Integrated Photonic Devices and Circuits
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Lecture - 24
Integrated Optical Components: Directional Coupler: Coupled Waveguide Contd...

Hello everyone, today's lecture we will continue integrated optical components of course and we have discussed directional coupler in the last lecture coupled waveguide basically we constructed differential equation 2 coupled equations between guided modes in either waveguides now from starting from that coupled differential equation will try to solve the power coupling between the 2 coupled waveguides.

And finally I will discuss about how to formulate a transfer matrix because this transfer matrix is the one, one can use for circuit simulations because directional coupler is an important component for photonic integrated circuit it is used for many you know circuit demonstrations. So, it is an important circuit element so for circuit simulation this transfer matrix is a very important formalism and we will be discussing that.

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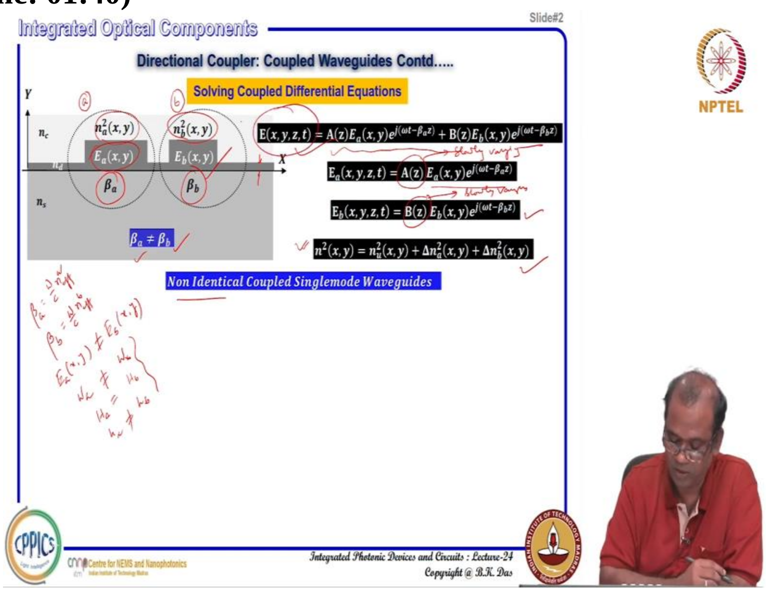

So, to solve coupled differential equation we just recap a bit how we got the couple differential equation we consider 2 waveguides they are parallel waveguides, waveguide a and waveguide b and this is a representative waveguide structure in silicon on insulator and n a square x y is the refractive index distribution to define waveguide a n square b x y that is corresponding to waveguide b.

And individually if you solve the mode field distribution dominant electrical distribution is E x, y for waveguide a and E b x, y for waveguide b and the Eigen value propagation constant for waveguide a is beta a and waveguide b is beta b and we should keep in mind that this beta a and beta b we are calculating for same frequency that is beta a = omega by c n effective for waveguide a and beta b = omega by c n effective b.

So, if we do not consider that this waveguides are identical that means non identical waveguide then obviously beta a not equal to beta b obviously E a x, y will not be equivalent to E b x, y the distribution may be different. And that that can be due to waveguide dimensional variation one is W a not equal to W b waveguide width obviously device layer thickness it is considered same H a, H b.

But the what you call that this is your slab height, slab height you can vary also this side waveguide a you can have a slab waveguide a this can be not equal or can be identical one of them can be not equal and in that case beta a not equal to beta b and we know that when they are coupled we cannot really consider that amplitude of electric field distribution that is propagating through the waveguide a will not be constant.

That will have some kind of slowly varying amplitude Az in waveguide A slowly varying amplitude we will be considering that the presence of waveguide B is weakly influencing the guided mode what is there in the waveguide A and this is vice versa this is the waveguide mode B which is slightly or weakly influenced by presence of waveguide A and that amplitude will be slowly varying as your function Z slowly varying.

So, this is actually your total field we can say for the system of coupled waveguide which is defined by this type of refractive index profile we have discussed in the previous lecture and if this is the refractive index profile we can say that they are superposition of slowly varying field amplitudes in waveguide A and waveguide B this is the 2 thing this is your total electric field. This is what we can just we have considered to construct the coupled differential equation which is nothing but this one and this one.

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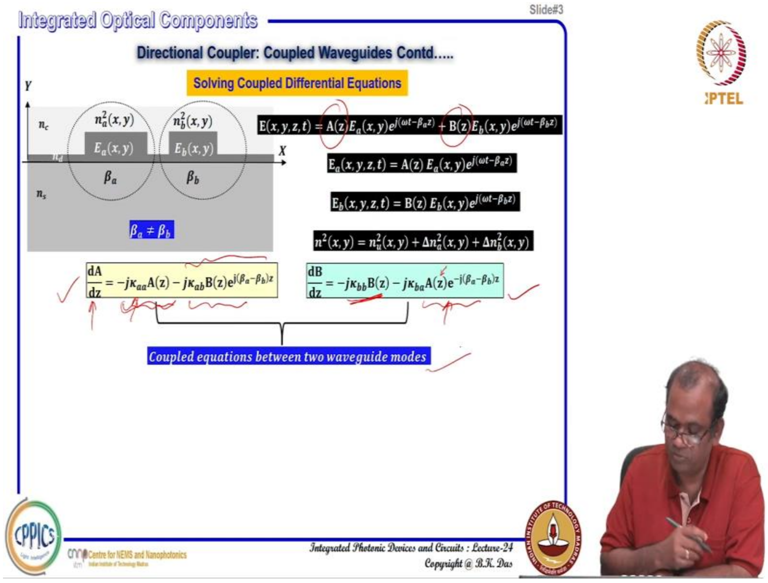

So, basically this A and B how they are coupled that actually will define the power or amplitude component or presence in waveguide A and waveguide B. So, this is something the first term and this is second term the first term you will note we have kappa aa and it is Az that means evolution of mode A in waveguide a amplitude A because of some kind of coupling coefficient kappa aa in that waveguide itself.

So that is Az because something contribution or can be considered as a somehow this term related to its own perturbation in waveguide a similarly this one here also you see this one but this is the term actually coupled term that is actual this term if this is non 0 that means something from waveguide b is even contributed to waveguide a. Similarly here if you see whatever it is there same way we can consider waveguide b amplitude will be involved.

Because of the presence of waveguide A amplitude Az so this is basically second term is the coupling terms that energy can transfer from one wave guide to another waveguide. So, this is the coupled equation between 2 waveguide modes we have derived earlier in the previous lecture we just put down here to follow up.

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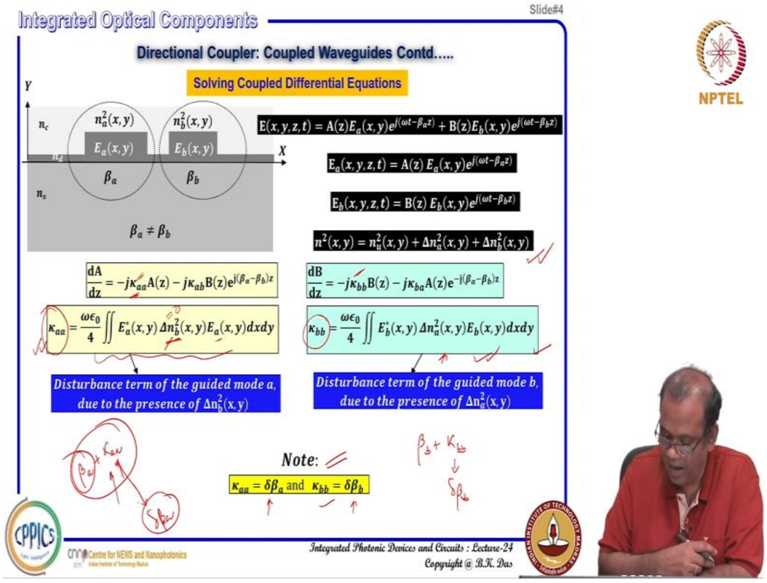

Now if you see this kappa aa that is what I said that in our coupled mode equation when we derived we found that, that expression is looking like this where this is like a coupling strength though but you see E a star E a star and delta n b square presence in the integration that means we say that disturbance term of the guided mode in a waveguide a due to the presence of delta n b squared they be due to the presence of waveguide b.

So, if this delta n b square if equal to 0 then kappa aa would have been equal to 0 that means something contributing the presence of because evolution tail is actually from waveguide a it is penetrating to waveguide b that is why it is delta n b square the presence of second waveguide will disturb the waveguide mode in waveguide a same way for kappa bb this term and this term is defined like that.

We assume that these 2 terms once we know the field distribution in waveguide b and waveguide a individually they are simulated using your full vectorial solutions or whatever mode solver you use and once you solve that and once you know delta n b square and delta n a square the vicinity where the waveguide a and waveguide b they are mutually interacting then you will be able to compute this too.

So, we can assume that this kappa aa kappa bb can be computed once you know the waveguide profile that when so once you know this refractive index profile. So, now note this is the important thing kappa aa = delta beta a, kappa bb = delta beta b because you see this is the term actually disturbed waveguide mode because of the presence of the waveguide b. So, how it is disturbed it can be shown that.

That is actually the value actually is nothing but your propagation constant or beta a will be modified as kappa aa that means if you just create that how the phase of the filed phase of the field in waveguide a how that will be changing as a function of length that is actually propagation constant that you can actually consider like that if there was no waveguide second waveguide then propagation constant was beta a.

But presence of second waveguide this kappa aa will be adding same is true for here. So, beta b should be added with kappa bb will show very quickly how that is feasible very clearly. So, we can say that this kappa bb = delta beta b that is what I have written here and kappa aa = delta beta a that means propagation constant how much it is being added because of the perturbation due to the second waveguide.

