Integrated Photonic Devices and Circuits
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Lecture – 28
Integrated Optical Components: Distributed Bragg Reflector (DBR)

Hello everyone, today I am going to discuss about distributed Bragg reflector that is another important integrated optical components and I will discuss that this DBR structure using coupled mode theory which is as I said that coupled mode theory is somehow it is a good theoretical analysis you can understand most of the integrated optical component and we have already developed coupled mode theory power code directional coupling and contra directional coupling.

So, as we already mentioned earlier that distributed Bragg reflector that is actually works best on the mode coupling in the contra direction when they are propagating and contra direction that means, one will be in the positive direction another mode will be in the reverse direction negative direction. So, considering that I will just highlight some kind of recap what we have learned so far based on contra directional coupled mode equations.

And then I will discuss how to solve this coupled mode equation particularly for distributed Bragg reflector and then I will discuss about a design of an integrated optical DBR filter passive bandpass filter or band rejection filter you can use a DBR structure.

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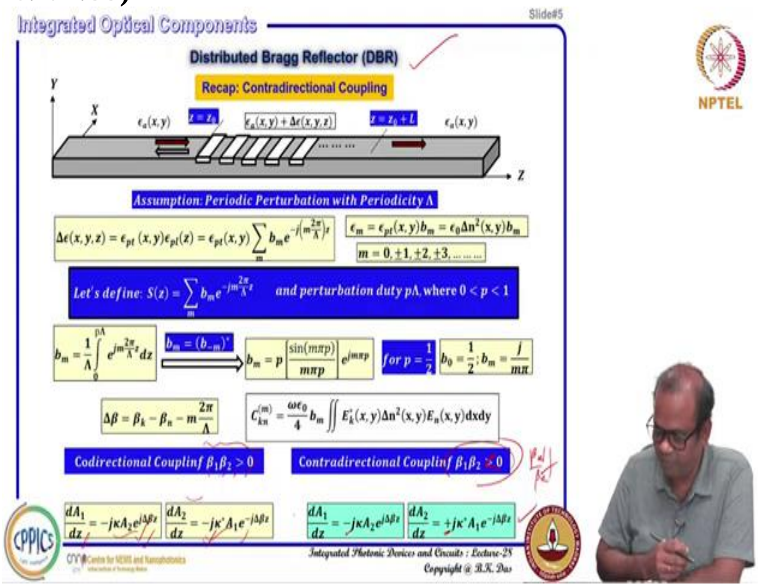

So, just go back to our distributed grating structure we discussed earlier that if you have a waveguide that is propagating along Z direction mode is propagating along Z direction it can be single-moded it can be multi-moded and waveguide you can consider this is the core whatever it is shown here in X Y direction X Y plane this core is there and surrounding cladding will be there and as long as the dielectric constant distribution for the waveguide epsilon a x, y is maintained.

Then you can see all the guided modes they are orthogonal and they will be traveling independently they will not interact each other. However, if you have a certain kind of periodic perturbation here for example given from z = z 0 to z = z 0 + L with a perturbation so, epsilon a x, y which is added with another perturbation term that is delta epsilon x, y, z. So, in that case what we have taken that this perturbation is periodic with a periodicity of lambda capital lambda this is the periodicity.

So, if you just start from here to here this is one period then it is repeating again another one and then it is repeating another one. So, it is a periodic structure that means, this white region we can consider that the height of the waveguide is reduced you can consider the width of the waveguide also can be reduced or increased periodically that can be also considered a periodic perturbation and that periodic perturbation.

We can actually define as like this you have some kind of x y profile will be there that is actually called transverse perturbation E pt x, y and then longitudinal perturbation that is actually z we can decompose into 2 functions and then E pt x y we can define what type of perturbation it is in that cross section waveguide cross section how it is perturbation and longitudinal direction since it is periodic we can decompose into Fourier harmonics.

So, if you see the spatial periodicity is lambda and Fourier harmonics it will be 2 pi over lambda times integer and it can be plus minus and so on and this b m is called Fourier coefficient. So, in that case I can define that this total perturbation we can together this E pt x y plus Fourier coefficient if I just consider that can be considered as a perturbation of m th Fourier harmonics along Z direction.

So, we can write E pt x, y as a delta n square x y that is the refractive index you know dielectric constant if you know just square of the refractive index is equal to dielectric constant so, you if refractive index perturbation is delta n if you just square it then you get the dielectric constant if you multiply epsilon 0 that the permittivity or the free space in the free space and b m is written this we have discussed earlier and it can be 0 plus minus 1 plus minus 2 plus minus 3 and so, 0 means that is the DC component of the Fourier transform.

Now, so, if you just think about that, let us define this longitudinal part we define as a S z defined function is there this one you are defining as a S z or you can write this as a E pl z longitudinal direction so that is we can write something like this b m e to the power the same thing we are just repeating here and perturbation duty is p lambda. So, this periodicity, you can consider this perturbation where the height is changed that fraction of the entire period that is called duty.

So, I can consider p lambda is the duty where p is the 0 less than greater than 0 and less than 1. So, that means, it is something this duty is less than lambda p can be up to close to 1 also. So, that is the duty and in that case we can find out this Fourier coefficient directly from this formula. So, we integrate 0 to p lambda e to the j m 2 pi / lambda z dz 1 over lambda and then we can find out b m and also we have seen that b m = b - m star that is actually property of this transfer function for the Fourier transform.

And then you just integrate this one we get the Fourier coefficient b m in terms of duty cycle you remember that if duty cycle is just p = 0.5 then it will be half and then half and then you can find out what is the value p if you are putting half for example, here it is given if you are putting p equal to half then b 0 that m = 0 will be just half that means the DC component of the Fourier transform is just half and then other than DC component.

If it is other value 0 not 0 plus minus 1 plus minus 2 and so on b m will be something you can just directly put here that p equal to half then b m value will be just j over m pi. So, that means b 1 will be j / pi and b - 1 would be also j / pi b - 1 will be j / minus this will be minus because m equal to minus but b - 1 star if you put then it will be j / pi again so, that is how we get b 1 equal to b - 1 star.

So, that is how we get b 1 equal to j over pi we have written and since, you have one component coming like this if you are just introducing this one in your coupled mode equation, then longitudinal page matching condition will appear like delta beta = beta k - beta n - m 2 pi over lambda and then kappa so, called coupling coefficient C coupling between kth and nth mode because of the mth Fourier harmonics.

