

Integrated Photonic Devices and Circuits
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Lecture – 31

Tunable Device and Reconfiguration Circuits Phase Error Interference

Hello everyone in the last lecture we have completed almost the discussion of passive integrated optical components which includes power splitters to Mach Zehnder interferometer, ring resonator distributed Bragg reflectors etcetera. So, those are we consider as a passive device because as you fabricate you do not need any external control to change your performance as designed as fabricated you want to use them for a different type of functionalities and also for integration of large scale integrated circuits.

Now today I want to start a new chapter that is actually tunable devices and of course reconfigurable circuits. So, sometimes a device which is designed and it is fabricated and the performance also passive performance you want to use but occasionally or whenever needed you may want to reconfigure or you may want to detune its performance from one particular operating wavelength to another operating wavelength per one; particular functionalities to another different type of functionalities.

All those type of things you can actually program you can reconfigure. So, those types of circuits are very much important sometimes we call them as an active devices or actively configured reconfigurable circuits. Before going into that detail I would like to give you some information about phase error interference. So, when you fabricate a device fabrication has its own challenge own limitations.

It may not happen that whatever you design after fabrication the device parameter design parameters is not coming properly or maybe for large scale integrated circuit you know wafer scale integration whenever you are doing in that case one particular location of the wafer it can have different type of fabrication tolerances because of the dimensional error, because of the process related errors.

And if you compare with another location because you know in a foundry in industry when you fabricate integrated circuit CMOS compatible integrated photonics circuits you are fabricating in wafer scale 200 millimeter, 300 millimeter diameter of wafer entire wafer you will be fabricating. So, similar type of dyes or devices you will have in different locations but because of the process error and maybe wafer related errors can cause some kind of dimensional error in your fabricated devices.

So, those type of errors how to correct how to get uniform yield of all devices all circuits across the wafer or a certain wafer certain device you wanted certain performance but after fabrication you see because of the deviations of the dimensions in at different locations the performance may be different. So that difference of performance is mainly because of we call as a due to the phase error.

Because you know all integrated optical devices photonic integrated circuits everything if you consider the performance the functions actually heavily dependent on interfering different waves coming out of different waveguides for example if you want to have a Y junction where 2 waves to mode guided mode can come together and can combine and they can interfere and depending on their phase relationship you can see what is the output intensity or power there itself.

So, phase plays an important role very, very important role. So that is why if some phase error comes because of the fabrication defects fabrication related process errors that phase error actually interfere your device performance. That is why we just give this subtitle as phase error interference. So, I would like to give you an idea some ideas with examples that how precise phase control is needed for any passive waveguide devices to have some kind of desire performance.

After that I will explain that what I mean that the different process error, different fabrication, different wafer level dimensional error etcetera they can cause this type of phase error or dimensional error. So, I will just touch upon some of the sources various sources of phase error and for those phase errors what would be the consequences in device performance these 2 things I will be covering in this lecture today.

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Tunable Devices and Reconfigurable Circuits Slide#2

Phase Error Interference

Precise phase control needed for passive waveguide devices

(1) Mach Zehnder Interferometer

$$A_0 = \frac{1}{\sqrt{2}} \left(\frac{A_i}{\sqrt{2}} \right) + \frac{1}{\sqrt{2}} \left(\frac{A_i}{\sqrt{2}} \right) e^{i\phi}$$

$$P_o = \frac{P_i}{2} (1 + \cos \phi)$$

$$P_o = P_i \cos^2 \left(\frac{\phi}{2} \right)$$

$$\phi = \phi_1 - \phi_2 = \frac{2\pi}{\lambda} (n_1 L_1 - n_2 L_2)$$

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So, let us take an example of Mach Zehnder interferometer we have explained this before it is just a simple Mach Zehnder interferometer it is one input is here and you are accessing output here and this is you can be considered as a single mode waveguide. You know we know that any guided mode that can be defined as some kind of amplitude A and then $E \propto y$ if it is a single moded waveguide field distribution.

And $e^{-i(\omega t - \beta z)}$ if it is unpartted waveguide that electric field can be defined as this one. So, this can be a vector this can be something like that, so z direction propagation. So, this A_i when I consider that is the amplitude of the incoming field of the guided mode that represents and then in the y junction we know that amplitude splitting happens and following the energy conservation you will have $A_i / \sqrt{2}$ in the upper branch $A_i / \sqrt{2}$ in the lower branch.

If these waveguides are symmetrically splitted through y junctions and these field again the amplitude reduce identical waveguide this waveguide looks identical and this is also identical waveguide comes here and then it goes through here and you assume that there is a ϕ phase difference happening in the phase introduction additional phase introduction in the upper arm compared to the lower arm.

How that can happen suppose this length l_1 and this length l_2 suppose l_1 not equal to l_2 and then additional phase difference between these 2 arms you can have. Even you can consider that this waveguide width for example here it is w_2 and this is actually w_1 and if suppose this w_1 not equal to w_2 you can consider that $l_1 = l_2$ but $w_1 \neq w_2$ then also you can say depending on the waveguide dimensions etcetera this phase it comes.

At different regions I will discuss that what could be the reasons if there is a phase error is there in the upper arm ϕ for example then we can say that at the output this comes with $A_i / \sqrt{2}$ along with whatever the propagation phase e to the power the total length if it is this one I can consider this one is $l_1 = l_2$ you are just considering l and then along with that e to the power $-j\beta l$ you have e to the j/l phase will be introduced.

Now at the output if you just calculate the output amplitude by superposition of this field from the upper arm and this field from the lower arm. So, we just according to the superposition principle again the transfer functions gives you $1/\sqrt{2}$ factors. So, this one will be this part it is coming from lower arm and this one coming from upper arm and if you do just $A_{\text{naught}} A_{\text{naught}}^*$ we can just this would be proportional to your P_{output} power output.

So, you can say that it is a power output can be defined by P_I , P_i is nothing but P_i we can consider A_i , A_i^* all these we have discussed earlier. So, like this so if you see if $\phi = 0$ then what happens $1 \cos 0 = 1$, so P_{out} will be P_i so if there is no phase difference between these 2 arms then you supposed to get whatever you are launching here power same power you will be getting here this is supposed P_i this is supposed P_{naught} .

