Integrated Photonics Devices and Circuits
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Lecture - 05
Fundamentals of Lightwaves: EM Waves: Maxwell Equations and Plane Wave Solutions

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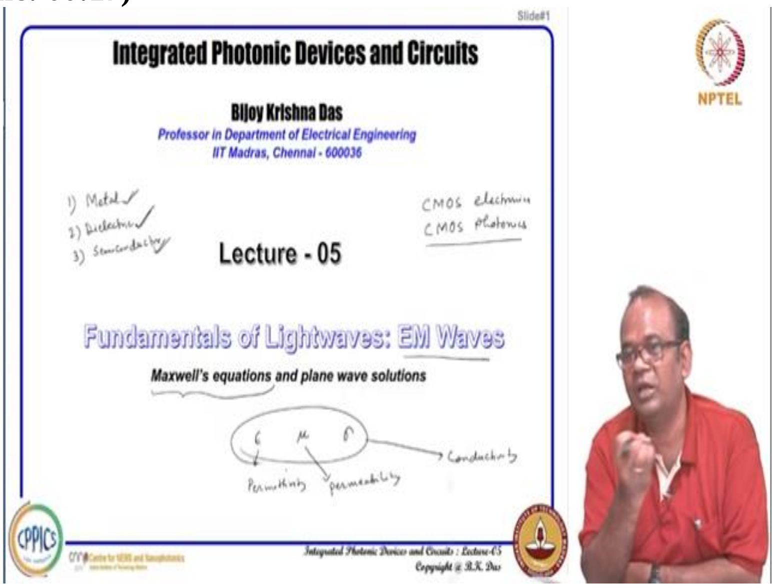

Hi everyone, so far we have given some overview about so called CMOS technology CMOS electronics and overview on our evolution on CMOS photonics technology meaning we have just discussed about how CMOS fabrication process can be translated directly to fabricate photonics chip that is what CMOS photonics. And we have seen that in CMOS electronics if you see the electronics and semiconductor is basically the main platform.

But apart from semiconductor we have seen that metal also used for interconnect. And then dielectric you use for dielectric material like silicon dioxide or silicon nitride etcetera used for isolation or insulation purpose and then third thing is semiconducting material use for actual transistor devices. So, these are the 3 different types of materials you use. And since our CMOS photonics also is directly we are considering that similar technology similar process lines.

So, they are also we have the availability of metal, availability of dielectric and availability of semiconductor. So, all these things we use for photonics devices, photonic circuits and obviously, we need to know we need to understand when you talk about photonics devices, photonics devices means basically we talk about photonics integrated circuit means it is basically light wave circuit.

So, light waves circuit meaning how light you are controlling in the circuit light wave you are controlling light flow, photon flow you are controlling the photonic circuit. This is similar to electronics flow control in electronic circuits. So, we need to understand how light wave behaves in these 3 materials. That is the fundamental thing if we know how that behaves the light waves in these materials metals, dielectrics, semiconductor then it will be easier for us to design photonic integrated circuit components.

So, today we will be trying to understand bit of fundamentals of light wave. Light wave is nothing but it is electromagnetic waves and this electromagnetic waves how it behaves in these 3 material platform I will be discussing today. And you know when we talk about electromagnetic waves you cannot just get rid of the Maxwell's equations because Maxwell's equations actually is the key to discover electromagnetic waves.

And we will try to understand the electromagnetic waves how it propagates in the space and then how it interacts with the material medium. When you talk about material medium, the electromagnetic wave in material medium light matter, you can say light matter interaction or electromagnetic wave material interaction. For that purpose, we need to consider 3 different parameters so called epsilon, mu and sigma.

So, these parameters it is you know that this epsilon is basically called as a so called permittivity and then, this is called permeability and this is called conductivity any material if you are considering in terms of electronics or in terms of photonics, you need to consider only these 3 parameters how what is the value of these 3 parameters in the material medium. Normally the epsilon permittivity, mu permeability it is a magnetic material permeability etcetera come sigma conductivity.

Whether depending on the conductivity you can actually distinguish whether it is a metal, it is a dielectric and it is a semiconductor or not? So, we need to understand also how these electromagnetic waves behaves in presence of some values material to material, how that they behave so, that also we need to know. So, what we would like to do that we will just start with Maxwell situations.

And then slowly, slowly we will try to understand that how these Maxwell's equations help to understand that electromagnetic wave exist even in the free space and they can travel also in the free space first. So, after we understand these basic things, then we will try to understand how these electromagnetic waves behave inside a material medium? And if that material media is a dielectric, how it behaves?

And if that is a semiconductor, how it behaves? And if it is a conductor, how it behaves? What is the nature? So, that is very, very important to understand the photonic integrated circuits starting from waveguide to device components etcetera everything you will understand or you can design if you know certain material having this type of property and how they are technologically viable along the line of CMOS fabrication process so, let us move on.

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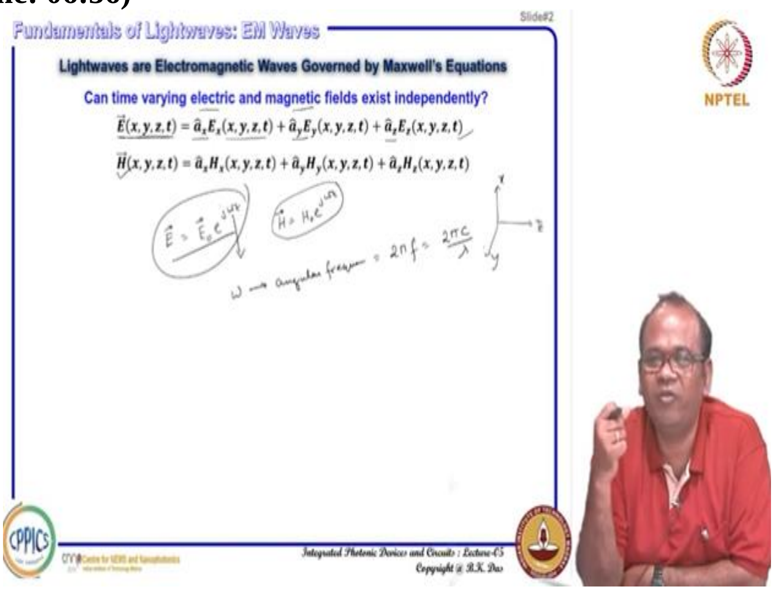

So, first question I start with a question can time varying electric and magnetic field time varying electric field or magnetic fields exist independently? So, you can create a electric field for example, if you have a electric field of source say E = E 0 e to the power j omega t just a sine wave it can be a field it can be a certain direction electric field it can have time varying field.