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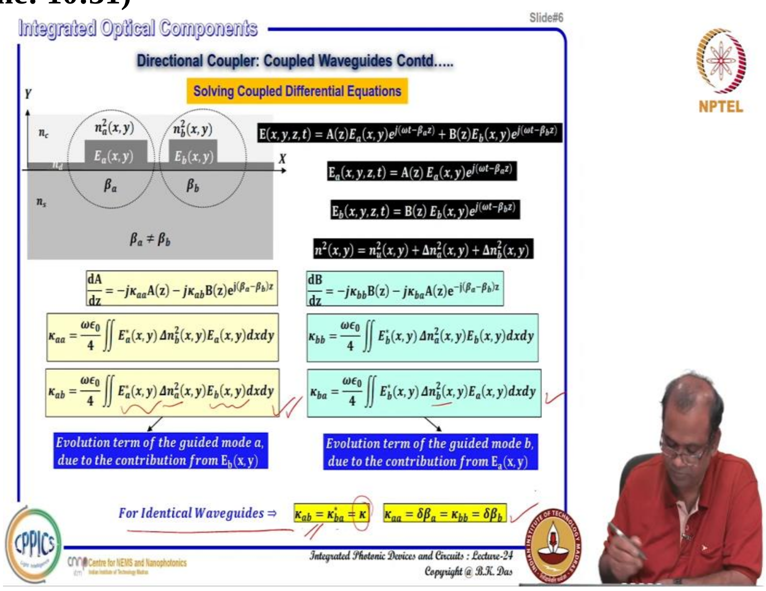

Now another coupling strength that is actually the term contribution coming from the second waveguide how much energy or amplitude even coupled contributing to waveguide a this is the kappa a, b that is actually defined if you see kappa a, b here E a star E b delta n a square we call this one something like that evolution term of the guided mode a due to the contribution from E b x, y.

So, because of this field E b x y that one how much energy is being tunneled to mode in the propagating mode in the waveguide a and vice versa is kappa b, a you can see their term just a, b will be interchanged a will become b here they a will become b here this b will become a here. So, this is how we can say that how energy mutually coupled energy so these contribute to d A d Z this part kappa ab and kappa ba contributes to dB dZ.

So, once kappa ab present obviously kappa ba will be present also we will just discuss that normally for identical waveguides if waveguides are identical then this thing E b E a star they will be the field distribution will be identical and delta in a square and delta in b square will be identical only thing is that you have to see E a star E b star that is change only. So, normally kappa ab will be equal to kappa ba star that can be considered as a kappa.

So, for identical waveguide this condition if you see this compare these 2 thing if waveguides are identical so mutually coupling strength from waveguide a to waveguide b and waveguide b to waveguide a kappa ab and kappa ba star they are just complex conjugate. So, one coupling this side takes complex conjugate and other side just if you know one way or the other way if that is coming as a complex the other way will be just complex conjugate that is it and this one we have discussed here.

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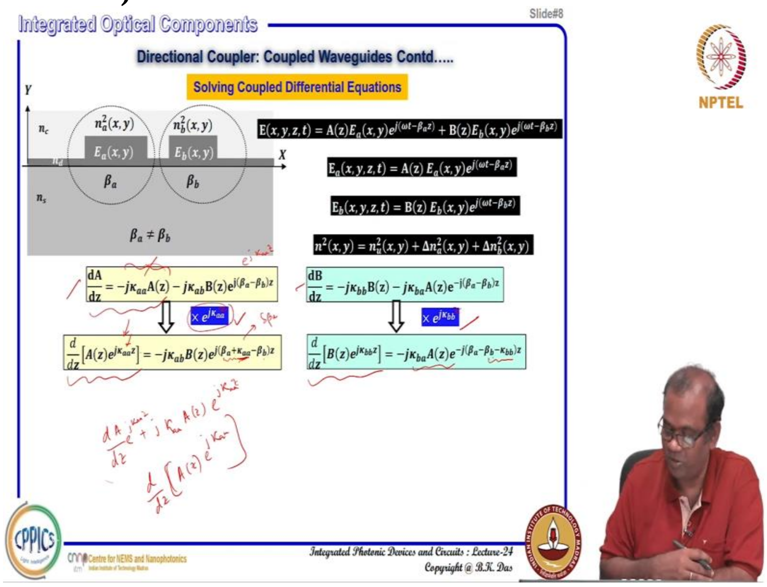

So, now start from here coupled equation keep in mind that kappa aa, kappa ab, kappa bb, kappa ba they are known they can be calculated using your normal mode solver or you can use any other method to know that in this case it is known now I would like to solve how is Az and Bz involving? If I know an analytical equation for Az and Bz then that is how we can call that the coupled differential equation is solved.

So, towards that end what do we do? This is just we take some trick we just both sides we multiply by e to the power j kappa aa we will come to know it will be more and more clearer why we are doing so and this side we are multiplying both sides by e to the power j kappa bb. So, this is some trick to solve this differential equation then what I do here if you see these 2 terms can be written like this.

How you see if I take this one they first term to left hand side then it will be like dA by dZ + j kappa aa Az. And then you multiply k kappa aa here also e to the power j kappa aa so this you can get in a compact form d dz Az e to the power j kappa aa. So, after multiplication both sides the first term if you take left hand side that can be written in compact form to z dependent function. If you just make a derivative first one it will remain the by dA by d e to the power j kappa Az.

And here also here it should be z also if you are multiplying e to the power j Az that is a typo error. So, if you just multiply here and is correctly multiplied here then you are getting like this and right hand side you also multiply e to the power j kappa aa Az. So, if you multiply that one that is added here so you had a beta a - beta b that beta a beta b is there in between I just put kappa a I clubbed kappa a with beta a because you know this kappa aa it will appear like a how much propagation constant is modulated in waveguide a.

Similarly here kappa bb z I am multiplying so I wrote same thing here in this form and right hand side same kappa ba Az it and it will be minus j beta and then kappa bb I just put I have just clubbed with beta b because beta b + kappa b v that means modified propagation constant because of the presence of your other waveguide so for good.

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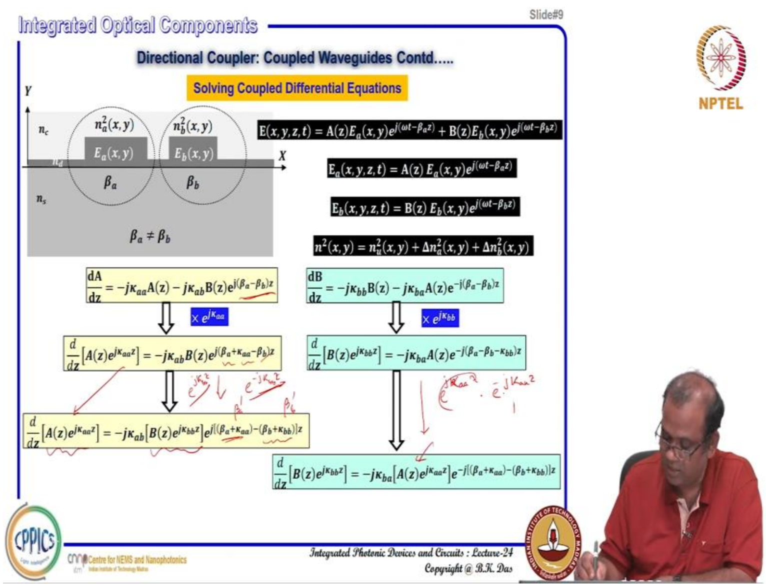

Now I just to do a little bit simplification so these one simplifying what I do left hand side I kept as it is right hand side what I did I just multiplied e to the power j kappa bb z and here I multiplied e to the power - j kappa bb z if you multiply these and these it will be basically one together. So, this one I multiplied with the b z I kept clubbed here just to make some kind of Az along with that e to the power j kappa aa here b z e to the power kappa bb I have kept.

And these one if you multiply here then you will have beta a kappa a and one more kappa bb will come that can be clubbed b beta b kappa bb. So that means instead of these one I now have modified beta a + kappa aa beta b + kappa bb. So, this can be written as beta a prime and this can be written as beta b prime that is modified propagation constant because of the mutual interaction between the waveguides.

So, same way I have also simplified here what we did here we have just multiplied e to the power j kappa aa and e to the power - j kappa aa z if you multiply these 2 meaningless one as if you are multiplying one but this one I club with Az and this one I club with Bz this simplification help us to consider further.

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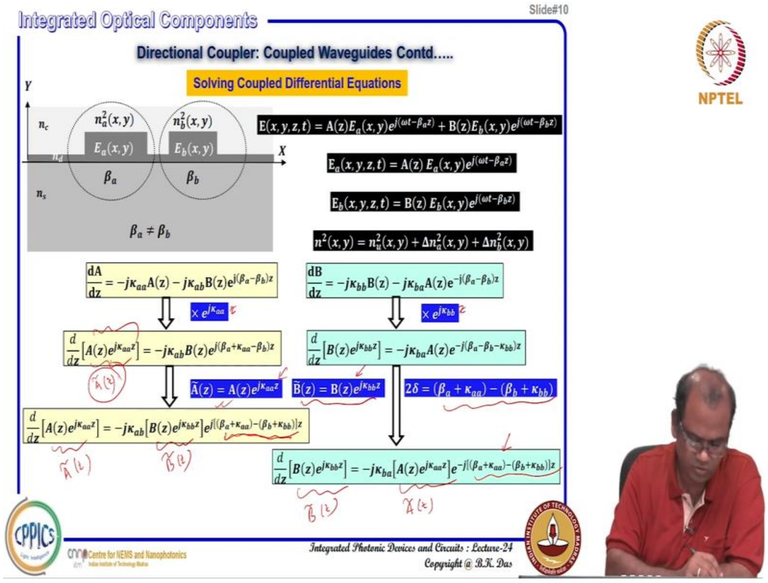

They should not forget that e to the power kappa bb z multiplied there. So, what do we get here? I use this one as the A tilde z together this is Az function only thing is that as your function z you have additional phase factor is there. So, I just club them together with a single z dependent modulation amplitude along with some kind of z dependent amplitude I just added here I just consider here and similarly for b I will be writing this one as B tilde as like this.

