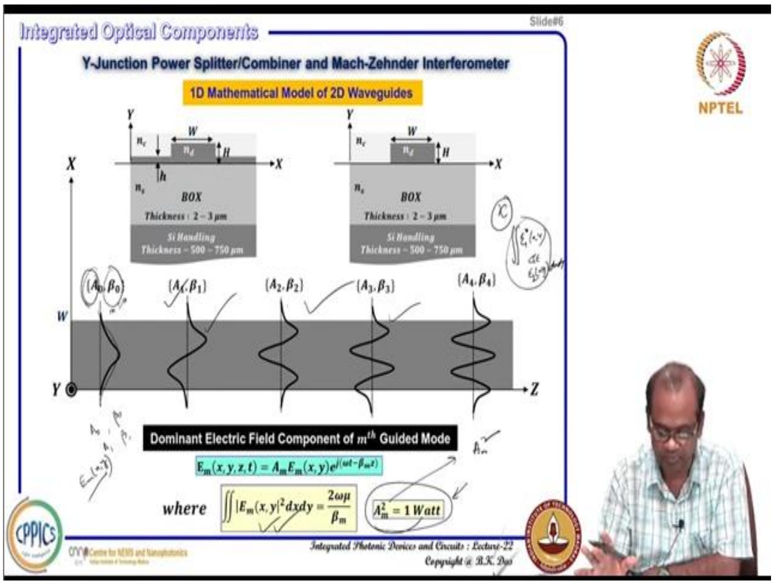


So, now instead of filed distribution because I have the inter power associated with the mode is symbolized by A m square and rest of the things are expressed by this that was normalization condition. So, probably as far as energy propagation is concerned power propagation is concerned we may forget the field distribution actual field distribution E m x y, you may not need to consider all the time instead if I know if some if there are some coupled mode theory or something in between some perturbation is there, this A m is going to change.

But we always assume that the field distribution is not going to change that is assumed same. So, that means we do not need to consider for our mathematical model, what is the field distribution actually, as long as you can just equivalent all the results other parameters we can calculate based on that. So, we do not need to calculate need to see the field distribution for example, you can see that we have defined Kappa once we know Kappa is defined something like this.

So, E 1 star x y then delta epsilon multiplied by E 2 x y and dx dy, so that is actually Kappa defined. So, once you define Kappa you do not need to bother about what is the filed distribution, further analysis and a coupled mode theory we have seen earlier. So, same way we can all always consider that fundamental mode will have amplitude A naught propagation constant beta naught A naught actually gives the power it is carrying.

Similarly, B naught will give the how fast the phase is moving phase velocity will be or disperse and relation actually you can come to know from beta naught. And then A 1 similarly, that is the power it carries the amplitude of the field and beta 1 is the propagation constant and so on all the modes. So, now we can reduce the number of parameters, all the field distributions omega t x y dependencies everything we can forget as long as we know what is field of amplitude value A naught beta naught for fundamental mode.

A 1 beta 1 for first order mode, second order, third order and so on. So, this is corresponding to mode index 0, mode index 1, mode index 2 and so on mode index we start from 0. So, this is how we can understand when we say that fundamental mode then we get if we just sketch like this we can say that this is a fundamental mode its amplitude we can always say it is A naught and propagation constant whatever I compute from the software or simulation software then we can write this beta naught more solver.

Then similarly, here amplitude A 1 and I can write beta 1 and so on. So, this information is good enough for understanding many of the devices how it works in integrated optics.

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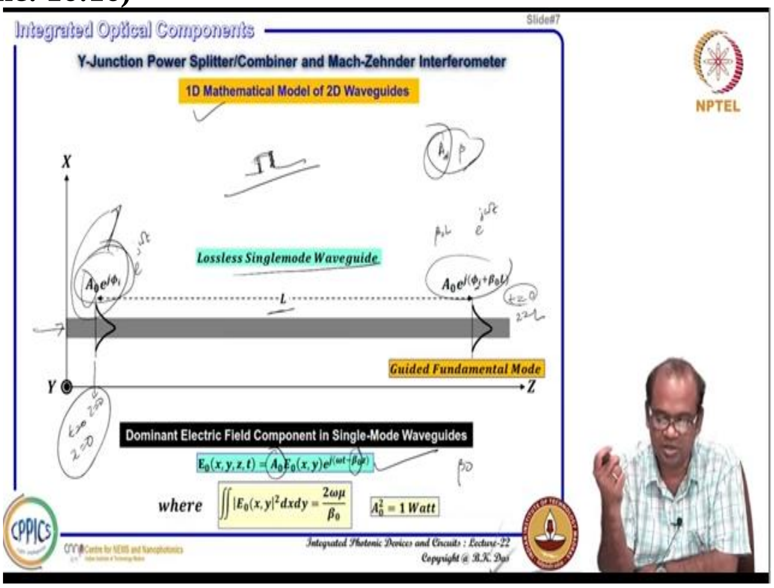


So, next, let us move on to a waveguide top view it is one single mode waveguide now, we are just considering single model waveguide. So, 1D mathematical model of 2D waveguide that is somewhat we express in terms of A 1 and beta A and beta, amplitude and beta that is so called I mean for 1D mathematical model for 2D waveguide. So, we were just told is considering the propagation direction how it is changing phase is changing cross sectional direction.

You do not need to bother about anything as long as you can quantify A naught and beta that is it, but a single model waveguide. So, we are considering suppose your lossless single mode waveguide just only one more support fundamental mode. So, what I need to consider field distribution will be like this I know that I should keep in mind at least I should not forget the filed to send in the x y direction only mathematically this part is most important here and beta naught is the important frequency is known to you.

And this is the case, if I just consider this is actually say t = 0, I am considering and also I am considering t = 0 z = 0, I am considering what is the value t = 0 and z = 0, I am just writing A naught and if here whatever light is coming from this side, if it has some kind of phase phi 1 I just added. So, this is the phase and amplitude at t = 0 z = 0. So, this is the z starting point for example, I can write this one. And then if there is no loss etcetera is there after travelling length of L.

So, then what I will be getting A naught e to the power j phi is there along with that it has a propagation constant beta naught. So, that means traveling beta L length, so, it will acquire phase of beta naught L, so that means I will acquire phase phi i + beta naught L so that means, this is also exactly t = 0 and z = L. So, that means t = 0 this is the situation here t = 0 this is a situation at z = L.

However, if I want to restrict my discussion only at z = 0 then what I can say that this thing will keep on varying as your function of e to the power j omega t sin omega t cosin omega t this value will be as if I just restrict if I want to just track what is happening at z = 0. So, that means z = 0 this value, this is the amplitude whatever phi value is there you just find as your amplitude and as a function of time it will be oscillating sinusoidally are the frequency of light, so 200 terahertz and so on that.

So, similarly, here also at t = 0 I consider this thing now, if you want to roll over time at this particular point, it will be oscillating with e to the power j omega t. So, that is how it is travelling wave it is a phase and time dependent variations how it will be that you can express in terms of only amplitude and whatever phase is there that is good enough and beta naught. So, we are no more considering the field distributions whatever the x y etcetera is there I do not need to consider this thing.

So, this is the lossless case if there is no loss, but you know this type of waveguide you have for example, if you are making it this type of waveguide structure then you know this is some roughness will be there this your fabricating with the CMOS industry, some lithography defects will be there, lithography is not always perfect you cannot get perfect plane both side there will be some roughness will be there.

So, there will be some kind of scattering loss you can expect also and depending on the wavelength and as well as also depending on what you call that some defects in the waveguide structure, you can see some kind of scattering losses also waveguide propagation losses there.

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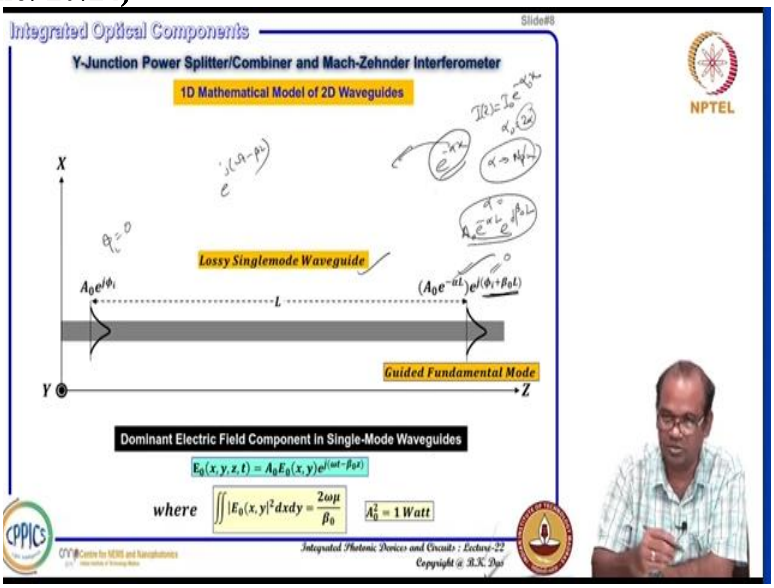


So, in that case how we will represent this one. So, Lossy single mode waveguide I can say that if this is the case we already discussed earlier if there is a loss electromagnetic wave it will be exponentially decaying. So, e to the power -alpha x that is actual alpha if it is represented in neper per meter in SI unit. So, that is how you know normally if it is amplitude this alpha we have already discussed earlier that this so called what you call that Beer's law so, that Iz = I naught e to the power alpha naught x.