And you can think of magnetic field also you can have a magnetic field source different ways you know how to create that magnetic field can be also time varying. So, first question is that we will try to understand that whether this electric field if it is time varying with a certain angular frequency omega is angular frequency, that is basically 2pi f when f is the linear frequency and sometimes you can write like this 2pi c by lambda wavelength.

So, called things we know that, that is how you can do this. So, thing is that here when I just mentioned this one electric field independently it is at a certain point in space, you can consider that this is vary. And you can say that at a particular point your magnetic field a particular point in the space magnetic field oscillating sinusoidal. Now, question is that, can they be related for example, since electric field is a vector.

If I just try to express this electric field vector is space dependent as well, you can have a electric field space dependent for example, if you had a point chair somewhere, you can calculate your electric field every point according to the coulomb’s law etcetera, you can find out what is the field surrounding it? That is x, y, z dependent, we can consider a Cartesian coordinate system and of course, you can have that field every point can be time dependent. So, I can express the electric field space dependent x, y, z, t.

So, I can say that at any point you can have 3 components x component, y component, z component and every where you can have the x component also space dependent, it can happen that in 1 point x component is high and another point y component is higher. So, that can happen that is why we can say that point to point I can actually decompose what are the electrical components x, component y, z component 3 orthogonal directions you can just think of and they are time dependent also you can think of.

Similarly, I can also think of magnetic field, this magnetic field also x dependent, x component, y component, z component space dependent x, y, z code in it. So, you can think of this if I consider this your x, this is z and this is the x and this is z, y, x, y, z, you can just consider. The frame reference frame and every point I can define what is the electric field and time dependent things.

So, if you define electrical magnetic field like that, it is fine you can just think them independently, but Maxwell actually showed that if electric field is time varying, before Maxwell it was known if electric field is time varying then you can generate magnetic field and magnetic field each time varying then you can generate also electric field. So, try to understand what is that actually and how that can be explained using your Maxwell's equations? If they are existing with time dependent variation in space then obviously they should be related.

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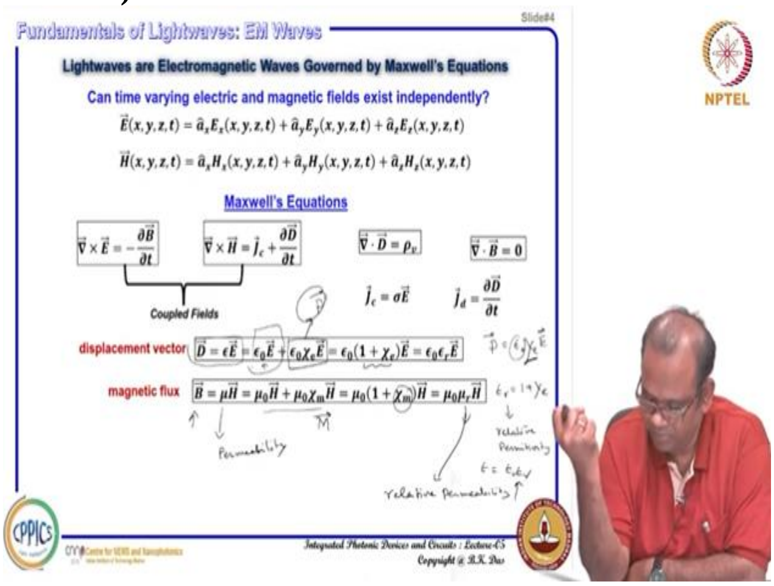

So, let us see this is Maxwell's equations, it is basically combination of law existed earlier some laws, physical laws already known before and Maxwell combined them together with some modification. How that is first law is that this one curl of E is the electric field vector is related to rate of change of magnetic field B is magnetic field induction and E is the electric field at a time varying magnetic field is there, then you can find that electric field curl of electric field will be generated, rotation of electric field you can find out.

And similarly, if you have magnetic field rotation is there then obviously that magnetic field can generate because of the conduction current is defined by this if you have a electric field then sigma conductivity is there then conduction current density and conduction current density if it is there, then you can see magnetic field, but, Maxwell's thought that if time varying magnetic field can give you electric field.

So, time varying electric field also this current density can be time invariant that means time independent, but you can have additional component which can be time varying that can contribute to magnetic field this D is related to electricity by the way, it will be clear a little later that is called for the moment you know that D is known as displacement vector. So, this displacement vector means you are just rate of change of displacement vector is normally we can call it displacement current J d.

So, that means, Maxwell actually added this J d term displacement current if the electric field oscillating in a material medium then you should have a displacement vector to be considered and that displacement vector time variant of displacement vector can be considered as your current component. So, he actually added this particular component and he tried to match whatever existing curl = del D by del t.

So, basically this is actually called Faraday law and this is actually ampere circuital law, ampere circuital law was existing up to here. So, Maxwell added this part and this one is the Gauss divergence law for the electric field divergence D and D again is related to electric field I will just explain and rho v that is the charge density and since there is no monopole exist in magnetic field magnetic monopole cannot exist.

So, divergence B = 0, so, we can write that simply divergence B could B again, I will be discussing how it is related to magnetic field. So, this is actually Gauss law, this is also Gauss law this is for electric field and this is for magnetic field. Now, we will just look this Faraday law and modified ampere circuital law which is modified by Maxwell's equation, then you see that electric field and magnet field they are coupled here also electrical is there, magnetic field, electrical field, magnetic field, electric field will be there.

So, J c = sigma E, this is basically it is coming out of ohm's law material medium if some conductivity or resistivity is there and field is there then you can see them current density fine, so, far so, good. Now, as I said what is that displacement vector? That is actually defined like this and D = epsilon E epsilon I have just mentioned earlier that it is the permittivity of any this permittivity can happen in the free space.

This permittivity can happen in the medium and that can be expressed in terms of this one where this particular term we just represent here we call it polarisation density. That means, if you apply electric field that electric field will have 2 components one is just the electric field and if it is a material medium in the material medium you know material medium it is composition of electrons, protons all those type of things.

So, any electric field is there, there you can create some kind of dipole moment and that dipole moment can create also some kind of so, called dipole moment per unit volume, it is called polarisation. So, that polarisation that dipoles can also create some contribution to the external electric field. So, you add them together this is as if nothing is there epsilon 0 is corresponding to permittivity corresponding to free space normally.

