

Integrated Photonics Devices and Circuits
Prof. Bijoy Krishna Das
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 29

Integrated Optical Components: Distributed Bragg Reflector (DBR): Device Design - Part 1

Hello everyone, in this lecture today we are going to discuss distributed Bragg grating reflector, distributed Bragg reflector especially the design aspects I will be discussing today. So, first I will consider using distributed Bragg reflector one can demonstrate various kinds of devices including laser, active devices, adddrop multiplexers but one of the important aspect is that it is just a reflector a band of wavelength around Bragg wavelength can be reflected that means that band will be stopped in transmission.

So that is why it is called bandstop filter. So, I will be discussing bandstop filter how one can design using a single mode waveguide structure. Similarly, I will be discussing the aspects of bandstop filter with a multimode waveguide structure.

(Refer Slide Time: 01:30)

The slide, titled "Distributed Bragg Reflector (DBR): Device Design", illustrates the design of a bandstop filter using a single-mode waveguide. It features a schematic of a waveguide with a DBR section between $z=0$ and $z=L$. The electric field components are defined as $E_f(x, y, z, t) \rightarrow$ and $-E_b(x, y, z, t)$. The DBR section is characterized by the equations $E_f(x, y, z, t) = A_f(z) \tilde{e}_0(x, y) e^{i(\omega t - \beta z)}$ and $E_b(x, y, z, t) = A_b(z) \tilde{e}_0(x, y) e^{i(\omega t + \beta z)}$. The Bragg condition is given by $\Delta\beta = 2 \frac{\omega}{c} n_{eff}(\omega) - \frac{2\pi}{\Lambda}$, with $\omega = \frac{2\pi c}{\lambda}$ and $n_{eff}(\omega)$. The slide also includes logos for CPPICs, NPTEL, and IIT Madras, along with the text "Copyright © B.K. Das".

So now, let us again recap the DBR structure it is basically top view of representative silicon on insulator waveguide structure. So, this is your waveguide you can consider this is your single mode waveguide, this is also single mode waveguide output and input also single mode waveguide but in between from $z = 0$. So, from here to here $z = L$ this is z direction, there is a periodic perturbation.

This periodic perturbation you can see that rectangular perturbation structures are there in both sides in the sidewall it is just defined in the sidewall and in this type of structure you know that the forward propagating wave will have electric field associated to forward propagating mode is this one and we can define also backward propagating wave because of the distributed reflections you can generate backward propagating wave.

If you are considering only launching from this side also only and this E_f and E_b backward propagating wave you have discussed earlier that how that can be expressed this $A_1(z)$ and $A_2(z)$ that is the variation evolution of forward propagating wave and backward propagating wave respectively along as a function of z propagation direction. And you know this $\Delta\beta$ that means a phase mismatch factor $\Delta\beta$ that is defined by $2\omega/c - \beta$ ω is the operating frequency.

ω as we know that is defined by $2\pi c/\lambda$ and $n_{eff}\omega$ that time n_{eff} effective λ you can express. So, as a whole you can express $\Delta\beta$ that means propagates difference in propagation constant that phase mismatch factor is can be expressed in terms of λ and this λ capital λ is the period of the grating structure from here to here this is the λ period. So, all these we have discussed in earlier classes.

(Refer Slide Time: 04:01)

The slide content includes:

- Slide#3** (top right)
- NPTEL** logo (top right)
- Distributed Bragg Reflector (DBR): Device Design** (title)
- Design of bandstop filter with a singlemode waveguide** (subtitle)
- Diagram:** A waveguide of length L with a grating. Forward wave $E_f(x,y,z,t) = A_1(z)E_0(x,y)e^{j(\omega t - \beta z)}$. Backward wave $E_b(x,y,z,t) = A_2(z)E_0(x,y)e^{j(\omega t + \beta z)}$. Grating period Λ . Phase mismatch $\Delta\beta = 2\frac{\omega}{c}n_{eff}(\omega) - \frac{2\pi}{\Lambda}$.
- Boundary Values:**
 - At $z=0$: $A_1(z=0) = A_1(0)$, $A_2(z=0) = 0$
 - At $z=L$: $A_1(z=L) = 0$, $A_2(z=L) = 0$
- Reflection Coefficient:** $R = \frac{|A_2(0)|^2}{|A_1(0)|^2}$
- Transmission Coefficient:** $T = \frac{|A_1(L)|^2}{|A_1(0)|^2}$
- Equation for R:** $R = \frac{-jk' \sinh(\beta L)}{\cosh(\beta L) + j(\frac{\Delta\beta}{2}) \sinh(\beta L)}$
- Equation for T:** $T = \frac{1}{\cosh(\beta L) + j(\frac{\Delta\beta}{2}) \sinh(\beta L)}$
- Handwritten notes:**
 - "Amplitude reflection Coeff." (under R)
 - "Reflection" (under R)
- Logos:** CPPICs (bottom left), Integrated Photonic Devices and Circuits: Lecture-29 (bottom center), Copyright © B.K. Das (bottom center), NPTEL (bottom right)

Now, as you know if we just consider boundary values something like that $A_1(z=0) = A_1(0)$ that means I am launching here in forward direction with a amplitude $A_1(0)$ that means here a fixed value you are launching at $z = 0$ if you put. And then the reflection coefficient that is r is

represented as amplitude reflection coefficient that also we have derived using coupled mode theory amplitude reflection coefficient.

So, this is the amplitude reflection coefficient or that can be defined as whatever backward direction amplitude is there that is actually we call A_2 divided by whatever A_1 launched here. So that is your reflection coefficient for the entire DBR structure and that is defined by this one here when $\Delta\beta$ if it is 0 that means this one this second term in the denominator will not be there and then in that case, we know the reflection coefficient is a tan hyperbolic so on $\Delta\beta = 0$.

And reflectivity that means intensity power reflection coefficient, so called reflectivity, this will be called as the reflectivity just you take it this is a complex you take complex conjugate multiply then you get reflectivity and where s is defined as $\kappa^2 - \Delta\beta / 2$ whole square κ how can be estimated we have also discussed in the previous class.

(Refer Slide Time: 05:57)

The slide, titled "Integrated Optical Components" (Slide#4), focuses on "Distributed Bragg Reflector (DBR): Device Design" for a "Design of bandstop filter with a singlemode waveguide".

Diagram: A schematic shows a DBR structure of length L along the Z -axis. Incident and reflected electric fields are given as $E_f(x, y, z, t) = A_1(z)E_0(x, y)e^{j(\omega t - \beta z)}$ and $E_b(x, y, z, t) = A_2(z)E_0(x, y)e^{j(\omega t + \beta z)}$. The detuning is defined as $\Delta\beta = 2\pi n_{eff}(\omega) - \frac{2\pi}{\Lambda}$.

Boundary Values:

$$A_2(z=0) = A_1(0)$$

$$A_2(z=L) = 0$$

$$r = \frac{A_2(0)}{A_1(0)} = \frac{-j\kappa^2 \sinh(sL)}{s \cosh(sL) + j\left(\frac{\Delta\beta}{2}\right) \sinh(sL)}$$

$$R = \frac{|A_2(0)|^2}{|A_1(0)|^2} = \frac{\kappa^4 \sinh^2(sL)}{s^2 \cosh^2(sL) + \left(\frac{\Delta\beta}{2}\right)^2 \sinh^2(sL)}$$

Calculating Bandwidth of a DBR Filter:

$$R_{max} = \tanh^2(\kappa L)$$
 For $\Delta\beta = 0$, $s = \kappa$.

Graph: A plot shows R versus $\Delta\beta$. The reflection coefficient is maximum at $\Delta\beta = 0$ and decreases as $\Delta\beta$ increases. The graph includes curves for $\cosh(x)$, $\sinh(x)$, and $\tanh(x)$.

Now, if you see just analyse this R that means, you are just taking complex conjugate of this one this R means $r r^*$ that is actually reflectivity and you are getting this equation. And you know if you just put $\Delta\beta = 0$ $\Delta\beta = 0$ s will become κ and then $\Delta\beta$ a second term will go and then reflectivity normally at $\Delta\beta = 0$ that will be maximum that is why we write $R_{max} = \tanh^2(\kappa L)$ that also you have discussed in the previous lectures.