And this you are getting the output that is simple and the assumed that there is no loss in the entire structure waveguide losses etcetera we just ignore here, if some losses is there so you have to consider maybe if it is total length is something l then you can consider P_{out} will be $P_i e^{-\alpha l}$ to the power $-\alpha l$ and so on α is the power extinction coefficient power loss coefficient.

And you see this can be a little bit simplified in terms of single term instead of 2 you can write $\cos^2 \phi = \frac{1 + \cos 2\phi}{2}$ - $\sin^2 \phi = \frac{1 - \cos 2\phi}{2}$ - \sin^2 again \cos^2 so it will be 2, so it

will be getting this values power. Now we are talking about phi this phi where prime it can comes, so phi you can consider total phase in the upper arm suppose effective index in the upper arm we consider as n effective 1 that is what this one.

And lower arm n effective 2 this is not square this is just this 2 is just for waveguide 2 that means lower arm considering, upper arm length is l 1, lower arm length l 2. So, upper arm total phase will be 2 pi over lambda times n effective in the upper arm times l 1 that is the phase acquired in the upper arm in the Mach Zehnder Interferometer that is actually we are calling as a phi 1 and similarly 2 pi / lambda n effective 2 effective.

If they are not identical waveguide I mean to say it and lengthy also not identical that will be phi 2 of course we are considering lambda is the operating wavelength that is same for both the arms because same way we wave splitting into 2 arms. So, this would be your phase so for example if you see somehow if this pi is actually instead of 0 it is phi for example then 1 - 1 that means p out will be 0. So everything will be lost to them. So that is what you do not want you want something a certain phase to be introduced here and accordingly certain power use should be getting at the output.

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The slide, titled "Phase Error Interference", discusses the need for precise phase control in passive waveguide devices. It features a diagram of a directional coupler based power splitter with handwritten mathematical derivations. The input signal is A_0 . The output signals are given by:

$$A_1 = A(l) = e^{j\beta_1 l} \left[\cos(\delta l) - j \left(\frac{\beta_2}{\beta_1} \right) \sin(\delta l) \right] A_0$$

$$B_1 = B(l) = -j \left(\frac{\beta_2}{\beta_1} \right) \sin(\delta l) A_0$$
 The phase difference is defined as $\Delta\phi = \delta \cdot l = \frac{\beta_1 - \beta_2}{2} \cdot l$. The effective index is given by $n_{eff}^2 = n^2 + \delta^2$. The propagation constants are $\beta_1 = \frac{2\pi}{\lambda} n_{eff1}$ and $\beta_2 = \frac{2\pi}{\lambda} n_{eff2}$. The coupling coefficient is $k = \frac{\pi(n_{eff1}^2 - n_{eff2}^2)}{2}$. The slide also includes logos for CPPICs, NPTEL, and IIT Bombay.



So, similarly if I just think about; another example of a directional coupler based powers splitters. So, as we saw before that a phase errors can actually deviate the output power in the

Mach Zehnder interferometer because it relies on interference between 2 waves and these 2 waves if slightest variations of phase is there that will be reflected in your output. Here also if we just consider a directional coupler.

Which we have discussed earlier as well you just consider the amplitude of the incoming wave coming through this waveguide A_{in} of course again power we considered here in this case input power maybe $A_{in} A_{in}^*$ the little bit difference here I just put input power as A_{in} here. And after interacting length of L of the directional coupler 2 waveguides they can be identical they can be non identical.

So, these 2 waveguides can be different also different dimensions and different propagation constant can be and they can come closer maintain parallel distance if you want then in this region that is the interacting length basically as they propagate it will see the presence of the second waveguide similar thing also happens if you are launching here it will see the presence of the first waveguide.

So, we are just giving an example of just launching or the one of the input ports in the upper arm and then you see some field you will get here in the A_1 at the so called we call it as a bar port this is your bar port and this is your cross port because this is crossing here that is a cross port. So, A_1 that means after interaction of length L whatever survives here that will be going here and whatever lift tunnel to the second waveguide that will be coming here.

So, whatever survive that expression we have actually given earlier using couple mode theory, this is the expression $e^{-j\delta L} \cos \delta L - j \delta / \beta \sin \delta L A_{in}$, A_{in} is the input that is your amplitude. And where we know that we define the phase actually here in this case this δL that we are just considering as it phase δL so differential phase we are considering.

So that is nothing but actually $\beta_a - \beta_b$ so this one propagation constant if we are just considering β_a , this is β_b we are considering. So, $\beta_a = 2\pi / \lambda n_{eff} A$ that means effective single mode a guide but effective index can be different depending on the

dimension and waveguide B that means lower waveguide $2\pi/\lambda$ and $n_{\text{effective}}$ if they are regional differences there.

I think this we have solved earlier normally you know this δ we have just defined as 2δ we have defined as you can recall it from the couple mode theory that the $\beta_a + \kappa_{aa} - \beta_b + \kappa_{bb}$. So that means this is the term κ_{aa} is considered that modification of propagation constant in waveguide A because of the presence of second waveguide that is what we call it as κ_{aa} that is modification of propagation constant.

And here modification of propagation constant in the second waveguide β_B propagation constant because of the presence of the first waveguide that is why we call it κ_{bb} . So, if you can just approximate $\kappa_{aa} = \kappa_{bb}$ they may not be equal but if we consider that one then simply we can write $\beta_a - \beta_b / 2$, 2 is there that is what we have used in the coupled mode theory.

Accordingly we have derived whatever the amplitude is expecting the bar port and the cross port. So, in this phase on if we just define $\delta n = \delta\phi$ that will be like this where this s also we have defined $\kappa^2 \delta^2$ this κ_a is actually we call it is κ_{ab} and that we consider as a κ_{ba} for example that is actually κ_a that means how much coupling strength whatever per unit length whatever coupling.

Because of the perturbation or because of the presence of the second waveguide that all these we have discussed earlier I am not repeating just takes place and I am just introducing here. Now if you see another way we looked into it and we have shown also that you can think of super mode excitation symmetric mode with a propagation constant β_s and asymmetric mode with a propagation constant β_s that is the solution coming out of combined structure.