So, with that you just add this one that is actually polarisation density and together we call displacement vector and if you are little bit simplify this epsilon 0 1 over chi E this chi E is representation is nothing but basically p we are writing p = epsilon 0 chi E is electric field basically polarisation density is proportional to electric field and proportionality constant epsilon 0 we are considering just free space multiplied by some factor which is actually responsible for the material property that is actually called electrical susceptibility.

And if you do that, then you just put down this epsilon r where epsilon r you are putting 1 + epsilon so, this one is your epsilon r. So, you are writing basically epsilon 0 times epsilon r that means, if I consider epsilon 0 is the permittivity of the free space epsilon are actually contributing because of the presence of material how it behaves with electric field. So, that actually being represented by epsilon r and it is known as the relative permittivity sometimes it is called relative dielectric constant of the material.

So, that means, I can say that this D = epsilon E where epsilon E is simply we can write epsilon 0, epsilon r forget about all the susceptibility, et cetera. So, material properties consolidated in epsilon r and as a whole it is by epsilon. Similarly, we defined B which is called magnetic induction density that can be defined as similar to epsilon you just define as a mu times magnetic field strength mu H.

And similar to that one if I just follow up you can think of some kind of magnetic dipole also that can create some certain kind of so called magnetization. It is something like polarisation we can call it here magnetization and then this thing will be called a magnetic susceptibility. And 1 + chi m can be called as a mu r and this will be mu is called permittivity permeability and this one will be called a relative permeability.

So, do not confuse with permittivity and permeability, permeability is related to magnetic field and permittivity related to your so called electric field. In another word, this thing actually characterise the medium in terms of distributed capacitance and distributed inductance, inductance is related to magnetic field capacitance related to electric field. So, if you can consider the entire medium even at its free space or material medium you can think of they have certain kinds of distributed capacitance.

And distributed inductance that is actually directly related to your electric field and magnetic field capacitance can hold electric it can somehow hold some electric field inside it and inductance also it can hold some magnetic field inside it so far so, good. So, we have defined D, B used in Maxwell's equations.

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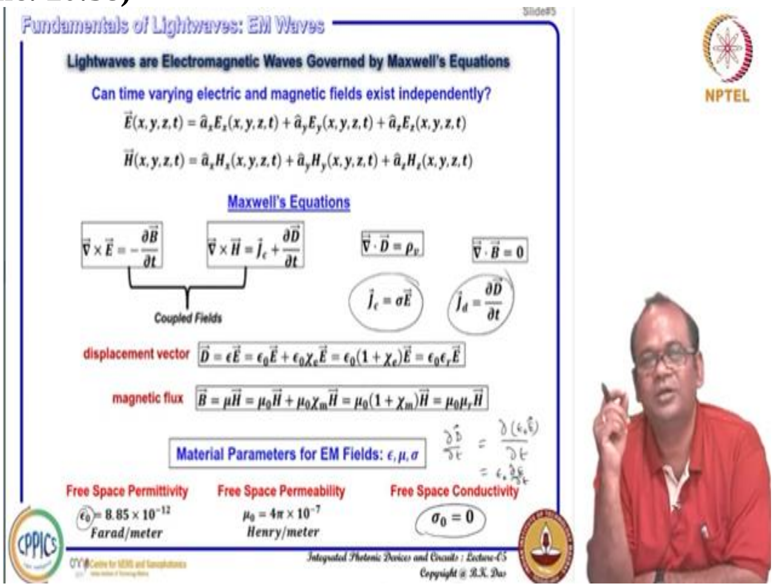

Let us move on this material parameter I said that, you can find that there are many ways to experimentally measure the value of so, called epsilon 0 in free space that is actually 8.85 into 10 to the power minus 12 Farad per metre, Farad do you know it is a unit of capacitance and per metre. It is something distributed per unit length what is the capacitance for example, that is what we can interpret like epsilon 0.

Similarly, mu 0 you can interpret like say inductance Henry inductance you can measure by Henry that the unit of induction is Henry, Henry per meter and that is 4pi into 10 to the power -7 in SI units all these attributes. And free space of course, the sigma conductivity free space you cannot just think of current because current you need to have electron flow conduction. So, that is why you can think of that too conductivity particularly this conduction current actually absent in the free space.

Though you can think of in free space del D del t, you can think of delta D if it is free space that can be written as epsilon 0 E by del t that means, del D delta t equal to this one obviously, vector epsilon 0 del E del t that means, you can think that this in play space this J d can present somewhat displacement current can present but conduction current is absent in free space you cannot just flow current in free space otherwise it would have been a huge problem.

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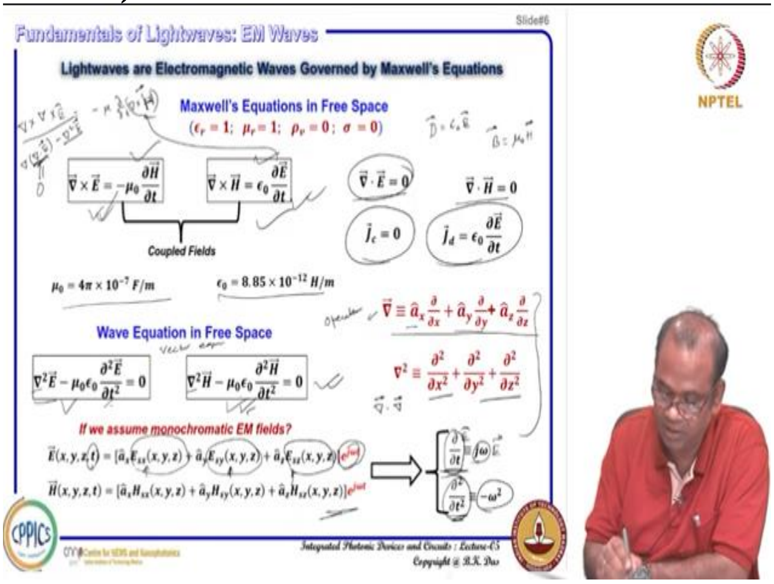

So, I just consolidated here in free space just Maxwell equation I have written down in free space coupled to Carl equation so, called Carl equation and if I just say in the free space D will be quantities divergence D, D = epsilon 0 E and epsilon 0 you take out that means, ultimately we are getting divergence equal to 0. Similarly, divergence B = 0 can be converted into B equal to say mu 0 H divergence be equal to 0 so, mu 0 can be taken factor out.

So, it can be 0 sigma = 0 in free space J c = 0 and J d will be equal to this one that is what I explained. And this numbers you just keep in mind sometimes it is useful to solve some problems also. And then we derive a wave equation in free space how it is that we try to decouple electrical and magnetic field from this coupled equation how it can be done. So, it can be done very simple suppose, you do this curl of E = - mu 0 del del t H.