Now, you see in this function we do have 2 different type of functions sin hyperbolic something x and cos hyperbolic x , x means here s times L , L is the length of the DBR structure and if I want to analyse how these R varies with a $\Delta\beta$ or any other parameter like κ etcetera or length

we need to know how the sin hyperbolic function, cos hyperbolic function looks like you know if you just plot sin hyperbolic, cos hyperbolic function this is a blue curve this is cosine hyperbolic.

So, this is minimum cosine hyperbolic $x = 1$ when $x = 0$ so that will be minimum, but in case of sin so you have to follow this red one. So, red one if you see that at $x = 0$ sin hyperbolic $x = 0$ and then as you go as you increase your x value both sides then it is just increasing to infinity at just a little bit slightly above $x = 1$ it is going very large and at certain value you will see it is almost merging to the value of cosine hyperbolic x .

So now, since it is tan hyperbolic function also we have represented tan hyperbolic function is nothing but it is a simple trigonometric formula we know that tan hyperbolic $x = \frac{\sin \text{ hyperbolic } x}{\cos \text{ hyperbolic } x}$. So that means, if you divide these 2 you get this green curve that is actually for tan hyperbolic x . So, if x means if sL , s means this one kappa or delta beta defined by these if they are detuned. Then we can we know how the sine hyperbolic function and cos hyperbolic function varies with that parameter and we will be eventually getting the results for power reflectivity of the DBR structure.

(Refer Slide Time: 08:54)

Integrated Optical Components Slide#5

Distributed Bragg Reflector (DBR): Device Design
Design of bandstop filter with a singlemode waveguide

Diagram showing a waveguide structure with fields $E_1(x, y, z, t)$, $E_2(x, y, z, t)$, and $E_3(x, y, z, t)$ at different regions. Boundary conditions are given at $z=0$ and $z=L$.

Boundary Values

$$A_1(x=0) = A_1(0) = -jk' \sinh(sL)$$

$$A_1(x=L) = A_1(0) = s \cosh(sL) + j \left(\frac{\Delta\beta}{2} \right) \sinh(sL)$$

$$A_2(x=L) = 0$$

$$R = \frac{|A_2(0)|^2}{|A_1(0)|^2} = \frac{\kappa^2}{s^2} = \kappa^2 - \left(\frac{\Delta\beta}{2} \right)^2$$

Calculating Bandwidth of a DBR Filter:

$$R = \frac{\kappa^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + \left(\frac{\Delta\beta}{2} \right)^2 \sinh^2(sL)}$$

For $\Delta\beta = 0$, $R_{max} = \tanh^2(\kappa L)$

$R = 0$ for $L \neq 0$ and $\kappa \neq 0$ $\Rightarrow \sinh(sL) = 0$ & $s \cosh(sL) \neq 0$

$\kappa = \kappa L \rightarrow \infty$
 $R_{max} = 1$

Graph showing $\cosh(x)$, $\sinh(x)$, and $\tanh(x)$ functions.

NPTEL logo

Copyright © 2012

So now, we look a certain issue R_{max} can be tan hyperbolic square kappa L, so tan function is this one this is tan hyperbolic function x . So that means, this kappa L tends to infinity that means, it becomes tan hyperbolic square becomes R_{max} become 1 that is known. So, maximum value of reflectivity can be one that is pursued, reflectivity it is a fraction of power reflected from the DBR structure with respect to the input power so that can be maximum 1.

So, as you reduce the κL means, this is x this x is reduced, then your reflectivity will be reducing and when $x = 0$ your reflectivity will be 0. So, when $x = 0$ means this κL value but $\Delta\beta = 0$ that is possible either $\kappa = 0$ or $L = 0$. In that case your reflectivity will be 0 that is like a homogeneous waveguide no perturbation nothing is there. Now, I will looking for some other issue, let us consider what could be the minimum value R must be 0 that is what it says.

So, in that case suppose L not equal to 0 and κ not equal to 0, L has a DBR structure as a finite length and κ also has a finite value. So, I will investigate whether R can be 0 for any $\Delta\beta$ value, $\Delta\beta$ can be expressed like this or for any other $\Delta\beta$ means ω or n effective or whatever is there any possibilities there I have learned that $\Delta\beta = 0$, reflectivity become maximum.

But then if it is maximum there can be some other point $\Delta\beta$ not equal to 0 obviously, that is should be $\Delta\beta$ not equal to 0, there may be a chance that reflectivity can be 0 ultimately you see the reflectivity \sin hyperbolic, cosine function is there. So, I would like to know the nature of the reflectivity when $\Delta\beta$ not equal to 0. You see $R = 0$ if you just try to see just looking into the expression it is easy to say that if \sin hyperbolic $sL = 0$ if this one equal to 0 then R will become 0.

But not always it can happen that if this is equal to 0 and these value denominator parts should have some finite value, if denominator itself becomes 0, then $0 / 0$ it is in determinant. So, we should be careful for choosing $R = 0$, so for $R = 0$ we have this is 0 and this value \sin hyperbolic also will be 0, I am just considering that $R = 0$ if this one 0 means this one 0, but at the same time this should not be equal to 0 that is what it is mentioned here.

(Refer Slide Time: 12:15)

Integrated Optical Components Slide 88

Distributed Bragg Reflector (DBR): Device Design

Design of bandstop filter with a singlemode waveguide

$E_f(x, y, z, t) \rightarrow$ at $x=0$ and $x=L$
 $E_f(x, y, z, t) = A_1(z)E_0(x, y)e^{i(\omega t - \beta z)}$
 $E_f(x, y, z, t) = A_2(z)E_0(x, y)e^{i(\omega t + \beta z)}$
 $\Delta\beta = 2\pi n_{eff}(\omega) - \frac{2\pi}{\Lambda}$

Boundary Values

$A_1(x=0) = A_1(0)$
 $A_2(x=L) = 0$
 $r = \frac{A_2(0)}{A_1(0)} = \frac{-j\kappa \sinh(sL)}{s \cosh(sL) + j\left(\frac{\Delta\beta}{2}\right) \sinh(sL)}$
 $R = \left| \frac{A_2(0)}{A_1(0)} \right|^2 = \kappa^2 - \left(\frac{\Delta\beta}{2} \right)^2$

Calculating Bandwidth of a DBR Filter:

$R = \frac{\kappa^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + \left(\frac{\Delta\beta}{2}\right)^2 \sinh^2(sL)}$
 For $\Delta\beta = 0$, $R_{max} = \tanh^2(sL)$
 $\beta_p = 0$, for $L \neq 0$ and $\kappa \neq 0 \Rightarrow \sinh(sL) = 0$ & $s \cosh(sL) \neq 0$
 $\sinh(sL) = 0 \Rightarrow \frac{e^{sL} - e^{-sL}}{2} = 0 \Rightarrow e^{2sL} = 1 \Rightarrow sL = j p \pi$
 where $p = 1, 2, 3, \dots$

CPPICs
 Center for NER and Nanotechnology
 Integrated Photonics Devices and Circuits: Lecture-29
 Copyright © S.K. Jha

So, looking into that we can say that \sin hyperbolic $sL = 0$ let us consider numerator then I know \sin hyperbolic $x = e$ to the power $x - e$ to the power $-x / 2$ that means, \sin hyperbolic 0 means e to the power $sL - e$ to the power $-sL / 2 = 0$, this is a little bit simplify then what you get e to the power $sL - e$ to the power $-sL$ that is actually equal to 0, e to the power $sL = e$ to the power $-sL$ that means, e to the power $2sL$ that is equal to 1 that is what is written here. And one can be written as e to the power $j2p \pi$ that is what it is written.

So, this p you could consider this is true per $p = 0, 1, 2, 3$ integer values, but this 0 value I will not consider why is that, so this turned into the picture that $sL = jp \pi$ from this equation if I compare this to these if I put $p = 0$ L not equal to 0, if I put $p = 0$, $s = 0$. So, if $s = 0$ this part will become 0 $S \cos$ hyperbolic $sL = 0$. So, while choosing $p = 0$ and for \sin hyperbolic sL to make 0 if you choose $p = 0$ then simultaneously this value will become also 0.