And one more thing I will be considering these longitudinal phase factor whatever we are considering that is defined by 2 delta so I will be using this one as a 2 delta here instead of this 0 delta instead of that 0 delta and instead of that I will be putting A tilde Z and instead of that I will be putting B tilde Z till here I will be putting but I should keep in mind how we have defined here I could put in A tilde Z and B tilde Z that is how we will be writing. So, it is straightforward simple just you have to use some kind of trick to get a suitable analytical solution.

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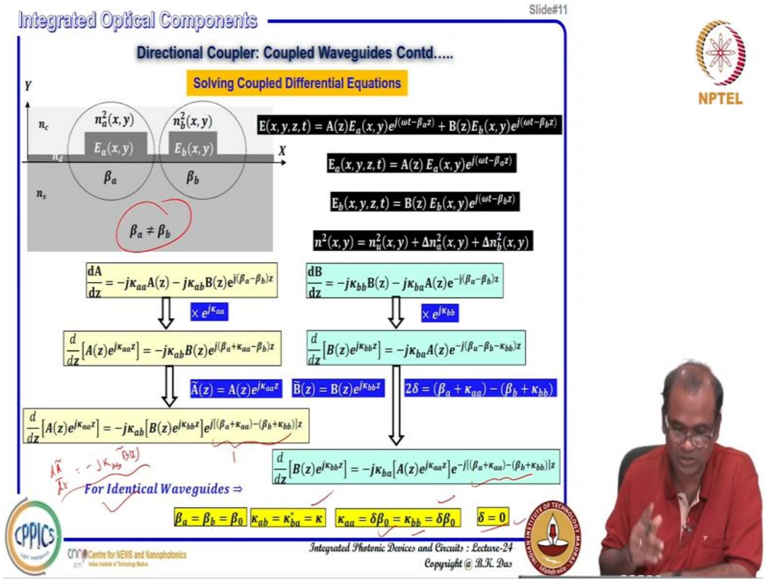

Now what do we do just once again I try to remind you if they are identical waveguide that means beta a = beta b = beta 0 kappa ab will be equal to this one kappa b star this one and delta beta 0 kappa a kappa bb delta beta 0 and delta will be equal to 0 that means I can directly put this one equal to 1 because the exponential 0 is 1 here also it will be 1. So, our solution will be very simple straightforward I can write dA by dZ = - j kappa ab B titled z.

So that will be our simple exponential equation sinusoidal solutions you will be getting how it will be coming but normally we want to consider dissimilar waveguide non identical waveguide so we will continue but anytime if you want to see what is the solution for the identical waveguide you have to just consider these things that is it that is the conceptual things.

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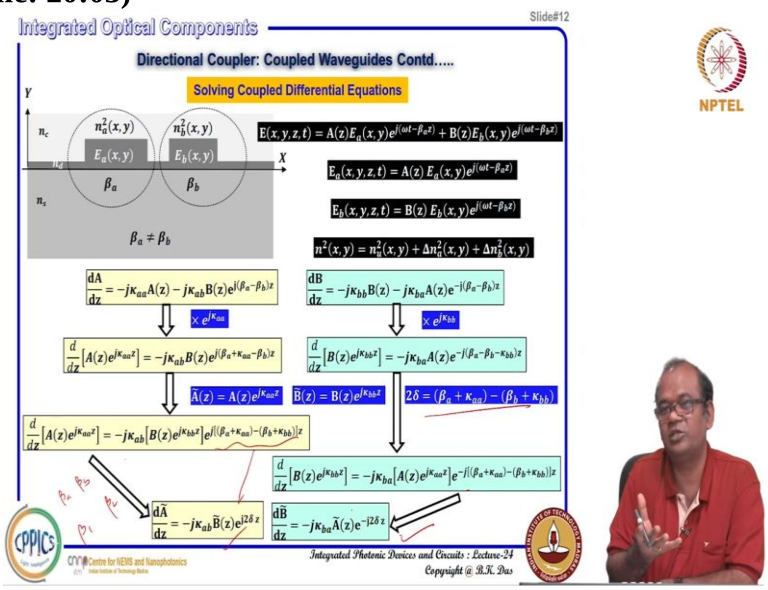

So now we go we simplify this one this way this one is delta we have written j 2 delta z this is actually 2 delta they have consider phase factor here put just simplification and this one simplified like this. So, it is just identical equation this type of identical coupled equation we have also seen earlier when co propagating modes are coupled in the same waveguide we have discussed earlier.

So, but there also we have just solved up to this point differential equation we are done but we have not solved this differential equation we derived this differential equation but we have not solved this equation. But now we want to solve this equation the solution method here whatever we are using that can be considered same for coupled mode equation when modes are guided in a single waveguide only.

Thing is that here we have a little bit modification of propagation constant like this and there this 2 waveguide mode instead of beta a and beta b I would be considering beta 1 and beta 2 both are in the same waveguide how they are coupled in some perturbation introduced and if they are coupling the same type of differential equation we will be also creating. So, it is just so far we have shown that. That this coupled differential equation is in between 2 waveguide modes is almost identical with the coupled difference equation between 2 modes in a single perturbed waveguide.

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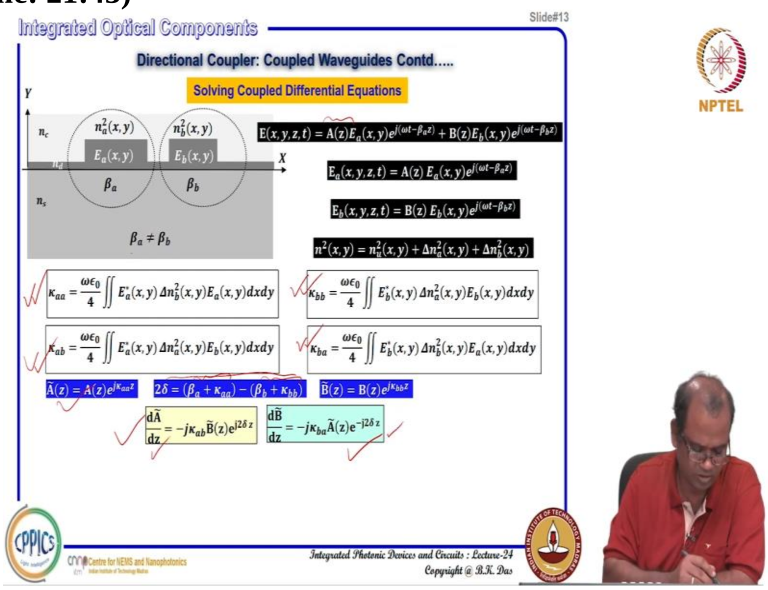

So, now we will be moving for solutions I have just summarized whatever so far important thing we need. We need this 2 thing for solutions and we have to always keep in mind what is kappa aa what is kappa ab what is kappa bb and what is kappa ba and what are the parameter we have used this is we can consider some kind of page mismatch this is the part of waveguide propagation constant for waveguide a part of waveguide.

Because of the presence of second waveguide what was the propagation constant and since beta a 0 = beta b kappa aa, kappa bb whatever it contributes that also may not be they are equal but after perturbation it can happen that 2 delta = 0 that can be considered if you can design that is fine and we have considered A tilde z that is instead of Az we have added one more kappa Az in that equation that as a result you are getting this one and this one. So, straightforward whatever we have discussed we have written. Now again for identical waveguide you just put delta = 0 just to remind you.

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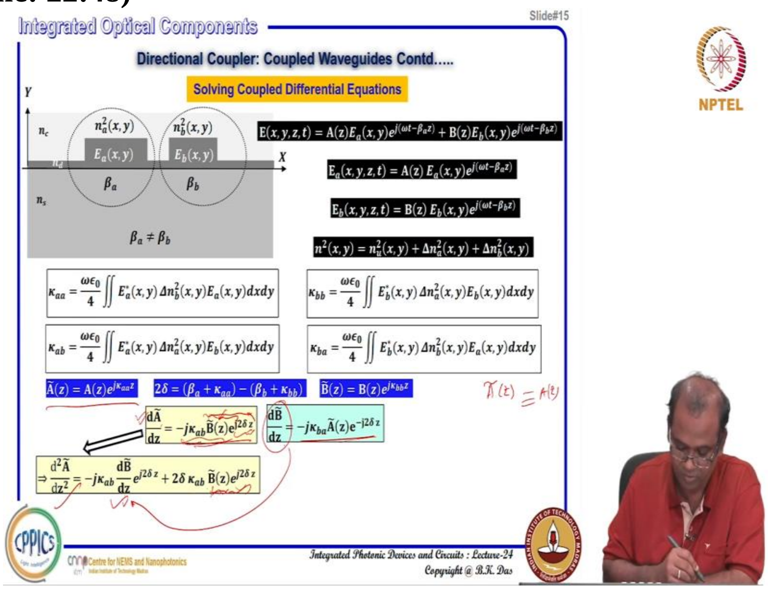

If you see now we will just move on we want to solve here A tilde z or Az just if you solve this one that means you will be directly getting Az also because you know the relationship. So, Az and this one how they are related you know so what do we do? We do second derivative of this one both side you do derivative with respect to z 1 more; then first term will be this one and second term you have this is also z dependent function this is also z dependent function.

So, we can define like that minus j kappa b, kappa ab, dB dz and this one I kept then I kept up to this one and this one I will be making derivative. So, I will be getting j j - 1 - 1 so plus so it is the real part 2 delta kappa ab of course kappa ab complex that can be considered beta z 2 can be complex this is also complex so all the j is not there j, j will be canceled so you will be getting second term.

Now in this equation you see still it is coupled you are getting second derivative of a but you have first a derivative of b and b is there. So, what you can do I can take this one equal to this one right hand side only Az I can put here and b I can b to the power this one this value I can get from here and I can put down here then what I will be getting this equation will be completely independent of b it will be only A tilde or dependent so that is what I have written.