We have expressed earlier that omega epsilon 0 / 4 b m that is a Fourier harmonics and then you can integrate E k star E n through delta n square that is actually so called coupling coefficient. So, you remember that this is actually dielectric perturbation and dielectric perturbation you have longitudinal term is there that longitudinal term actually clubbed with the longitudinal phase because that is also the beta k z beta n z will be there in the all the coupled equations.

So, they are clubbed this one and that is why only the transverse direction whatever perturbation is there delta n square x y and field distribution for the kth mode fields distribution for the nth mode you integrate over wherever these 3 values are nonzero, you get the value and b m you can consider which Fourier harmonics is being considered for involved in coupling between 2 modes and that is how we can calculate we discussed this in details earlier already.

So, here we can just repeat here just to take up take it forward, we know that for co-directional coupling, we consider evolution of the mode 1 then we can write this equation and this equation for the other one and in that case, for co-directional coupling, we considered beta 1 and beta 2 greater than 0 because the beta 1 if it is a positive direction, mode 2 also either will be traveling also positive direction plus both are traveling in the negative direction then also plus.

So, we consider that a beta one for co-directional coupling this is the condition that needs to be fulfilled. If this is the condition fulfilled If you have coupled equations are like this these 2 equations and a beta 1 and beta 2 greater than equal to so, this would be less than or equal to 0 that means contradirection this is a type error this is less than or equal to less than 0 that means one of them will be positive direction another will be negative direction.

So, in that case, so, we get because this combination is in picture so, coupled equation one will be minus sign and other will be plus sign that is the difference actually, we said that for co-directional coupling the both is a minus sign and contradirectiona coupling both are opposite sign because you have beta k / beta k this term is there. So, depending on the positive and negative direction the coupled equations will be looking like that.

So, this is required and we are talking about distributed Bragg reflector that is our discussion point then in that case we will be coupling light from a mode propagating in the forward direction to a mode propagating in the backward direction. So, mode will be coupled from energy will be coupled from forward propagating mode to backward propagating mode. So, that thing it at all some phase matching condition is fulfilled.

Then we can see some kind of energy transfer and that energy transfer can be understood can be explained analytically using this 2 coupled equation if we can solve this 2 coupled equation, then we will be understanding how the mode 1 is evolved longitudinal direction and how mode 2 is evolved in longitudinal direction meaning along the z direction.

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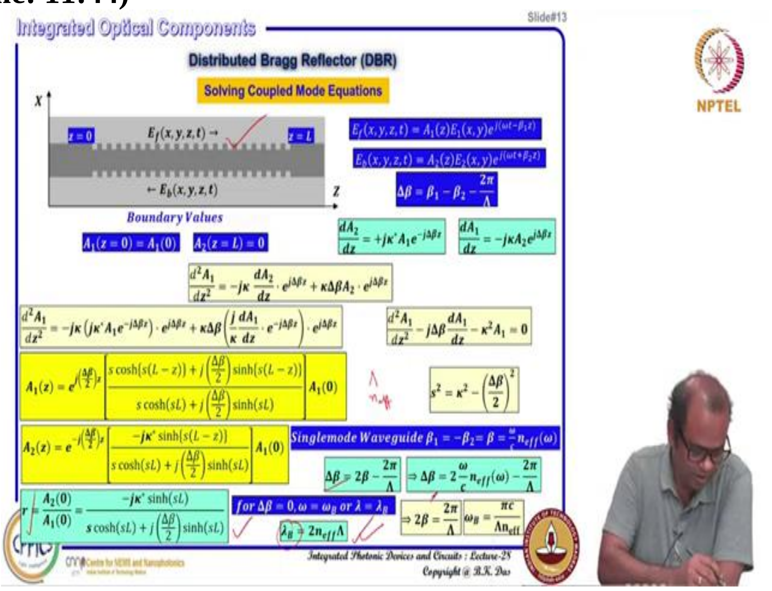

So, now, we will be discussing about solving coupled mode equations. So, we said that our this is something we are considering our getting structure top view we are saying and instead of height perturbation here for understanding purpose we consider understanding and technology implementation purpose normally instead of height modulation particularly for silicon photonics.

People used to go for width Modulation waveguide width modulation so, here if you see width waveguide width it is periodically modulated from z = 0 to z = L. So, in that case, we are talking about coupling between forward propagating mode and backward propagating mode, forward propagating mode the associated electric field can be space dependent x y z dependent and time dependent we mentioned.

Here f that means forward propagating mode and backward propagating mode similarly, we just write E b x, y, z, t. b for b stands for backward f stand for forward direction and this is your direction obviously, x direction and vertical direction your perpendicular to the screen is y direction. So, top view we are showing so, before that pre project z less than 0 the waveguide is uniform.

So, all the orthogonal modes will be there if it is a multimode waveguide mode suppose fundamental mode is guiding field distribution will be like these and it will be propagating here and once it is entering into the grating structure there you have a perturbation periodic perturbation. So, there is a chance that mode will be coupled if it is a single mode fundamental forward mode and fundamental backward propagating mode they will be coupled.

So, we can define 2 modes in the forward direction E f x, y, z we can write because of coupling there will be z dependent amplitude variation will be there we introduce A 1 z and field distribution is E 1 x, y and since it is forward propagating wave, so, phase will be e to the power j omega t - beta 1 z this is positive z direction propagating and if I consider another mode having field distribution of E 2 x, y.

And propagation constant of beta 2 that is propagating in the reverse direction we can define backward propagating wave is E b x y z like that, in this case we have just consider general situation so, that it need not be the forward propagating fundamental mode needs to be coupled to the backward propagating fundamental mode it can be multimode waveguide any forward propagating mode that is characterized by beta 1 and E 1 will be coupled to the backward propagating mode having propagation constant beta 2 and field distribution E 2.

And we know that this longitudinal phase constant difference delta beta phase difference what are the delta beta value we consider actually beta 1 - beta 2 / 2 pi over lambda and this is the coupled equation we discussed and we have to solve this coupled equation considering these boundary values. What is that boundary values A 1 z = 0 A 1 means this one forward direction at z = 0.

We have some certain value this can be 1 also we can consider normalized or any value such that you can consider A 1 0 square equal to 1 watt also that means you can consider 1 watt I am launching then how much fractions will be coupled to the other modes, we need to find out you can consider one here we have considered the amplitude A 1 0 and another boundary value we must know that if something at all propagating in the backward direction you are launching from this side.

So, there will be nothing coming from the left side. So, only thing is that because of the perturbation entire region, you can see that light will be can be propagating in the forward direction that can be coupled to the reverse direction. So, reverse direction if it is coupled and it is constructively building up then you can see that this direction reverse direction backward propagating mode, you can see some kind of energy it can be excited because of the perturbation.