For example alpha naught x will be 2 alpha because alpha is the loss coefficient for the amplitude this is sometimes called loss coefficient for the field amplitude. So, at this point of time whenever we are representing that that field is exponentially decaying because of the loss this alpha the dimension should be this is neper per meter. So, accordingly the amplitude will be dropped along with that you will have this phase initial phase is there although you can set initial phase equal to 0 if phi = 0 here you can put equal to 0 here.

Then it will be A naught e to the power -alpha L e to the power j beta naught L that is it that would have been the phase. So, you might be wondering why plus sign is there because you know this traveling wave is e to the j omega t - beta L forward direction beta z, but since we are just considering how much phase is there how much additional phase is acquired here. So, that is why we are writing beta naught L you can always write minus j also plus minus that depends on you. So, as it propagates we are just adding the phase otherwise, here this phase will be you can write e to the power -beta L that is also fine so far so good.

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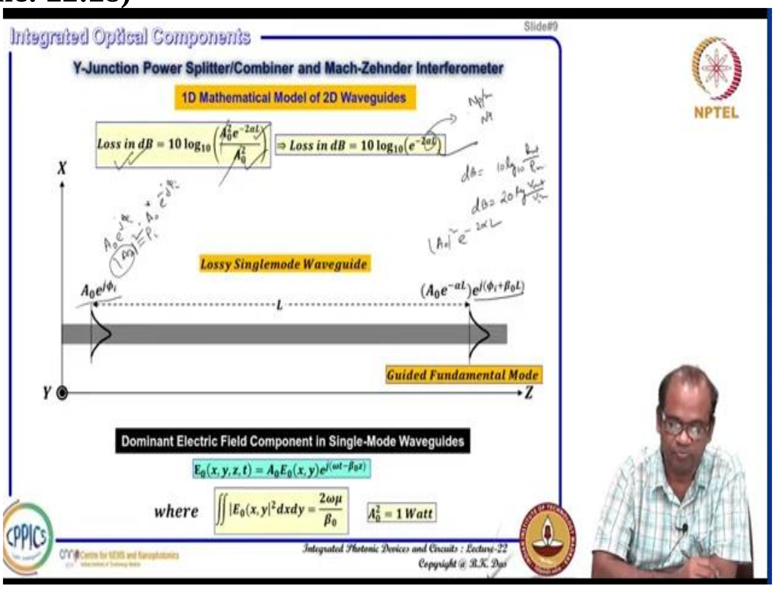


Now, alpha sometimes you know in the waveguide in photonic integrated circuit or integrated optical circuit, the waveguide loss normally represent in decibel. So, I tried to just explain here suppose, if it is neper it is there you whenever you are considering in the amplitude scheme because normal integrated photonic devices we have to analyse in terms of the amplitudes because amplitude and phase is very important because you have to always consider some kind of interference etcetera so, that is why that is important.

But sometimes you can see that people are industry or foundry they are saying that a great loss is decibel per centimetre or dB per centimetre something like that. So, that is actually nothing but dB definition you know that anything gain or loss in dB we say 10 times log of 10 base P output / P input that is actually decibel power and if it is voltage normally that same dB will be written like 20 log of V out / V input. So, V square where you are able to power in terms of circuit point of view.

So, here also we can say that loss in dB I can say that what is the power here and what is the power here? Here power I can write A naught e to the j phi i times A naught star e to the power -j phi i. So, that means complex conjugate you have to take and then you have to multiply that will be your power this is A naught square. So, A naught square is nothing but you are considering like a phi.

So, this A naught square is the input power and output power how much it will be? It will be A naught square e to the power -2 alpha L and this one really complex conjugate if you take that that will be cancel like j phi one to be. So, whenever you are considering power the phase information is lost basically. So, nevertheless that is what energy point of view we can understand that one, so, I can say that the loss in dB you can certain log of power in the output by taking ratio power in the input. So, A naught square, A naught square will cancel.

So, ultimately this will reduce to this one loss in dB 10 log 10 e to the power -2 alpha L. This alpha will keep in mind that is actually expressed in the neper per meter or neper per centimetre depending on your L, if it is your represent and millimetre. So, you can say that neper per millimetre if it is micron neper per micron then ultimately this alpha L is neper alpha L together it is called neper.

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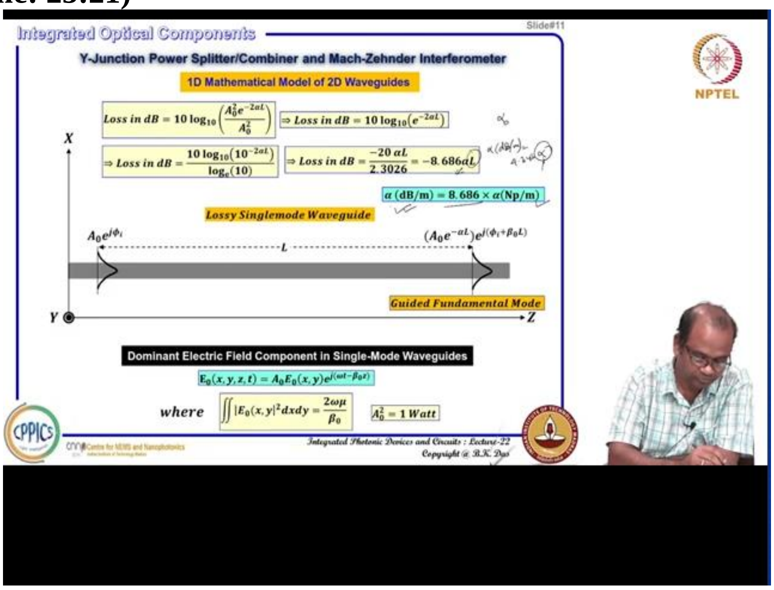

So, now if you just move on loss in dB, this little bit simplification here what do you have? So, this is actually decibel, this is your normal consideration exponential thing is there, but what we can do instead of exponent e to the power -2 alpha L, if I can write 10 to the power -2 alpha L we can express we have to divide it by log e 10, log 10 to the base e if I divide it then this instead of exponential power I can dictate power decibel power I can put 10 to the power this thing.

So, if we write that this is a simple logarithmic relationship if you do that what you can get log of 10 to the base e comes like 2.3026. And here you see base 10 and 10 to the power -2 alpha L. So, log of 10 to the power something x means it is basically x. So, that means, this x should be multiplied by 10 that means, whatever you are getting in the numerator 20 2 alpha L multiplied by 10 minus sign is that 20 alpha L / 10.

So, if you just simplify then it will be you just divide 20 / 2.3 and so on, then you will be getting 8.686 alpha L minus sign of course minus sign that because of loss minus sign because here power is reduced that loss. So, whenever you are considering loss, you probably do not need to write minus you can just simply write plus. So, in that case alpha if you want to represent in terms of decibel per meter SI unit this L like remove then 8.68 multiplied by alpha neper per meter.

So, if you are just L if you are taking here alpha loss dB per meter then I can write that if neper per meter is given if you just multiply 8.686 then you can get loss in decibel per meter. So, if alpha instead of neper if it is intensity loss is given then alpha dB per meter can be written as 4.34 because 2 times this 10 is there and 20 will be coming instead of 2 alpha that will be only alpha will be coming power relationship. So, that will be 4.343 times alpha that means intensity loss coefficient.

If it is intensity loss coefficient you multiplied by 4.343 then you get loss in decibel per meter and if this alpha is the neper then you have to multiply 8.68686 then you will be getting loss decibel per meter that is just a technical thing sometimes you may need to follow sometimes people will be talking about loss neper per meter and decibel per meter you should be able to distinguish that.