And then β_s can be written as operating wavelength λ . So, $n_{\text{effective}}$, so symmetric mode what is the effective index and then $2\pi/\lambda$ times $n_{\text{effective}}$ as. So, we can show that this κ can be expressed if you know this $n_{\text{effective}}$ base and $n_{\text{effective}}$ aa is then we

can show that this κ whatever κ we are considering κ_{ab} κ_{ba} that can be expressed as $\kappa = \pi \Delta n / \lambda$.

Where $\Delta n = n_{\text{effective } s}$, $n_{\text{effective } a}$ as they are the solution of the super modes, so Eigen modes solution of the combined structure and corresponding $n_{\text{effective}}$ index. So that means we know that this $\Delta \phi$ actually involves Δ and that actually contributes how much amplitude which will be coming here and how much amplitude will be coming here depending on s value κ value and Δ^2 value.

So, ultimately we see you have to see that a $n_{\text{effective } a}$, $n_{\text{effective } b}$ or $n_{\text{effective } s}$, $n_{\text{effective } a}$ as when I talk about the ineffective a that means effective index of the guided mode in the waveguide in the absence of first waveguide in the absence of the second waveguide. Similarly $n_{\text{effective } b}$ is the effective index of the guided mode of the secondary guide in the absence of the first waveguide.

But when you talk about symmetric and asymmetric super modes then we can solve combined structure then individually if they are guiding single mode collective combined structure that can have 2 modes we call them supermodes and those supermodes will have a effective index so called $n_{\text{effective } s}$ we can define like that and $n_{\text{effective } a}$. So, in this way we can find κ value from there.

And this κ value can be used here for s and Δ value can be expressed in terms of β_a and β_b and then we can find out what is amplitude here? What is the amplitude here? So, suppose $\Delta = 0$ this would become just like a \cos . If $\Delta = 0$ this would be coming like a $\cos \kappa l$. And this would be coming like a $\Delta = 0$ this will come like a $-j \sin \kappa l$ A_{naught} of course A_{naught} here.

So, this will be cause value this will be the value $\Delta = 0$. If in any case Δ has some nonzero value out of the variation of $n_{\text{effective } a}$ or $n_{\text{effective } b}$ then this variation can cause the output power different then what you do here. So, these are actually it is coming all this effective index

changes differences etcetera that actually introduce phase. So that means this phase here are actual phase has to be precisely controlled.

If you want a delta specific delta that delta should be there because of the fabrication error it should not differ so that whatever you desire you are not getting for example, so this precise control again needed for directional coupler operation if you want to use that as a passive device.

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The slide, titled "Phase Error Interference", discusses the need for precise phase control in passive waveguide devices. It features a diagram of a Microring Resonator (MRR) with input port B_i , output port B_o , and internal nodes A_1 and A_2 . The diagram includes the following elements:

- Equation:** $A_1 = A_2 e^{-\alpha l} e^{-j\beta l}$
- Resonance Condition:** $\phi_{\text{cavity}} = \beta l = 2m\pi$ where $m = 0, 1, 2, 3, \dots$
- Phase Equation:** $\phi_{\text{cavity}} = \frac{2\pi}{\lambda} (n_{\text{eff}} \cdot l)$
- Transfer Function:** $T = \frac{B_o}{B_i}$
- Handwritten Note:** "All-Port Configuration"

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Now third important component and so I would say that the most important component for photonic integrated circuit called microring resonator sometimes we call it as MRR as you know in a micro ring resonator it is given the all pass configuration where you have a ring and one pass waveguide interacting where light will be tunneled here bit and round a pass again comes back in steady state you know that how to solve the transfer function.

And what would be the B naught and we can find out the transfer function T can be B naught / B i square that will be the power transfer function we know that in the power transfer function also we have seen that the resonance condition should be that is called phi cavity round trip phase because of this round trip travel beta l beta is a propagation constant of the waveguide that must satisfy 2 pi times integer.

Then that beta value gives you corresponding lambda beta = 2 pi / lambda n effective is resonant in the ring and that can store the field will be enhanced and that particular lambda will be missing in the output and you will know that that in a ring resonator as a function of lambda if you see in the transmission T you can normalize transmission 1 then you see that repeatedly periodically some wavelength that should be same width some wavelength will be missing in the transmission that means that lost energy will be stored in the ring.

That is how you get a certain wavelength range if you see this would be like a notch filter m is the integer it can start from obviously it should not be 0 if m = 0 round trip phase convert be 0. So, 0 need not be considered it can start from 1 and so on. And then this phi cavity we can say that 2 pi / lambda n effective l. Again if you see if there is any error in n effective compared to your desire design then you can see that you may not get desire wavelength average resonate I will explain more details in this next couple of slides later.

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Tunable Devices and Reconfigurable Circuits Slide#5

Phase Error Interference
Precise phase control needed for passive waveguide devices

(4) Distributed Bragg Reflector (DBR)

For Bragg Reflection: $\phi_p = 2\beta\Lambda = 2m\pi$ and $\phi_p = 2 \frac{2\pi}{\lambda} n_{eff} \Lambda$

Longitudinal Mode Resonances: $\phi_{FP} + \phi_1 + \phi_2 = 2m\pi$ $m = 0, 1, 2, 3, \dots$

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Now the fourth important device we have discussed so far is the Distributed Bragg Reflector we call it a DBR. You see we have discussed very recently that a periodic perturbation is there, a rectangular perturbation you are launching here, then you have forward propagating wave and backward propagating wave will be generated and that backward propagating wave will be grown will be enhanced, if you maintain a certain condition so called phase matching condition.

You remember that phase matching condition if it is a single mode waveguide that phase matching condition or that $\Delta\beta$ we defined by $\Delta\beta$ that is actually $m \cdot 2\pi / \lambda$ that is the phase mismatch. So, if $\Delta\beta = 0$ then it is called longitudinal phase matching condition in that case $2\beta = m \cdot 2\pi / \lambda$, λ is the periodicity. So, now if you see that you can consider $2\beta\lambda = 2m\pi$ that is what I have written as a ϕ_B that is actually writing as a ϕ_B .

