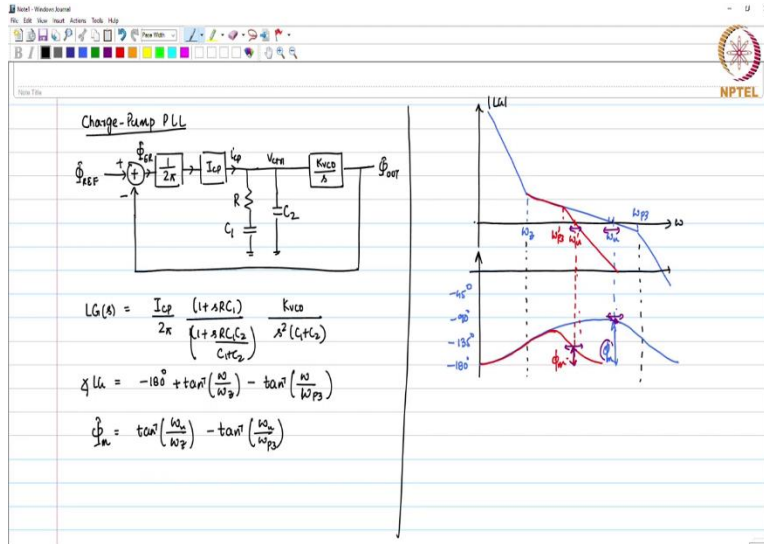


Phase-Locked Loops
Dr. Saurabh Saxena
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture – 28
Design Procedure For Type-II Order 3 Charge Pump PLL

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Hello everyone. Welcome to this session. We were looking at charge-pump PLL with Type-II and Order 3 in the PLL loop. So, let me draw the small signal block diagram of the PLL which we were discussing in the previous session. So, we had this phase error detector PFD whose gain was $\frac{1}{2\pi}$ that goes to a charge-pump with UP and DN signals.

The gain for the charge-pump combined with PFD was $\frac{I_{CP}}{2\pi}$ and that goes to a loop filter which has R and C₁ and a ripple bypass capacitor C₂. In the previous session, we saw the need of the ripple bypass capacitor. The control voltage actually changes the frequency of the oscillator with a gain of $\frac{K_{VCO}}{s}$ for the phase. This is the small signal block diagram of the PLL.

Here you have the input or the reference phase, this is your output phase, the phase error can be seen here as ϕ_{ER} , the output of the charge-pump is i_{CP} and this node voltage is V_{CTRL} . The loop gain of this PLL has been the following:

$$LG(s) = \frac{I_{CP}}{2\pi} \frac{(1 + sRC_1)}{\left(1 + \frac{sRC_1C_2}{C_1 + C_2}\right)} \frac{K_{VCO}}{s^2(C_1 + C_2)}$$

So, for the given loop gain, earlier we found that we need to worry about the phase margin for this and I wrote angle of loop gain as,

$$\angle LG = -180^\circ + \tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p3}}\right)$$

$$\omega_{p3} = \frac{1}{\frac{RC_1C_2}{C_1 + C_2}}$$

I will just rewrite that again and phase margin is given by,

$$\varphi_m = \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

So, the magnitude of the loop gain in dB with respect to ω happens to be like this. At zero frequency, it changes and then at unity gain frequency and this particular frequency is ω_{p3} , let me just extend this line. So, this is ω , this frequency is ω_u and this frequency is ω_{p3} , this is ω_z .

With respect to this plot, the phase plot happens to be like this. So, this is going to be -180° , -135° , -90° and -45° . The phase plot will be like this, and then it will again become -135° . In this case, the phase margin happens to be this. I showed the other plot also where let us say ω_{p3} location is something like this. So, you will have a pole ω_{p3} , let us call this as ω_{p3}' , you have new ω_u' and based on your ω_{p3} , you can have something with the ω_{p3} , it may not actually increase that far and it will again come back and this will go to -180° . So, the phase margin in the second case is only this much.

So, the question is now, which of these two plots or how would you like to have the position of ω_{p3} and ω_z such that you get the phase margin which you desire, and which of these plots is a better phase plot to use for your design. When I say better, it refers to which is the most optimum design.