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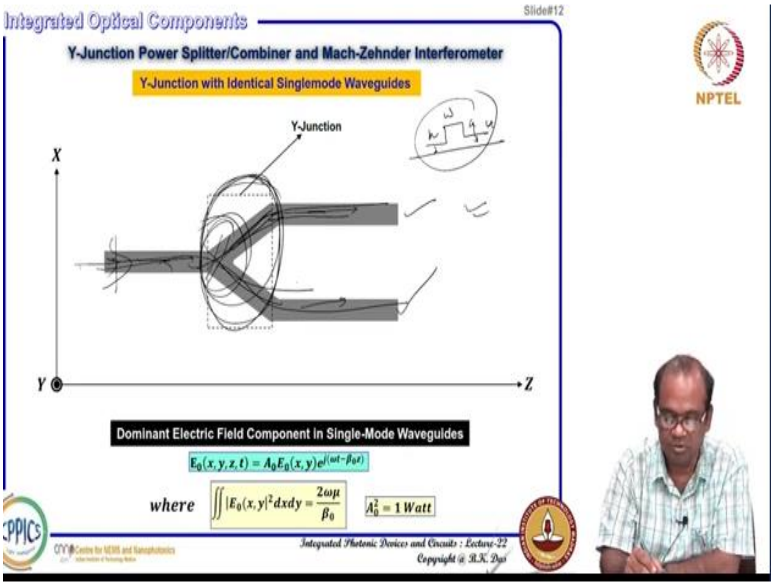


Now, let us move so we have so far represented this 1D waveguide, 2D waveguide, 1D mathematical model and also we have represented how amplitude loss figure and propagation constant is sufficient to understand the wave propagation in a single mode waveguide on multiple mode waveguide. Now, we will be discussing about Y-Junction. So, Y-Junction meaning you have to have one input waveguide and it will be splitted into 2 waveguides output waveguides through a Y-Junction type structure like this.

So, it was very popular long time ago when waveguide cross sections are a little bit higher lithium niobate waveguides and a lot of polymer waveguides people used to design one is to 2 power splitter in this part, just one waveguide single mode waveguide it is the this one and another single mode waveguide another single mode waveguide they are identical, this waveguide also identical.

Identical in the sense if you see the waveguide dimensional this w value H value and h value silicon on insulator waveguide all these are same, only thing is that in the XZ plane if it is their Z propagating waveguide. So, if you are just considering in the surface of the substrate, top view you are looking, so you can say that in the top view in the XZ plane propagation direction is changing. Light is propagating and then if it is propagating area should go like this or it should go like this.

So, this type of Junction you can design such that you can have a very, you can have no almost no loss in this junction you will need to design because of the junction and this type of perturbation will be there now. So, your waveguide whatever fundamental mode propagating like a orthogonal mode no power will be dissipated anywhere it will be carrying but suddenly it sees some kind of adaption some kind of perturbation, so they are actually orthogonality condition breaks.

So, when orthogonality condition breaks, that means, you have to think about the coupled mode theory that means, that particular mode will be disturbed and all other existing modes including radiating modes energy will be there is a possibility of energy transfer as long as some kind of mode matching happening and phase matching condition happening to conditions required for coupling.

So, this is actually the result of coupled mode theory you can see that any power you are launching here that may actually get somehow can get lost completely also very minimal can come here that type of situation happened. So, you need to design appropriately, so that you can minimize your loss to the outside world.

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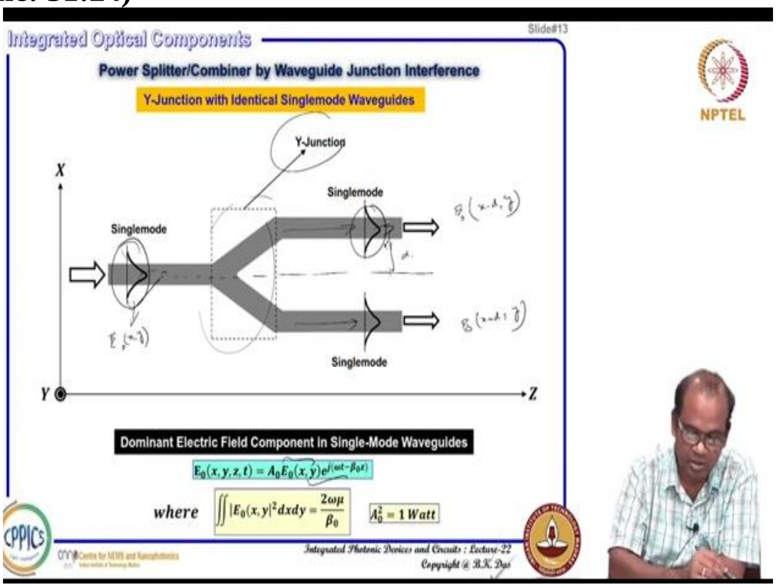


So, what is that if it is something Y-Junction like that you would like to have the single mode waveguide fundamental mode you showing like this and this junction; suppose something is there lossless or whatever, then you could see that mode single mode waveguide here also to will be there here will be there. So, situation you see the field distribution here that is actually we are showing in X direction.

But ultimately that should be the fundamental mode that is x y distribution profile Y direction is there Y direction it is not changing for any other modes, it will be also similar type of situation in the Y direction probably depending on your structure symmetric, asymmetric cladding or so you can have this thing. So, single mode waveguide you are representing, but you should keep in mind this one this is a 1D model mathematical model and you can always represent with amplitude as I said.

And here also this has same profile, but you see if you carefully see the X coordinate here and X coordinate here is not same. So, you have to appropriately whenever you are just relating this mode to this mode you have to appropriately change the coordinate for example, if this is E naught x y here, then this one will be I can say that some distribution is there I can write E x - d for x greater than d. So, suppose this is the d distance the centre d distance x - d and y.

Similarly here I can write E naught field distribution x + d so, and then y so, this type of coordinate will be shifted only field distribution is same, but they are laterally sifted. So, whenever you are doing some mathematical derivation or etcetera and you need a field profile distribution also that time you should keep in mind that the coordinate also be adjusted appropriately.

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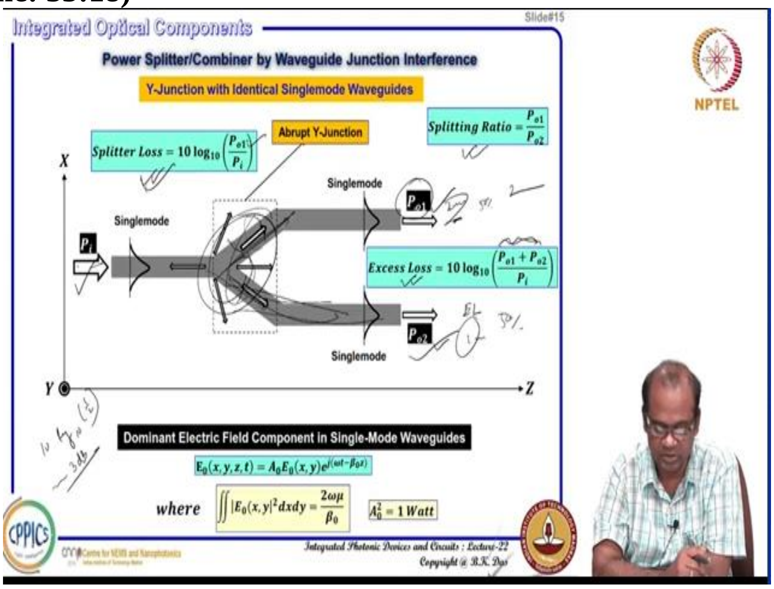


Now, you see it this junction is not propellant angle it is abrupt what can happen this is scattering as I mentioned this a lumped region we have huge perturbation it can happen depending on the angle how big angle you are creating and also you see carefully if you see the width here it is no different. So, this type of suddenly it sees different types of waveguide and it is kind of huge perturbation lumped, in a lumped region is there.

So, you can expect something will be scattering backward direction and by outside also it is scattering the side or scattering and maybe a little bit it will go here and whatever little bit go into the waveguide that will take the shape single mode field distribution, shape will be similar, but only thing is that the power here that will be actually will be reduced because of the loss acquired here loss actually because of the scattering losses in the abrupt junction.

Here normally we define some kind of figure merit if this Y-Junction practically even though you just theoretically design a good Y splitter Y-Junction splitter, but practically you may see some kinds of losses also. So, because of the some kind of abruption, it is very difficult to design also good model, good Y-Junction but nevertheless, whatever the value experimental you get? You get a power here P naught, P o1 P output 1, P output 2 here input power.

We define one figure of merit excess loss, we call it as excess loss sometimes it is represented by E L excess losses you whatever power you are extracting in port 1 and port 2 you sum them and take a ratio the input and take log 10 10 log 10 of that one. So, this is the total power you are availing at both outputs and you are taking ratio of the input ports and take an issue that is called excess loss in dB.

And then another thing is that if there are losses everything is there it can happen that this types of waveguides these waveguide and waveguide 1 and waveguide 2 they may not be symmetric. So, their width may be slightly different also it can happen you can design that also in that case even though loss is there and something is coupling here, something is coupling here depending on the waveguide dimension their angle maybe this angle is a bigger, this angle is a bit lower.