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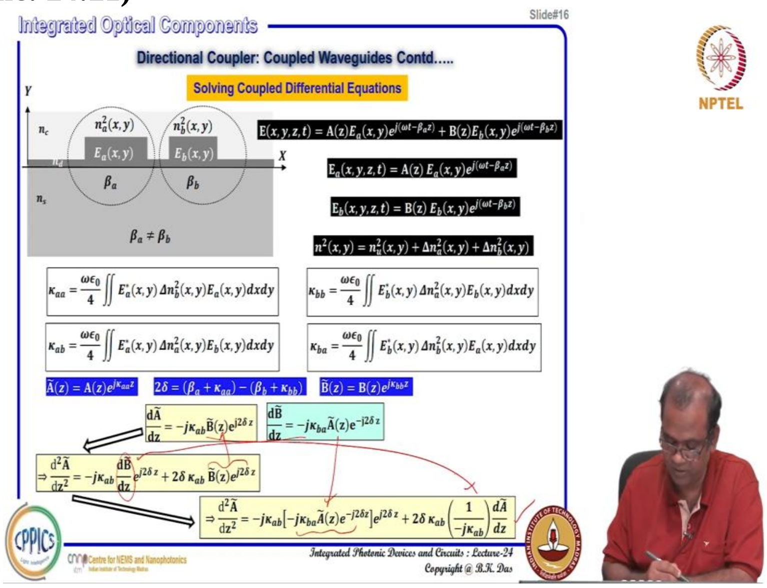

So, this one I have taken from here and this one I have taken from here then I get this thing so dB instead of dB dz I have just put this one directly and 2 delta kappa ab and b Z this one is over - j kappa ab dB tilde as dZ. So, this is straightforward we just put here mathematically so it is a very simple.

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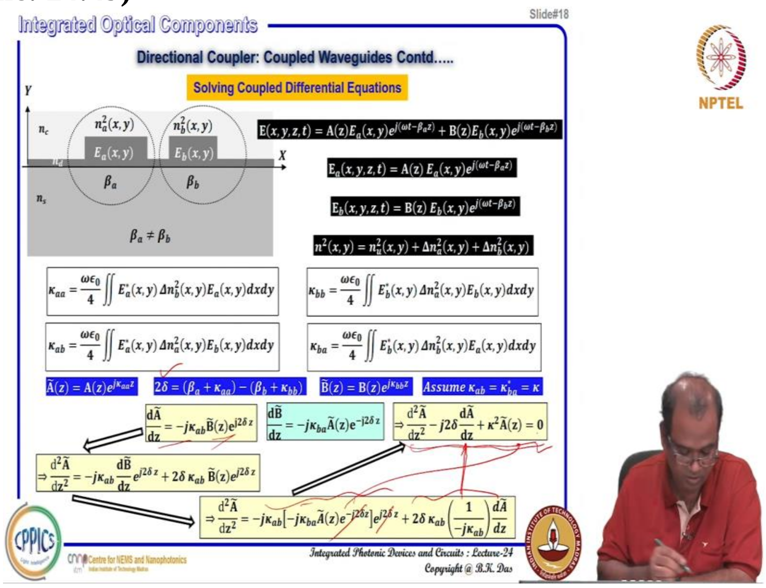

Now again to proceed further I think it would be better because you have kappa ab, kappa ab that will be cancelled of course here but here kappa ab and kappa ba 2 terms are there if they are different you can keep on using that for solution but for to get a complete analytical solutions most of the time also practically you will see kappa ab will be equal to kappa star ba = kappa.

If you just consider this concept kappa ab is equal to kappa b star kappa ba star = kappa and in substitute they are the minus minus plus j, j minus and then kappa square it will be so this will be kappa this will be kappa star so kappa square and minus minus plus j, j minus. So, one more minus if you take the side that would be plus and if you take this to the side that would be also minus if you take j upward this minus j you can cut you can write plus j here on the top.

So, it will be plus j so if you take this side that will be minus so what we get? We get this equation so this term goes to like this and this term goes like this because this one this one cancel e to the power - z 2 delta z 2 delta and z delta is different delta is defined by this one. So, this one will become this one so this one, this one and this one so ultimately we get this equation second order differential equation involving only a amplitude. Once we get solution a here then I will be able to solve b from here just to make a derivative I can relate what is the value of b. So, this now challenge is that how to solve this equation.

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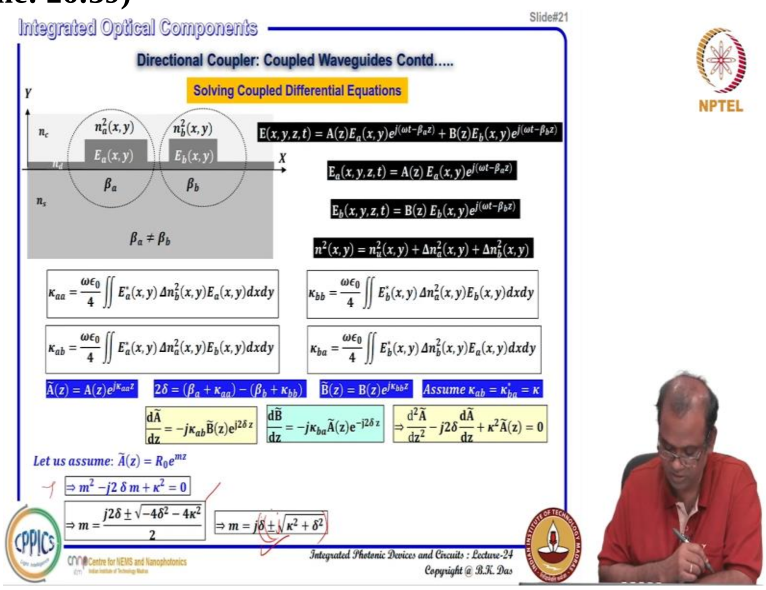

Let us move on so let us assume that the solution of the second order differential equation to solve this I think normal method is that you will just assume a solution exponential solutions and then use a substitute and get some constant and the constant value solve from the initial boundary condition that is the standard way of second order differential equation how you solve so you consider A tilde z equal to something R 0 e to the power mz.

If you substitute here what you will get second order derivative you will be getting m square and this first order derivative you will be getting m and this one simply you will be getting this thing so you will be getting after substituting here you will be getting this equation to solve for m. So, if you solve the m so you get m = - b + - root over b square - 4 k square by 2 standard way. And then you find n value is you are getting its of course complex here it will be this j will be common to both. So, because here minus sign so this will be j also here and j will be there just typo mistake.

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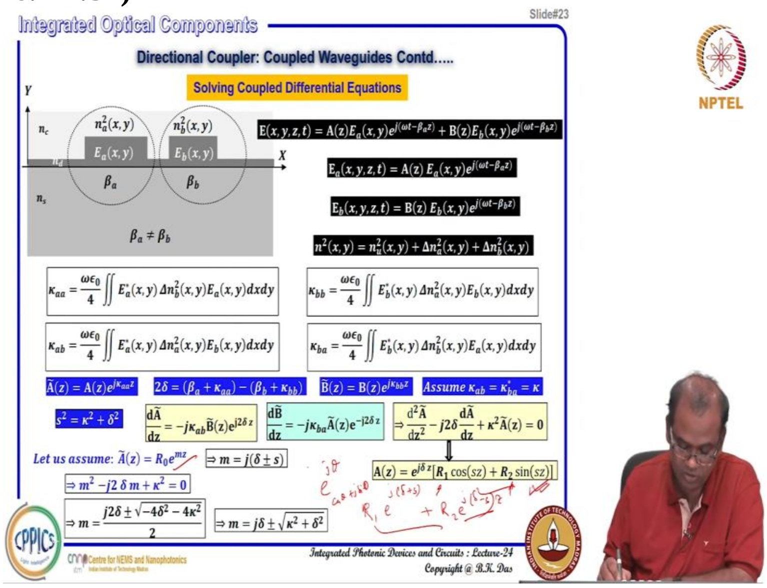

So, in that case we will be just considering here it is written delta plus minus s square root of kappa square and delta square whatever you are getting that you are assuming that is equal to s square they are positive value kappa square plus delta square positive value and square root whatever you are getting they are related by s square. So, basically a is actually depends on kappa d actually in this square form.

So, one access kappa another access so diagonally to base so that way you can represent now we know this is the solution we consider and this m value can be 2 different value that is actually imaginary complex so for this purpose for this type of route we can consider solution of this kind basically I can consider this a R 1 we are considering R 1 e to the power j delta + s + R 2 e to the power j delta - s.

So, if you just a little bit modify you can actually represent because usually this is actually e to the power something j theta something like that you can write cos theta + j sin theta so in little bit simplify we can introduce one more constant R 1 and R 2 and R 1 cos sz z of course z is there so solutions you are considering that it is z. So, R 1 cos sz R 2 sin sz that we can consider. So, this is the solution I can consider but we have introduced R 1 and R 2 that can be solved by using an initial condition. So, now once you know this Az I can just use this Az here and I can find Bz.

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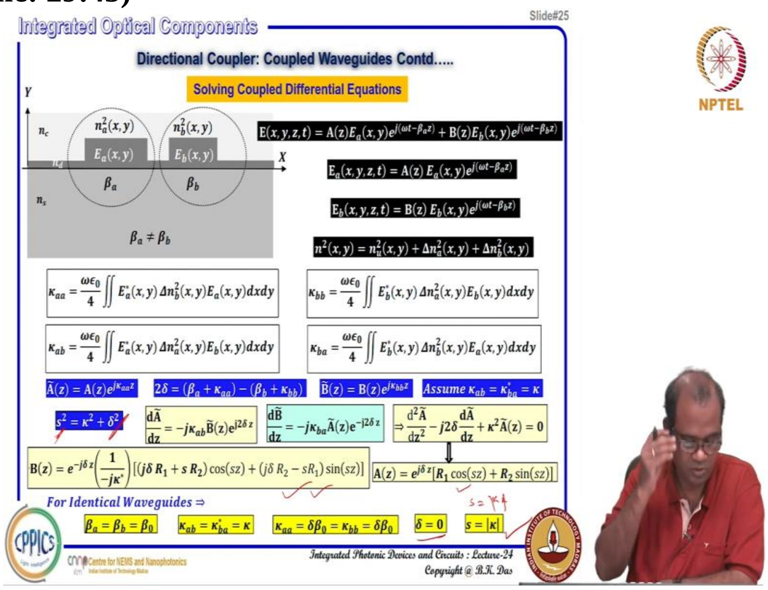

So, I have written here how we will be getting Az, Bz solutions in this case I have just consider that e to the power j k this tilde what about e to the j kappa aa that term I have just ignored ultimately you will getting Bz and Az whatever you get along with that you supposed to multiply e to the power j kappa aa and here it will be getting e to the power j kappa a entire time you have to multiply kappa bb z and a then you will be getting tilde.