H I am writing here if we take one more curl then what happened right hand side I can put down like this curl of H. And then curl of H you have the expression here you just put this curl of H instead of curla of H you put this one. Then at left hand side is only electrical right hand side also will be electric field and left hand side what you can do you can expand this with a vector identity like this one minus gad square E.

This is left hand side and again divergence E you know that in free space that is equal to 0, so this will be 0 so it is leftover del square E. So that del square E we are writing and minus mu 0 epsilon 0 del del E 2 that is how you get your decoupled equation. Similarly, for magnetic field you can start with this equation take curl from both side and then substitute curl E here then you get this equation and if you see carefully what is this del square?

Del is basically a vector quantity it is an operator it is basically called a deal operator it can operate on any scalar if it operate on a scalar the scalar will become a vector and it can also operate on vector also you have x component you have to see del del x how it is varying along the x direction and along y direction how it is varying and along z direction how it is varying. So, this variation in all 3 directions you clap together that is actually vector and if you try to see the del square, del square means it is nothing but del dot del.

So, del dot del means this one you can represent this del and del squaring other systems also reference system like spherical coordinate system, cylindrical coordinate system, but most of the time we will be dealing in photonic integrated circuit this Cartesian coordinate system is better it is all right. Sometimes you know if you are dealing with fibre optics, normally it is better to have a cylindrical coordinate system in that case you can represent cylindrical coordinate system I am not going into that details just you remember this one.

Del operator is like this and del square operator will be like this. So, del square operator here, del square is a scalar by the way and del square you are operating on vector it will be a vector of course, and this is also vector. So, this is basically the vector equation differential equation vector and if you see that this can be solved basically impact if you solve this equation this one independently or this one you can independent you can solve you can find the value.

You can find a possible solution once you find a possible solution in validated by this differential equation, then you can find magnetic field by using one of these equation. If you know electrical then you can find magnetically to this equation this equation or this equation you use you can find. So, it is sufficient to solve one of them then you can use this one to find other component if you are solving electric field, then you can find magnetic field using this equation, if you are solving magnetic field you can get electric field using this equation.

We just try to consider that we assume the monochromatic electromagnetic fields instead of time dependent this part I just consider the time dependent oscillation e to the power J omega t every component oscillates with e to the power J omega t I just put here space is just consider that space dependent x component, this is space dependent y component, this is space dependent x, y dependent z component.

Each of them is packed with e to the power omega t that means, which electric field is oscillating with a particular sinusoidal manner. That is why we are writing e to the power omega t you can consider just cos omega t or sin omega t, but we will be using it to the e to the power omega t. So, that mathematically it would be easier to handle. Similarly, magnetic field if I just consider like that, in that case this del del t and del to del t.

Suppose you are just considering del E del t then we will be getting J omega e that means, instead of del del t operator I can write J omega and del del 2 I will be writing minus omega square because I have these operators basically I want to use them.

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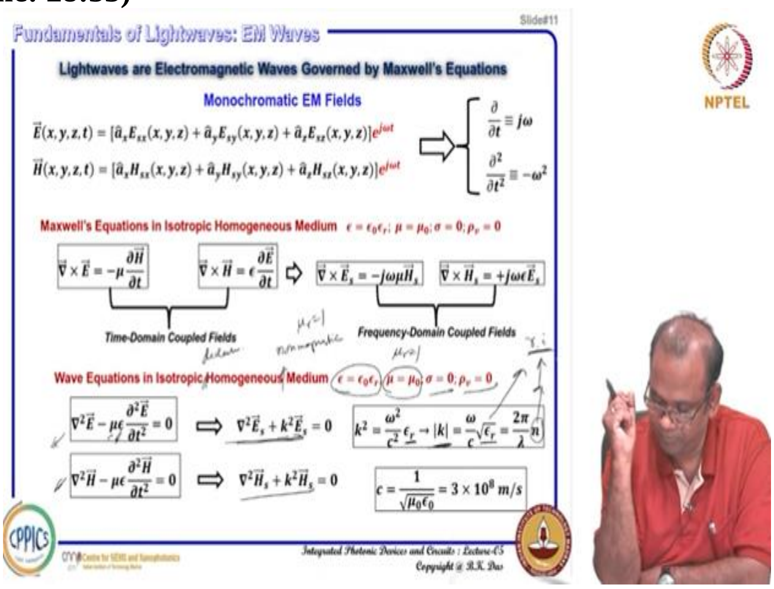

Then if I use this thing these 2 in this Maxwell's equations then for this one if I use then I can write this Maxwell scale equation in this form here if you see the time dependent part is replaced by j omega and that is why it is called frequency domain coupled fields. Sometimes, it is called Fourier domain coupled fields because frequency and time they are related with Fourier transform we know that sometimes it is called Fourier domain, if you are using this one in the wave equation.

It will be this one will be represented by this one how it is in free space we are considering epsilon = epsilon 0, mu = mu 0, sigma = 0, rho = 0. That is how we got and if I just put this one is equal to minus omega square if I just put minus omega square then I write this will be written as something del square E + mu 0 epsilon 0 omega square E so, mu 0 omega 0 epsilon 0.

I can have introduced certain constant I just put this omega square mu 0 epsilon 0 as K square starting constant omega monochromatic with omega is there, mu 0 is constant epsilon is constant. So, I can put this one and that means, this mu 0 epsilon 0 is just he will do the another constant defined by this one will come to know that you can remember that c = 1 by omega 0 epsilon 0 it is basically 3 into 10 to the power 8 dimension you put down metre per second. Normally we know velocity of light in free space.

So, in that case, I can get k square = omega square by c square if I just defined c equal to that like this and if I write k mod of k the square root I will just put in mod you will come to know why I am putting mod here, because the square root omega by c you can just put plus minus omega by c So, that you can have k = + and minus and so on. So, I can have electric field equation like this and electric field equation like this and this one I am just calling it a wave equation you know why?

Because, the solution is like a you will get like a wave solution that means, the field is something will be like a space dependent time dependent wave you will be getting a solution normally you can get something solutions like that, if it is only 1 dimensional we are considering you can get a solution one type of solution like this another type of solution you will be getting ct + x, fct + x.

So, if that wave is 1 dimensional propagating along the x direction, then we will be calling us this one positive direction and if it is negative direction, this will be fct + x. So, function of ct, ct is something velocity and time is there. So, we can have that type of solutions, but, we know that if it is homogeneous medium just free space that equation is like that the solution will be like a wave travelling I will come to that point a little while later.