So, in that case power coupling this waveguide and power coupling this waveguide they may not be same. So, if that is the case, we can actually find another figure of merit called splitting ratio just take ratio, how much it is splitting. So, if you are getting suppose you have 2 milli watt here and here you are getting 1 milli watt then splitting ratio is 2 and another thing is that split or loss you have waveguide and you have wave splitter here some losses is there whatever things there.

So, I have a splitter but I am accessing here. So, here I will be obviously I will be seeing less power and that less power because you have a splitter here. So, because of the splitting here you will be getting low power here that is why you are calling that is a splitter loss splitter loss P o1 divided by P input that is so, if it is something no losses there and they are symmetric then here you will be 50% power you will be getting at 50% power you will be getting there.

In that case, you will be getting splitter loss that means about same that means 50% 50% So, if you compare with the input that will be half P o1 / P i that will be half exactly. So, in that case, log 2 10 P half so, you will be getting P splitter loss equal to 10 log of 10 half so, this actually approximately you will be getting 3 dB. So that is actually called splitter loss if it is the ideal one that is actually 3 dB that is why it is sometimes called as a 3 dB or splitter Y-Junction power splitter.

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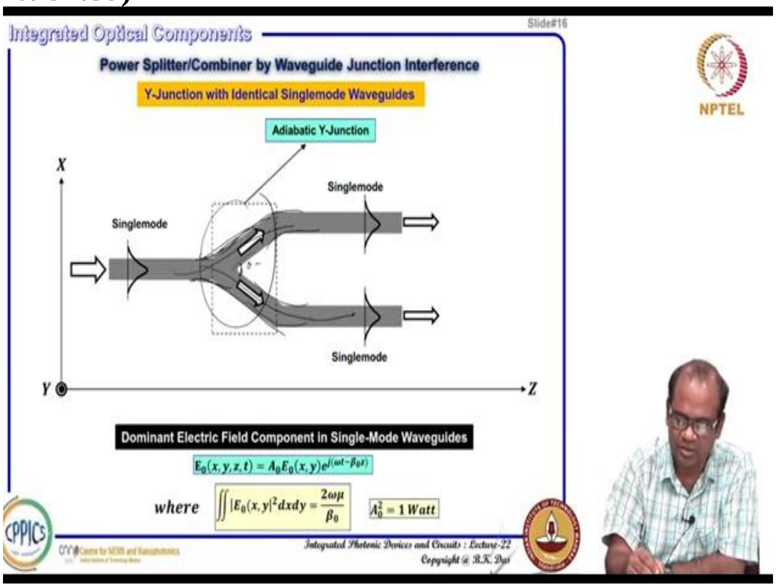


So, this thing now you can do some kind of junction instead of abrupt Y-Junction what you could do? You can have an adiabatic you can have a very small angle and this thing you can make little bit of smoother instead of random so adiabatic and angle is close to 1 degree, 2 degree or something like that if you make then it will be going like that and smoothly slowly it is bending is they are also adiabatic you are making here. Smoothly this changing that means it is like whatever we have discussed earlier that during taper waveguide.

So, dimensional change you cannot just interconnect 2 waveguides one smaller waveguide with another bigger waveguide with just directly because of abrupt junction you will see lot of losses will be there and all the modes will be excited in the multimode waveguide but if you do slowly you taper single mode waveguide into multimode waveguide then fundamental mode will be excited into the multimodal waveguide also as a fundamental mode.

Otherwise if it is abrupt all other modes will be excited that is sometimes may not be interesting to you may not be useful for you. So, that is why you can make it slowly tapering out and theta may be small so that losses here can be minimized. So, you can approximate you can design some of the device you can design the theta particular lithium niobate waveguide you can design the loss can be 0.001 dB or something like that. So, you can design that way assume that there is no loss here.

So, in that case power will be almost 50% going here and almost 50% coming here. So, if you can design such that this junction loss is minimum, then we can call it adiabatic actually practically you cannot get complete adiabatic that means no loss will be at all they are practically not possible. But you can actually reduce minimize the loss here. If you design properly, you can see almost all the power will be coupled to this one into the 2 output ports.

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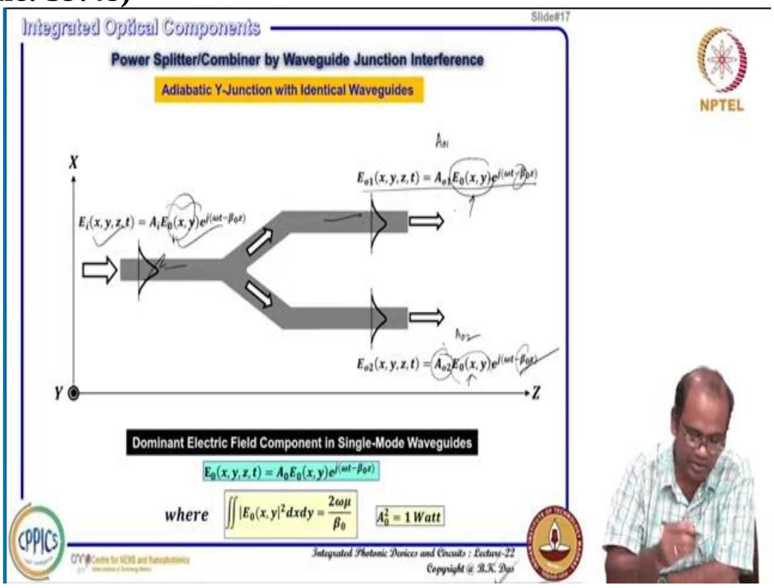


So again, come back to basics that input waveguide field distribution if you just represent like A i E naught x y this is a field distribution the x y dimension so that you can keep in mind always you should not forget that field distribution is there and here output also something amplitude will be A o1 output amplitude 1, output amplitude 2 output field distribution since this waveguide and this waveguide they are identical field distribution I have also used the same.

E naught x y, E naught x y field distribution same only coordinate a different that has to be introduced whenever you are doing some computation etcetera for other application. At this moment these are just you have to keep in mind that they are identical waveguide and same propagation constant here, same propagation constant I would this direction. Only thing is that here you will be getting A output 1 and A output 2 only difference here you do not need to bother about beta naught.

Because beta naught here and beta naught here as all our same identical waveguide, propagation constant same field distribution is also same. So, amplitude is the only difference you can make out here that is reason what we can do?

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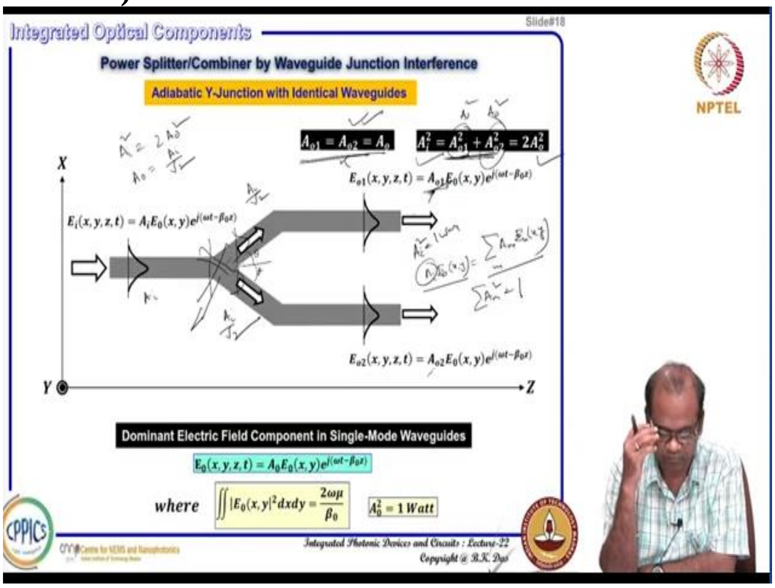


We can just simply say that that amplitude A naught 1 and A naught 2 = A naught so, this one if it is symmetry one also we can again another constant we can get actually that that will help you problem easier that amplitude A naught 1 A output 1 A output 2 this should be identical because waveguides are identical and they are symmetric this angle, this angle they are symmetric and they are identical so in that case this one will be this one.

So, that is what we have written and again we know what happens if it is adiabatic we know that the amplitude normally if I have E naught x y that should be A i that should be equal to A m E m x y that is what we have discussed earlier if it is waveguide coming into a part of region and in the multimode junctions or something like that, if there are many modes are there then we can actually say that superposition of all these modes should be equal to the input mode field distribution.