So, just factoring that one you are getting these solutions here so ultimately we know that how they are related but the this type of solutions you can get also the coupled solution coupled mode equation in single waveguide also when waveguide is perturbed it can be integrating or so on. Now for identical that just remind you that for identity upgrade one more additional thing is that delta will be equal to 0.

And I have introduced this s square so when delta 0 then s will be equal to kappa because simple kappa, kappa square actually mode of the kappa, kappa star kappa square mode kappa, kappa star. So, now we have this 2 equation now challenge is that we have to find out R 1 and R 2 value once we know R 1 and R 2 value then our thing is solved.

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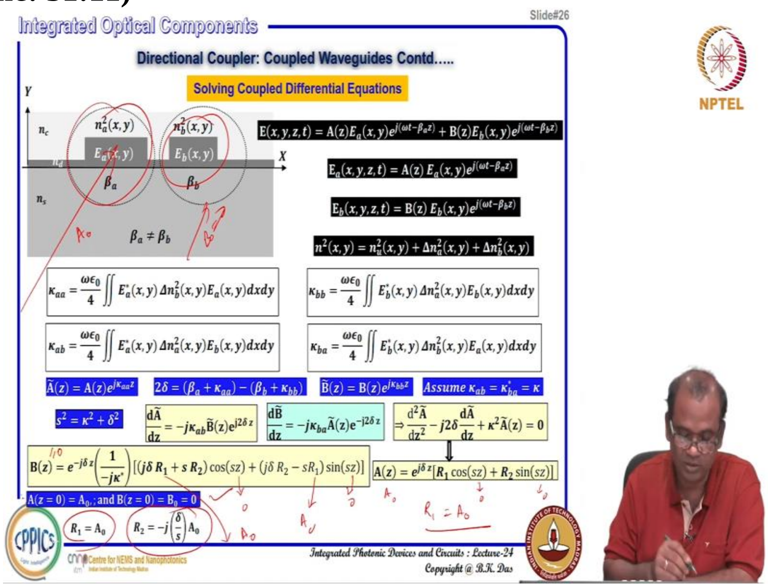

So, what we did let us consider an initial condition like this in waveguide a at z = 0 because you see this is z dependent waveguide z propagating waveguide this also z propagating when you are launching here something a 0 amplitude but here b 0 = 0 nothing I am just considering then I tried to find out at z = 0 I will be putting Az will be equal to A 0 and this will be z = 0 this will be z = 0 if I put then what I will be getting R 1 = A 0.

So, simple R 1 = A 0 we will be getting but when I am putting Bz = 0 B 0 that means I will be putting 0 for z = 0 I will be putting z equal to z 0 z = 0 and R 1 of course is A 0 I am finding then I can find R 2, R 2 expression will be like this simple straightforward you just have to consider this one you are just launching in one of the waveguide then you get the solution for the entire all the 2 constants.

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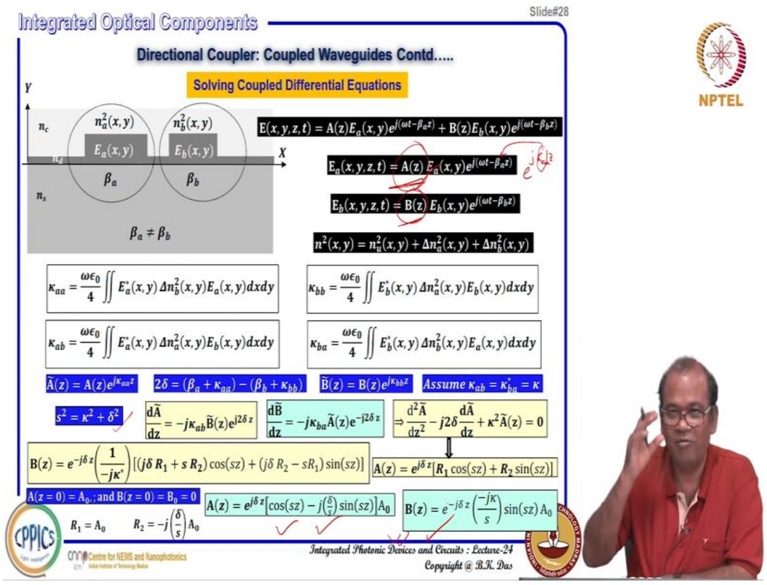

So, just introducing these 2 constant I get Az will be equal to this one the in this type of form and Bz it will be in this type of form I can write this is this is the ultimate solution. So, only thing is that in this waveguide what is unknown only unknown thing is that delta and kappa once you know delta and kappa is also known. So, kappa you can compute delta also you can compute so once you know numerical you can compute.

So, you know that how long suppose z dependent if it is propagating then how this Az will be evolving amplitude will be evolving and Bz will be evolving you know whatever it will be evolving you can also add that any additional phase kappa aa z also has to be multiplied then that will give you some kind of complete things but this kappa a you can club a here actually beta a take plus kappa a then only thing is that you have to solve Az here also Bz.

So, kappa a means modified propagation constant whenever you are considering modified progression constant just deal with Az, Bz that is it. So, these are the solutions so we are almost done this solution so once we know that solution so we can use this to design our directional coupler with desire specification.

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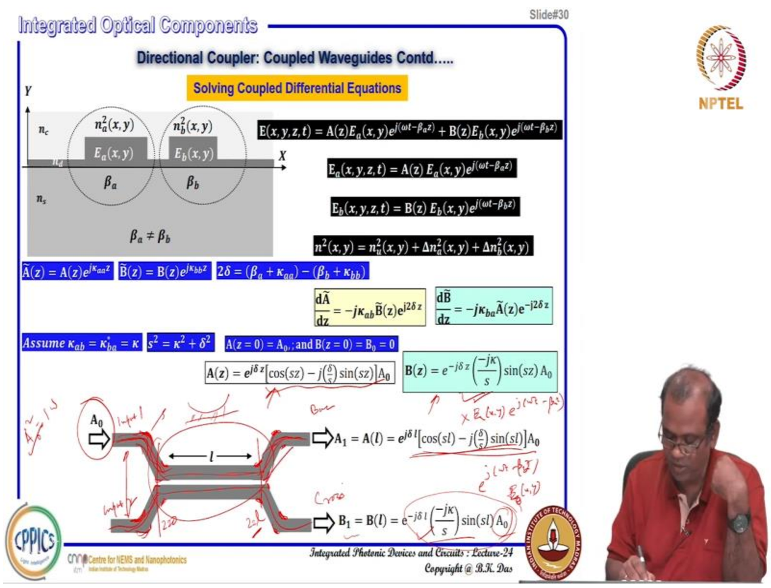

So, we have again written down everything whatever we have consider so for coupled equation so far assumption kappa ab = kappa ba star and this is also assumed and this is the initial condition assumed that means one of the waveguide you are launching another object is nothing is there or the beginning at z = 0 then your solutions as it propagates what will be the amplitude in the waveguide a and whatever the amplitude in the waveguide b you can get.

So, now we just take help of this structure this cartoon this is actually a standard representation of a directional coupler so what is that you have 2 waveguide here input waveguide here input one input 2 essentially they are decoupled for away decoupled and then you can slowly bend this type of abrupt bending is not possible if it is abrupt bending lot of losses will be there but when you are designing waveguide you can make it smooth.

Then waveguide can be add a smoothly coming towards waveguide 2 same situation you can consider here smoothly slowly without any loss this module can come here this waveguide can come here and then here also this can be smooth this can soon not be sting or something like this and this one also should be smooth. Here all this you just consider smooth otherwise in the junction you will be getting if it is not adiabatic then you will be getting losses.

So, something like this so this is your waveguide smooth it is kind of s bend type structure you just make so that you can get almost no loss adiabatic so adiabatically both the waveguides come closure and when you see that they are even is a tail is almost touching other one according to the gap then you maintain this length parallel make it parallel. So, this is the region you can start z = 0 and z equal to sail l, l is the length.

So, interacting length l and in the interacting length this l is basically waveguide is a parallel actually if you want more complication because of the some kind of design fashion so you can have this one can be bend structure another can be straight something like waveguide this another waveguide like this and it can be but appropriately you have to model this one so that you can analytically get good solutions.

So, for the moment our intention is to find out a just comprehensive analytical solutions, so we consider that both the coupled waveguides are parallel they can be fabricated of course. Now I know that this is the 2 equations if I have A 0 I am launching here one of the waveguide then I know Az, Az means here z = l so everywhere z = l if I put then I will be getting whatever at the output of this is actually called bar waveguide.

And this is whenever you are launching with respect to this one this would be called cross waveguide in the bar you know the expression will be here once it is distort after l it will remain constant assume that waveguides are lossless and similarly here whatever to be crossed coupled that will be like this. So, this equation basically e to the power - j delta l so depending on the l you can find out what is the amplitude.

And of course A 0, A 0 can be 1 A 0 square can be 1 watt it can be 1 milliwatt it can be anything. So, if you know the amplitude here then you know after interaction length of l and then decoupled then whatever value it is coming out how much amplitude will be there here we know that and then again it will be propagating like E a E b x, y and then e to the power j omega t - beta b z So that way it will be propagating but this is amplitude after here.