If I just replace the homogeneous medium that epsilon 0 will be replaced by epsilon 0 epsilon r and mu we are considering like a mu 0 as if mu r = 1. So, what is that mu r = 1, I am considering isotropic homogeneous medium when I consider isotropic homogeneous medium one additional inherent thing is that non magnetic material need not be non magnetic, but most of the photonics devices we will be handling that will be non magnetic material where mu r should be equal to one.

So, if you consider that then mu simply equal to mu 0 or mu you can write simply whenever necessary we just put for free space mu 0 and homogeneous media. Since it is an isotropic homogeneous media it must be it should consider dielectric, dielectric medium charge free dialectic means no conductivity no free carriers are there to conduct current it is a silicon dioxide type dielectric material if you consider that and sigma = 0 and charge also is not there this is a special assumption that you are considering wave equation in isotropic medium.

So, your mu 0 will be replaced by mu if it is magnetic mu r = 1 nonmagnetic otherwise, mu epsilon I am just simply writing epsilon and mu and then I can write also similar like free space same equation, but this k definition slightly will be modified because of the presence of epsilon r. The presence of epsilon r inserting here you can define k square equal to instead of omega by c square you have to multiply epsilon r will be coming here hope this is clear.

Then the k will be omega by c times square root of epsilon r and the square root of epsilon r that is square root of dielectric constant or relative permittivity is defined as a refractive index or I can say that refractive index this one this is a material property so, dielectric material if it is nonmagnetic then square root of epsilon relative permittivity we can represent by another physical parameter called n typically it is defined by n variable and it is known as refractive index. So, in that case k square will be like this, you just keep in mind that thing.

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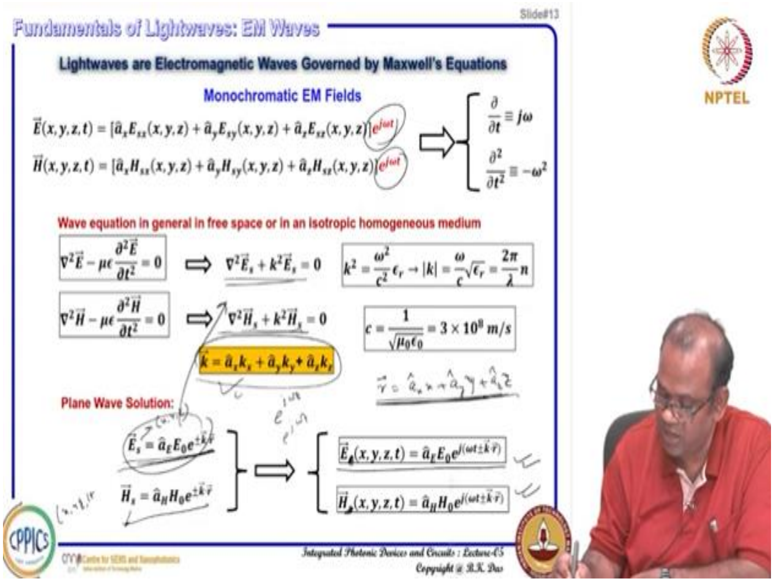

Now this k you see here electric field is a vector E s is a vector magnetic field is also vector. So, k I have just defined here if you just make a square root you could will write it like this one plus minus plus sign and minus sign how to define how to interpret that plus sign and minus sign for k. So, we can say that this plus sign means it is a wave so, you are supposed to get this wave solution which is actually have certain value of k.

For that wave solution k will be whenever you are getting a solution for E s or H s we will be getting solution in terms of k, k is a constant so far, but when we use plus it will signify something and when you use minus it will it will be signifying something before that we just think that since it is a vector all are vector E magnetic field we are concerned like a vector if we define k is also like a vector then if you write k like a vector I can say that k square will be just simply k x square k y square k z square that can be scalar.

So, if I just simply think about k is a vector then also this equation satisfies well indeed in reality the k you will find it is indeed a vector you can decompose into it has a particular direction also how is that let us a little bit more bit will be clearer. So, if you are just considering k is a vector one of the solutions of this equation you can write like this because del square r means here we are writing like a x + a y x, y coordinate y + a z.

So, in that case I just simply write the solution plus minus k I can write and if I consider vector that vector will have a some kind of position del square I will be writing so, this can be a solution for that. So, this will be a solution one of the solutions you can consider there will be multiple solutions which consider this is one of the solutions where the electric field I am just considering the unit vector a E that is that direction electrical field will be the E 0 is a constant some amplitude similarly, magnetic field will be like that.

So, what I consider that r is x dependent, so, that means this x is x, y, z dependent will be there I can define quickly and H s also can be x, y, z dependent and along with that we have a time dependent function each magnetic field magnetic field and electric field we consider they are originating from harmonics single monochromatic. So, you can think of e to the power j omega t, e to the power j omega t both cases.

So, in that case total solution time dependence I should not write here E 0 here this is because it is time dependent but I have just considered here. So, that will be actually you are writing a E, E to the power omega t + - kr we are writing plus minus kr k, dot r. So this is actually solution so, ultimately I just converted these one into frequency domain after frequency domain we get a solution and then that will be a space dependent solution.

And then I know that we are considering you e to the power j omega t monochromatic wave electromagnetic fields then we can just multiply this one then you get this solution. So, this is the typical solution you can consider one of the solutions you can well there can be many ways you can represent differential equation you can have a solution in many form here I just consider that one of the simplest form sinusoidal form, it will be a g omega t is there.

And here are through this type of solution if you just substitute this one here. Here it satisfies with considering r equal to this one and k equal to this one this will satisfy, that is why I do not have any problem to consider k is a vector and r is also a vector position vector. And this is some kind of indicating something which is appearing in the solution also, we have to interpret this one what is this?