So, here you have to field distribution will be there one mode here one mode here so, superposition of these 2 modes will give you the input field distribution and in that case, we know that this A m square that should be equal to 1 watt A 1 A i square = 1 watt then all this together will be giving you 1 watt or you can write that if there are 2 modes are there that is why we have written A i square that is a power associated with the input field input mode and this is a power associated to the output 1 and this is the power associated with output 2.

So, these 2 if you add because there is no loss at all adiabatic lossless Y-Junction then in that case we can write A naught 1 square equal to plus this one and this is also A naught square this is also A naught square that is what we are a assumption symmetric waveguide then we can write A i square = 2 A naught square that is what it is written here. So, if this is the case then what I can write A naught should be equal to A i / root 2.

So, that means, amplitude division when it is happening in the Y-Junction if it is A i at the input here you are supposed to get A i / root 2, here you will be getting A i / root 2 that ends use your energy conservation assuming no loss is there this is possible to design last almost like a waveguide loss type of situation which will be created. So, you can assume that thing.

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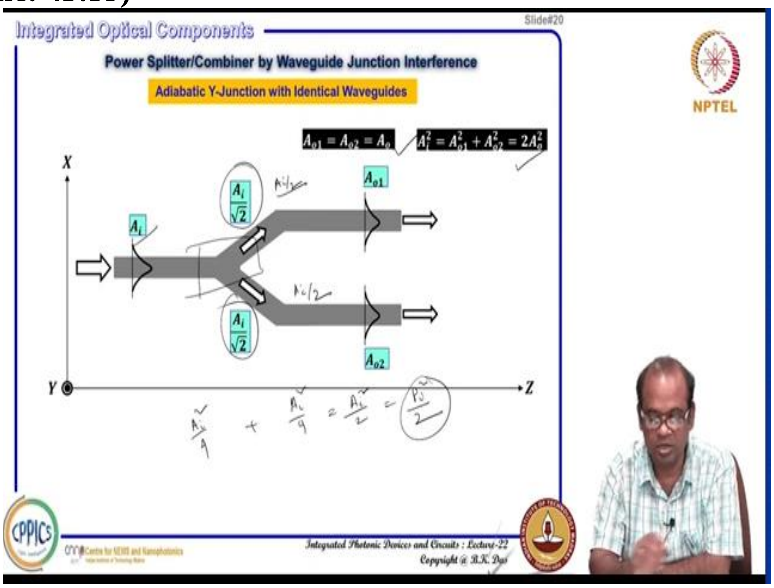


Now, you see I have now written A i you know A o1, A o2 that means output amplitude 1, output amplitude 2 that is what I forgot everything about x y dependent profile only thing is that I know that they are identical waveguide, since they are identical waveguide So, I am not also considering what is the beta involved also. So, anytime needed I know that beta value is known to me I can take that help for further calculation.

So, according to our concept this thing we derive, so, amplitude will be here, here will be the amplitude straightforward. So, you see it is do not actually mistake take that if it this is the amplitude will be A i / 2 sometimes you do mistake that way, because if you do A i / 2 just simply amplitude division is there you may end up with a problem with the fundamental law of physics.

Because you see what is the energy here it will be A i square / 4 power and here if it is A i 2 here also amplitude division, it will be A i square / 4 if you add them you will be getting a square / 2 you will be getting just P i square / 2 and you are considering this is lossless. So, if it is lossless and you are only getting half of the power launched at the input waveguide, so that is not correct. So, if you are considering this is a lossless model lossless Y-Junction, then amplitude when it is divided that should be transfer function will be 1 / root 2.

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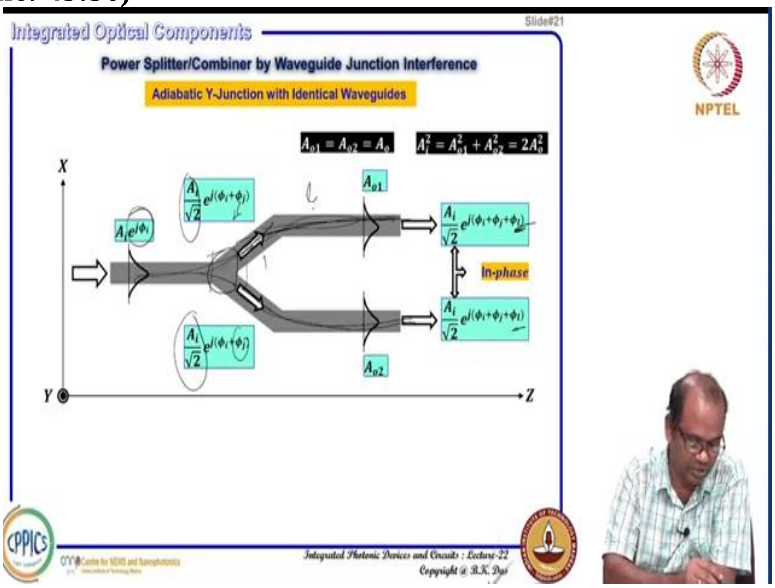


Now, if you consider that in the input, if you have a phase like this some phase initial phase is there phi i, so I can write this one. Now when it is coming after splitting I know that amplitude will be this one amplitude will be this one. Now, because of the splitting I can say that that particular region because of the junction you may get some kind of junction phase phi j and phi j since both the waveguides are identical this phi j and phi j will be same.

If they are not identical little bit different asymmetric or something like that this phi j, this phi j would have been different. So, I need 2 things amplitude and phase for integrated photonic circuit simulation or modelling. So, I have used amplitude and phase profile I am not considering as long as I understand they are identical waveguide and here this one. Now, as it propagates here it travelled some length.

Suppose overall length it is travelling that is why whatever phase here it will be acquired by this upper on that will be additional phase phi l and if it is symmetric again additional phase phi l. So, that means, whatever phase is there if everything identical symmetric everything is there. So, phase also if you just compare they are also identical. So that phase also for some understanding purpose we can also forget sometimes people say that if it is amplitude this one and amplitude here will be A i / root 2.

And then if you ask what about phase obviously phase will be there, but here in this case normally phase will be identical afterwards if you process them any interference or anything you see they are in phase basically. So that is why that phase is not so, important if it is this type of Y-Junction possibility you were considering.

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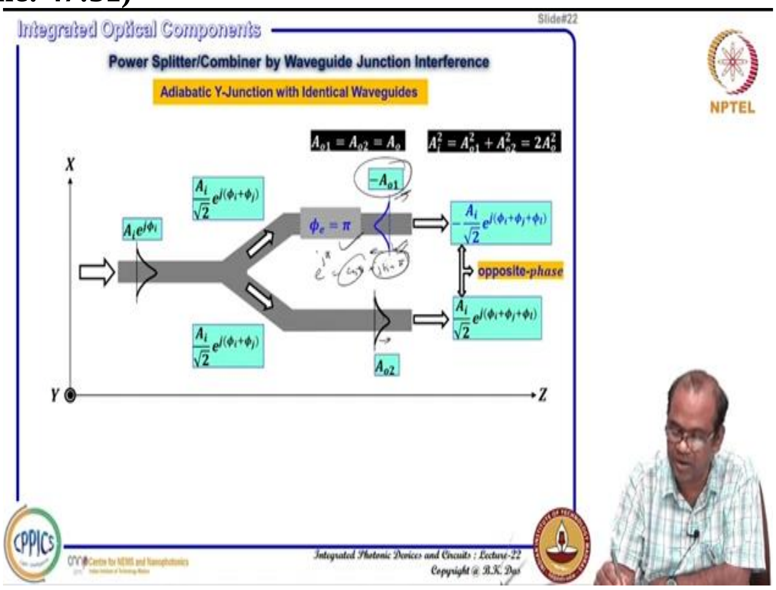


Now, if you are considering suppose, you have a one arm of pi phase shift is there, pi phase shift means this field is coming pi phase shift means I am adding additional pi phase shift to represent that I just reverted reverse the field you said field positive by field strength I am considering if it is positive this direction it will negative these direction because pi phase shift, it is just reversed it is shown amplitude to be negative here, pi phase shift means amplitude to be negative e to the j pi it will be a j phi means it is cos pi + j sin pi, sin pi = 0 cos pi = -1.

So that pi phase shift means it will add minus sign to your amplitude. So, in that case what will be there so in that case you will be getting whatever phi L will be there in addition to that minus sign I added here minus sign, minus sign and other things are same.