But beyond z = L you do not have any perturbation. So, you would not see any backward propagating mode here because you are launching from this side so backward propagating mode beyond z = L is 0. So, that is why if I have a forward propagating mode is having amplitude this one and backward propagating mode is this one A 2 z then A 2 z = L must be = 0. So, these 2 boundary values we must be knowing.

If you want to solve how much power is being exchanged between these 2 modes, this boundary condition is known we have to find out that one based on that we have to solve these 2 differential equations. So, how will solve that first what do we do we just take this one I want to solve for example A 1 first A 1 z. So, here A 2 z is there this A 2 z I have to replace from here.

So that I know A 2 equal to A 1 something is there so what I do this equation I do differentiation this equation with respect to z once more. So then we can get this one d 2 A 1 / dz 2 and then - j kappa and dA 2 / dz and I keep as it is this one and then next one I have to differentiate this one. So I will be getting j delta beta, j delta beta will be multiplied by - j so it will be + 1 + kappa delta beta A 2.

So, just make a differentiation this equation with respect to z we get this one now, you know these value I can bring from here and these value A 2 I can bring from here A 2 I can represent in terms of differentiation on A 1 and dA 2 / dz I can take here I can substitute here then ultimately this one will be complete equation for A 1 that will be completely decoupled that equation will be completely decoupled from A 2.

But inherently more 2 modes are coupled that that much we know but mathematically we can get independent equation for A 1 now. So, that equation will be looking like this I just substitute this one dA 2 this one dA 2 / dz equal to this one I substitute here - jk and then kappa delta beta A 2, A 2 I take from here I substitute here then I get this one and move on to that on a little bit simplify then you get this nice equation.

So, this equation in works only A 1second order simple differential equation, but it is a complex differential equation the solutions A 1must be complex because j involve here delta beta is given here 2 modes here. So, we need to solve this one the solving this equation and we know that solving differential equation you will be getting constant and that constant values can be derived by these boundary values.

So, we know that how we have solved similar type of differential equation for co-directional coupling but here only contradirectional coupling this - kappa square it will be kappa square there so, that is the only difference we consider this one after solving this one using this condition I get A 1 z same passion I have just little bit I did here few step I skipped because it is a procedure the same, the second order differential equation, you can just simple method you can use and use the boundary conditions boundary values.

Then you get A 1 z will be like this and A 2 z will be like that, just a few steps you can try a similar like whatever we have solved for co-directional coupling case and in this case, S we have represented as a kappa square delta beta / 2 you remember that in co-directional coupling is was defined by kappa square + delta beta / 2. So, contradirectional case 1 minus sign plus sign minus sign is there that is why this minus sign translated here and we get a solutions A 1 z, A 2 z.

So, that means, I now know how this A 1 z, A 2 z will be evolved as a function of z as far as if I know delta beta value and if I know the kappa value, because delta beta and kappa value if you know then we will be knowing s once we know s then I will be able to find out how z dependent amplitude will be varying for forward propagating wave and how z dependent value will be varying for a backward propagating wave.

So, we have now solutions so, this solution from this solution if you see both A 1 z and A 2 z in terms of A 1 0 because A 1 0 is the value we know A 2 z = L that is actually 0. So, once we know A 1 0 value, how much amplitude I am launching in the forward direction I know what is the z dependent variations will be there for forward propagating wave and this is also A 1 0 as a function A 2 z as a function of A 1 0, A 1 mode means A 2 z will be just proportionally it will be there.

So, I can say that reflection coefficient suppose this length is L that means I can say what is happening up to L this one and what is the value at z = 0 for example, z = 0 if I just z = 0 I will be getting A 2 z = 0 that means I have here I have launched here A 1 0 this direction and then these direction I will be getting A 2 0 that means if I put z = 0 I will be getting whatever field amplitude is reflected in the backward direction at z = 0.

So, if I just put A 2 z = 0 and A 1 0 take ratio once I put z = 0 then numerator will be sL and this will be sL this will be sL, so sL is written A 2 0 / A 1 0 what it means it means A 2 0 what is reflected from the system entire system A 2 at 0 whatever value and A 1 0 what is I have launched that means that can be considered as a reflection coefficient. So, I can find out for the entire structure what would be the reflection coefficients in this expression again you see unknown delta beta and s.

So once you know delta beta once you know kappa then I can find s once you know s then I can find what is the reflection coefficient? So, this is how we can solve how the amplitude will be varying if we know all these any electric field propagating mode propagating from the forward direction or from the left to right, how much energy or amplitude will be coupled energy will be transferred to the backward propagating mode.

So, far we have considered beta 1 and beta 2 I have not mentioned that without the waveguide is a single mode or multimoded only the condition we have imposed that the mode one of the forward propagating mode will be coupled to the one of the backward propagating mode and in that case, the backward propagating mode I am launching in one of the forward propagating mode and one of the backward propagating mode will be coupled depending on the delta beta value.

Choosing delta beta value and then if I know that how much energy is being coupled to the how much amplitude is grown to the backward propagating mode and take a ration with the forward propagating mode exactly at z = 0 that will be giving you the reflection coefficient. Similarly, if I tried to make say A 1, A 1 z = L whatever the value you get at z = L and if you divided by A 1 0 whatever you have launched that will be actually your transmission function.

So, I can find out what is the reflection what is the reflection coefficient what is the transmission coefficient and if you take our R R star then you get reflectivity and if I get tt star I get t that will be transmission coefficient transmission fraction of power will be transmitted that will be defined by t and if the structure entire structure is lossless then we know that r + t should be = 1 fine.

So, the solution so, far we are getting very good if it is single mode waveguide that means beta L = - beta 2 only 1 mode will be guided fundamental mode then we can say that the coupling happening for the forward propagating fundamental mode to backward propagating fundamental mode. So, both are same they are propagation constant will be same only direction will be different because only fundamental mode beta L = - beta 2 is defined as beta and again we know beta = beta can be defined from the dispersion relation.

Again and again we had discussed omega / c n effective or omega / beta equal to phase velocity c / n effective. So, that is known now, if I just substitute here beta L = - beta 2 equal to beta that means beta - beta - 2 pi / lambda that is your delta beta. So, delta beta will be delta beta we are getting 2 beta - 2 pi / lambda that is what I have written here. So, forward propagating wave you want to see if there is a coupling between forward propagating fundamental mode to backward propagating fundamental mode.