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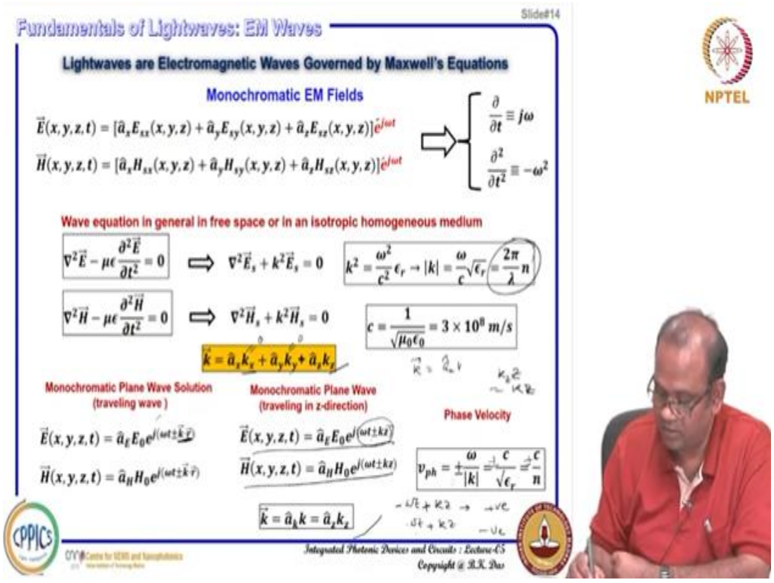

How to interpret that one? That can be interpreted, if we just do a little bit proceed further what do you do this is a monochromatic plane wave solution, it is called travelling wave why this is travelling wave? It will be clear if you just simply monochromatic plane wave travelling only z direction that means, it is only z dependent validation is there for example, suppose you have a k is a x I here it is written k x, k y, k z.

Suppose this one equal to 0 this one equal to 0 only k z is there then this term will become what that will be k z times z and k z since k x k y = 0 so, k means I can write simply like k only z direction k is there. So, that is called actually wave vector this k k is equal to a vector is equal to 2 pi by lambda n so, in that case we can write this one. So, in this it will be our z omega term this is called phase term in phase term we have when we have omega plus minus t k z can consider like this e k, k and only z direction.

Then we can simply find that if you just see this is something like that the travelling wave with this is the phase this phase part omega t plus minus k z I think all these you have learned in electromagnetic theory course, I just put down here and from here you can find out if you are using a plus sign and minus sign it will show that the phase is travelling will phase motion will be in the direction of negative z direction or positive direction if you are just considering omega t - k z that means it is positive z direction.

It will be travelling and if you are just considering omega t the phase will be phase change will be happening at every instant of time in positive z direction and then this will be negative this is positive and this will be negative this is sorry this will be positive direction and this will be negative direction. So, phase travel and that phase it will constant phase if you track how that constant phase is travelling in the positive direction or constant phase certain phase suppose a phase as you consider phase can vary 0 to 2 pi.

So, you considered you are just considering when 0 phase is coming appearing again right now 0 phase is here and this 0 phase will move in positive z direction this 0 phase will move in negative direction as a function of time. So, this phase how it travels if you just track then you can find that that phase velocity is basically nothing but omega by k and that can be plus minus k is plus minus if you write it will be again you can write plus minus c by epsilon r and c by n.

So, that means, phase velocity you are ultimately indicating whether it is positive direction propagating or negative direction propagating. And of course, sometimes some people use minus omega t here. Conventionally it is not used in electromagnetic case, but if you use minus sign, then in that case if I put this one and this one minus omega t + k z that also that represent positive direction propagation and this minus sign will be putting and this one when you put minus that will be representing minus direction propagation.

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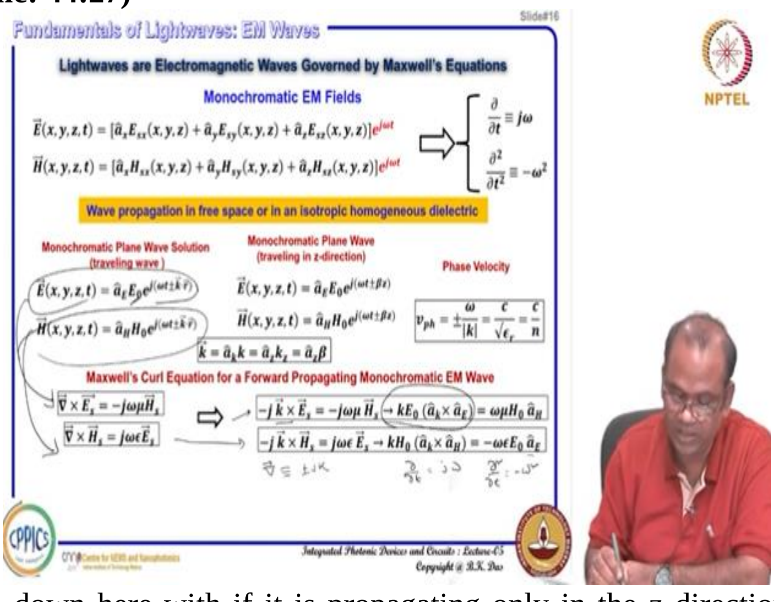

So I just written down here with if it is propagating only in the z direction if you are just representing position k vector is along z direction, then this k z component we can represent sometimes with beta that is a traditional in textbook is just represent like this beta and then same thing I am writing here and one more thing you should keep in mind that curl question for a forward propagating monochromatic electrical wave electric electromagnetic wave.

So, if you just use this one these equations or this equation, this 2 equation and you substitute here and this one you substitute here, then what you get after such this is a solution we were using and directly were substituting in current equation, then you get this one and you get this one after simplification, you get this one this left hand side kE 0 and k direction is unit vector a k electrical direction is a so a k cross aE and E 0 will be there is usually some kind of amplitude we have put and right hand side will be omega mu H 0 a H.

And time dependent part will be packed up from both sides. And similarly, I get from this you are getting this one. So, that means del is basically equivalent to minus j k del operator if you operate on this, it is basically minus j k as I said that del-del t is = j omega and del 2 del t 2 = - omega square and del operator if you do then it will be plus minus j k depending on positive direction or negative direction and then if you are putting del square that will be actually k square equivalent.

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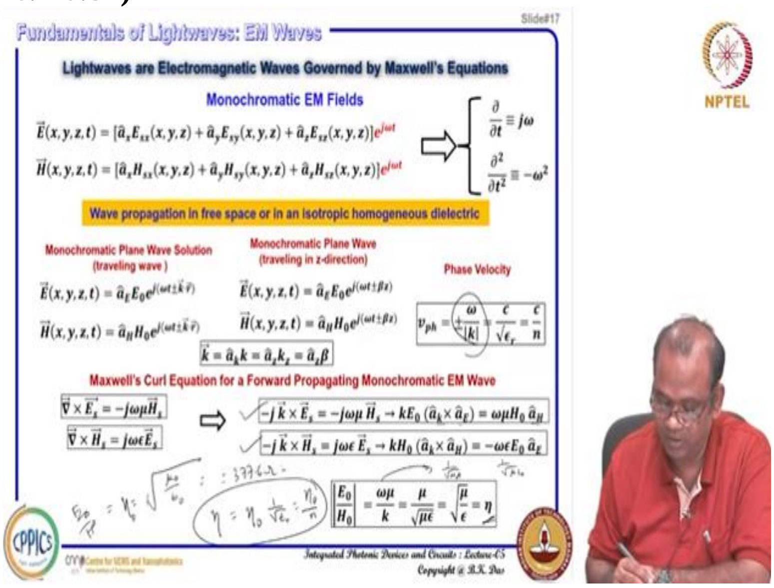

Next you see, if I just follow this one these are unit vectors follow one of these equations this equation or this equation, then I will be able to see that this is the unit vector this unit vector a H. So, E 0 by H 0 if I see that is omega by mu omega mu divided by k omega k is a velocity mu 0 epsilon 0 or material medium we can write mu epsilon omega by k is equal to phase velocity we have as shown here.