So, here it will be this will be amplitude and it will be multiplied by EA x y that is a profile this one multiplied by e to the power - j omega t - beta Az but whatever happening here that actually decides your amplitude variation because that is the interacting region. So, they exchange energy that is why their amplitude exchanged when they are decoupled that is how we can derive using the coupled mode equation.

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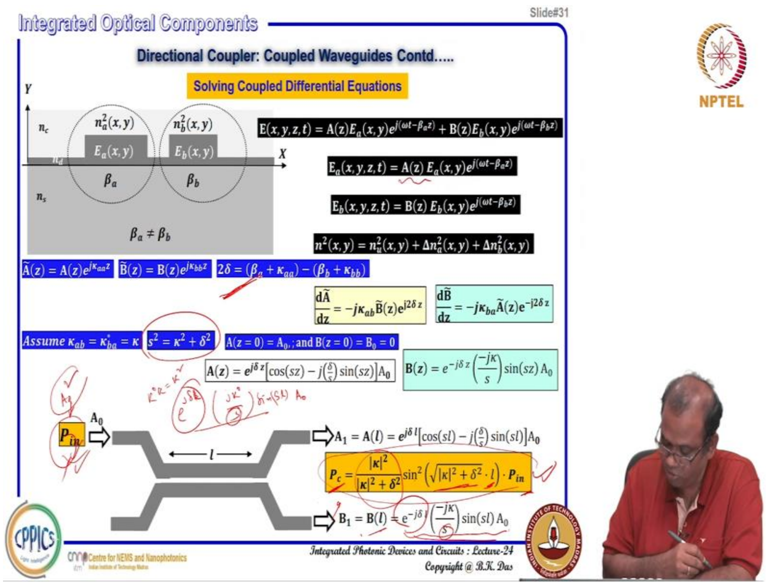

So, this method actually very powerful has a basis to have a compact model of any directional coupler that you need for circuit simulation let us see what is the power that means A amplitude A 0 square I can consider as a corresponding power amplitude. So, this is the entire amplitude it is normally we assumed we have normalized our mode profile such that the amplitudes is actually carrying all the power expression.

So, A 0 square this is a P input so if it is P input then output side this is the coupled or cross port P c here whatever I will be getting here how much I will be getting there. So, what I will be doing this is amplitude you take complex conjugate of this one that will be your multiply with this one that in your coupled power if you take complex conjugate of this one then you will be getting Az delta l e to the power z delta l.

And that will be j kappa star by s sign s l that is a complex when you get multiplied by A 0 so this one and you multiply with this one e to the power j delta l this phase information will be canceled j k - j k that means it will be j j minus so it will be k k star basically k square kappa not k it should be kappa, kappa square. So, numerator will be kappa square and denominator will be one s is here another s is your complex conjugate you multiply.

So, s is square s is square means we know s is square means kappa square delta square so this would be your kappa square delta square and sin s l that will be sin square s l. So, sin square s means this one l is this one and A 0 square means P input. So, if this is the P input cross port power will be expressed by this one. So, I know how much power is coupled so power splitting you can find out.

So, this is interesting this is the expression important expression that if you have 0 non identical waveguide having some kind of delta is there if they are not identical this non identical this delta will be non 0 and you will be getting this expression.

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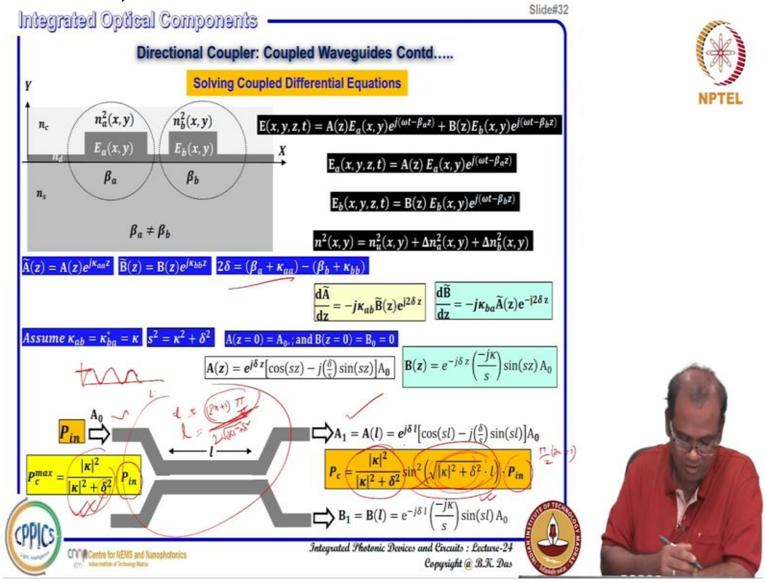

Now we just a little bit analyze this thing what will be the maximum power here how is that this value you see this sin square. So that will be bearing if you increase your length so depending on the length this value will be going sin square function this is a constant for a parallel waveguide kappa square delta square they can be whatever the parameter is a constant. So, the power coupled in the cross port that is actually varying a sin square with P in.

So, I can say that if l = pi by 2 n + 1 pi by 2 divided by this kappa square plus delta square. So, if your l value n is integer l value is this one maybe 2 we can write here it 2 n + 1 pi divided by this one then you will know the sin square value will be 1 n = 0, 1, 2, 3, 4 so on that means odd integer multiplied by pi by 2 this one that will be cancelled so it will be 1. So, normally in the cross port I can see that power is varying maximum and then reducing maximum and then it is reducing according to the sin square function.

So, if you just as a function of length if you just plot it will go up to some maximum value and then comes back according to the sin then again go maximum value and comes back again goes maximum value and comes back and if you see what is this maximum value so sin square value this much for any integer value of pi by 2 if we just use that will be 1 so, what are the maximum value? This would be your maximum value.

So that is what we have written in cross port maximum power coupled can be this one depending on the input you can take an ratio that can be transferred function or the maximum value. So, you can design your l such that this l equal to this multiplied by pi by 2 this entire thing should be coming as a pi by 2 times 2 n + 1 such that particular length can ensure maximum coupling but that maximum coupling is how much this one.

Just note that delta square is a positive value kappa square kappa square plus delta square so it will be never equal to P input as long as it is not equal to 0 if delta is not equal to 0 that means you will never be able to transfer complete power launched into directional coupler can be cross coupled to the second guide cannot be appeared in the cross port. So, if this is important conclusion that if you are waveguides are not identical.

You will never be able to transfer complete power to the second waveguide whatever length you increase it will go up to a certain value this one according to the delta this value but it will never become P input whenever it is coming down that means power is going back to bar putting in so you can plot basically as a function of delta or as a function of length how power is actually transferring from input to output cross port.

If you are considering no other losses are there only waveguide whenever some power is lost by one waveguide that will be gained by another waveguide if that is the condition some loss additional loss will be there that can be treated separately individually you can consider that thing.

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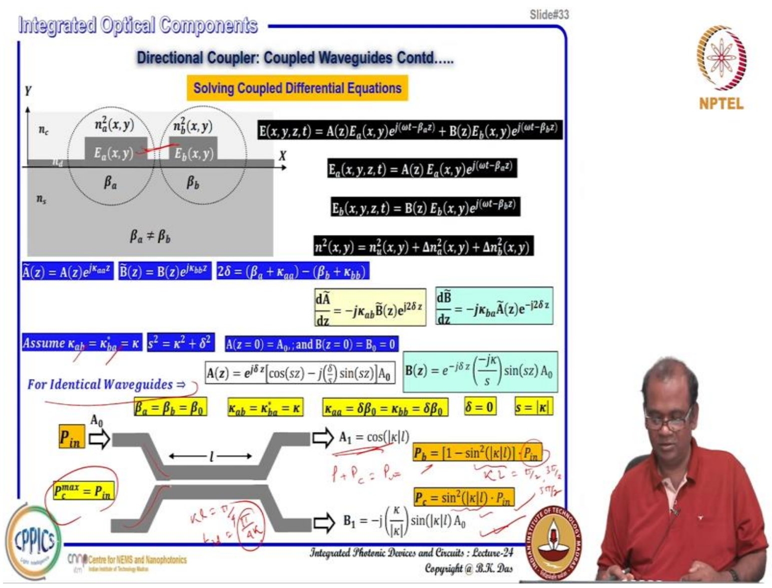

So, in this case we can say that the P c max for example identical waveguides if you are just considering just take delta = 0. So, if delta equal to 0 if you just put here in the previous slide delta equal to 0 if you see this will be kappa l and this will be delta = 0 kappa square kappa square 1. So that means it is basically sin square kappa l P input that is what the sin square kappa l P input that is the power coupled in the cross port.

So, if you see now if you are l just changing kappa l can be pi by 2 can be 3 pi by 2 can be 5 pi by 2 and so on. So, for those length actually you will be getting power completely transferred to the second waveguide. So, for identical waveguide you can indeed transfer power to the second waveguide completely theory says and obviously if it is lossless whatever it is coupled to the second waveguide whatever will be there in the bar port P b that means 1 - sin square kappa l p input.

This is obviously this will be less than 1 so together P b + P c should be equal to P input lossless. So, power splitting will be distributed like that and in that case maximum power coupling to the cross port obviously equal to P input. So, in this case power complete will transfer to the second waveguide in that case in the bar port 0 power and if you keep on increasing the length you can reach to another link frame power will be coupled back to the again bar port. So, it is actually varying with a sin function sin square function or cosine square and in this case sin square function is better representation.