Because of the past Fourier harmonics m = 1 by the way we have considered when I put 2 pi / lambda not m that means, we are considering only past Fourier harmonics because we can consider that you can design your structure so, that this delta beta will be close to 0 when beta 1 - beta 2 - 2 pi / lambda consider as m = 1. So, in that case we consider this one m = 1 we will consider and delta beta again beta we know that this one this value we just put beta instead of beta omega / c n effective 2 pi / lambda.

So, this is your delta beta now, you note suppose fundamental mode and single mode fundamental mode propagating the forward direction and only coupling possibilities there if you do not want to lose any energy outside then energy can be coupled to the backward propagating fundamental mode. So, the backward propagating fundamental mode or whatever the reflection coefficient you will be getting that is depends on delta beta.

So, delta beta how this delta beta can vary the delta beta can vary by changing frequency, by changing effective index, by changing periodicity. So, once you have your waveguide fixed periodicity fixed modulation fixed that means the periodic perturbation is fixed that means, delta beta can vary with the frequency. So, that means, you can have delta beta equal to zero you can find a solution for one frequency and if you detune the frequency your delta beta will be nonzero.

So, this delta beta you can vary as a function of frequency and add a function of frequency you can see reflection will be there. So, that means is r will be frequency dependent so, you get a reflection coefficient which is frequency dependent it is similar to ring resonator you know the transmission characteristics the transfer function is a frequency dependent particular wavelength is resonant into the ring resonator that particular wavelength the field will be stored inside the ring.

Similarly, in that output that particular wavelength will be missing, but here also we see reflection is a frequency dependent. So, certain frequencies reflected back in completely that frequency will be absent in the transmission. So, we are now ending up with a device again which can be a transfer function can be a reflection can be a function of frequency. So, if you can design properly you can actually use this device for various applications.

So, now, we consider if delta beta = 0 then we can consider if there delta beta = 0 means these value equal to these value. So, for that particular omega whenever you are solving that is actually omega b 2 beta will be 2 pi / c and beta will be omega / c n effective then we consider omega b that is the angular frequency corresponding to delta beta = 0 that angular frequency delta beta = 0 we consider Bragg frequency.

Bragg angular frequency pi c / lambda n effective and if you consider again you know omega = 2 pi 2 pi c / lambda if omega B we can write lambda B. So, if you just substitute 2 pi c / lambda B then what we can get we can get one more equation lambda B lambda B = 2 n effective lambda. So, that means, if we know the periodicity of the periodic structure and if we know the effective index of the average effective index of the structure then we know this lambda B particular lambda B we can find where delta beta = 0 that is what we have solved.

So, delta beta will be 0 for a given waveguide structure and periodicity under measure, that particular wavelength will give you delta beta = 0. So, what happens to delta beta = 0 for the reflection coefficient reflection coefficient and transmission coefficient and we know that when delta beta = 0 that is the longitudinal phase matching condition satisfied in that case coupling will be maximum that means, we can consider this r value will be maximum reflectivity will be maximum.

That means, when lambda B exactly matched to this expression then that particular wavelength will be seeing maximum coupling in the backward direction and if you are just detuning from that lambda B then delta beta is nonzero. So, what you will see that your reflection will be dropping that is the common sense qualitatively we can understand. So, delta beta that is the 0 means longitudinal phase matching conditions satisfied.

So, coupling will be maximum that we have learned from the coupled mode theory and if delta beta is nonzero coupling will be less. So, you will see less reflectivity for those wavelengths. So, that means your reflection is a certain band will be reject it will not propagate in the forward direction it will be reflected in the backward direction around lambda B that is a beauty of a DBR structure. So, around a particular wavelength some band if you want to reflect back use DBR structure.

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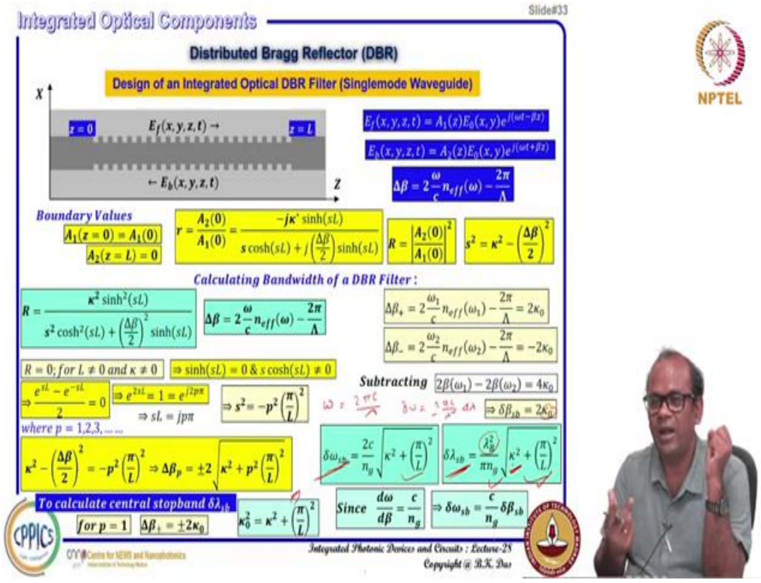

So, now, what we will do, we will just try to discuss about how to design an integrated optical DBR filter with a detailed specification, I want certain DBR structure which will reflect a certain band of wavelength at a certain lambda we can define that as a lambda B that can be 1550 nanometer and I want around 1550 nanometer delta lambda some bandwidth about 1 nanometer to be rejected to be reflected and rest of the wavelength will be traveling without any problem.

Can we design such DBR structure if I need suppose 5 nanometre can we design such structure and if I have how much it will be reflectivity, I need 50% reflectivity in that band, can we is it possible to design that if I want a very high extinction reflectivity, maybe everything should be almost 1 reflectivity 1 can we design that So, that is what we are going to discuss now.

So, we just summarize this is a single mode guide we consider that means forward direction propagating E f same E not x. y j omega t - beta z and backward direction A 2 E not because it is fundamental mode in the backward direction also field profile will be same and backward direction + beta z we consider and delta beta we have defined that beta 1 - beta 2 - 2 pi / lambda. So, beta 1 = - beta 2 = beta we consider then delta beta will be in this one so, we know what is the delta beta and from here delta beta = 0.

Then omega corresponding to omega B and we know that boundary conditions boundary values that we are launching from this side and no backward propagating wave is there at z = L backward propagating wave will be here and forward propagating wave will be here. So, since backward propagating wave coming here that means backward propagating wave will be present also in the input side because whatever reflected back that will continue to propagate in this direction.

And reflection coefficient we have derived by solving differential coupled differential equation and if we want to know reflectivity just take complex conjugate you will be getting that this one complex this is a complex value complex conjugate then we will be getting and s we have defined s is actually depend on kappa and delta beta. Now, we want to design what first of all we need to know kappa.