So, if you just substitute that will be mu over epsilon square root that is actually known that eta impedance or the medium and if you are writing say free space it can be mu 0 by epsilon

0 that is actually 377.6 or something like that E 0 by H 0 in free space and in material media this eta mu 0 by epsilon 0 if it is non magnetic dielectric material medium then you can consider that will be nothing but eta 0.

This is actually if it is free space we are considering eta 0 this eta 0 times we can write 1 by epsilon r or you can write eta 0 by that actually refractive index. So, free space to material medium the intrinsic impedance related by this equation. So, it will be packed or by refractive index in material medium impedance intrinsic impedance it is typically it is lower everything is clear these are all basic things, but very much important for this course, we have to again and again use these terminologies for understanding photonic integrated circuits.

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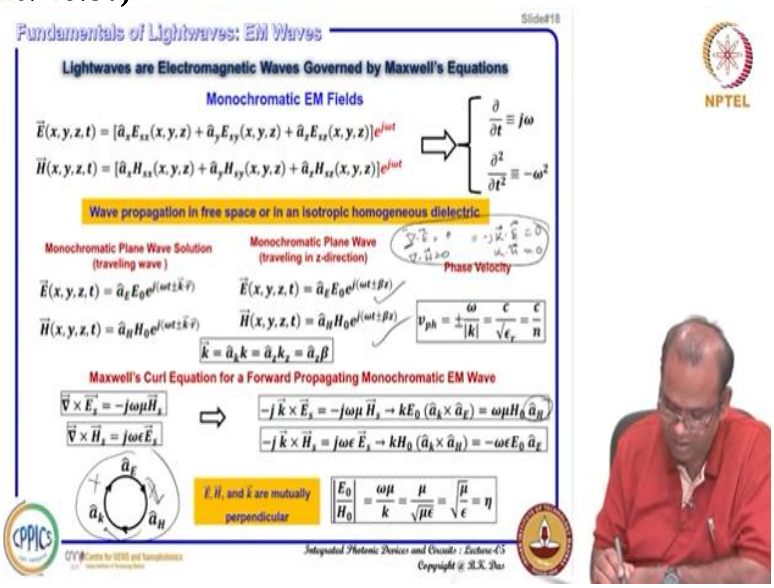

Now, try to follow that if you see this thing a k, a E is a vector if you take cross product of a unit vector along wave travelling direction k direction and electrical direction, then you get age. Similarly, a k cross a H it is basically a E that means, electric field is perpendicular to the both propagation direction and magnetic field. Similarly, here we can say that magnetic field direction is perpendicular to the propagation direction and electric field.

And another thing you know that divergence E if you just see divergence E equal to say 0. So, this one this del operator you can replace by say minus it is propagating in the polar direction j k dot E = 0 that means, electric field, if you are taking a dot product with k they just become 0 that means E and k is perpendicular. Similarly, if you are just using divergence H = 0 that means, also k dot H = 0.

So, that means what I say that electric field and magnetic field both are perpendicular to the k vector. So, that is why we are propagating in the k direction and electric fields and magnetic fields they are perpendicular direction and electric field magnetic field also they are orthogonal to each other. So, that is why in the propagation wave is propagating in the k direction, your electric field can be this direction and magnetic field can be this direction.

So, that is how we can propagate like magnetic field will oscillate in this direction and electric field can oscillate in this direction and it can propagate the disturbance can propagate in this direction z direction. So, that is what we understand from the Maxwell's equation and Maxwell's equations tells us that even there is no material medium it is a free space absolute free space just simply epsilon r = 1 and mu r = 1 then also.

This thing valid this solution also you can have and everything can be fulfilled that means, in free space also electromagnetic wave can actually existed that is the discovery of Maxwell. So, that is how he interpreted that light wave comes from sun in through free space without any disturbance it can travel and from this actually, we can just to remember it, we can just put like this a E H k.

So, they are actual orthogonal if you are taking a E cross H, then you can get a k direction again you are just considering a k cross a E then you are getting magnetic field. So, just to remember we can just use this picture and conclusion is that E, H, k are mutually perpendicular to each other.

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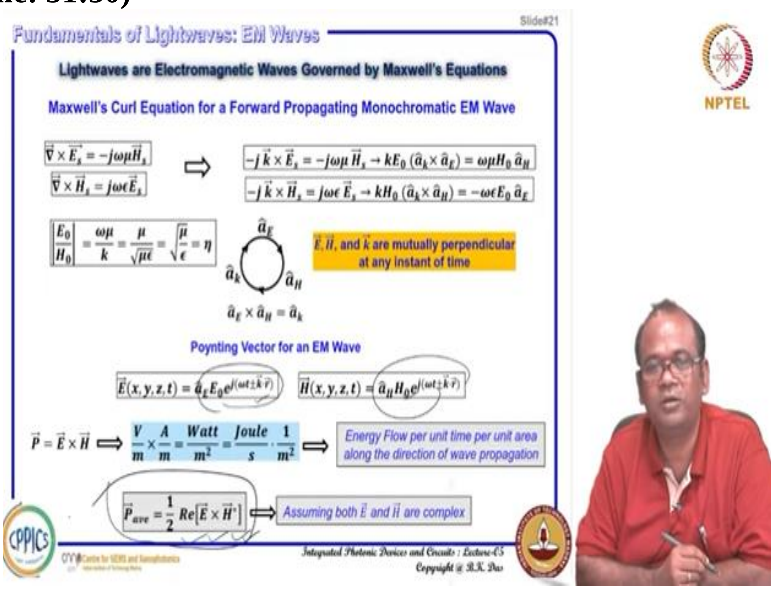

And one important thing here just think about poynting vector that is also you learnt earlier poynting vector in electromagnetic wave equation maybe in the basic courses. So, if you have electrical defined by this as we have solved magnetic field defined by these if I just try to find a solution for E cross H then it is called pointing vector and E means you know electric field dimension is volt per meter magnetic field is ampere per meter.