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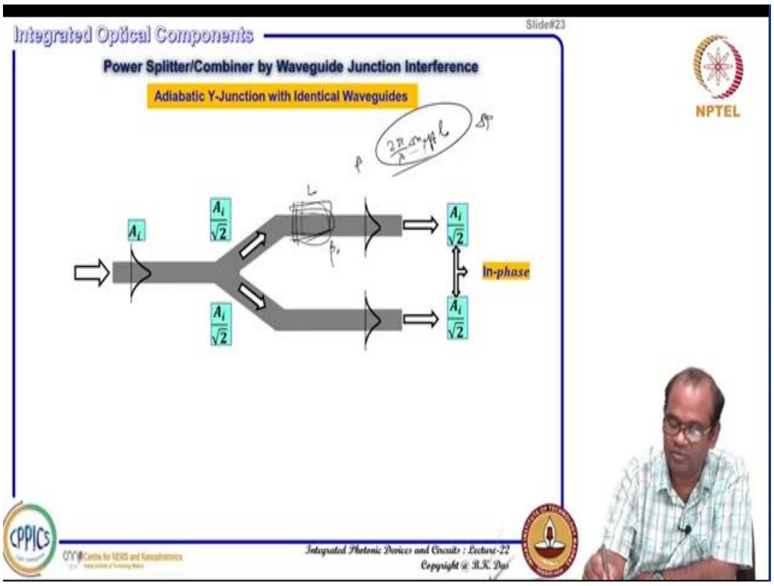


So, in that case also you can consider simply that in phase situation when there is no phase shift you are adding here, phase shift can add by external influence or you can little bit changed by waveguide with dimension anyway whatever way you can introduce some phase you can suppose it was propagation constant beta naught locally you just change the width a little bit so, that you get additional pi phase shift.

Because of the additional refractive index change you can get total phase will be 2 pi / lambda and say delta and n effective change for a length of L. So, this will be your additional delta phi. So, this delta n effective for that length, if this is the length some may be 1 micron 2 micron you can consider and delta n effective because of the width some additional effective index will be changed, if you multiply that will be if it is that is phi that I was discussing if no phase is there, just I have simply considered the adding phase.

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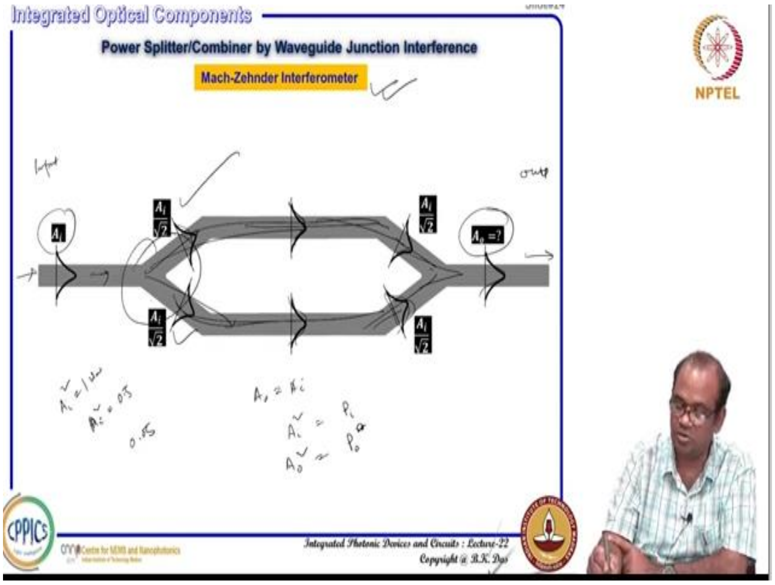


Now, we will be discussing that Y-Junction we have it is not complete yet Y-Junction to understand a little more about Y-Junction, I just take help up Mach-Zehnder Interferometers very good device important device as I mentioned again and again, it is used for many applications, including a high speed modulator lithium niobate modulator and silicon modulator we will use this type of structure Mach-Zehnder interferometer.

What it is, if you see from the top basically identical waveguide you have an input waveguide you have a Y-Junction splitting them and coming here and then another Y-Junction there to combine into another one waveguide. Now if you see I am launching here this is one by one Mach-Zehnder interferometer 1 input and this is input and 1 output this side this is your output. So, one by one Mach-Zehnder interferometer and if you were launching here light with amplitude A i, it may not be normalized.

Suppose it was normalized then A i square would be equal to 1 watt but if it was not normalized to suppose it can happen that A i square = 0.5 that means 500 milli watts launched and if it is this one then 50 milli watts is launched. So, depending on that A i is there that value is known to you for example A i. Now we consider Y-Junction is lossless so lossless junction means your amplitude root 2 and they will be propagating with same phase. So, to represent same phase I have a shown electric field, both cases in the same direction.

But you know, as they propagate they will keep on oscillating, but I want to see what is the relative changes happening or not overall whatever change is happening that is fine. So, relatively this mode is looking like this this mode will be looking like that, because they are travelled same path, at any instant if this mode is looking like this, this mode will be looking like this the propagate strength will be that particular incentive looking like that now, they are combined here.

Now, my question is what would be the not amplitude at this mode something will be there. I think obviously, if you just see they are lossless you can say that A naught will be exactly equal to A i is not it? Nothing will be because A i square is the P i and A naught square is the P naught. So, these are lossless, this is lossless, this is lossless, whatever power you are launching here everything will be appearing here. So, normally A naught if it is asked it should be A i could be at least it should be shown.

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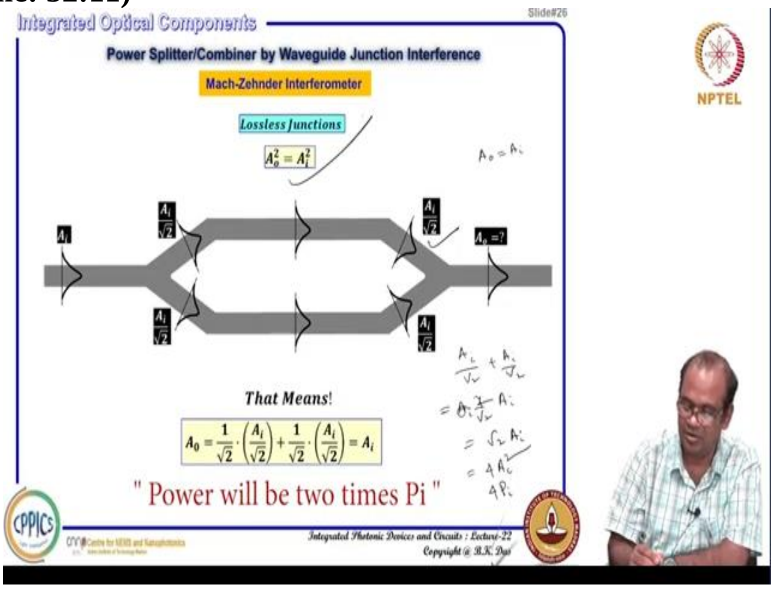


That is what A naught square = A i square that means P output should be equal to P input that is true. Now, you see what that mean? If A = A i, but here you are getting this one A i root 2 and A i root 2 if you just simply add them A i root 2, A i root 2 then you would get something like this 1 / root 2, 2 / root 2 A i that means, what you would get root 2 A i, root 2 A i means power will be 4 A i square, if you just simply add then power will be 4 times P i where from the power will come energy conservation fields.

So, if it has to be like this then what you can do that whatever this is coming into here that will be whatever we do again root 2 fraction. So, from here to here when it is going transfer function 1 / root 2 and here to here again coming that is also 1/ root 2 here to here is also coming 1 / root 2. So, 1 / root 2 here and 1 / root 2 times will be added there then if you add them, then this will be 1 / root 2 A i / root 2 that means A i / 2 + A i / 2 that should be equal to A i.

So, if it is a lossless case so, any junction from here to here, it is 1 / root 2 again from here to here also A i / root 2. So, reverse way also A i / root 2 that is actually thing happening this is also A i / root 2.

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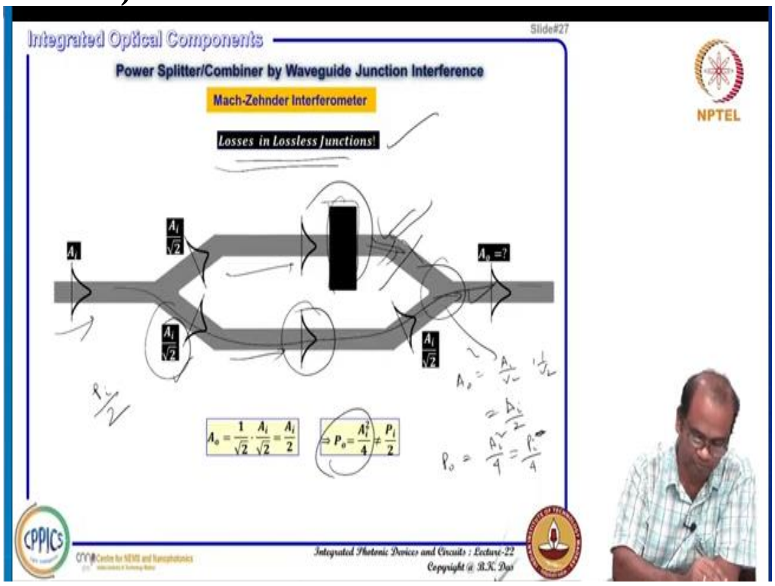


So, now you think more interesting thing Mach-Zehnder interferometer now, what happens your power launching amplitude soon A i / root 2 here A i / root 2 here it propagates identical path identical things and suddenly in the path for example, you block 1 that means I want to stop it will absorb, it will not reach it will not come out here then you have only this one then what is happening if you launch here coming here then after here when it is reaching what would be A naught.