So, this is required ϕ_B called Bragg phase ϕ_B , if this match the corresponding $2\pi / \lambda_{\text{effective}}$ that particular λ will be reflected back called λ_B Bragg wavelength. So, we write this one this one again a little bit modified one 2β means $2\pi / \lambda_{\text{effective}}$ and then λ that is your ϕ_B . Again you see you have to have if you want to have this λ as a λ_B Bragg wavelength to be reflected this ϕ_B has to be matched.

This phase condition this type of phase is needed at any cost your $n_{\text{effective}}$ is deferring because of the dimensional error and fabrication error etcetera coming due to your process related error I mean to say then you can say that that λ_B these are λ_B Bragg reflection you would not be able to see. Similarly again we show with Fabry-Perot cavity Bragg reflection this Fabry-Perot cavity using.

That also we have discussed Fabry-Perot cavity DBR1, DBR2 reflection spectrum r_1 reflection and spectrum r_2 and length is l then we know that if you want to see a resonance within the suppose Bragg 1 is giving this type of reflection and Bragg 2 is giving this type of reflection then both sides you get a positive feedback then you can see depending on the round to phase condition you can see some kind of longitudinal modes within the Bragg response.

And in that case you see longitudinal mode resonances that we need the ϕ_{FP} Fabry-Perot that means whatever round trip propagation whatever the phases there and π may be introduced by the Bragg wave DBR 1 and phase 2 introduced by the Bragg by DBR 2 that should be $2m\pi$ we have discussed earlier $m = 0, 1, 2, 3$ that also discuss integer. So, this is the condition for longitudinal mode resonances.

Now again phi FB if we are just considering let us consider that phi 1 and phi 2 whatever comes we know phi 1 and phi 2 they can give some kind of phase difference as you know DBR can give introduce some phase exactly at Bragg wave length that is $\lambda / 2$ and so on. And if you just simply that is somehow defined with your DBR that phase also can control there is any variation in this phase because of the dimensional error etcetera.

You may get different resonance wavelength similarly only if you just consider Fabry-Perot just phi FP that means $2\beta L$, phi FP will be $2\beta L$ if L is the this one length then $2L$ is the round trip length and round trip propagation constant multiplied by round trip length that will be the phi FP that Fabry-Perot related phase we are considering that must be equal to $2\beta L$ $\beta = 2\pi / \lambda n_{\text{effective}}$ L is multiplied again you see $n_{\text{effective}}$ comes here.

So, if there is any error $n_{\text{effective}}$ index that either can come because of the again fabrications or any other influences that will cause variation in phi FP and then phi FP variation will actually derail your resonance wave length detune your region your resonance wave length which may not be desire for you. So, we have given poor examples say Mach Zehnder interferometer, directional coupler and then microring resonator and then DBR.

So, 4 different important devices we have discussed where we have found that the precise control of phase is required to have certain kind of output function if it deviates then you are operating wavelength to resonance wavelength everything will be changed and that will be a catastrophe. This is actually important part phase control is very, very important. Now I will try to show try to give you some quantitative value that how this $n_{\text{effective}}$ can vary because of the fabrication related, technology related, process related either is there in terms of dimension and etcetera of the device.

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Tunable Devices and Reconfigurable Circuits Slide#6

Phase Error Interference
Various sources of phase errors and consequences

(1) Waveguide Phase Shifter

$$\phi = \beta l = \frac{2\pi}{\lambda} n_{eff} \cdot l$$

$$\Rightarrow \Delta\phi = -\frac{2\pi}{\lambda} \Delta n_{eff} l + \frac{2\pi}{\lambda} \Delta n_{eff} l + \frac{2\pi}{\lambda} n_{eff} \Delta l$$

$$\Rightarrow \Delta\phi = \frac{2\pi}{\lambda} n_{eff} \left[\frac{\Delta n_{eff}}{n_{eff}} + \frac{\Delta l}{l} - \frac{\Delta\beta}{\beta} \right]$$

For $\Delta l = 0$ and $\Delta\lambda = 0$, $\Rightarrow \Delta\phi = \frac{2\pi}{\lambda} \Delta n_{eff} \cdot l$

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Let us see a waveguide let us consider a waveguide silicon on insulator waveguide and silicon on insulator waveguide you know one type of this is your box layer and then device layer is structure like a rectangular things in the top maybe top oxide layer is there and this is some kind of photonic waveguide structure it will have a diamond some like W and H but if you have a rib type waveguide structure like this then you can have a waveguide parameter will be W H and h.

So, according to your design suppose you wanted certain waveguide width certain height I have and certain slab height I want that particular parameter that should be maintained throughout the waveguide anywhere anyplace any deviations of W, H, h resulting to deviation in your mode solution or in another one deviation in your effective index calculation. So, as we saw that effective index variation can cause a lot of problem in your integrator optical device performance.

That precise control or n effective is very important to have certain resonance wave length certain performance power output and so on. So, here we try to show that if you just consider waveguide length so suppose this length about l and if it is perfectly design pure according to your design and it is uniform pure mean I mean to say that uniform throughout the waveguide length then it will have a beta propagation constant.

And for a length l travelling whatever the phase this is $\phi = \beta l$ if I just operating wavelength is λ $2\pi / \lambda$ $n_{\text{effective}} l$ that is fine. Now if I see that if this phase is slightly varying deviates differential change happening and what are reason either λ should vary or $n_{\text{effective}}$ should be vary or l should vary this 3 different parameters you have to think about oaky. So, if just do so that what cause differential change of $\Delta\phi$.

So, just make a differentiation than you can get first term you can $2\pi / \lambda^2$ minus value $\Delta\lambda n_{\text{effective}} l$ that is what things I would like just differentiate with respect to λ and then if we differentiate the any differential change for $n_{\text{effective}} 2\pi / \lambda$ constant l constant Δn similarly variation length is vary in $2\pi / \lambda n_{\text{effective}} \Delta l$. So, this little bit modify this one which is take a factor of $2\pi / \lambda n_{\text{effective}} l$ from all the 3 term.