So, thing is that obviously, this reflectivity depends on kappa as well as delta beta. We said that any coupling possible when phase matching condition is close to 0 delta beta close to 0 and kappa is nonzero. So, delta beta tends to 0 and coupling coefficient must not be equal to 0 higher the coupling coefficient more strength coupling strength will be more. So, first thing is that calculating coupling constant for this type of DBR structure.

How to do that, we just try to see the cross section, any cross section if you see in the DBR structure, we will see similar to this one. So, you see, this is the entire waveguide width that means I am talking about this one, that is traced along X direction entire width X direction, this is w and then you see width is modulated, the modulated width shown here this side and the side I have shown here one site that means x 1 to x 2 and y 1 to y 2 this is a re waveguide structure silicon on insulator just example we have given earlier also.

This is some refractive index substrate that is buried oxide box oxide and n c can be here can be oxide also and this is n d that is the device layer refractive index normally we know that n d greater than n s greater than n c or equal to n c. This is the condition and we know that any 2 mode involved in coupling the coupling coefficient defined by for a periodic structure kappa it is 2 mode, mode 1 and mode 2 coupling between mode 1 and mode 2.

Because of the 1 Fourier coefficient m = 1 omega epsilon 0 / 4 Fourier coefficient b m instead b m you remember that b m = 2 / m pi we consider for rectangular periodic perturbation, we consider 1 and mode 1 field distribution mode 2 field distribution and then refractive index modulation. So, we see that in the cross section this region refractive index is modulated this radial refractive index is modulated that means, this region instead of silicon now, n d you are getting n c.

So, in this 2 region I have delta n = n d square instead of n d square you are having n c square. So, that is actually delta n square. So, delta n square means refractive index square how much refractive index square is changed in this region and in this region so, delta n square I know but that that is true only x 1 to x 2 and y 1 to y 2 region this type of refractive index modulation you have done in your periodic perturbation.

So, that means, this one I can write simply n d square - n c square and that is happening which region x 1 to x 2, y 1 to y 2. So, I can write x 1 to x 2 I should integrate y 1 to y 2 I integrate. So, whatever the E 1 x y E 2 x y in this region is there how much overlap is there with the grating modulation that will be your kappa value simple only thing is that you need to know propagation a field distribution of guided mode if it is single mode you can solve numerically.

What is the full vectorial method we have discussed? So, use that and you get your field distribution. So, now what you get I just simply C 12 I am writing here and I know that C 12 equal to C 21 - 1 star that is a Fourier coefficient we discussed that complex conjugate of the Fourier harmonics that is backward m equal to plus and minus they will be equal and we define that kappa this is the kappa value we write omega epsilon 0 / 4 and b 1 b m we have written as j / m pi.

So, b 1 = j / pi I have written and I will come to this point here a little later and then delta n I said that in that n d square / n c square that will be their integration will be x 1 to x 2, y 1 to y 2 I have written that one, but these one what is this. This you know that whenever we just use a modes mode field distributions we consider this mode field distribution E 1 star x, y, E 2 x y dx dy = delta 12 2 omega mu / beta.

So, that type of non while developing coupled mode theory we have used that 1 mode is actually can be associated with 1 watt normalized to 1 watt and that integration is if it is the same mode than 2 omega mu / beta we have used that orthogonality condition utilized and since now I am calculating numerically I have to consider there this individual mode will be normalized and normalization will be first mode will be normalized to if you write it will be in field it will be omega mu naught / beta 1.

And another will be 2 omega mu naught because this is a square comes. So, when you are getting individual field that will be 2 omega mu naught / beta 2 and if you multiply that one that is coming like that, if 2 modes are different, but our interest is that mode 1 and mode 2 will be same that means beta 1 = beta 2 should be called omega / C n effective that is the case So, I can write here beta 1 beta 2 omega / c.

So, if I just substitute n effective here then it will be simpler equation. So, for single mode waveguide E 1 x y equal to E 2 x y = E naught. So, just put here all the values I am putting simplifying here and of course this 2 why this 2 is coming because I am integrating x 1 to x 2, y 1 to y 2 only this region if your perturbation is the other side that means, you have to 2 times you have to multiply if perturbation only 1 side then this 2 is not required.

So, there are also DBR structure people demonstrate having grating structure in 1 side only. So, this is kappa value we can simply calculate if we know the field distribution and if we know what is the perturbation region and we just multiply n d square - n c square and n effective of the guided mode. So, that is that straightforward whatever we have developed using coupled mode theory from there we can find out what is the kappa value.

So, kappa value is independent of periodicity but it is dependent on duty cycle depending on the duty cycle you have this expression b m = j / m pi. So, it is actually considered for 50% duty cycle b m we consider this is actually this expression is for 50% duty cycle if duty cycle is differing then whatever the value comes the b m expression that has to be considered. So, in this case we consider the duty cycle is 50%.

So, p equal to half we consider and duty is p lambda you remember we have discussed on that one. So, kappa value we know how to calculate you have a single model guide and you will know how much perturbation normally you know coupled mode theory developed based on the weak perturbation. A perturbation is very strong this type of kappa calculation may not match actually it can differ.

If you calculate like this way in that case if it is a strong perturbation then you have to go for direct solution of Maxwell's equation you may not get any analytical formula. So, you have to solve Maxwell's equation for the entire structure so called FDTD method finite difference time domain method and then you can get how much you can find out what is the field reflecting backward direction then you can find you do not need coupled mode theory in that case.

Coupled mode theory works for normally for small perturbation, but most of the perturbation we use for DBR getting structure somehow this kappa calculation matches very well. So, this is what it is shown. So, for example, a waveguide dimension of 500 nanometer and device thickness is this one this device layer thickness is 220 nanometer fondly used and h that means the slab height is 150 nanometer. So, if it is 220 nanometer that means this one is just 70 nanometer raised. So, if this is the case then we can consider x 1 - x 2 = delta W.

This side delta W this side delta W and both sides this delta W if you are varying an x function here and if you calculate kappa by solve numerically they need to close like that. So, as you increase the modulation that means width variation periodic width variation both side if you are considered 10 nanometer means 10 nanometer this side x 2 - x 1 and 10 nanometer this side you are modulating both sides.

So, in that case kappa value it is coming you see per micrometer 0.00 pi per micrometer kappa expression if you see dimensionally that comes with per micrometer. So, as the modulation increases the periodic modulation periodic perturbation that increases your kappa value increases. So, stronger and stronger kappa will be stronger because this integration value will be increasing more and more.