If you just multiply them then watt per metre square watt means joule per second one per metre that means energy Joule is energy, energy flow per unit time per unit area energy per unit time per unit area along the direction of wave propagation that means, if you can find this one E cross H that actually represents that how much energy it can carry in the propagation direction a E you can write a E cross a H that means kth direction that a k direction that is actually a k direction.

So, that means this thing actually represents something along propulsion direction and what is that something that is actually energy flow per unit time per unit area. Now, since we are using E and H in this complex form, so, we can show that actual if you are just using real part of it here and real part of it here and if you are using complex form here and complex form here.

So, using complex form, if you just do that one half of real part of E cross H star that is equivalent to the average power flow along the direction of propagation and you have to take half, this can be also shown that that if you are just directly using real part and you take E cross H and then take our H then you will be getting this value same thing as e and H are complex value. So, you have to remember this also for time to time to understand wave guiding energy flow in the photonic integrated circuits.

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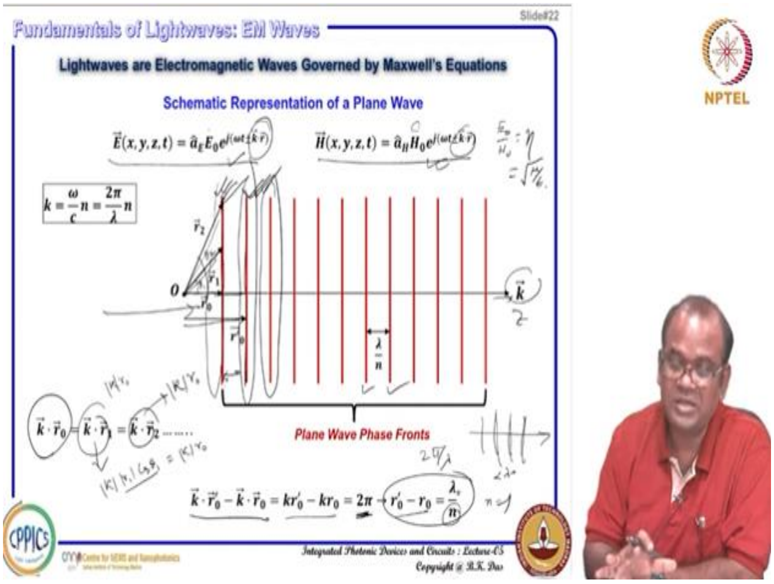

Now, the last thing in this stage, I would like to explain these equations whatever the solutions we got here for electrical and magnetic field they are coupled though, because E 0 and H 0 is related E 0 by H 0 basically eta = square root of mu by epsilon. So, if you know electric field then you can find out what is the magnetic field and H 0 and E 0 can be also complex they can have some kinds of phase relationship that we will discuss later.

But for the moment, free space and one is homogeneous medium typically, they are not complex you can consider real and they cannot go out of phase these are called basically plane waves solution why that is plane wave? If you just consider this k direction and you just consider a plane perpendicular to the k this is propagating direction and it was just considered a plane just perpendicular to k direction.

This is a red line means plane perpendicular to the; this good direction discipline, if it is z direction then this red line is indicating xy plane. So, in the xy plane suppose I define that this is the origin this point is the origin and from here to here this is propagation direction and just saying that up to a distance r 0 I consider a plane and in that plane I plot another position vector r 1 another position vector r 2.

Now, if I try this our position vector and this is a k now, if I try to find out k dot r 0 what is that? That is basically k r 0 so, that is basically kr 0 because k and r 0 in the same direction and if you are just considering this is theta 1 and this is theta 2 then these one is what I can write k r 1 cos theta 1 r 1 cos theta 1 = r 1 cos theta one = kr 0 and kr 2 similarly, I can write kr 2.

So, that means, this k dot r value in this plane in this xy plane anywhere if you just consider if wave is propagating in this direction in that plane you can have the phase part this part is constant. So, it can emerge in a plane perpendicular to the propagation direction where you can find everywhere at any instant of time phase is constant that is why it is called plane wave every plane perpendicular to the k propagation direction phase its constant at an instant of time next instant of time it is changing everywhere it will be changing same way.

So, that is it is a plane wave again if you see you can imagine another line at a distance certain distance you can consider whatever a phase, phase you know 0 to 2pi it can vary 0 to 2pi and if it is anything beyond 2pi you can scale again from 0. So, you can consider at a particular distance also they are also the phase the same whatever phase is here, here also same phase let us consider this one if you try to find out what is the k dot r prime?

This is the R prime distance and this is r 0 distance k dot r prime k dot r 0 you are just subtracting what is the difference between them? So, if you see the phase difference, there will be this one and if they are same, I can consider 2pi obviously there is a distance you can imagine 2pi, pi is dependents. So, that means, this point to this point if it is 2pi, pi is different than what is happening here?

At the same time, you can say that same thing is happening here at the same time it is same thing will be happening here in terms of phase because phase can vary from 0 to 2pi. So, if that is the case, then we can say that r prime 0 r 0 because k = 2pi by lambda if you are just putting that one this one will become like this r prime 0 so, at this point to this point the distance is lambda by n lambda is the wavelength.

So, whenever plane where we represent normally in textbook or whenever we will be discussing that sometimes plane wave we just said that this is a plane wave propagating in this direction that means, these lines we are sketching just to represent the phase fronds equiphase fronds at a distance lambda, lambda over a means, that is the wavelength in the material medium, if it is free space n = 1 normally free space will be this lambda 0 and if it is material medium wavelength will be different.

So, that is how plane wave we can get plane wave solutions though this type of plane wave is very, very practical you can approximately create this type of plane wave, but this type of plane wave consideration it actually helps to understand very well, how the Maxwell's equation works, how we can deal with Maxwell's equations. So, we just stop now for this lecture.

And this is what we can understand that how electromagnetic solution can be found from Maxwell's equation in free space and homogeneous dielectric medium homogeneous dielectric, non magnetic media. Now, next thing is that we will be just discussing about other material medium for example, if it is a lossy medium, if it is a completely conducting medium if it is a semi conducting medium, how it will work.

So, we need to understand how light wave behaves propagates along the when it propagates inside the material medium or when it is incident in a material medium when it comes from one material medium to another material medium, what happens those things we need to learn also. And these basic things made important whatever we discussed today will be just anything we will talk in terms of all these light waves in future. Thank you very much.