Then what will be getting first term will be this $\Delta n_{\text{effective}} / n_{\text{effective}} \Delta l / l - \Delta l$ this is the minus $\Delta\lambda / \lambda$ you will be getting. So, this is the as useful actually what every you get suppose to get your β value and if you have any deviation in $n_{\text{effective}}$ or Δl or $\Delta\lambda$ then you have to $\Delta\phi$ change will be calculated accordingly fine. So, let us assume for our general discussion.

This Δl is 0 that means length variations is not happening because you want at per certain length of waveguide device you can fabricate that more or less accurately. And suppose you are operating wave length is fixed λ , so that means $\Delta\lambda$ also will be 0, so Δl is 0 and $\Delta\lambda = 0$ we can just assure that. So, if we do so then $\Delta\phi$ only change is $\Delta n_{\text{effective}}$.

That is what repeatedly I am saying that any phase variation that is a most important culprit is that any $n_{\text{effective}}$ change effective index change $\Delta n_{\text{effective}}$ change Δl length we are just assuming not so critical. So that means $\Delta n_{\text{effective}}$ change can cause your phase deviations that is phase deviation in term cause you device performance because all the devices photonics integrated device they rely on interference, interference or need certain phase relationship to have certain kind of interference.

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The slide, titled "Phase Error Interference", illustrates the relationship between waveguide dimensions and effective index. It shows two cross-sectional views of a waveguide on a silicon substrate. The left view shows a waveguide with width W , height h , and core index n_c . The right view shows a similar waveguide with width W , height H , and core index n_c . The substrate is labeled "Si Handling Thickness - 500 - 750 μm". The core is labeled "BOX Thickness - 2 - 3 μm". The refractive index profile is given as $n^2(x,y) = n_c^2(x,y)$. A handwritten equation states $\Delta n_{eff} = \frac{2\pi}{\lambda} \Delta n_{eff} l$. Below the diagrams, a green box contains the derivative equation: $\Delta n_{eff} = \left(\frac{\partial n_{eff}}{\partial W}\right) \Delta W + \left(\frac{\partial n_{eff}}{\partial H}\right) \Delta H + \left(\frac{\partial n_{eff}}{\partial h}\right) \Delta h + \left(\frac{\partial n_{eff}}{\partial T}\right) \Delta T$. Handwritten notes include $n_{eff} \rightarrow n_{eff}(W, H, h, T)$, $n_{sil} = 1$, $\Delta n_{sil} = \frac{\partial n_{sil}}{\partial T} \Delta T$, $\Delta n_{sil} = 1.4 \times 10^{-4} \Delta T$, $\Delta n_{sil} = 8.4 \times 10^{-4} \Delta T$, $\Delta n_{sil} = 1.9 \times 10^{-4} \Delta T$, $\Delta n_{sil} = 2.3 \times 10^{-4} \Delta T$, $\Delta n_{sil} = 1.9 \times 10^{-4} \Delta T$, $T = 300K$, and $\Delta T = 10K$. The slide also features logos for CPPICS, NPTEL, and a copyright notice for R.K. Das.

Now let us see what causes this ineffective change because of the fabrication error process related error. So, you have waveguide width, a waveguide height and you have your slab height h also this is your slab height, slab height is there and if it is your photonic layer waveguide structure when slab height $h = 0$ then it is kind of rectangular waveguide structure that we have earlier we have discussed.

So, dimensional related the n_{eff} if you see and n_{eff} if you just think of that n_{eff} must be function of waveguide width how much you are designing waveguide height, how much you are designing slab height of course this slab how much device layer you are living here for defining your waveguide and of course temperature of the device sample. So, n_{eff} because silicon is a very good thermo optic material.

And you can say that for a range ΔT amount of temperature increased then we can say that the Δn change in the bulk silicon, bulk silicon if you are just considering that will be something how much it is changing per degree Kelvin centigrade and then what is the amount of temperature you are changing. So, for the bulk silicon this value is not n_{eff} I should not say n_{eff} bulk then it is Δn_{sil} for example.

But delta in silicon that is called temperature coefficient that is actually about 1.86×10^{-4} . It looks very refractive index change per Kelvin that is per Kelvin one Kelvin temperature change cause this much refractive index change, it is simple it is looking very small 10^{-4} order wherever if you see silicon bulk is about 3.477 something like that 776, this type of refractive index at $\lambda = 1550$ nanometer wavelength. With respect to this one you can have 10^{-4} changes in temperature fluctuation of 1.

So that means if bulk refractive index change is happening instead of n_d you are just varying it is not constant over temperature then you can say that this effective index also temperature dependent. So, we have just on the top of it you can consider if it is a bare structure like this Tox top oxide layer is not there it is just open then you can think of that it is exposed to the air and air also you can have some kind of in the open ambience it can have certain kind of humidity.

So, humidity can change over time and depending on the humidity surface layer air normally n_{air} you consider a refractive index is 1 but if humidity changes also you can see that refractive index change can be in the order of 10^{-4} or so. So, those types of humidity everything can also play a major rule however when you have a silicon Tox layer or silicon dioxide top layer Tox so called Tox we are considering in that case you would do not get really the your devices not really directly exposed to your ambient humidity so we can ignore that.

So, in turn what I mean to say that your effective refractive index or the guided mode that can vary if you are varying w , if you are varying H if you varying h because all this thing calculation of effective index in your mode solver and you will solve the guided modes etcetera they are basically all this parameter we have shown that they can change the refractive index. So, mathematically if we just look into it just see how much differential refractive index can happen.

Because of different parameter variations, so we can just define this one that means this is one type of coefficient per unit changing waveguide width how much refractive index change can happen? Refractive indexing can happen for the guided mode. Similarly per unit change of the device layer thickness how much effective index can change because if you have a wafer like 300 millimeter diameter.

Like this 300 millimeter diameter for example 30 centimeter or it is can be 12 inch 1 foot diameter and you know that if you see it is a 3 layer system you have a substrate mostly 700 500 micrometer to 500 micrometer to 700 micrometer then you have a box layer about 2 to 3 micrometer and then a device layer that is about 220 nanometer 300 nanometer and so on this is your SOI layer.