Because nothing is coming from this side, A naught will be in order we have 1 / root 2 A i / root 2 A i / root 2 times because transfer function is again 1 / root 2, 1 / root 2, 1 / root 2 A i / 2 then what is the output power A i square / 4, A i square / 4 this is output power that means P i square / 4. In principle here what is the power here power was just P i / 2 but when it is coming back here power is P i / 4 what is the reason that means something has been lost here and you can we say that this is lossless Y-Junction.

But you have coming here and when it is adding here, you see power is off, so that means losses in lossless Y-Junction that is what I mean you have losses in lossless function. So, something happening something appearing that when some power is 50% power is lost here somehow. So, this is a paradox you see if you just remove this one, then you would get everything here. And when you have you are stopping here you are not getting all the power in the output whatever power you are supposed to get 50% is loss.

To get that 50% last one you need help of this power this field that is the beauty that is the beauty of integrated optics we are actually interferences happening in the waveguide I will just explain that quickly.

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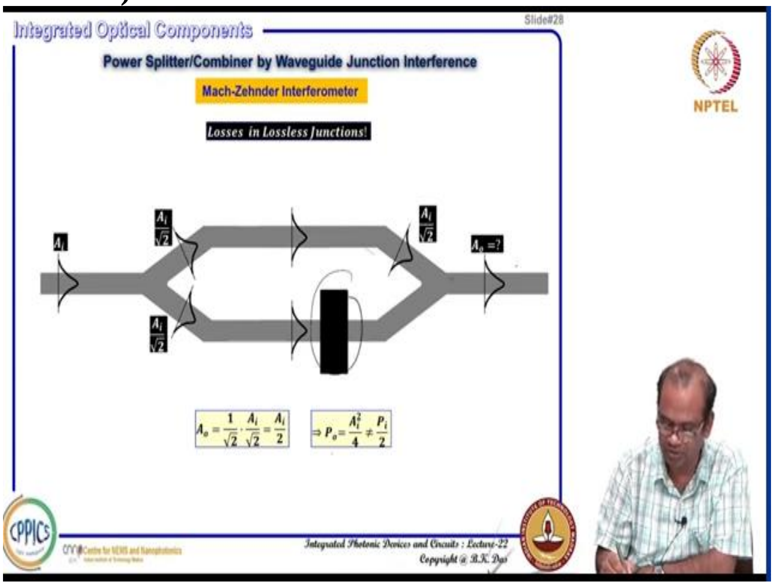


Since centering it happens, so, whenever it is blocked here this is coming these A naught also it will be like this not equal to P i / 2 say same situation what the unity same situations as soon.

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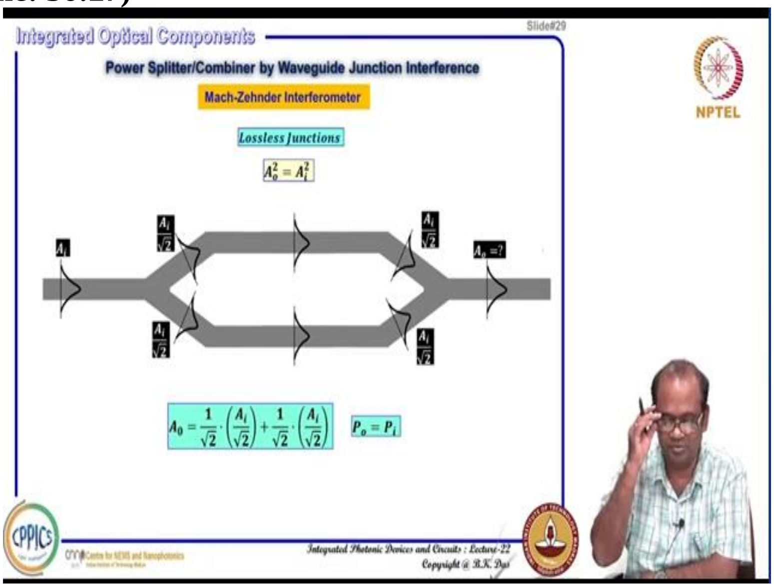

Now let us see I repeat again that nothing is blocked with the arcs so, in principle when it is coming. So, all the energy conservation point of view I do not have any paradox everything fine.

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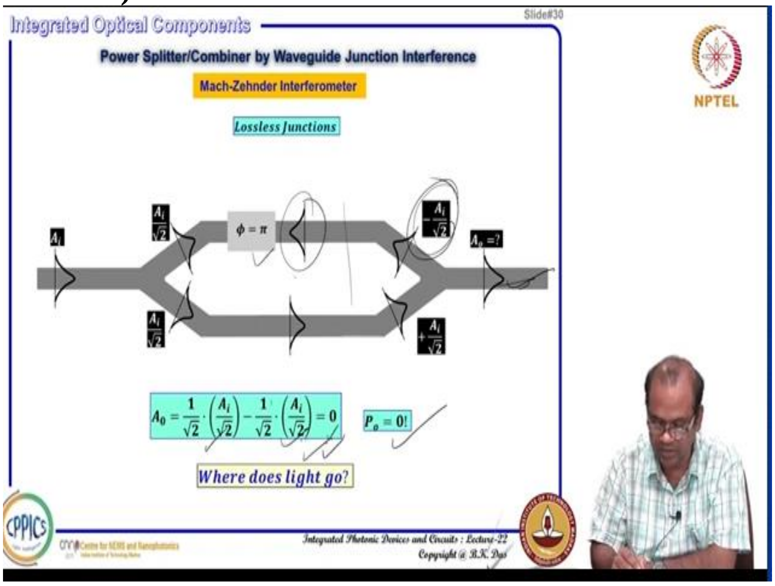


Now, you see suppose you are giving pi phase shift here one arm we were introducing pi phase shift, I discussed that earlier when another junction was not there, that time pi phase shift is there with respect to this one I have a soon the field is flipped here field distribution and when it is propagating everywhere relatively pi see that is flipped and this is coming like that. Now, this flip means I can just say that phase shift means minus sign will be introduced.

So, again you see you just add them together it simply if you add that is also 0 even 1 / root 2 if it is coming here 1 / root 2 A i / root 2 coming from there and 1 / root 2 A i / root 2 coming from here, if you add them they are 0 that means power will be 0. So, there are also happening. So, if you block there also something happening or pi phase shift we are giving that also happening that power is completely missing here A i will be 0 P naught will be 0 here.

So, beauties that if you block one place, you are getting 50% power lost whatever coming from one arm 50% of that is lost in the Y-Junction and whenever you are introducing a phase phi 1 of the arm then everything is lost no power is reaching here. P naught according to this principle, which is lost, where does like go then question. So, that is important interesting.

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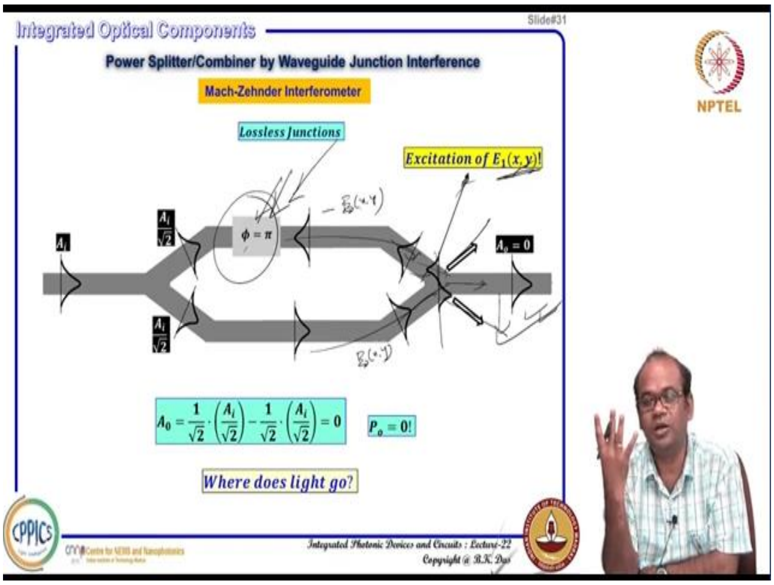


If you see whenever pi phase shift comes when it is coming, if you just merge them together slowly this mode and this mode will be it is adiabatic, and they will try to come slowly and in this region if you say the waveguide which is a little bit more. So, in this region it is happened that the higher order mode beta = beta 1 that may be existing and that solution is existing.