So that means $0 / 0$ will come that is that region $p = 0$ value should be omitted because that value is coming indeterminate. So that we have written p should vary from 1, 2, 3 and so on. So, for p if you just follow this equation and with that equation p must not be equal to 0 and p should be starting from 1, 2, 3 then whatever value you get for that sL value you get that will result into reflectivity equal to 0.

(Refer Slide Time: 14:53)

Integrated Optical Components Slide#7

Distributed Bragg Reflector (DBR): Device Design

Design of bandstop filter with a singlemode waveguide

$E_1(x, y, z, t) \rightarrow$
 $E_2(x, y, z, t) = A_1(z)E_0(x, y)e^{i(\omega t - \beta z)}$
 $E_3(x, y, z, t) = A_2(z)E_0(x, y)e^{i(\omega t + \beta z)}$
 $\Delta\beta = 2 \frac{\omega}{c} n_{eff}(\omega) - \frac{2\pi}{\Lambda}$

Boundary Values

$A_1(z=0) = A_1(0)$
 $A_1(z=L) = 0$
 $A_2(z=0) = 0$
 $A_2(z=L) = A_2(L)$

$R = \frac{|A_2(0)|^2}{|A_1(0)|^2}$
 $\kappa^2 = \kappa^2 - \left(\frac{\Delta\beta}{2}\right)^2$

Calculating Bandwidth of a DBR Filter:

$R = \frac{\kappa^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + \left(\frac{\Delta\beta}{2}\right)^2 \sinh^2(sL)}$
For $\Delta\beta = 0$
 $R_{max} = \tanh^2(\kappa L)$

$R = 0$ for $L \neq 0$ and $\kappa \neq 0$ $\Rightarrow \sinh(sL) = 0$ & $s \cosh(sL) \neq 0$
 $\sinh(sL) = 0 \Rightarrow \frac{e^{sL} - e^{-sL}}{2} = 0 \Rightarrow e^{2sL} = 1 = e^{j2p\pi} \Rightarrow sL = j p \pi$
 where $p = 1, 2, 3, \dots$ $p \neq 0$

$\Rightarrow s^2 = -p^2 \left(\frac{\pi}{L}\right)^2 \Rightarrow \kappa^2 - \left(\frac{\Delta\beta}{2}\right)^2 = -p^2 \left(\frac{\pi}{L}\right)^2 \Rightarrow \Delta\beta_p = \pm \sqrt{\kappa^2 + p^2 \left(\frac{\pi}{L}\right)^2}$

CYP © 2005 for DBR and Semiconductors
 Integrated Photonic Devices and Circuits: Lecture 29
 Copyright © S.K. Das

Now you try to see that this one this equation with square because it is j imaginary term is there. So, if you just square it then you get j square, j square = -1. So, minus 1 p square pi / L, I have taken this site. So, from this equation I get this one and again you know s square we have defined like this while developing coupled mode theory. So, if we just put instead of as I just put kappa square - delta beta / 2 then it will be p square pi over L square that is true.

We get delta beta we have just writing for p, p is the integer value we are considering plus minus this one kappa square + p square pi square so I can find out for these delta beta value if it is satisfying either plus or minus of this one when p is running over 1, 2, 3. So, discrete values of delta beta solution will be getting where R can be equal to 0 reflectivity can be 0. So that means, I said that delta beta = 0 your reflectivity can be maximum.

But as you keep on increasing delta beta value both plus direction and minus direction delta beta can be plus and minus depending on this expression if this one is larger than 2 pi / lambda then it is a plus if it is less than 2 pi / lambda capital lambda then it will be negative. So, both direction it can have solutions like these and periodically you can see R equal to minimum. So, you can see that in principle I will show that that the reflectivity normally if you just put delta beta when delta beta = 0 reflectivity can be maximum.

And then it can come 0 and then again another 0 will come that means in between there will be maximum. So, slowly reducing this type of characteristics you will be getting that oscillations will be there is a main band and then both side you can have periodic maximum minimum and that maximum will be slowly in the reducing order if you just plot this one as a function of delta

beta you will be getting this type of characteristics and so that. So, delta beta not equal to 0 reflectivity can become 0 for other values periodically and that depends on using this equation.

(Refer Slide Time: 17:37)

Integrated Optical Components

Distributed Bragg Reflector (DBR): Device Design

Design of bandstop filter with a singlemode waveguide

Boundary Values

$$r = \frac{A_2(0)}{A_1(0)} = \frac{-j\kappa \sinh(sL)}{s \cosh(sL) + j\left(\frac{\Delta\beta}{2}\right) \sinh(sL)}$$

$$R = \left| \frac{A_2(0)}{A_1(0)} \right|^2 = \kappa^2 - \left(\frac{\Delta\beta}{2} \right)^2$$

Calculating Bandwidth of a DBR Filter:

For $\Delta\beta = 0$

$$R_{max} = \tanh^2(\kappa L)$$

To find central stopband ω_0 , we substitute $p = 1$

$$\Delta\beta_1 = \pm 2\sqrt{\kappa^2 + \left(\frac{\pi}{L}\right)^2} = \pm 2\kappa_0 \text{ (say)}$$

So, to find central stopband $\Delta\omega$, ω_0 stands for ω we should substitute $p = 1$ because $p = 1$ you will get $\Delta\beta$ because $\Delta\beta$ can be expressed in terms of ω and if I want to find out stopband then I must find that there will central maximum and then positive side minimum and negative side minimum then oscillations will be there, so this is your $\Delta\beta$ running this is your 0.

So, these value, these minimum comes because a $p = 1$ and then next minimum come because of the $p = 2$ and so on. So, backside reverse direction also $p = 1$, 1 maximum minimum and then again $p = 2$ and other minimum something like that, but this $\Delta\beta = 0$ reflectivity can be maximum. So, our interest is that this is the main stopband that is the main reflected spectrum.

So, if I want to know the stopband then I need to know the value of $\Delta\beta$ here at corresponding ω value, $\Delta\beta$ value and corresponding ω value I will find and here also I will be getting the $\Delta\beta$ value and corresponding ω value, then 2ω you can consider ω_1 this is supposed to come ω_1 and this is ω_2 , $\omega_1 - \omega_2$ is that central stopband that is what to find the central stopband we have to put $p = 1$ then you get $\Delta\beta = 2$ values plus minus κ^2 $p = 1$ you put π/L .

So, these 2 value it just mention that this inside square root whatever the value is coming that can be κ . So, κ is a coupling coefficient we know that, but you have to add π/L

square obviously if L is very large this value can be ignored, the second term can be ignored in that case it will be just simply to kappa plus minus 2 kappa. Otherwise, if L is comparable to pi you are something like that comparable to kappa so with this second term if it is compared to kappa square then I think you cannot ignore that.

(Refer Slide Time: 20:03)

Integrated Optical Components Slide#10

Distributed Bragg Reflector (DBR): Device Design

Design of bandstop filter with a singlemode waveguide

Diagram: A waveguide with a DBR section of length L. The electric field components are given as $E_f(x, y, z, t) \rightarrow$ and $E_b(x, y, z, t)$. The boundary conditions at $z=0$ and $z=L$ are $A_1(z=0) = A_1(0)$ and $A_2(z=L) = 0$.