Now, you take two cases. One case is where the phase plot is actually peaking near the ω_u frequency and let us say this is the phase margin which you want, then if you think about it, the variation in ω_u around this frequency ω_u given, due to some parameters in the design, the change in the phase margin around this point is actually very less.

On the other hand, if this is the phase margin which you desire, whether it is low or high, that is another case, but if this is the phase margin which you desire, then a slight change in your unity gain frequency because of your change in a pole frequency, zero frequency based on your RC and other parameters, the variation in the phase margin will be a lot more.

So, what we need is actually we need a design where we need to design this particular PLL in such a way that the change in the phase margin at the unity gain frequency is much less if you change the unity gain frequency by some amount. So, this appears to be one of the ideal choices. So, whatever unity gain frequency you want, based on the unity gain frequency, you should place your poles p_3 and your zero in such a way that you get your phase plot variation around that unity gain frequency to be very low. So, we can do that in a systematic manner. Let us just look at it.

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The slide displays a circuit diagram of a PLL. The feedback loop consists of a resistor R and a capacitor C_1 in parallel. The forward path has a zero at ω_z and poles at ω_{p1} and ω_{p2} . The transfer function is given as:

$$LG(s) = \frac{I_{cp}}{2K} \frac{(1 + sRC_1)}{(1 + sRC_1C_2)} \frac{K_{vc0}}{s^2(C_1 + C_2)}$$

The phase plot shows the phase margin at the unity gain frequency ω_u . The phase margin is defined as the difference between the phase and -180° at ω_u . The phase plot shows a peak in the phase margin near ω_u .

The presenter is a man in a blue shirt, sitting in front of the slide.

Design Procedure

The image shows a digital notepad with handwritten mathematical derivations and a design procedure. The derivations on the left side are as follows:

$$\dot{\phi}_m = \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

$$\frac{d\phi_m}{d\omega_u} = 0 \Rightarrow \frac{1}{1 + \left(\frac{\omega_u}{\omega_z}\right)^2} \times \frac{1}{\omega_z} - \frac{1}{1 + \left(\frac{\omega_u}{\omega_{p3}}\right)^2} \times \frac{1}{\omega_{p3}} = 0$$

$$\Rightarrow \omega_u^2 = \omega_z \cdot \omega_{p3}$$

$$= \omega_z^2 \left(1 + \frac{C_1}{C_2}\right)$$

$$\Rightarrow \omega_u = \omega_z \sqrt{1 + \frac{C_1}{C_2}}$$

$$\dot{\phi}_m = \tan^{-1}\left(\sqrt{1 + \frac{C_1}{C_2}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{1 + \frac{C_1}{C_2}}}\right)$$

$$K_c = \frac{C_1}{C_2} = 2 \left(\tan^2 \phi_m + \tan \phi_m \sqrt{1 + \tan^2 \phi_m} \right)$$

The design procedure on the right side is:

Design Procedure

1. Given ω_u and ϕ_m
2. $K_c = \frac{C_1}{C_2} = 2 \left(\tan^2 \phi_m + \tan \phi_m \sqrt{1 + \tan^2 \phi_m} \right)$
3. $\omega_z = \frac{\omega_u}{\sqrt{1 + \frac{C_1}{C_2}}}$
4. Choose R . $C_1 = \frac{1}{\omega_z R}$, $C_2 = \frac{C_1}{K_c}$
5. $|u| \geq 1$, K_{vco}

$$I_{op} = \frac{2\pi C_2}{K_{vco}} \omega_u^2 \sqrt{\frac{\omega_{p3}^2 + \omega_u^2}{\omega_z^2 + \omega_u^2}}$$

So, before I just write that phase margin, let me just write those ω_z and ω_{p3} also though you know it. So, they are given by,

$$\omega_z = \frac{1}{RC_1}$$

$$\omega_{p3} = \frac{1}{\frac{RC_1 C_2}{C_1 + C_2}}$$

So, if I make my phase margin to be maximally flat near the unity gain frequency, then the variation in the phase margin with respect to the unity gain frequency will be much lesser around unity gain frequency.