So, 3 layer and this device layer is supposed to be 220 nanometer for example but if you see when it is you are buying SOI wafer 300 millimeter this they will say that it is everywhere may not be 220 nanometer there may be some variation of plus minus 5 nanometer and so on thickness is there. So, H thickness is there if you are defining a certain waveguide with certain width thinking that it is 220 nanometer.

But actually your device layer thicknesses more than that or less than that. So, you can also think a considerable contribution of this device layer thickness variation. So, Δn effective ΔH if it is some variation is there then we can find that and similarly for h slab height and temperature. So, these 4 parameters we need to know suppose I want to operate I design a waveguide $W = 500$ nanometer.

I want to design with a device layer thickness equal to 220 nanometer, I want to have a device layer thickness or for example 150 nanometers and I want to have a temperature operating temperature maybe around 300 Kelvin room temperature 27 degrees centigrade or 300 Kelvin. Somehow if you are width varying somehow plus minus something these varying plus minus something and that is coming from your passes related error. And temperature may be fluctuating maybe 1 degree while operation or something likes that. So, what could happen?

(Refer Slide Time: 40:45)

Tunable Devices and Reconfigurable Circuits Slide#9

Phase Error Interference

Various sources of phase errors and consequences

$\Rightarrow \Delta\phi = \frac{2\pi}{\lambda} \Delta n_{eff} \cdot l$

Thickness: 2-3 μm
Si Handling Thickness: 500-750 μm

$\epsilon_r(x,y) \rightarrow n^2(x,y)$

$n_{eff} \rightarrow n_{eff}(W, H, h, T)$

$\Delta n_{eff} = \left(\frac{\partial n_{eff}}{\partial W} \right) \Delta W + \left(\frac{\partial n_{eff}}{\partial H} \right) \Delta H + \left(\frac{\partial n_{eff}}{\partial h} \right) \Delta h + \left(\frac{\partial n_{eff}}{\partial T} \right) \Delta T$

For a rib waveguide of $W = 500 \pm 20$ nm, $H = 220 \pm 20$ nm, $h = 150 \pm 10$ nm

$\frac{\partial n_{eff}}{\partial W} = 3.4 \times 10^{-4} / \text{nm}$ $\frac{\partial n_{eff}}{\partial H} = 2.1 \times 10^{-3} / \text{nm}$ $\frac{\partial n_{eff}}{\partial h} = 2.0 \times 10^{-3} / \text{nm}$ $\frac{\partial n_{eff}}{\partial T} = 1.7 \times 10^{-4} \text{K}^{-1}$

For a rectangular wire waveguide of $W = 500 \pm 20$ nm, $H = 220 \pm 20$ nm

$\frac{\partial n_{eff}}{\partial W} = 1.6 \times 10^{-3} / \text{nm}$ $\frac{\partial n_{eff}}{\partial H} = 3.7 \times 10^{-3} / \text{nm}$ $\frac{\partial n_{eff}}{\partial T} = 1.7 \times 10^{-4} \text{K}^{-1}$

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So, we have estimated simulated for a re-waveguide of W equal to this one 220 H equal to consider plus minus 10 nanometer variation can happen in slab thickness plus minus 20 this is just a old example, today you can get much lower values of course variations device layer thickness 220 nanometer plus minus 220 nanometer and whenever you are defining your waveguide in photolithography or even lithographic or advanced lithography system.

Then you can think of that waveguide diamonds can vary plus minus 220 nanometer that is a probabilistically it can vary maybe one wafer location to another wafer location it can be different. And we have seen that if we just around 500 nanometer 220 h 150 nanometer if you see if slight as there is per unit length per nanometer waveguide width you just think about 500 nanometer if it is 501 or 500 - 1 then you can see that 10 to the power -4 refractive index change can happen.

Similarly height thickness device layer thickness 1 nanometer variation around 220 nanometer, so you can expect refract index change of 2.1 times 10 to power - 3. Similarly then del n effective / del H if you see refractive index variation of 10 to the power - 3 you can see. So, basically this device height device layer thickness which is lower, slab height which is lower, there variation can cause you actually higher refractive index change compared to width sensitivity.

As I say that if you are considering waveguide temperature variation our calculation shows that it is in the order of 1.7×10^{-4} per Kelvin this is the coefficient refractive index coefficient with respect to width variation, with respect to over height variation, with respect to slab wide variation, with respect to temperature variations. Now if you just consider instead of slab a height you can consider this type of we get where $H = 0$.

So, in that case again you consider $W = 500 \pm 20$ nanometers = 220 nanometer and so on h variation you do not have a h. So here also you see almost $\frac{\Delta n_{\text{effective}}}{\Delta W}$ in this case relatively higher, it is instead of 10^{-4} you are getting about 10^{-3} because width you know because you do not have this bulk, this bulk actually contributes a lot for temperature stabilization etcetera.

If it is not their actual temperature stabilization width variation sensitivity etcetera all those things can happen and height variation own change much maybe 1.5 times or so compared to re-waveguide structure and for temperature variations almost same we can consider. So that means your fabrication has to be really high-tech really a nanotechnology require but you can always ask question when you refractive indexes changes in the order 10^{-4} and 10^{-3} what is the big deal?

I will show you that this is really a big deal even if it is 10^{-4} , 10^{-3} that is a big deal big problem for your device performance. So, I mean to say that even 1 nanometer variation in width height or slab height variation that a huge problem in your device performance. So that means you have to be that much careful that your device should not deviate even 1 nanometer 2 nanometer in width wise on height wise or so on. If you want them to use as your passive devices of course. So that is very important conclusion.

(Refer Slide Time: 44:45)

Tunable Devices and Reconfigurable Circuits Slide#10

Phase Error Interference

Various sources of phase errors and consequences

Performance degradation for a MZI

$$P_0 = P_1 \cos^2\left(\frac{\phi}{2}\right)$$

$$\phi = \phi_1 - \phi_2 = \frac{2\pi}{\lambda} (n_{eff}^1 \cdot l_1 - n_{eff}^2 \cdot l_2)$$

Let us assume: $\lambda = 1550 \text{ nm}$, $l_1 = l_2 = 100 \mu\text{m}$, and $n_{eff}^1 - n_{eff}^2 = 10^{-2}$

$\Rightarrow \phi = 1.3\pi$ $\Rightarrow P_0 = 0.35P_1$

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So, I would like to say that does it matter that 10 to the power -3 - 2, 10 to the power -4 variations really matters. Let us see example of again our Mach Zehnder interferometer we have shown that phi equal to this one we have expressed, let us consider that the operating wavelength is 1550 nanometer and we consider this l1 and l2 they are identical balanced Mach Zehnder interferometer 100 micrometer. However because of the diamond signal variation for example n effective 1 and n ineffective 2.