Boundary Values:

$$r = \frac{A_2(0)}{A_1(0)} = \frac{-jk' \sinh(sL)}{s \cosh(sL) + j \left(\frac{\Delta\beta}{2}\right) \sinh(sL)}$$

$$R = \left| \frac{A_2(0)}{A_1(0)} \right|^2 = s^2 - \left(\frac{\Delta\beta}{2} \right)^2$$

Calculating Bandwidth of a DBR Filter:

For $\Delta\beta = 0$, $R_{max} = \tanh^2(kL)$

For $R = 0$, for $L \neq 0$ and $k \neq 0$, $\sinh(sL) = 0$ & $s \cosh(sL) \neq 0$

$$\sinh(sL) = 0 \Rightarrow \frac{e^{sL} - e^{-sL}}{2} = 0 \Rightarrow e^{2sL} = 1 = e^{i2p\pi} \Rightarrow sL = jp\pi$$

where $p = 1, 2, 3, \dots$; $p \neq 0$

$$\Rightarrow s^2 = -p^2 \left(\frac{\pi}{L}\right)^2 \Rightarrow k^2 - \left(\frac{\Delta\beta}{2}\right)^2 = -p^2 \left(\frac{\pi}{L}\right)^2 \Rightarrow \Delta\beta_p = \pm \sqrt{k^2 + p^2 \left(\frac{\pi}{L}\right)^2}$$

To find central stopband $\Delta\omega_1$, we substitute $p = 1$

$$\Delta\beta_{\pm} = \pm 2\sqrt{k^2 + (\pi/L)^2} = \pm 2k_0 \text{ (say)}$$

Subtracting: $2\beta(\omega_1) - 2\beta(\omega_2) = 2k_0 \Rightarrow \Delta\beta_{\pm} = 2k_0$

$\beta(\omega_1) - \beta(\omega_2) = 2k_0$ $\beta_1 - \beta_2 = 2k_0$

So, in that case what do we get delta beta plus I am writing I know delta beta expression and delta beta plus I am writing for a given omega 1 frequency here 2 omega over c n effective we write n effective again n effective so must be at omega 1 - 2 / lambda that will be happening because of the delta beta value of 2 kappa naught and another delta beta minus where R = 1 comes corresponding frequency must be omega 2.

So, we know this expression omega 2 I am writing 2 pi / lambda, lambda is fixed and that is happening because of the - 2 kappa naught. So, I have at this point plus 2 kappa naught 2 kappa naught means this one you can have reflectivity 0 and - 2 kappa naught delta beta = - 2 kappa naught. So, delta beta as you know it can be plus and minus and these 2 value you can get first minimum within that minimum what is the frequency range I know the omega 1 here and I know the omega 2 here.

I will concentrate this expression kappa not known just subtracting these 2 I will be getting the stopband that is what we do? Subtract this one and this one. So, if you subtract what do you suppose to get omega / c n effective normally it is 2 times the beta 1 omega and we just this one in principle omega / c n effective that is beta we have written beta omega 1 - beta 2. So, it is easy to write down earlier we have defined again and again beta = omega / c n effective.

So, what I write here this is beta omega 1 whatever the value beta coming at 2 times omega 1 beta omega 1 and 2 times beta omega 2 that is subtracting and this one when you subtract 2 pi / lambda here it will be plus that will be removed cancelled and then 2 kappa naught - - 2 kappa naught that will give you 4 kappa naught. So, from here we get beta omega 1 - beta omega 2 that is giving you just 2 kappa it is 2 kappa naught. These one I write as a delta beta stopband that is what we have written delta beta stopband = 2 kappa naught that is what the expression I have written here.

(Refer Slide Time: 22:39)

The slide, titled "Integrated Optical Components" (Slide#12), focuses on "Distributed Bragg Reflector (DBR): Device Design". It describes the "Design of bandstop filter with a singlemode waveguide".

Diagram: A cross-section of a DBR structure is shown with a central waveguide layer of thickness L and refractive index n_1 , sandwiched between two cladding layers of refractive index n_2 . The structure is periodic along the z -axis. The electric field components are labeled as $E_f(x, y, z, t)$ and $E_b(x, y, z, t)$.

Boundary Values:

$$A_1(x=0) = A_1(0)$$

$$A_2(x=L) = 0$$

$$r = \frac{A_2(0)}{A_1(0)} = \frac{-j\kappa^2 \sinh(sL)}{s \cosh(sL) + j(\frac{\Delta\beta}{2}) \sinh(sL)}$$

$$R = \frac{|A_2(0)|^2}{|A_1(0)|^2} = \kappa^2 - \left(\frac{\Delta\beta}{2}\right)^2$$

Calculating Bandwidth of a DBR Filter:

$$R = \frac{\kappa^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + \left(\frac{\Delta\beta}{2}\right)^2 \sinh^2(sL)}$$

$$\text{For } \Delta\beta = 0 \Rightarrow R_{\text{max}} = \tanh^2(\kappa L)$$

Subtracting:

$$2\beta(\omega_1) - 2\beta(\omega_2) = 4\kappa_0 \Rightarrow \delta\beta_{\text{sb}} = 2\kappa_0$$

To find central stopband $\delta\omega_{\text{sb}}$, we substitute $p=1$:

$$\Delta\beta_{\text{sb}} = \pm 2\sqrt{\kappa^2 + (\pi/L)^2} = \pm 2\kappa_0 \text{ (say)}$$

The slide also includes the NPTEL logo and a small video inset of a person in the bottom right corner.

So, now again this is the expression we should keep in mind I will follow this one again we know another expression d omega, omega beta relationship is there for any given waveguide structure we have omega beta curve how to generate we know that at a given frequency I can find out what is the slope they are how it is varying d omega / d beta that is actually nothing but c / n g called group velocity. So, from here if I just simply do a little bit delta omega sb then delta beta goes this side.

So, in this way I can write a differentially I can write a delta beta is detuned how much delta omega will be detuned, this is a simple omega beta relationship gives if I know n g group index of the waveguide then if I determine my frequency a bit how much delta beta will be changed I know that if we know the n g or group velocity c / n g, from this expression I can write down that this delta beta can be replaced with this expression because delta beta term is related to delta omega.

So, we write $\Delta\omega = \Delta\omega \Delta\beta$ instead of $\Delta\beta$ I will be writing here $\Delta\omega$ stopband = $c / n g$ times $\Delta\beta$ naught. So, $\Delta\beta$ sb, so $\Delta\beta$ is nothing but 2κ naught. So, I write $c / 2 n g^2$ is there and κ naught expression is this one. So, 2κ naught $c / n g$, $c / n g^2 \kappa$ naught $\Delta\beta^2$ expression I have written here. So, I know what to do if I know the group index, if I know the κ , if I know the length then I will be able to actually calculate what is the central stopband of a DBR structure.

So, $n g \kappa L$ these are the design parameters for the DBR structure. So, if you can design properly if you want to get a certain stopband certain bandwidth you do not want to pass through the waveguide DBR structure. So, you can actually design a waveguide such that certain $n g$ you can get and κ , mainly $n g$ for a given waveguide structure single mode waveguide structure $n g$ does not change much.

So, only you can control κ how κ can be controlled, how much perturbation you are doing in the waveguide structured more the perturbation more the κ value. So, depending on that I can calculate stopband centering around Bragg wavelength of Bragg frequency we can just convert it into ω λ relationship we know this $\omega = 2\pi c / \lambda$. So, you can write $\Delta\omega = 2\pi c \lambda^{-2} \Delta\lambda$ minus sign will be there.

So, this $\Delta\omega$ if you just convert into λ then we can find out stopband in terms of λ , λ_B^2 Bragg wavelength $\pi n g$ and this one so you can control your stop bandwidth, how much bandwidth you want to reflect you do not want to transmit them through the waveguide such a particular band, it can be a channel particular channel information channel you want to stop carrier frequency carrier wave length you known then you can actually design your DBR structure.