So, what I am going to do is, I am going to take the derivative of the phase margin with respect to unity gain frequency and make this derivative to go to zero. If you do this, we will get,

$$\phi_m = \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

$$\frac{d\phi_m}{d\omega_u} = 0$$

$$\Rightarrow \frac{1}{1 + \left(\frac{\omega_u}{\omega_z}\right)^2} \times \frac{1}{\omega_z} - \frac{1}{1 + \left(\frac{\omega_u}{\omega_{p3}}\right)^2} \times \frac{1}{\omega_{p3}} = 0$$

You solve all these terms, what you are going to get is,

$$\Rightarrow \omega_u^2 = \omega_z \cdot \omega_{p3}$$

It is like here your unity gain frequency is a geometric mean of your zero and the pole frequency.

Now, let us look at it. What does it reflect? That you have $\omega_z = \frac{1}{RC_1}$, I can also write this ω_{p3} as given below.

$$\omega_{p3} = \omega_z \left(\frac{C_1}{C_2} + 1 \right)$$

So, we get,

$$\omega_u^2 = \omega_z^2 \left(1 + \frac{C_1}{C_2} \right)$$

$$\Rightarrow \omega_u = \omega_z \sqrt{1 + \frac{C_1}{C_2}}$$

So, you got the relationship between ω_u and ω_z in terms of the capacitor ratio $\frac{C_1}{C_2}$.

So, I am going to substitute this back in the first equation to find our phase margin which is given as,

$$\varphi_m = \tan^{-1} \left(\sqrt{1 + \frac{C_1}{C_2}} \right) - \tan^{-1} \left(\frac{1}{\sqrt{1 + \frac{C_1}{C_2}}} \right)$$

So, what you see here is that your phase margin is just a function of the two capacitors which you are choosing.

So, in that way, I can just take tan on both the sides and you will solve that equation which turns out to be,

$$\frac{C_1}{C_2} = 2 \left(\tan^2 \varphi_m + \tan \varphi_m \sqrt{1 + \tan^2 \varphi_m} \right)$$

I will call this capacitor constant as K_c . So, we have,

$$K_c = \frac{C_1}{C_2} = 2 \left(\tan^2 \varphi_m + \tan \varphi_m \sqrt{1 + \tan^2 \varphi_m} \right)$$

So, given the phase margin, you can find the ratio $\frac{C_1}{C_2}$. Once you know the ratio $\frac{C_1}{C_2}$, you can find the other parameters also. Let us look at the design procedure using our phase margin analysis.

So, first, you need to know the bandwidth and the phase margin of the PLL. So, given ω_u and phase margin, how do we know what is ω_u and what is phase margin? That you are going to see later. There will be some other parameters, system level design requirements where you will get the unity gain bandwidth and your phase margin.

So, given ω_u and phase margin, because you know phase margin, so, you can find out $K_c = \frac{C_1}{C_2} = 2(\tan^2 \varphi_m + \tan \varphi_m \sqrt{1 + \tan^2 \varphi_m})$. Once you know K_c and you know ω_u and because phase margin will give you $\frac{C_1}{C_2}$. So, you can find out that $\omega_z = \frac{\omega_u}{\sqrt{1 + \frac{C_1}{C_2}}}$.

When you know ω_z , you need to choose R, this is a choice which will be dictated by the noise in the system but you need to choose R. Then once you know R, $C_1 = \frac{1}{\omega_z R}$, and $C_2 = \frac{C_1}{K_c}$. So, now see you made a choice over R, you know now your C_1 and C_2 . Once you know your C_1 and C_2 , using the fact that the $|LG|=1$, you can find I_{CP} as,

$$I_{CP} = \frac{2\pi(C_2 + C_1)}{K_{VCO}} \omega_u^2 \sqrt{\frac{1 + \omega_u^2 / \omega_{p3}^2}{1 + \omega_u^2 / \omega_z^2}}$$

Here, you need to know K_{VCO} .

So, you can find I_{CP} . So, you look at it, given ω_u and phase margin, you found $\frac{C_1}{C_2}$, then ω_z , then you chose R, you could find C_1 and C_2 and you can find I_{CP} . If all the parameters are chosen in such a way, then you will place your ω_z and ω_{p3} in such a way that whatever phase margin you want, your phase margin plot will always be having the derivative of the phase at the desired point to be equal to zero. Thank you.