So, again I said that this kappa this coupled mode theory everything it matches very well when kappa is smaller here we have considered up to 50 nanometers that means 500 to 50 nanometer perturbation 500 is the width 50 nanometer is a perturbation that means 10% modulation. So, up to 10% we say that somewhat it can match what our actual scenario is that.

The coupled mode theories are highly approximation you have considered first condition is that the field is amplitude that A 1 z and A 2 z that is actually slowly varying amplitude second order derivative we have ignore. So, that is how stronger perturbation it will not be very much useful to develop coupled mode theory but you can get an idea trained if you use this one even if it is kappa is more you calculate using coupled mode theory.

You get a trained at least you can say that okay I am going to get this much reflectivity or this much reflection coefficient. So, kappa is the first thing we need to decide how much kappa we want higher the kappa I can get stronger the reflectivity reflection coupling from the forward propagating mode to backward propagating mode. So, kappa I can decide according to our DGR specification.

Now, next thing is that calculating field amplitudes a 1 and A 2 z I would be now interested to know how A 1 z actually varying as a function of z for a given kappa I know that A 1 z expression I have derived earlier with this boundary values A 2 z I had the analytical formula. So, I know now kappa value I can with a certain modulation I can estimate what is the kappa value I can now put delta beta = 0.

For example or some value some detuned from omega B or lambda B some value will be there I can substitute here and then as a function of z I can find a 1 0 I can consider so z = 1 as a function of z I can find how A 1 z it is you see cos hyperbolic sine hyperbolic cosine so hyperbolic cosine it is coming. So, now if I plot it you can use your MATLAB program to plot this one to see how it is varying whether this A 1 z is significantly reducing as you function of z or it is slowly varying.

Something like that you can find out similar thing can be happening A 2 z you see this is the plot for delta beta = 0 I have consider exactly page matching condition and then I find delta W I consider about 25 nanometer corresponding kappa is approximately 0.01 per micrometer and we consider periodicity about 292 nanometer. So, periodicity 292 nanometer we consider to match the lambda be exactly equal to 1550 nanometer.

That is the communication band exactly middle of the C band optical C band then I see that A 1 z at z this is equal to 0 this is z = L 100 micrometer 100 micrometer long grating I consider with a periodicity 290 nanometer duty cycle 50% modulation 25 nanometer this side 25 nanometer this side 25 nanometer A 1 z that means this one I am just talking mode of that one you say starting from L I am consider that A 1 0 this one actually equal to 1.

Say it slowly almost exponentially it is decreasing and up to here it is reaching something like that and then what do you see A 2 z as I said that A 2 z = L = 0 that is 0. Then that will be actually increasing, increasing this direction, backward direction property so backward direction propagating field strength will be more and more towards z = 0 because it is building something like that.

So when it is coming this one at z = 0 backward direction the field will be at this amplitude and this will be amplitude. So, in this direction here I will be getting the backward propagating wave mode will be around say 0.55 amplitude and forward direction it will be 1 so forward in this region I will be getting forward direction amplitude is 1 and backward direction mode also will represented that will have 0.55.

So if you take ratio that means you can find out the reflectivity is 0.55, 55% will be reflected. So, that is what we get. So, that means if we just consider kappa equal to this one n = 100 micrometer then you get about 55% will be reflected or something 50 means amplitude 0.55. So, R square will be less 25% or so, it will be reflected back. So, now we know that if I use for a given kappa value if I use 100 micrometer, whether everything will be reflected or not we find that not everything will not be reflected.

So, what do we do? We do 2 things we can increase the kappa value or we can increase the length mode we have increased to just show here. So, here if you see now delta W that means modulation we have increased to 50 nanometer both side this side 50 nanometers that side 50 nanometer. This is 50 nanometer. This is 50 nanometer. Kappa is increased to 0.23 kappa calculation we have shown earlier how to do that and lambda same periodicity and length you are considered 200 micrometer.

Then we see interesting because kappa increased you see within 100 micrometer length z = 0 to z = 200 you see, this is actually your A 1 z, A 1 z a rapidly falling reducing and if you see if you do here you have backward wave also you see as it falls backward propagating wave will be also increasing that means almost 100% it will be reflecting back A 1 z will be decreasing and as you decrease backward propagating mode will be picking up.

So, you get almost 100% reflectivity and that even you do not need to go up to 200 micrometer, but this kappa if you just terminate here about 120 micrometer getting length, then your entire field whatever you are launching from this side, that will be reflected back. So, you can either increase L or increase kappa. So, depending on that you can find out how much you want to reflectivity around delta beta = 0 corresponding lambda B you can just find out lambda B = 2 n effective period that is what we have derived earlier.

So, one thing is that reflectivity we can estimate kappa value depending on the kappa value I can just predict what is the reflectivity? I would get; for a given length if I see that we I cannot increase kappa more because of loss is or some technological problem then I can go for longer length to get more reflectivity. So, that is what we get. Now, next thing is that calculating reflections and transmission spectra I said that delta beta = 0 you coupling will be more maximum and kappa must be not equal to 0 more kappa is better.

But again I said that delta beta close to 0 just a little bit detune that also will give you some effect in coupling. So, around the delta beta = 0 and corresponding lambda B = 1550 nanometer I would see some kind of band. So, band will be rejected maybe reflectivity may not be as high as for delta beta = 0 but you may get a band. So, to get a transmission and deflection spectrum how to process it you know we have R equal to here we have written and capital R this is the amplitude deflection coefficient.

This is the power reflectivity just complex conjugate if you take this one is the value you will be getting and we know that delta beta equal to this one this delta beta wave just reproduced here. So, just omega you vary then you can get when matching exactly delta beta 0 that means, I can find this one if you are putting 0 then I can find it omega B corresponding to this value for a particular period omega B I will be getting lambda B I will be getting now.

You just detuned your frequency or wavelength from that omega B that means delta beta I am detuning from 0. So, in that case but delta beta = 0 if I put I know reflectivity equal to this one if we just put delta beta = 0 that means S will be delta beta 0, S will be equal to just kappa, S = kappa delta beta 0 then what I will be getting here I will be getting sin hyperbolic and distributed this term will go once this term will be go weighing then find S square will become kappa square.

So, sine hyperbolic square divided by cost hyperbolic square you will be getting tan hyperbolic square for delta beta = 0 reflectivity will be tan hyperbolic square compile and we know delta beta = 0 corresponding omega B I derived earlier again repeating here and lambda B corresponding to this one and if it is lossless case because this DBR structure no loss there will be some loss technically because there will be some kind of roughness etc will be introduced.