(Refer Slide Time: 26:37)

Integrated Optical Components Slide#13

Distributed Bragg Reflector (DBR): Device Design

Design of bandstop filter with a singlemode waveguide

$$E_f(x, y, z, t) \rightarrow$$

$$E_r(x, y, z, t) = A_r(z)E_0(x, y)e^{j(\omega t - \beta z)}$$

$$E_t(x, y, z, t) = A_t(z)E_0(x, y)e^{j(\omega t + \beta z)}$$

$$\Delta\beta = 2\frac{\omega}{c}n_{eff}(\omega) - \frac{2\pi}{\Lambda}$$

$$E_f(x, y, z, t) = A_f(z)E_0(x, y)e^{j(\omega t - \beta z)}$$

$$E_r(x, y, z, t) = A_r(z)E_0(x, y)e^{j(\omega t + \beta z)}$$

$$E_t(x, y, z, t) = A_t(z)E_0(x, y)e^{j(\omega t + \beta z)}$$

$$\Delta\beta = 2\frac{\omega}{c}n_{eff}(\omega) - \frac{2\pi}{\Lambda}$$

Calculating Bandwidth of a DBR Filter:

$$r = \frac{-jk^2 \sinh(sL)}{s \cosh(sL) + j\left(\frac{\Delta\beta}{2}\right) \sinh(sL)}$$

$$s^2 = k^2 - \left(\frac{\Delta\beta}{2}\right)^2$$

$$R = \frac{k^2 \sinh^2(sL)}{s^2 \cosh^2(sL) + \left(\frac{\Delta\beta}{2}\right)^2 \sinh^2(sL)}$$

$$\delta\omega_{3dB} = \frac{2c}{n_g} \sqrt{k^2 + \left(\frac{n_g}{\lambda}\right)^2}$$

$$\delta\lambda_{3dB} = \frac{\lambda_0^2}{\pi n_g} \sqrt{k^2 + \left(\frac{n_g}{\lambda}\right)^2}$$

$$k = j\left(\frac{1}{\lambda}\right) \frac{n_d^2 - n_{eff}^2}{n_{eff}} \int_{x_1}^{x_2} |E_0(x, y)|^2 dx dy$$

$$\text{For } \Delta\beta = 0 \quad \Delta\beta = 2n_{eff}\left(\frac{\omega}{c}\right) - \frac{2\pi}{\Lambda}$$

$$R_{max} = \tanh^2(sL)$$

Integrated Photonic Devices and Circuits : Lecture-29
Copyright © S.K. Das

So, let us go for certain example as a mentioned I repeated this amplitude reflection coefficient square defined and power reflectivity and then stop bandwidth and then you have a stop bandwidth in terms of angular frequency stop bandwidth in terms of wavelength I know and lambda obviously, you know how to calculate lambda B that is for delta beta = 0 lambda beta = 2 n effective period. So, you can control the period you can find the wave but if you know n effective index of the waveguide guided mode.

And if you design a certain period depending on that certain wavelength would be a Bragg wavelength around that point actually delta beta will be equal to 0. Now, let us consider as it is shown top view here if you go to cross section then you can say that while you can have this is your total waveguide width and periodically part of this region this re-waveguide structure slab region this thickness is called h slab and this thickness is called H device layer thickness and this one will be calling as a waveguide width.

And then you are getting a perturbation from x 1 to x 2 lateral direction and obviously, y 1 this is x y coordinate. So, this is the x coordinate and y 1 x coordinate and y 1 this is x 2, y 2, y 1 and so on coordinate system is there this is the region it is part of it periodically. Periodically it is removed the silicon this is device layer if it is silicon on insulator this is silicon and this is your box layer buried oxide layer.

So, using this structure you will know how to calculate kappa value earlier we have expressed using your so called rectangular periodic perturbation and for that purpose if we calculate kappa using coupled mode theory you know that if it is a single mode waveguide only one mode will be

launched from this side and same mode will be reflected backward direction. So, in that case κ will be $n_d^2 - n_c^2 / n_{\text{effective}}^2$ times overlap integral in this region you have to see how much field strength suppose you have field like this defined like these.

So, these fields strength covering in the part of region that is actually comes in your κ calculation and you need to integrate only x_1 to x_2 y_1 to y_2 in the xy plane because beyond that no perturbation is there. So, normally to calculate κ you need to know field strength for the 2 coupled modes and as well as you are the region you are getting perturbation, 2 times is there because you can have 2 sides.

If I can calculate one side that means x_1 to x_2 y_1 to y_2 that means you are calculating κ per one side and similar κ you will be getting in the other side that side 2 times comes. If it is only one side perturbation the side is completely field then you can have this then you do not need to write this term just one side. Using this κ value using this $\Delta\beta$ expression and using also that expression $s^2 = \kappa^2 - \Delta\beta^2 / 2$ whole square.

And $\Delta\beta$ is expressed like in terms of ω ; ω can be expressed in terms of λ . So, as your function of wavelength if I define a period up to 92 nanometre for $L = 200$ micrometre device length and Δw means this perturbation suppose this is the perturbation so called $x_2 - x_1$ can be called as a Δw . So, one side perturbation Δw is 50 nanometre other side another 50 nanometre, then we can use this κ calculation I get κ calculation about 0.023 per micrometre.

Obviously, this one will be per micrometre λ is here, per micrometre expression is dimension. So, consider $\lambda = 292$ nanometre and corresponding $n_{\text{effective}}$ if you calculate for these waveguide dimension certain waveguide dimension you consider then what you get typical value the Bragg wavelength you are getting exactly at 1550 nanometre for a given design.

And then what you see as you detune $\Delta\beta$ meaning you are detuning ω or λ here it is λ is 1, then you will see the centralize maximize there that is actually equal to 1 for $\Delta\beta = 0$ it is 1, you are simulating this is a plot using MATLAB code. So, you can do also you can try how it is actually happening. So, you can find that you see after certain from 1550 that is we are actually phase matching condition satisfied where $\Delta\beta = 0$ that is why you are getting maximum.

Now, other than $\Delta\beta = 0$ there are other values you are getting where the minimum occurs according to our previous discussion minimum means $R = 0$ occurs. So, our interest is that first 0 that side and this side so this is as a function of λ you are calculating whatever λ you are getting here, whatever λ you are getting here you have to subtract to get $\Delta\lambda_{sb}$ that is the stopband.

So, I can have very narrow stopband depending on the design parameters like κ or Δw of course, length is also involved how narrow how broad or you want to get so longer the length you can get a narrow r the reflection bandwidth because l is in that denominator here. And when you see that κ is small, small κ means $\Delta\lambda_{sb}$ will be smaller. So, you can get a narrow channel narrow stopband.

Some application you need broad stopband and some application you need maybe very narrow linewidth channel you do not want to allow you want to get reflected. So that can be actually designed that you can find out just you have to design your waveguide calculate $n_{\text{effective}}$, calculate n_g and then calculate the part just decide what type of perturbation is required for having a Bragg wavelength at λ_B .


Once you know $n_{\text{effective}}$ and period you can give it a λ_B and decide λ_B you can calculate and accordingly the period you can set, once it is set then you can actually find as a function of length or κ how the bandwidth is related. Suppose, you need a very narrow bandwidth then κ has to be reduced to a smaller value. So that accordingly Δw can control this Δ value how much perturbation you can control you can get a narrow r . If you want more broader and broader you will increase the κ value that means you are increasing the perturbation then you can get a broader stopband.

(Refer Slide Time: 34:34)

Integrated Optical Components Slide#14

Distributed Bragg Reflector (DBR): Device Design

Design of bandstop filter with a singlemode waveguide



Experimental Results of Fabricated DBR Filter:

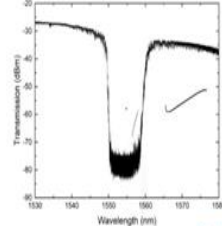
$$\delta\omega_{\text{DB}} = \frac{2c}{n_g} \sqrt{\kappa^2 + \left(\frac{\pi}{L}\right)^2}$$

$$\delta\lambda_{\text{DB}} = \frac{\lambda_g^2}{\pi n_g} \sqrt{\kappa^2 + \left(\frac{\pi}{L}\right)^2}$$

For $\Delta\beta = 0$ $\lambda_g = 2n_g L \Lambda$ $R_{\text{max}} = \tanh^2(\kappa L)$

$\delta\omega = 526.7 - 524.1$
 $\sim 200 \text{ meV}$

$W = 575 \text{ nm}; H = 220 \text{ nm}; h = 160 \text{ nm}$
 $\Delta W = 190 \text{ nm}; L = 450 \mu\text{m}; \Lambda = 290 \text{ nm}$



Transmission (dB) vs Wavelength (nm)

Copyright © NPTEL

Copyright © CPPICs

Copyright © IIT Madras and Contributors

Integrated Photonic Devices and Circuits : Lecture-29

Copyright © S.K. Das

Now, you see such a device actually we have developed we have understood the coupled mode theory now, that this is possible. Now, I have shown that some device we fabricated in our lab in IIT Madras. I see this is the how waveguide a scanning electron microscopic image it is shown here. And the DBR Design you define in a periodic perturbation and fabricate then finally we end up with this type of structure both side details perturbation is there.