So, this mode will be repeating to the first order mode also this mode also will be repeating to the first order mode, because they are phase matched here it will be few just mach specially it will be looking like a E 1 x y it was coming as a E naught x y from this side and here also E naught x y this side, but this E naught x y is minus specified it is there. So, when their distribution is there, it will be looking like a hierarchy it will be exciting to the first order mode.

So, when first order mode is excited here. So, pi phase shift is there, pi phase shift is there so fundamental mode is now converted together converted into first excitation mode. But that mode again you have a single mode waveguide that cannot be supported because your hierarchy mode and we supported this single mode waveguide, so that will be lost. So, that is the reason why I phase shift is there then power will be lost. So, this is how one can actually if you actively tune this phase phi 0 phi 0 phi 0 then here light will be on off on off.

So, you can make a switch you can make a modulator digital optical signal you can get highlight lowlight highlight lowlight, you can just if you just somehow externally electronically you can control this phase that means you are getting a electro optic module if this is a data coming that according to the data, you can just modulate your phase and you can get your optical data input output.

So, this is the principle of electro optic modulator, electro optic modulator is used we have discussed again and again for electrical to optical data conversion, electrical data will be converted into optical data. So, E to O conversion takes place so that is how it is used. So, for that purpose we need to know we need to find out the exact transfer function of this structure.

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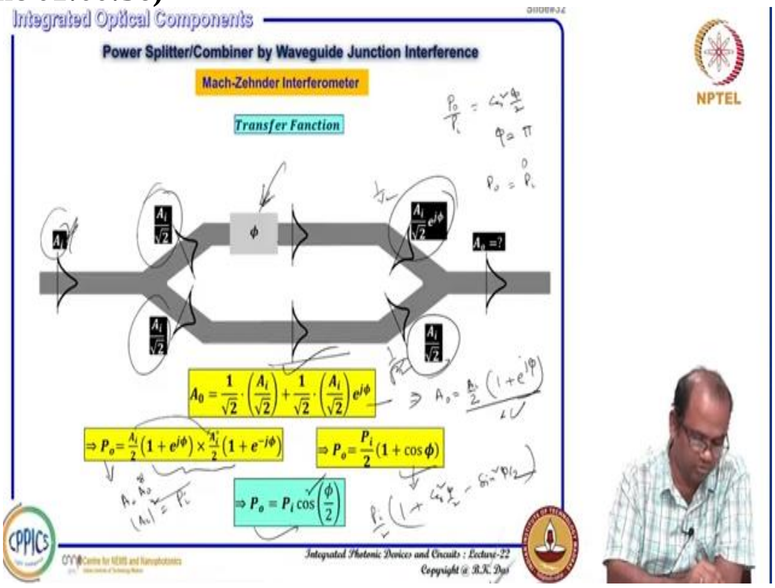


Let us try to see what is the transfer function let us say that this phase is phi not pi just arbitrary phi then what happens? Amplitude comes here root 2 phi then e to the power j phi amplitude comes here, no phase change no phase change coming like that. Then what will be the A naught? A naught will be have to multiply 1 / root 2 here, 1 / root 2 here and then you have to add them that is what we have done. If you add them together then what we are getting this one now, this is nothing but A naught = A i / 2 1 e to the power j phi that is actually A naught.

Here amplitude will be this one that phi dependent amplitude will be there. Now, if I want to see power what is the power here that means, I have to check P naught P output will be that means A output and A output star complex conjugate this a complex now, this complex conjugate if you multiply then we are getting this one A i / 2 1 + e to the power j P hi A i star that complex conjugate A i can be complex also maybe they are these are initial phases there we can represent that in terms of phase etcetera complex form.

So, A i star writing by 2 and 1 + e to the power –j phi that is the star if you simplify that one just multiply then you will be getting P naught will be equal to A i star that will be there that can be written as P i input power. So, input power phi, so that means as a function of phi you can actually output power you can modulate. So, if this is some kind of phase modulation is coming so, output power you can also intensity phase modulation to intensity modulation is conversion taking place.

So, if you do this one this will be square because this can be written as cos square phi / 2 - sin square phi / 2 and then 1 is there cos phi can be written like this P i / 2 then 1 - sin square phi equal to 2 cos square phi / 2 2 cos square phi / 2 these 2 will be cancelled P i cos square phi by 2. So, that means just simply to relate transfer function so, that means if I try to find out P naught / P i that is the power transfer function is nothing but cos square phi / 2. So, if phi = pi it is phi / 2 cos phi / 2 0.

So, P output will be 0 if phi = 0 then P naught will be equal to P i and phi equal to intermediate any value it will be changing like a cosine function. So, it is a lossless Y-Junction cascaded oppositely then you can get Mach-Zehnder interferometer that can be converted into modulator so, this is very important a device architecture.

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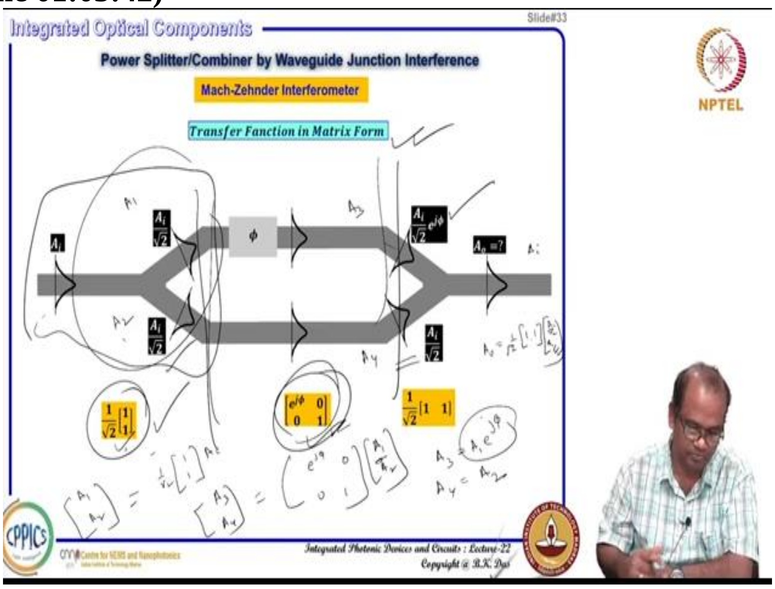


So, whenever you would sometimes for large scale integration you may be interested to know that represent the transfer function in a matrix form. You can just represent the transfer function in this Y-Junction what is the transfer function you can write this type of transfer function if you see that 1 over root 2 1 1 and your input is this one and this one maybe your A 1 and A 2 for example.

So, I can write A 1 A 2 I can write like this matrix form, A 1 will be 1 / root 2 A i A 2 will be 1 / root 2 A i. So, this A 2 will be there so, this can be a transfer matrix for this Y-Junction. So, it is just column matrix 1 / root 2 will be there and if you have a phi then we can write this type of transfer matrix how it is coming e the power j phi 0 0 1 suppose this is your coming as a A 1 A 2 A 3 A 4 if you want to get A 3 A 4 you want to get and then A 1 A 2, so whatever A 1 we are getting A 3 will be A 1 e to the power j phi.

If we just do that and A 4 will be A 2, whatever A 2 is there that will be your A 4 and whatever A 1 is there that will be your A 3 and e to the power j phi, we have written e to the power j phi. Now, that means this matrix you can use as a transfer matrix from these to these. So, first Y-Junction one transfer matrix and then second one because of this phase shift you can represent a phase shift transfer function is this one.

And then another Y-Junction instead of row matrix it will be just column matrix because you are adding these 2. So that means you can get A naught should be equal to 1 / root 2 1 1 and you can write A i / root 2 A i / root 2 so that is what A naught you will be getting that time A i getting. So, you can actually represent Mach-Zehnder interferometer in terms of matrix also. So, this matrix ultimately whenever you will be using large scale.

For example, quantum photonic circuits, as well as you want to have some kind of programmable photonic integrated circuits, that time this transfer matrix will be very important will be using and so, I considered here in this lecture and I can conclude for today's lecture and following lecture, we can discuss about the other important components. So, this is the first thing Y-Junction Mach-Zehnder interferometer we have discussed.

And next thing will be trying to discuss about the coupled equation to coupled waveguide can also act as a power splitter and you can use also to coupled waveguides into 2 by 2 matrix form 2 by 2 device. And that can be also another type of Mach-Zehnder interfered with I would be discussing that in the following lecture. Thank you very much.