Some loss will be there but for understanding for analytical discussion purpose we consider that it is a lossless case. So, in that case reflectivity and transmissivity R and T just add they must be equal to 1. So, once you know R, then you can find t = 1 - r that means, energy conservation whatever power will be transmitted that actually related to whatever power is reflected R + T will be reflection plus transmission total will be 1 if you are launching power equal to 1 watt for example, you can normalize to that.

Now, we see you plot for a delta W = 25 nanometer that is corresponding kappa = 0.01 micrometer we calculated earlier for these waveguide parameters obviously and lambda = 252 nanometer periodicity and 100 micro meter grating length you see, this is your transmission characteristics, this is your reflection characteristics that means, this is your R and this is your T. T = 1 - R basically you see at exactly around 1550 nanometer that is what our period is matched because lambda B = 2 n effective lambda.

You know the effective index of the waveguide and perturbation periodicity is lambda. So, n effective periodicity 292 and n effective of this waveguide if you calculate and multiplied by 2 that would be lambda B we matched exactly 292 nanometer and after MATLAB simulation it can it is showing that around 1550 nanometer you see you have a not this one is not R this is actually T and this is actually R.

This is the reflection you see this is the reflectivity this part it is reflected back and what is reflected back that will be missing in that transmission. So, this is the transmission characteristics blue one here it is transmission characteristic red one is a reflection characteristics. So, you see reflectivity here if you see how much you are getting nearly 60% and 60% at 1550 nanometer.

Then transmission there you are getting 40% that means, exactly at 1550 nanometer wavelength lambda B = 1550 nanometer wavelength you can expect 60% reflectivity 60% of the power will be reflected back 40% will be transmitted but again you see since delta beta around 0 is 1550 nanometer corresponding to 1550 nanometer need to be adjusted delta B, but if you detune by detuning the wavelength it will detune the wavelength omega will be detuned omega will be detuned means delta beta will no more be 0.

So, if you can increase the wavelength from lambda B positive direction or negative direction delta beta will be increasing compared to 0 when it is increasing coupling will be less coupling means forward coupling between forward propagating mode and backward propagating mode and then you see your reflection will be dropped rapidly and transmission also you will be seeing back.

So, specific band particular band you are getting in the reflection and that will be missing in that transmission. So, it is a very nice device you can design suppose I want this much reflectivity this much bandwidth, I can design that. So, suppose what decides this bandwidth, we will discuss that how much band what is the 3dB bandwidth for example, F W H m that actually very important for application point of view.

So, let us move on to that before going into that if I just see, if you are just going for a little bit stronger coupling both side 50 nanometre modulation this side width is 50 nanometre this side 50 nanometre. So, corresponding kappa if you calculate that is actually 0.23 micrometre per micrometre period I have not kept same and I have considered 200 micrometre long need not be 200 micro meter because we have shown that for this kappa even 150 micrometres is enough.

But we consider 100 micrometer, just to get reflectivity and transmittivity transmission reflection and transmission. So, you see now, this reflection is the red curve this is R and this is your transmission so, along with the main peak, this is actually lambda B corresponding to lambda B around 1550 nanometer you get a particular band almost flat top reflection you are getting with a particular band, but you get also some side loops both side reflection side loops.

So, normally that side loops also sometimes very important sometimes it is very problematic also some applications. So, there are some engineering methods some design things are there you can actually design your DBR structures with some kind of upward digestion etc. so that you can actually remove this side loops, but here our interest is that how to decide a design a DBR bandwidth with very high reflectivity.

So, for that purpose we take help of only the reflection spectrum just concept transmission and I have just removed and reflections consider and we can consider this as your Bragg wavelength according to this one, the Bragg wavelength is defined 2 n effective lambda. So, lambda if you put any n effective waveguide if we calculate then lambda B exactly will be getting around 1550 nanometer and so on.

Now, you see as you go away from lambda B this side or this side that means your delta beta not equal to 0 then your reflectivity drops and it is 0 value, you see this will be minimum at first minimum and second minimum third minimum fourth minimum and so, it will be side loops will be coming like that. So, we consider that where is actually happening first minimum right side and first minimum left side that particular width if we defined delta lambda S B that is actually called stopband.

So, called S B stands for stopband delta lambda S B stopband that can be considered as a bandwidth another definition is 3dB bandwidth of course, but since we get a clear 0 in both sides you 0 reflections. So, I can consider this band between this 0 first 0 both side that can be considered as a bandwidth. So, how to do that I know this one all this expression I have written a little bit move on what we do R = 0, for L not equal to 0 kappa not equal to 0 is that possible.

Certain length is there getting length L not equal to 0 kappa also not equal to 0 because your grating is there period is also 292 nanometer is there. So, for that if you just inspect this one this characteristics carefully when it will be 0 you have a numerator denominator obviously anytime numerator 0 means R will be equal to 0. So, we can say that sin hyperbolic sL must be equal to 0 but sine hyperbolic sL if you just put must equal to 0, this will be putting equal to 0 at the same time sL square cos hyperbolic sL should not be equal to 0.

If this is equal to 0 then it will be indeterminant 0 by 0 indeterminant I want exactly 0 R equal to 0 when that is possible. Let us stick first if I put sine hyperbolic sL equal to 0 hyperbolic function if you just write sin hyperbolic function x is normally e the power x - e to the power - x / 2 and cos hyperbolic x = e to the power x + e to the power - x / 2 that is actually like a normal method.

Normally when you consider sin x, normally e to the power ix - e to the power - ix / 2 you define similarly because hyperbolic so x is the real here. So, in that case we can write this one so sin hyperbolic sL we can write like this. So if that is actually equal to 0 you can just little bit do little bit algebra e to the power 2sL must be equal to 1. So for R = 0 this is one condition numerator I am considering.

So, it will be j 2 p pi it should be equal to 2sL, 1 means instead of 1 I can write like that. So that means this sL if it equal to j p pi, then this one will be numerator will be 0 but I know that sL = j p pi and what is p, p is integer p can vary from 0, 1, 2, 3 that is your p value but once you get p = 0 s will be equal to 0 because L not equal to 0. So, that means this one also will be 0 this part also will become 0.

So, that means when I put sL = 0 for p = 0, that means this is 0 this is 0, this is also will become 0 that means indeterminant that is a p = 0 will not consider rather what you consider p instead of starting from 0 we can say 1, 2, 3 so on, so p is defined like this, so p = 1,2, 3 and so on the numerator will become 0 and denominator will become non-zero. So in that case I can get R values will be 0.