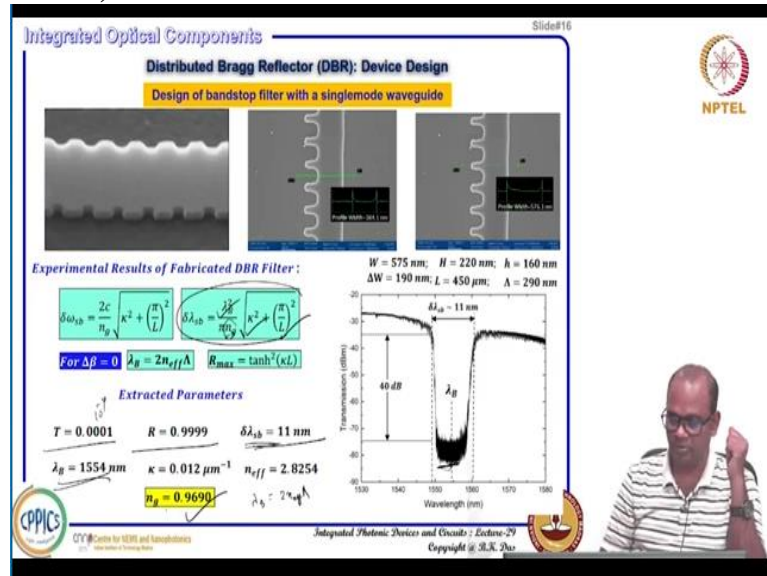
So, input side will be input waveguide single mode waveguide output side will be a single mode waveguide in between this is the DBR structure some section of the DBR structure is shown. And this will be your so called period and that will be your from here to here that can be your perturbation depending on the perturbation or it can be considered $\times 1$ and this can be considered $\times 2$.

If this is your x axis and this is the z axis vertical direction is the y axis and y_1 y_2 you can consider and you can calculate kappa value and you can have a DBR structure not necessarily that you need to have both side grating you can have one side also it for example, it is shown here that this region if you are just taking the scanning here as a mention this point to this point the width is 384 nanometre and this one around 150 or 200 nanometre.

So that way you can also design a DBR structure, it can be one side it can be both side another one sided we are actually the if you see this one here if you see this is totally 576 same structure, same device actually you are scanning here to see the width. So here 384, there 576, so delta w that means width perturbation is 576.1 - 384.1 nanometre. So, this is about then 192 about 200 nanometre around 200 nanometre perturbation is there.

So, such a device we have fabricated and we have actually got the transmission characteristics you are launching suppose this is your DBR structure and then grating coupler grating coupler here you will launch here and you will launch a routine the wavelength then you see certain wavelength in transmission is missing that means that part that lost energy in the transmission that must be reflected backward direction.

(Refer Slide Time: 37:14)



So, what do you see in the transmission you see put a DB extinction you get a bandwidth of 11 nanometre for a grating length of 450 micrometre. Delta w is given here 190 nanometre 575 nanometre all the other parameters given and you see almost nearly 1550 nanometre you get a stopband and bandwidth 11 nanometre and extinction that means this much 40 DB down suppose you have a laser line here whatever value you get the intensity and here laser line it is there it will be 40 DB down.

So, such high extinction filters add some applications especially in quantum photonic application and micro photonic applications we will discuss later. So, using the experimental results you can find out what is the transmission coefficient it is actually 10 to the power - 4 only transmission coefficient reflectivity is 99.99% R = 0.9999 delta lambda so on. So, from this experimental data we can extract lambda B = 2 times n effective this one. And also you know from the delta lambda sb stopband expression this one you have n g is there.

So, you know lambda B you know kappa you know L so you can find out n g extracted n g we can get something like that group index. So, from the experimental results you can find out what

are the design parameters whether that is matching with the calculations or not you can compare. So, in our design in this type of design and fabrication experimental results matches very well with the coupled mode theory results.

So, this is a good ploy to design a stopband filter using single mode waveguide periodically part of single mode waveguide or so called DBR structure distributed Bragg reflector integrated in a single mode waveguide structure.

(Refer Slide Time: 39:31)

The slide, titled "Integrated Optical Components" and "Slide#17", focuses on "Distributed Bragg Reflector (DBR): Device Design". It describes the "Design of bandstop filter with a multimode waveguide" with parameters $H = 250 \text{ nm}$, $W = 760 \text{ nm}$, and $h = 150 \text{ nm}$, supporting "TE like modes @ $\lambda = 1550 \text{ nm}$ ". The slide shows a 3D schematic of the waveguide structure, field profiles $E_0(x,y)$ and $E_1(x,y)$, and the following equations:

Phase Matching Conditions

$$\Delta\beta^{(0)} = \beta_0 - (-\beta_0) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \beta_0 = \frac{\pi}{\Lambda} = 2n_{eff}^0/\Lambda$$

$$\Delta\beta^{(1)} = \beta_0 - (-\beta_1) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \beta_0 = \frac{\pi}{\Lambda} = (n_{eff}^0 + n_{eff}^1)/\Lambda$$

Coupling Constants

$$\kappa_{00} = j \left(\frac{1}{\lambda} \right) \frac{n_d^2 - n_c^2}{(n_{eff}^0)^2} \iint_{x_1, y_1}^{x_2, y_2} |E_0(x,y)|^2 dx dy$$

$$\kappa_{01} = j \left(\frac{1}{\lambda} \right) \frac{n_d^2 - n_c^2}{\sqrt{n_{eff}^0 n_{eff}^1}} \iint_{x_1, y_1}^{x_2, y_2} E_0(x,y) E_1^*(x,y) dx dy$$

Source: P. Sah et al., JLT, vol. 35, pp. 128 - 135, 2017. Integrated Photonic Devices and Circuits: Lecture-29. Copyright © 2017.

Now, let us move on to how if suppose your device is multimodal, what happens if you are still doing a DBR structure whether you will be getting stopband filter or some completely different type of filter characteristics you will be getting. So, we have shown that if you fabricate a DBR structure in a multimode waveguide at least it can support 2 modes for example, earlier we are considering only single mode waveguide, it is 2 mode you get a very interesting results and that can be useful for many applications also.

I will show you some results let us consider this is the top view schematically it shown top view of the waveguide structure designed in silicon on insulator platform just top view showing. Initially input side it is a single mode waveguide, this output side and that single mode waveguide it is shown the propagation constant is beta naught prime both are same it only supports fundamental mode assume and in between you increase the width a little bit adiabatically.

So that any fundamental mode like this if you are launching that will be slowly adiabatically it will be expanded. So that even though it is a multi mode waveguide structure higher order mode

will not be excited assume at the beginning, but what happens as it propagates through the DBR structure. Since it can support both the mode fundamental mode as well as first order mode.

There is a chance that forward propagating fundamental mode can couple light into the backward direction to the fundamental mode and backward direction first order mode because it can support, so it can this fundamental mode whatever frequency wavelength you were launching here, that frequency wavelength can be phase match to the backward propagating fundamental mode and backward propagating first order mode.

So, once it is phase matched and if you find their overlap or coupling strength is non zero, so you can see that you can get one reflection band due to the coupling between forward propagating mode to backward propagating mode and another band you will be getting because of the coupling between forward propagating fundamental mode to with backward propagating fundamental first order mode.

So, you can expect it to stopband, so same device earlier if it is a single mode waveguide structure and DBR structure is there you could get only one stopband, but if it is a multimode waveguide then you can get it 2 stopband distinctly because they are phase matched at different wavelengths or different frequencies this is some 3D view here it is shown input waveguide single mode, output waveguide single mode taper such that anything launched here that is a single mode.

And as it is entered into the DBR structure, it will remain fundamental in the forward direction but because of the phase matching condition light can coupled to the backward propagating fundamental mode and backward propagating first order mode. So, to understand that, it is soon that for example, you have 2 modes in the forward direction in the grating structure you have the fundamental mode is having because you width is large.

So, this propagation constant, this propagation constant will not same you know β actually $2\pi / \lambda n_{\text{effective}}$. So, width is different means $n_{\text{effective}}$ is different so for a given wavelength β will be different here, but the fundamental mode β will be different here that is why here we are considering $\beta_{\text{naught prime}}$ and here we are considering β_{naught} and first order mode we are considering β_1 .