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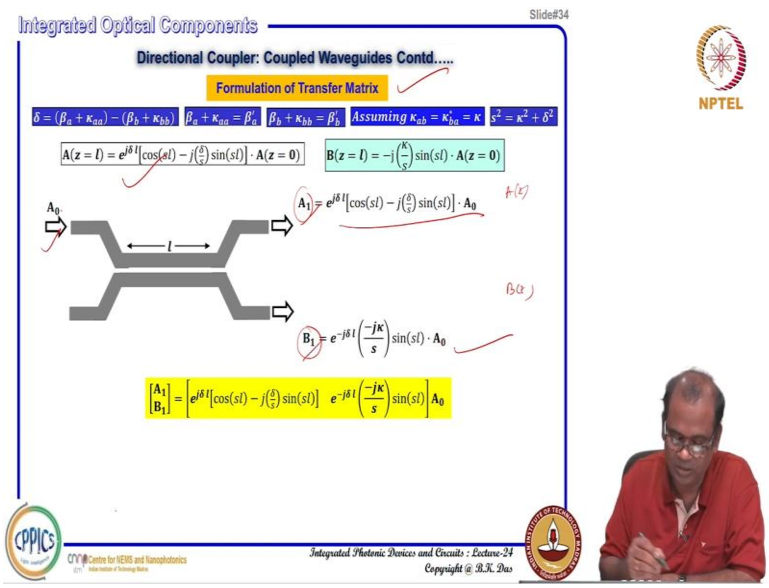

Now let us go for it we are now done that how to solve and how to design the waveguide obviously if you see that if you want a 3 dp power splitter kappa l should pi by 4 if it is pi by 4 that means sin kappa l = 1 by root 2 and sin square will be half. So, it will be 50% here and 50% here so you can design kappa l = pi by 4 or l 3 db = pi by 4 kappa. So, if you calculate kappa value.

If you know what is the kappa value you will be able to calculate what is the 3 db length so this kappa value depends on what is the gap kappa expression we know that kappa means it is basically we consider kappa ab or kappa ba that expression. So that expression depends on how closely these 2 fields are overlapped so more the kappa value you can make compact more the kappa value you can make length smaller compact link up 3 db possible later if you want.

So, you do not need y junction basically y junction designing adiabatic design is very difficult or rather adiabatic design of directional coupler is relatively easier this is only because it is a waveguide is basically very tightly confined mode and little bit perturbation etcetera If it is there then you can get a lot of losses. So, directional coupler is prefer most and that is why this is an important power splitter or amplitude splitter used for photonic integrated circuits.

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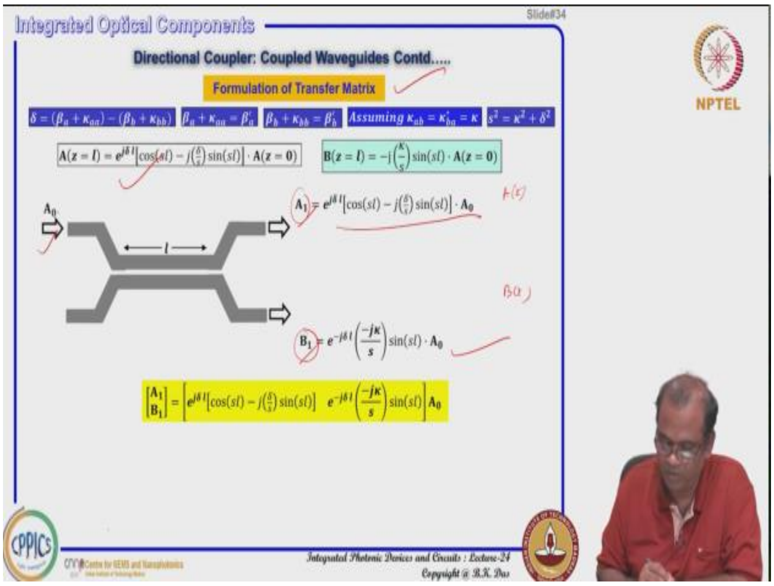

Now we are going for formulation of the transfer matrix I again repeat what is the amplitude whenever you are launching here you can get bar port amplitude and this is your cross port amplitude we represent A 0 and here we are writing A 1 and B 1. So, second waveguide any amplitude we are considering like a B and first waveguide any amplitude we are considering you remember that we are just considering Az and Bz.

So that is the reason I am considering A 1 and B 1 here and that is what is written here l length that is why l we have put here. So, according to the coupled mode equation whatever we get that is what we have written.

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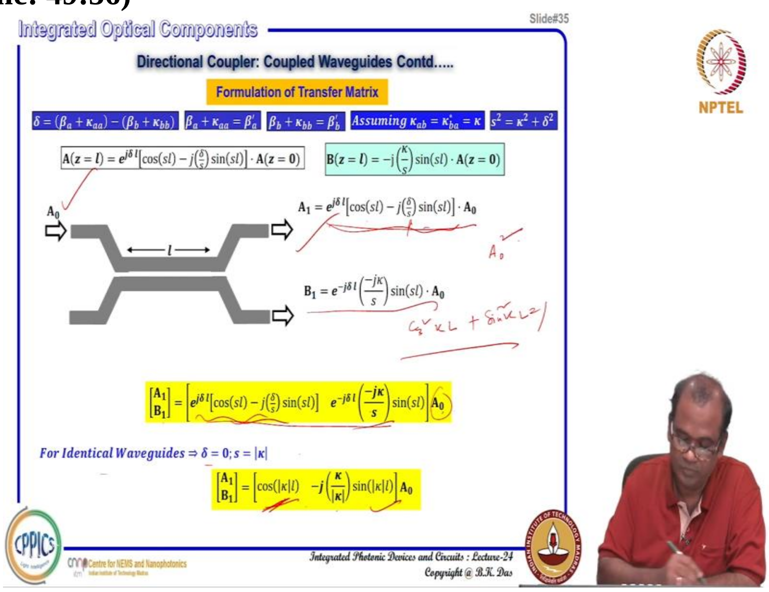

Now you see if I try to get a transfer matrix how it will be looking like I have only one input and I can have a row matrix so this is the value I have put first element and this is the second element. So, we can write A 1, B 1 something like this A 1 will be this one multiplied by A 0 and B 1 will be this one multiplied by A 0 that is in the matrix form we can write and if delta = 0 then it will be cos kappa l this one.

So, anytime at any length if you see that the value in the this one actually in the cross port because delta = 0 means this term will go and the s will become kappa. So, in that case total value will be cos square kappa l and this one will be kappa, kappa will be cancelled complex conjugate if you take sin square kappa l. So that should be equal to 1 normally if that is equal to 1 that means whatever power you are launching here you will be getting even delta equal to 0 you can solve even delta not equal to 0 also.

If you just take complex conjugate multiply and complex conjugate multiply add them together then you will get A 0 square here whatever power is coming this is a lossless solution.

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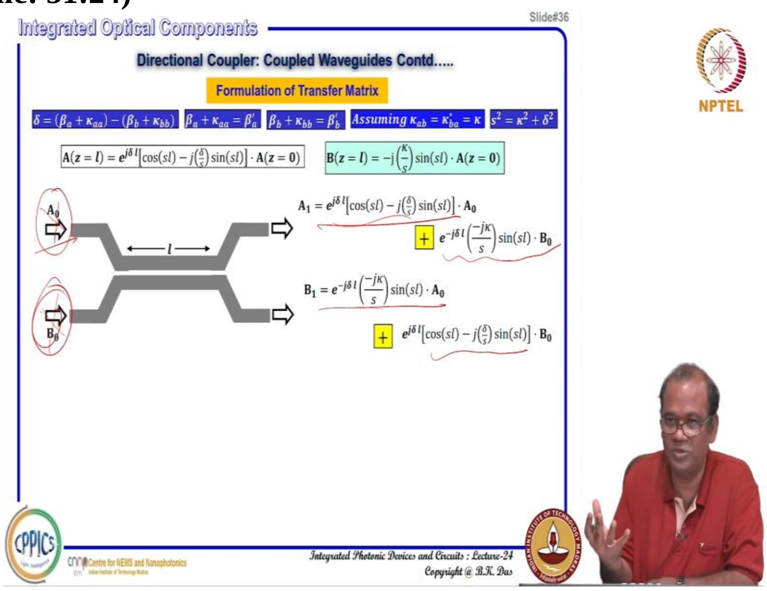

Now if you have a B 0 this waveguide first waveguide port 1 you are launching A 0 you got this one and you got this one now you have B 0 then you see what are you supposed to get in the course because of the B 0 here you will be getting almost similar. So, here you are getting this one now because a B 0 you will be getting similar thing in the cross port here plus A 1 will be adding this one.

And B 1 will be adding another type of this type of solutions here so this kind of superposition principle you just consider B 0 you just stop then you see whatever value you are getting now A 0 stop you see the B 0 whatever value you are getting and then total value each both are one such one then superposition its electrical circuit normally we use that even per case all these things superposition principle to use. So, here also this same superposition principle we are using

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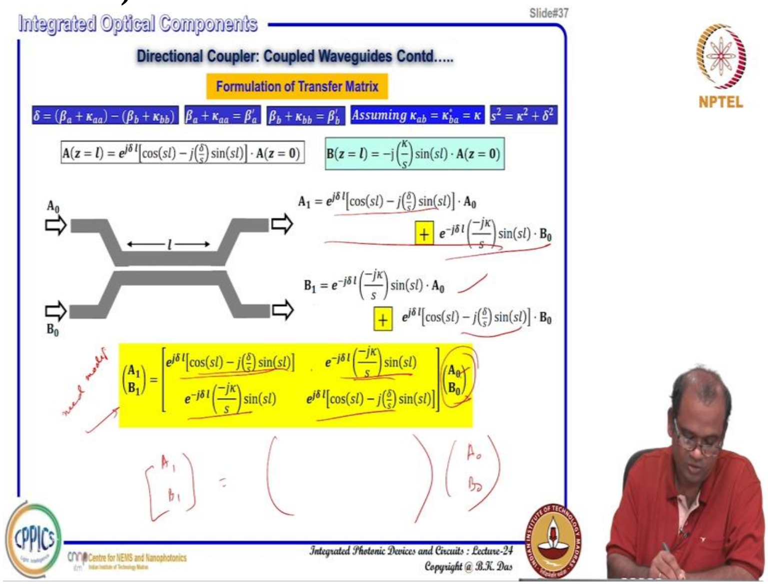

Now if I just try to make a matrix upon this is because A 0 B 0 is the input A 1 B 1 is output A 0 B 0 is the output I want A 1 B 1 that is this thing and some matrix form I will write so that I can get A 0 B 0 that is what do you need for circuit simulation. So, from this equation we can just simply write this is the A 1 component and this is coming from B 0. So, A 1 will be this one multiplied by A 0 plus this one multiplied by the one.