So p = 0 means p = 1 means we will be getting first minima p = 2 means another R = 0, p = 3 that means P I am varying means I am changing very basically delta beta if I am just changing s value sL changing means s actually directly depends on this one so s is varying means delta beta is being changed. So, delta beta changed means I am considering minima I can get periodically when p = 1, 1 minimum R = 0 p = 2 another minima p = 3 another minima and so on I will be getting so little bits replace it.

sL is this one means s square L square that means j = - 1 j square p square pi square L I take this side s square this one why I have written s square because I know the s square expression s square expression is kappa square - delta / 2. So, that means this s square I have written here and p square pi / L square I have written here. So, now, I can write delta beta now is discretized value for certain delta beta p value according to 1 I can write 1, 2, 3, 4 so on.

For that particular value R can be equal to 0 we remember that delta beta = 0 R cannot R is the maximum now, I consider R I am getting 0 for delta beta p this expression plus minus this one. So, delta beta kappa square I take this side so, I write so, p = 1 means I will be getting delta beta p 1 and there will be 2 values so, delta beta plus minus direction I also I will be getting minima's and left hand side that is the minimum all the minima whatever I am getting in the transmission spectra that I can actually express analytically.

For which delta beta but here delta beta is represented in terms of lambda we know how to convert delta into lambda or frequency, this one delta beta I just change omega then delta beta will be changing from 0, I have some value 0 omega B I will be there now I am just changing from that omega from omega B then I get delta beta non-zero. So, that corresponding omega can be lambda.

So, I can get corresponding delta beta value and corresponding delta beta value will give you these values. So, this is straightforward mathematical equations you can just try to do that. So, to calculate central stop band delta lambda B I have to consider p = 1 for p =1 I will be getting delta beta p 1 value plus another value minus plus minus because square root is there I just tried to find out delta beta plus minus value so, what we called delta beta p plus equal to + 2 kappa naught and delta beta minus.

I will be writing - 2 kappa naught what is kappa naught kappa naught I am just considering these one because kappa square plus you have to add pi / L pi square / L square that total thing I am just representing as a kappa naught square. So, that means delta beta plus when delta beta plus when delta beta is plus 2 kappa naught there I will be getting one R = 0 and - 2 kappa naught there also I will be getting 1 0.

So, that means this corresponding to delta beta = 2 kappa naught plus and this minima corresponding to delta beta - 2 kappa naught where kappa naught equal to this one kappa naught square is defined by this one. So, I need to understand what is this what is corresponding lambda here and what is corresponding lambda here and if you subtract then you will be getting the bandwidth of the filter.

So, you can find out what is the particular bandwidth you want to reject from this structure. So, now we go this delta beta plus corresponding to 2 kappa and I corresponding frequency I write omega 1. So, delta beta + 2 omega 1 / c n effective - 2 pi / lambda and 2 kappa naught kappa naught value I know here and delta beta minus I will be writing another frequency omega 2 2 omega 2 / c n effective omega 2 like this.

So, this thing basically what I am saying that this is corresponding to omega 1 and this is corresponding to omega 2 corresponding lambda you can calculate that is straight forward. So, I know that a certain omega 1 I supposed to get 2 kappa naught delta beta and another frequency I get - 2 kappa naught so, between omega 1 and omega 2 whatever the value band that particular band sees reflection from the structure.

So, if we subtract these 2 what I get this one basically we write beta omega 1 omega / c n effective for omega 1 frequency so, I write 2 beta omega 1 - 2 beta omega 2 this value this value will be cancelled and 2 kappa naught - 2 kappa naught it will become 4 kappa no naught t and 2 2 cancel it will be 2 and beta omega 1 and beta omega 2 whatever the value.

So, I am getting beta omega 1 - beta omega 2 that is what we have written as delta beta sb what I am writing because within that whatever beta comes that is actually sees reflection. So, that is why we called it delta beta stop band. So, if we just subtract this beta omega 1 and beta omega 2 within that so, beta omega 2 to beta omega 1 so, omega 1 to omega 2 whatever beta comes that beta values actually will not see any will see some kind of reflection in the main peak.

So, that delta beta sb I have written as 2 kappa not so, beta 1 beta 2 again I want to find out corresponding frequency. So, delta beta 2 frequency I have to converse all lambda I have to convert I know that omega = beta equal to I know that beta equal to omega / c n effective. So, if I do d beta / d omega then I know that this is n g / c that we have earlier already discussed. So, d beta / d omega equal to basically 1 / v g or d omega / d beta equal to group velocity and phase velocity.

So, from the dispersion relation I can write this one d omega / d beta equal to c / n g. c is the velocity of light and n g is the group index. So, I get the group velocity c / n g d omega / d beta now, I can find delta beta and delta omega relationship if we know the group velocity. So, I write delta omega if it is stop band corresponding delta omega stop band corresponding delta beta sb. So, delta beta sb if I scale to frequency.

That means, I have to multiply n g / c whatever delta beta whatever value 2 kappa naught is there if I multiply n g / c then you will be getting frequency domain stop band a straight forward so, if we do so, frequency movement that means 2 c / n g kappa naught, kappa naught value is this one square root of this one. So, delta omega is equal to this one and again you know omega equal to 2 pi c / lambda.

So, delta omega equal to I can get delta omega = 2 pi c / lambda square delta lambda minus sign will be there minus sign if we do not consider then this delta omega you can consider delta lambda sb. So, this is an important formula I can find out suppose I want a particular band to be reflected I have to define kappa value, I need a particular kappa value and if I know L and periodicity defines the Bragg wavelength.

I know the group index then I can find out how much band what is the bandwidth it will be reflected back. So, I can define it I can design a DBR distributed Bragg reflector which can actually reflect a particular band and that particular band is defined by length. So, longer the length longer the length it will be narrower L can be infinite very large then this can be 0 then in that case only kappa dependent for very long grating structure I think length does not matter what is the bandwidth it is reflecting what reflection bandwidth.

But for a shorter length of the grating it is actually related with the L we can define that one. So, that is how if I go back to this this structure so, this band this stop band particularly this whatever we have represented this delta lambda sb that actually depends on your lambda B square this lambda B square / n g proportional to n g and then square root of kappa square + pi square / L square this this is the expression already we have defined here.

We have discussed lambda B square pi n g with this I stop here for you know now you have learnt now how to design a DBR structure if it actually if you know that it is it has to be designed in silicon, silicon on insulator platform silicon photonics platform for photonic integrated circuits you know the waveguide dimension what is the technology limitations how much modulation you can do and depending on that, you can actually design your DBR filter for a certain bandwidth of spectrum to be reflected or to be stopped, thank you very much.