So, I can get a phase matching condition $\Delta\beta = 0$ that means forward propagating fundamental mode $\beta_0 - \beta_0$ backward propagating fundamental mode that is why $-\frac{2\pi}{\lambda}$ that gives rise to if you use this expression, I can find a Bragg wavelength that λ_{B0} satisfying this one period is known as $n_{\text{effective } 0}$ effective index of the fundamental mode node.

So, you can get a Bragg wavelength here around Bragg wavelength you can get a stopband and another phase matching condition you can consider $\Delta\beta = 0$ that means forward propagating fundamental mode and backward propagating first order mode minus β_1 that is the $\Delta\beta$ expression $\frac{2\pi}{\lambda}$ that must be equal to 0 then you will also get β_0 expression β_1 expression if you are just using wavelength in terms of wavelength then you get λ_{B1} .

At another wavelength where $n_{\text{effective } 0}$ fundamental mode and $n_{\text{effective } 1}$ first order mode because here you see you are putting $\frac{2\pi}{\lambda} n_{\text{effective } 0}$ and here you will be putting $\frac{2\pi}{\lambda} n_{\text{effective } 1}$. So, if you just simplify that one then you get this is the Bragg wavelength for forward propagating fundamental mode coupling to the backward propagating first order mode.

Now, question is that when something backward propagating mode the beauty here earlier what we have discussed in this case this much is lost in transmission, but where that should appear that should appear in the reflection. But in this case, here also fundamental mode when reflected coupled backward direction these waveguide can support fundamental mode. So that will appear that means λ_{B01} Bragg wavelength which is reflected here that will appear in the reflection.

Because it can support and guide back to the backward direction towards the source you can separate them in that case, but, if it is coupled to the fundamental first order mode, when the first order mode propagating in the backward direction when it enters into the single mode, you get it cannot support. So, what will happen? This will be coupling to the slab. So that means, if anything coupled to the backward propagating if forward propagating fundamental mode is coupled to the backward propagating first order mode.

That backward propagating signal or energy that will not be collected by the single mode waveguide because its shape is like this. So that will be lost here so that will be missing in the

transmission you can get to stopband one around λ_{B00} another is λ_{B01} , but in the reflection you will get only pick around λ_{B00} because that can be collected. So that is interesting if you are using certain this wavelength, this will be missing in the forward direction.

But, it will be also missing in the backward direction because of the waveguide filtering here, because this side when it is coming backward direction the single mode waveguide cannot support the first order mode. So that types of application we can also think of here it is shown that for a given waveguide suppose your waveguide width is 760 nanometre and device layer thickness is 250 nanometre, slab height is 150 nanometre here this is slab height, this is your width this is your h .

Then we can find the first 2 modes when you solve numerically using numerical or anything else any numerical software you use simulator if you use, then you get into fundamental mode 2 modes lower order mode that is supported guided one is fundamental mode another is first order mode, both the modes are TE like mode that means, electric field along y direction is dominating, for that guided mode.

So, E_y means you are meaning y component this is y component will be dominating that means electric field will be oscillating along y direction most, fundamental mode you see in the center it will be maximum and slowly decreasing and there will be some kind of discontinuity just to satisfy the boundary condition and first order mode if you see, you see this is a blue colour and this red colour that signifies that you are getting something like this.

So, this is a positive and this is a negative and here it will be positive it is shown here, but it will be oscillating as a function of $e^{j\omega t}$ as a function of time it will become positive negative positive sinusoidal it will vary. So, but here at any instant of time if you see one side is positive and other side will be negative field strength and the entire thing it is like a standing wave structure around lateral direction. And that will be oscillating as a function of time also.

So, I do not think all of you can understand by the time whatever we have expressed explained so far in this course. And the coupling constant that means forward propagating fundamental mode to backward propagating fundamental mode coupling, we defined as κ_{00} fundamental mode is indicating 0 here. So, it is standard whatever we discussed earlier coupling coefficient how to calculate we know.

If we know the fundamental mode in the grating region $E_{naught \times y}$ whatever you are getting that one who square and then integrate one sided x_1 to x_2 . So, if you are just suppose this region is what are theoretically that is what it is mentioned and you get κ_{00} and if you want to get to κ_{01} meaning forward propagating fundamental mode and backward propagating first order mode how they are coupling.

So, in this case instead of $n_{effective \ 0}$ here must be it is 0 you get $n_{effective \ 0}$, $n_{effective \ 1}$ square root mean value. So, this geometric mean if you use that from the couple mode theory comes, I think once we have discussed that and then we can have a overlap between E_{naught} to $E_{1 \ star}$ first order fundamental mode. So, just one thing you should keep in mind that if the same waveguide if grating perturbation is both side is there then this κ_{01} would be 0.

Because you have one side positive and other side negative here suppose this side is positive perturbation and this side is positive if you integrate you get a positive value, but if you multiply this one and this side multiply you will be getting negative. So, one side one half integration gives you positive overlap another side it will give you negative overlap if you calculate this one as a result the κ you will be getting 0.

So, even if it is a multimode structure to it can support 2 modes but if grating is symmetric, both sidewall perturbation is there in that case, you will you may get plasmatic condition with the back or propagating fundamental first order mode for a certain wavelength, but κ will be 0 that means coupling between forward propagating mode to backward propagating first order mode forward propagating fundamental mode to back or propagating first order mode is 0.

So that is the reason if you want to get an active stopband effective stopband because of the coupling between forward propagating fundamental mode with backward propagating first order mode in that case that grating you must fabricate define in one of the sidewall. So that is very important for silicon photonics especially any re waveguide structure that is true obviously, it is widely used in silicon photonics because the sidewall grating definition is very using lithographic technique, standard CMOS compatible lithographic techniques.

(Refer Slide Time: 52:59)

Integrated Optical Components Slide#18

Distributed Bragg Reflector (DBR): Device Design
Design of bandstop filter with a multimode waveguide

TE like modes @ 1-1550 nm

Effective index

Width [μm]

Phase Matching Conditions

$$\Delta\beta^{(0)} = \beta_0 - (-\beta_1) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \lambda_B^{(0)} = 2n_{eff}^0 \Lambda$$

$$\Delta\beta^{(1)} = \beta_0 - (-\beta_1) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \lambda_B^{(1)} = (n_{eff}^0 + n_{eff}^1) \Lambda$$

P. Sah et al., JLT, vol. 35, pp. 128-135, 2017

Integrated Photonic Devices and Circuits: Lecture 29

Copyright © S.K. Das

Now, how to design that I want the Bragg wavelength $\lambda_B^{(0)}$ at a certain wavelength $\lambda_B^{(1)}$ at a certain wavelength, you remember that if you try to find out which one will be higher $\lambda_B^{(0)}$ $\lambda_B^{(1)}$ definitely n_{eff}^0 greater than n_{eff}^1 we know that. So that means $\lambda_B^{(0)}$ is greater than $\lambda_B^{(1)}$, if you subtract that what you get? You get $n_{eff}^0 - n_{eff}^1$ times λ_B .

So, if you see different between the effective indices of the fundamental mode as well as first order mode that actually defined how much separation will be this stopband. So, we need to calculate we can design with this difference can be controlled by controlling the design parameters the waveguide structure. So, we have calculated as a function of waveguide width for a given $H = 250$ nanometre device layer.

And here we have calculated for 3 slab height effective index calculated this is for fundamental mode as a function of width effective index how it is increasing for $h = 150$ nanometre it is soon 10 nanometre below and 10 nanometre this is 10 nanometre below and this is 10 nanometre 3 different structures 3 different design of the slab height you can see that for a wider width even this is insensitive to the slab height all 140 nanometre 150 nanometre 160 nanometre slab height for a wider width that is almost closer to the bulk.

So that is the reason this 10 nanometre variations of the slab height you will not see n_{eff} index changes of the fundamental mode and then as you know up to this much width you would not see any second mode is supported if you just increase the width then you get another mode

appearing, obviously again for 3 different slab height, we can see the refractive index for the first order mode. And if you go up to 1.1 micrometre for $H = 250$ nanometre.