I have just a little bit exaggerated instead of 10 to the -3 we just consider 10 to the power -2 for example, 10 to the power -3 you can consider but it all depends on again length also, it will just consider delta n variation between these 2 arm is 10 to the -2 then if you calculate phi that is actually coming 1.3 phi that means wave propagating here and a propagating here they will see their phase difference is about 1.3 phi.

You know phi difference can cause complete disappearing in the output everything will be lost in the junction but it is 1.3 phi a little more than that. So, if we calculate what is a power we just put 1.3 phi here you get only 35% output here 65% will be lost. So, 10 to the power -2 if it is there you can have 10 to the power -3 and suppose you are considering l = 1000 micrometer, same effect refractive index variations difference in refractive index if it is 10 to the power -3.

And if you are using 1000 micrometer here length that they are also you see 35% of power will be released that means just 1 nanometer variation in device layer thickness or one nanometer variation in device layer width you can have huge problem you were ending with a 35% output power instead of 100% take another example ring resonator.

(Refer Slide Time: 46:51)

Tunable Devices and Reconfigurable Circuits Slide#11

Phase Error Interference
Various sources of phase errors and consequences

Performance degradation for a MRR

$$A_1 = A_0 e^{-\alpha l} e^{-j\beta l}$$

$$B_1 \Rightarrow A_1 \Rightarrow B_0$$
 (Diagram of a ring resonator with input B1, output B0, and internal points A1, A0)

For Resonances

$$\Phi_{\text{cavity}} = \beta l = 2m\pi \quad m = 0, 1, 2, 3, \dots$$

$$\frac{2\pi}{\lambda} n_{\text{eff}} l = 2m\pi \Rightarrow n_{\text{eff}} l = m\lambda_m$$

$$\Delta n_{\text{eff}} \cdot l = m \Delta \lambda_m \Rightarrow \Delta n_{\text{eff}} \cdot l = n_{\text{eff}} \left(\frac{\Delta \lambda_m}{\lambda_m} \right)$$

$$\frac{\Delta \lambda_m}{\lambda_m} = \frac{\Delta n_{\text{eff}}}{n_{\text{eff}}}$$
 (Handwritten notes: 150 nm , $150 + 35$, 1.6 , $m = \frac{n_{\text{eff}} l}{\lambda_m} = \frac{2.8 \times 100}{1550} = 1.8$, $\Delta \lambda_m = \lambda_m \frac{\Delta n_{\text{eff}}}{n_{\text{eff}}} = 1550 \times 10^{-3} \times \frac{10^{-3}}{2.8} = 0.55 \text{ nm}$)

Let us assume: $\lambda_c = 1550 \text{ nm}$, $n_{\text{eff}} = 2.8$ and $\Delta n_{\text{eff}} = 10^{-3}$

$$\Delta \lambda_c = 5.5 \text{ nm}$$

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We have shown that ring resonator cavity this is the resonance condition we have discussed again and again. And this if you just this you concentrate here that $\beta = 2\pi / \lambda m$, m I have written that resonant wavelength m th order resonant wave length just to respect m here, n effective is the guided mode effective index l is the round trip length of the ring resonator and this is total length l perimeter length.

So, this one so this equation can be modified to this one and if you just simplify the 2π 2π cancel then you can say that a n effective $l = m$ times λm . So, m we put 1000 for example 1000 order of longitudinal mode order and you know n effective is about let us consider n effective say 2.8 $l = 100$ micrometer for example, then 1000 times λm whatever you will be calculating that will be the micrometer because this is micrometer.

So, λm will be 2.8 times 1000 time 100 divided by 100 micrometer. So, 100, 100 cancel so that will be 2.8 micrometer I am considering if you are considered 1000, if it is 1550 nanometers the m th order would be lower. Little bit modification here let us see any differential

change in effective index. So, I differentiate l I am just considering maybe constant perimeter is constant nothing happening to length that is according to the region coming.

Only dimensional error cross sectional error W, H, h these areas there Δn effective l , Δl ineffective l that means m is same order of resonant wave length I am considering $\Delta \lambda$ l . So your wavelength same order but resonance wavelength will be varying $\Delta \lambda$ l . So, from here I am just writing here, so Δn effective times $l = m$ again I can write $m = n$ effective l / λ m .

So, n effective l / λ m instead of m n effective / λ m I am considering and $\Delta \lambda$ m we are writing. So, this is simply m I replaced and then if you just compare these 2 things l , l cancel I can consider Δn effective / n effective = $\Delta \lambda$ / λ m , so $\Delta \lambda$ m / $\Delta \lambda$ is this one. So that means slight differential change in your effective index are dimensional error then you can find out how much shift can happen in a resonant shift can happen in ring resonator.

So, in that case what we do? Let us assume I am expecting a resonant λ m a particular resonant as λ r 1550 nanometer and n effective we can consider about 2.8 fundamental mode single mode waveguide 2.8. And suppose your Δn effective change happening because of the dimensional error is 10 to the power -2 if you just calculate $\Delta \lambda$ m , $\Delta \lambda$ m will be $\Delta \lambda$ r that means resonance detuning will happen 5.5 nanometer.

So, 5.5 nanometer resonance detuning is a huge, suppose you want to operate at 1550 nanometer resonance now it is resonant giving you resonance at $1550 + 5.5$ nanometer you know ITU channels for communication etcetera if it is 200 giga hertz ITU channel that means it is about 1.6 nanometers it is 5.5 nanometer spacing 1.6 nanometer spacing of ITU channels but dw dm communication and you are getting 5.5% that means 3 channels will be hopping.