And B 1 will be this one multiplied by a 0 this one multiplied by B 0 so that is what this thing together these 2 equations together we can write in a matrix form. So, far it is good but little bit modification require this thing this needs some modification why? Clarify so whatever we have just so far discussed directly we are getting this is good.

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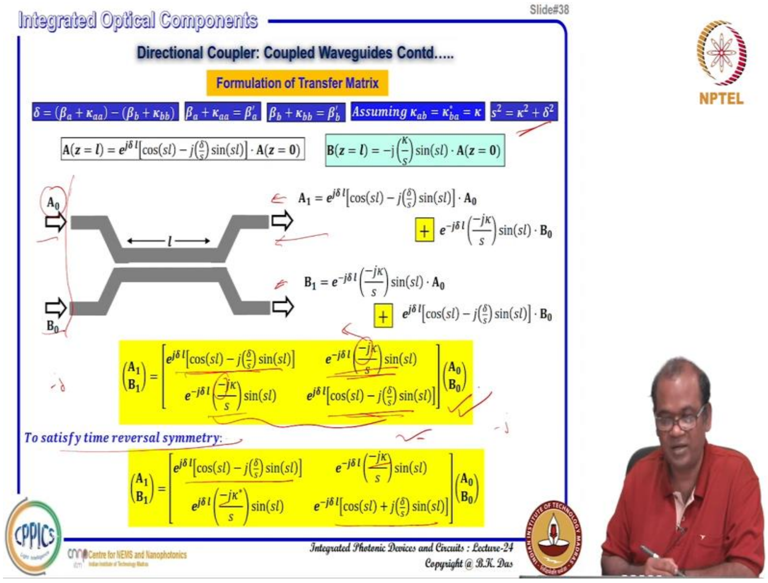

Now if you just think to satisfy time reversal symmetry. So, you see all these elements are complex now we are launching from this side and this side it can happen that we are launching this A 1 and B 1 and I should be able to get A 0 and B 0 from the same matrix? This I have to take inverse to this site and I should be able to get so to get this one correctly that this is a basically bi directional device.

I am launching this side whatever I am getting this one if I launch this side they are also I should be getting an identical mathematical matrix formulation we need. So, for that purpose what do we do? We just decompose this one we write here as it is. And these one I use the complex conjugate you see e to the j delta l I will be writing minus e to the power j delta l as I am considering like real cos sl, cos sl because sl square you see if we have defined this one that is a positive number just consider real.

And then this minus j plus j complex conjugate I am writing delta by s delta is the real number we consider and in this term I consider this minus j I just keep separated without minus j here and without minus j here rest of the time thing I can consider complex conjugant why this miners the I am not counting I will be discussing that. Now we consider the minus j k are you just consider minus j I am consider kappa here kappa star this one I just converted into kappa star.

And it e to the minus j delta l e to the power j delta l so if it is the case if this is the situation this type of equation if you just see other than this minus j if you just take complex conjugate then you will be get complex conjugate and then transpose that would be actually inverse. So, you will be getting same other than minus the complex conjugate minus j you have to just multiply diagonally then this one complex conjugate if you take.

And transpose that will be actually inverse so I will be getting A 0 B 0 not in terms of A 1, B 1. So that is the region to satisfy time reversal symmetry we were just using this matrix in this form. And in this form if you write one additional thing you will be getting.

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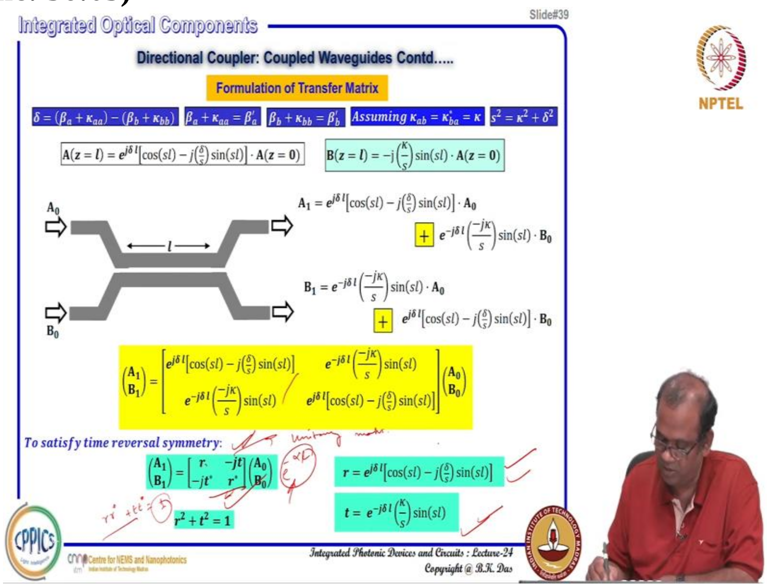

What is that? You see you are just representing this one this is something we have kept up from before but we can like minus j minus j i have kept and the t, t star and r, r star I am considering where r is this one t is this one if you just use this form if you just take the determinant of this one r, r star and you will be getting plus t, t star that is the determinant and that determinant you can you have to have one if it is a lossless waveguide r square.

So, determinant is actually equal to r square + t square if it is lossless this would be equal to 1 so that is why it is actually unitary matrix determinant is 1. So, for lossless case if it is losses they are also you just consider everything is propagative total length is the e to the power minus alpha l you can consider just ignoring loss practical situation loss will be there that can be considered separately.

Otherwise you can just simply consider this type of device as a unitary operation unitary transfer function unitary there is no loss. So, this type of unitary transfer operation that is very useful also quantum mechanical functions we will see later. So, this is actually your ultimate transfer function transfer matrix have a directional coupler you should keep in mind that is defined like this and t will be defined.

Like that whatever well you r put here r star food here t whatever you put here t star if you put here then this will be your transfer matrix for the directional coupler any directional coupler you design and you can represent once you get this can be calculated you know delta is known s can be calculated everything can be calculated in complex form it can be represented and then you can use that as a compact model compact transfer matrix for the directional coupler.

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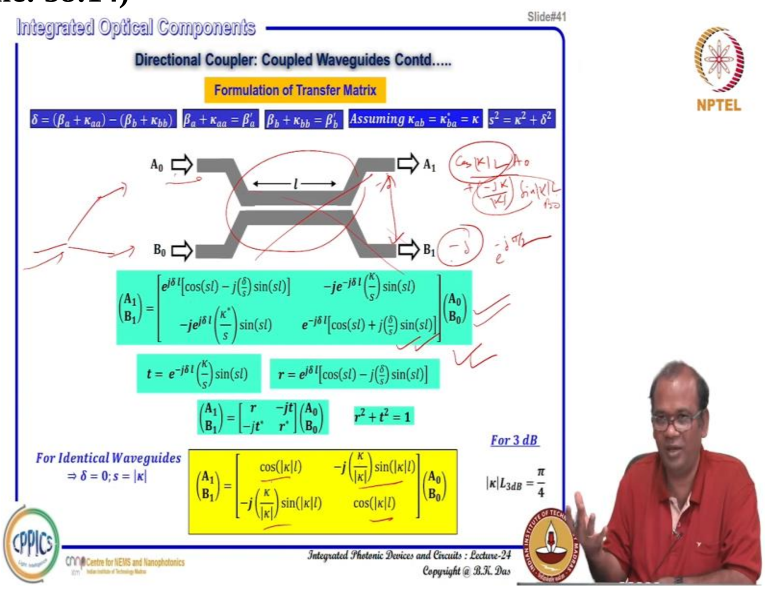

Now this is in summary so this is the complete transfer function this is your input this is your output A 1, B 1 you will be getting when you know A 0, B 0 but this is your like a black box you can use directional coupler if you know r and t. So, then you can get this is something input here input here you get 2 output ports here. So, input to output transfer function can be represented this; whatever we have discussed this is repeated here.

Repeated here just to discuss one more thing just for identical waveguide you know delta = 0 s = kappa delta equal to 0 s = kappa if you cos kappa his one this one and this one. So that means here you will be getting cos kappa l identical waveguide and into A 0 plus you will be getting plus means minus j kappa by kappa sin kappa l. So, one thing you should remember that suppose kappa is real.

So, anything you are getting this is for B 0 and anything you are launching here and whatever you are getting here you have a minus j here and whenever it is going from this to this also you have a minus j additional thing we are that is coming from the physics of the directional coupler when it is cross coupling you will have this minus j means e to the power minus j pi by 2 phase shift pi by 2 phase shift always will be there.

When it is tunneled through directional coupler to the other waveguide then if you see that these 0 this is your transfer function from input to bar port this is your transfer function from input to cross port. So, input to cross port always you are adding minus j that is coming because of the theory of couple mode equation theory of directional coupler where anything you coupled to the second waveguide you pi by 2 phase it e to the power minus j pi by 2.

That is unlikely y junction you know remember that whenever you are launching this one whatever you are getting here and whatever you are getting here they are actually same piece. But here you see light is going directly another it is just crossing once it is crossing phase shift it is pi by 2. That is actually inclusively used here in this equation it is inclusively used if this is general equation basically transfer matrix but the coupled directional coupler.

So, this is from here onward will be just discussing for ever so many other device complicated structures complex circuit but this directional coupler is actually a basic building block and we can take help of this structure to understand other component like Mach-Zehnder parameter also made out of as I said that this directional coupler is fabrication friendly for silicon on insulator platform rather than y junction.

So, we will be using this directional coupler for demonstrating Mach-Zehnder parameter again instead of y junction and because y junction transfer matrix is different directional coupler transfer matrix is different. So, you can use this one for understanding Mach-Zehnder inter parameter output you can again use this thing for finding out the transfer function of Mach-Zehnder inter parameter a transfer function of varying resonator and so on with this I stop today we will wait for the next lecture. Thank you very much.