And h equal to in this range the waveguide will support only 2 modes and this both 2 modes are TE like mode as I mentioned for this slab height. If you change slab height democratically if maybe $h = 0$ then you can see that TM modes also will be coming into picture not only TE mode. So, in that case I can actually define for example, if I define a waveguide width here, so I see that difference between effective index of the fundamental mode here and this much, but if I design the waveguide width here, so difference is this one.

So that means, depending on the waveguide width along waveguide width variation can control this value, by controlling that actually you can get the what is the separation between 2 stopband could be so here we have just shown that as a function of width were just calculating that λ_{B00} as a function of width where it will be there at a longer wavelength and λ_{B01} it will be shorter wavelength we find that this is the region where actually the bottom region actually called C band and above region about 1565 it is called L band.

About less than 1565 nanometre that is called C band optical C band and optical L band. So, we now learned that it is really possible to design a device using distributed Bragg grating in which you can get 2 stopband in transmission out of this 2 stopband 1 band will be reflected backward direction that is associated to the coupling between forward propagating fundamental mode to backward propagating fundamental mode.

But the coupling between propagating fundamental mode to backward propagating fundamental mode that will be missing in the reflection what because backward direction first order will not be supported.

(Refer Slide Time: 57:44)

Integrated Optical Components Slide#19

Distributed Bragg Reflector (DBR): Device Design
 Design of bandstop filter with a multimode waveguide

$H = 250 \text{ nm}, W = 710 \text{ nm}, h = 150 \text{ nm}$
 TE like modes @ $\lambda = 1550 \text{ nm}$

Phase Matching Conditions

$$\Delta\beta^{00} = \beta_0 - (-\beta_0) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \lambda_0^{00} = 2n_{eff}^0 \Lambda$$

$$\Delta\beta^{01} = \beta_0 - (-\beta_1) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \lambda_0^{01} = (n_{eff}^0 + n_{eff}^1) \Lambda$$

P. Sah et al. JLT, vol. 35, pp. 128 – 135, 2017

Integrated Photonic Devices and Circuits - Lecture-29
 Copyright © S.K. Das

So now, let us look simulation result, we have just consider H equal to device layer equal to 250 nanometre W = 710 nanometre supporting 2 modes h equal to about 150 nanometres slab height both the modes are TE like mode we know that we have simulated earlier we have shown the results earlier and the find that really to stopband this is a simple simulation using FDTD technique that is available for also commercial software's available finite difference time domain technique that means a solver Maxwell solver in time domain.

So, you find that 1 stopband around this one around 1600 nanometres on the stopband here. And beyond this shorter wavelength region you see it is not really picking something like this. What is the reason because at shorter wavelength the waveguide itself is somewhat DBR structure actual somewhat coupling light to the slab modes that is why it is being lost also in this region. But if you just design maybe waveguide so title is that little bit wider waveguide if you use instead of 710 nanometres or 1 micron something like that.

Then you could see something characteristics like this, you would be getting something like this and then this and then this like this 2 different stopband and after a long wavelength at towards this shorter wavelength, you will see somewhere that is coupling to the slab. So, wider bandwidth wider width of the waveguide helps to get a very good distinct stopband with some passband is both side and in between also but you have to be careful that if you are going for higher waveguide width.

Then you can eventually end up with guiding another mode. So, in that case you can expect one more stopband also that means, that particular stopband at some other frequency at some other

wavelength it will be phase matched and in that case you can get another multiple stopband you will be getting. So, depending on the number of modes and phase matching condition you can get multiple stopbands in that transmission but in reflection only fundamental mode will be supported.

Because input waveguide and output waveguide you are designing single mode waveguide structure because most of the integrated photonics or device structures, they are designed with a single mode waveguide structure only in the DBR structure if you want this type of some specific application I will explain also if you want to get this type of transmission spectrum, then you go for this device this is easy design basically.

And you see here there is another one curve also soon, if you just increase the device temperature a bit then refractive index of the silicon integers and because of that, your n effective also increases. So, $\lambda_B = 2 n$ effective period so when n effective slightly increased due to the temperature that is called thermo optic effects silicon is a very good thermo optic effect a 25 Kelvin temperature if few increase.

Then you see the Bragg wavelength, fundamental mode stopband and first order stopband both will be actually red shifted 2.2 nanometre red shifted why? Because n effective increases as you increase the temperature effective index or bulk silicon refractive index will be increased so that is why it will be like looking like that. And another thing it is also experimentally found that very interesting that instead of keeping the waveguide surface pre empty here you get a particular characteristic like this one.

But now if you put a drop of water so that top cladding is water and water refractive index you know n water is about 1.3 and n air is 1 so that means cladding refractive index is increased, so once the cladding refractive index increase n effective will also increase. So that is how when air is given as a cladding of the waveguide structure, you see the red shift happening this red thing this is the thing both fundamental stopband and first order stopband red shifted.

And red shifted at about 7.8 nanometres this case it is measured at one is about 6.6 nanometre because n effective for fundamental mode and n effective for first order mode they do not increase equally because of the cladding refractive index change. So, by changing the refractive index in the cladding material you can change determine the characteristics, but this eventually people

exploited for sensing application suppose you have certain liquid and you know that had certain liquid will have certain refractive index and you do not know what is that material.

So, you can guess just use that material in that liquid in the top or coat in the surface and then you see the Bragg wavelength, how much it is sifted and looking into the shift you can estimate that you can actually find you can try to guess what material could be in the surface. So, for bio sensing applications etcetera people are demonstrating different type of DBR structure in silicon photonics technology platform.

(Refer Slide Time 01:03:46)

The slide, titled "Distributed Bragg Reflector (DBR): Device Design", illustrates the design of a bandstop filter with a multimode waveguide. It includes a schematic of the waveguide structure with regions A, B, C, and D, and a 3D perspective view of the device on a BOX substrate. The phase matching conditions are given as:

$$\Delta\beta^{(0)} = \beta_0 - (-\beta_0) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \lambda_D^{(0)} = 2n_{eff}^0 \Lambda$$

$$\Delta\beta^{(1)} = \beta_0 - (-\beta_1) - \frac{2\pi}{\Lambda} = 0 \Rightarrow \lambda_D^{(1)} = (n_{eff}^0 + n_{eff}^1) \Lambda$$

The slide also shows a transmission spectrum plot of Transmission (dB) versus Wavelength (nm) from 1520 to 1620 nm. The plot features a central bandstop region with two sidebands labeled "C band" and "L band". A schematic of the device layout is shown, including an input grating coupler (GC), a DBR section, and an output grating coupler (GC) connected to a waveguide (W) and a photodetector (PD).

Now, I will show you that such a device also fabricated here it is so you see 560 nanometre and perturbation that delta w 150 so the about 710 nanometre, so this is $w = 710$ nanometre it is approximately supporting 2 modes and in this case it is 760 nanometre w so it also supports 2 modes and we have fabricated device this fabricated same structure it is shown here and it is characterized this input grating coupler, output getting coupler, DBR structured is there around 5 millimetre long waveguide fabricated.

And this is about 450 micrometre getting structure fabricated. So, when it is coming and going and if you are tuning your tuneable source wavelength you are tuning then you see if there is no DBR structure reference waveguide. So, this type of characteristics wavelength dependent coupling is there in the grating coupler that is why you see the difference as a function of wavelength and this is the transmission.

But when grating structure is there you see, there is 1 stopband here and other stopband is here and this region actually lost because of the coupling to this lab mode. So, this region actually your region were 1565 this side is so called this side is L band and this side is C band. So, you could see that L band will be passed through the DBR structure C band can be stopped completely.

So, if you want to have certain band broad band if you want to stop, you do not want to allow to transmits in a certain destination, then we use this device only this band wavelength range up to here it will be passed again this range it will be stopped. So, some unwanted band if it is there, you can stop here both sides only a selected band you can transmit. So, you can design a device something like that, that is selected band will be passed it is a DBR structure normally it is used for stopband.

But in this case you can have 2 stopband wide broad stopband here if you see it is more than 15 nanometre stopband this side also more than 15 nanometre and beyond. But in between you get about from 1565 to 1600 about 30 nanometre bandwidth it will be passed. So, you can selectively a certain bandwidth you can actually transmit through silicon waveguide structure or DBS structure. So, I will stop here for this lecture today.