So that is a huge problem 5.5 nanometer resonance detuning just because of the 10 to the -2 refractive index change, if it is 10 to the -3 refractive index change this would have been 0.55 nanometer that is a lot also that is also very much sensitive. So that means 1 nanometer

waveguide width variation can cause your resonant shift a lot half nanometers shift can happen also particularly for high curing regenerator this half nanometer shift means you are gone completely outside your of your resonant wavelength. So that is a huge problem.

(Refer Slide Time: 51:33)

Phase Error Interference

Various sources of phase errors and consequences

Performance degradation for a DBR

For Bragg Reflection

$$\phi_B = 2\beta\Lambda = 2m\pi \Rightarrow 2 \frac{2\pi}{\lambda_B} n_{eff} \cdot \Lambda = 2m\pi$$

$$\Rightarrow 2n_{eff}\Lambda = m\lambda_B \Rightarrow 2\Delta n_{eff}\Lambda + 2n_{eff}\Delta\Lambda = m\Delta\lambda_B$$

$$\frac{\Delta\lambda_B}{\lambda_B} = \frac{\Delta n_{eff}}{n_{eff}} \frac{\Delta\Lambda}{\Lambda}$$

Let us assume: $\lambda_B = 1550 \text{ nm}$, $n_{eff} = 2.8$, $\Delta\Lambda = 0$, and $\Delta n_{eff} = 10^{-3}$

$\Delta\lambda_B = 5.5 \text{ nm}$

Now last thing is that if slightest change in the DBR structure distributed reflector DBR we have discussed earlier. Here also the Bragg reflection happens we discussed that this condition has to be fulfilled 2 beta times lambda that means if it is a period this is your period for example you have just considering this to this period that is your lambda that means round trip phase enable every unit cell round trip phase that is beta times 2 lambda should be m times integer multiple of 2 pi.

So, this one can be expressed $2\pi / \lambda_B$ it is a Bragg wavelength of course this is the Bragg phase we consider we defined earlier n effective times lambda then this would be equal to this one. So, ultimately you get this is the Bragg wavelength we can calculate if we know the period, if we know the n effective and if we know the grating order refraction mostly it is 1 then we know to $\lambda_B = 2 n_{eff} \lambda$.

So, this is the grating order which order diffraction or you are using per your phase matching condition that we have discussed earlier you can just see back. Now if you do this one a differential you just differentiate both set both sides then what you see $2 \Delta n_{eff} \lambda$

2 effective keep constant period variation if you are just considering any variation happens in the period $\Delta \lambda$, n is the grating order we are considering same $m \Delta \lambda$ B.

So, $\Delta \lambda$ B is the deviations for the Bragg wavelength detuning from the Bragg wavelength because of the variation is Δn effective and $\Delta \lambda$. So, if you just a little bit manipulate here then you get $\Delta \lambda / \lambda B = \Delta n \text{ effective} / n \text{ effective} / \Delta \lambda$ a λ you can assume that period variation if it is 0 then you can see that this is exactly similar like your resonant wave length in the ring resonator.

So, resonance wavelength in the ring resonator the variation sensitivity refractive index variation sensitivity same as the DBR structure Bragg wavelength deviation sensitivity it is same. Let us assume $\lambda B = 1550$ nanometer your phase matching wavelength λB 1550 nanometer n effective again you consider 2.8 $\Delta \lambda$ suppose it is 0 and 10 to the power -2 refractive index change can cause again Bragg wavelength detune 5.5 nanometer.

You can have a Bragg transmission in transmission on this one stopband can be say 1 nanometer to 5 nanometer for example this is a stopband say around 1 nanometer 2 nanometer or 5 nanometer etcetera. But what happens if any way your 10 to the power -2 change happens then you get 5.5 nanometer deviation you will be getting here somewhere here even it is 10 to the power -3 variations because 1 nanometer width variation can give you 10 to the minus the width or height variation can give you 10 to the power -3 refractive index change.

So, then also you can get 0.5 nanometer variation suppose it is a 1 nanometer stopband then it will be just coming somewhere here suppose you are operating exactly at this wavelength that will be now it does not see much of the DBR response it will be like transmitting like a transparent medium. So that is huge problem. So it is a really you will need a precise control precise technology to control that.

And so far if we know that our CMOS technology even if it is a nanotechnology people are using advanced technology up to 10 nanometer technology etcetera, but they are actually you can go for precise fabrication but for CMOS compatible silicon photonics technology we typically use

180 nanometer technology or 90 nanometer technology CMOS technology node 45 nanometer technology the latest thing are coming.

So, far 180 nanometer technology, 90 nanometer technology and 45 nanometer technology as you go for advanced technology your this type of error will be reduced no doubt about it. But so far whatever boundary is giving you have the tolerance error fabrication error can happen up to 1 to 5 nanometer variation can happen in width whenever defining photolithography or using a bibliography something like that.

1 to 5 nanometer divisions can happen in full wafer side. Suppose you have a wafer you are designing your mask is suppose you are using to define a waveguide with a 500 nanometer here same design would give maybe about 495 nanometer width but here you can give you 505 nanometer may be middle you can have 500 nanometer. So, entire wafer if you see different region waveguide may come out with a different dimensions.

And different dimension 5 nanometer dimension means 5 times into 1.3 and so on that means about 10 to the power -2 also. So that variation can give you this much detuning in DBR structure, ring resonator, Mach Zehnder interferometer and phi phase detuning etcetera will happen. So, fabrication cannot ensure complete uniform yield throughout the wafer even though you are using 45 nanometer technology node CMOS technology node.

You can have also the device layer whenever you are procuring buying the silicon on insulator wafer as I said that device layer thickness cannot be you cannot expect device layer thickness through our 300 millimeter diameter uniform, there will be plus minus 1 nanometer is always obvious 1 to 5 nanometers. So, it is a basic precision of nanotechnology required to control all these and that is so far we are we do not have, so it can happen a lot of problems.

So, now if there is a problem, there is solutions people are coming out with the solution of how to solve all this will discuss that in the next lecture how post fabrication correction of devices once you fabricate full wafer, how to correct that if there is any problem in some device performance somewhere so you can correct that. And then you can just use for your different

circuit application and packaging all those types of thing can do that. So, I stop here for this lecture today.