

Phase-Locked Loops
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Lecture – 33
Noise Analysis in CP-PLL: Part II

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The diagram shows a CP-PLL system with the following components and noise sources:

- Reference:** R_{REF} with phase ϕ_{REF} and noise $S_{REF}(f)$.
- PFD:** Phase detector with gain $\frac{1}{2\pi}$ and noise $S_{PFD}(f)$.
- CP:** Charge pump with current I_{CP} and noise $S_{CP}(f)$.
- Loop Filter:** Resistor R and capacitor C_1 in parallel, followed by capacitor C_2 . Noise V_n^R is added at the resistor.
- VCO:** Voltage-controlled oscillator with gain K_{VCO} and noise $S_{VCO}(f)$.
- Divider:** Divider with gain $\frac{1}{N}$.
- Output:** Output signal S_{OUT} with phase ϕ_{OUT} .
- Disturbance:** Noise N added at the output.

Noise Transfer Function (NTF)

- $- NTF_{REF} = \frac{\phi_{OUT}(s)}{\phi_{REF}(s)}$
- $- NTF_{CP} = \frac{\phi_{OUT}(s)}{V_{CP}(s)}$

Loop Gain, $L_L = \frac{I_{CP} \cdot K_{VCO}}{2\pi N \cdot s^2(C_1 C_2)}$

- $L_L(s) = \frac{I_{CP} \cdot K_{VCO}}{2\pi N \cdot s^2(C_1 C_2)} \frac{(1+s/\omega_{z1})}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$
- $NTF_{REF} = \frac{N \cdot L_L(s)}{(1+L_L(s))}$

Noise Sources:

- Reference Noise: Phase domain
- CP Noise: Current domain
- Resistor Noise: Voltage domain
- VCO's Noise: Phase domain
- Divider Noise: ϕ

Hello, welcome to this session. So, after identifying the basic noise components in the PLL, we need to do the noise analysis to find out how noise from different PLL blocks adds to the output noise. So, for that, what we have to do is first identify all the noise sources in the PLL. So far what we have done is we have found out the basic noise sources like resistor and transistors. Now, what we are going to do, we are going to find the noise of each block.

So, PFD is a block and PFD adds noise. So, we will find out what is the total noise which the PFD will add and we will model it with its noise spectral density $S_{PFD}(f)$. So, the noise spectral density for the noise added by the PFD is modeled by S_{PFD} here. Then after the PFD, you have charge-pump. So, charge-pump adds its own noise and that noise is denoted by the noise power spectral density of the charge-pump $S_{CP}(f)$. Then you have loop filter. In loop filter, you have resistor and capacitor, capacitor does not add any noise, the resistor adds noise with respect to its voltage.

So, either you write this as a voltage or you write it as V_n^R , that is fine. So, then you have capacitor C_1 , this does not add any noise, you have capacitor C_2 , that also does not add any noise and you have VCO which adds noise at the output and that noise is the noise of the

oscillator which is defined in terms of phase noise. Let us just find out what is this phase noise and then you have a divider in the feedback and there is effective noise of the divider which gets added.

So, we identify all these noise sources in the system, S_{Div} , and by the way, you are feeding from the reference, the reference itself may not be that clean, that also adds noise to the input, so, $S_{\varphi_{REF}}$. Now, what I have done is I have just identified all the noise sources in the system. Now, we do one more thing. The thing is that in case of PLL, we are making sure that the output phase is locked with the input phase or reference phase.

So, the variables which we are looking at other than these voltage or current signals at the input and output of the PLL, the signal the PLL locks such that the phase error is equal to 0 which is like we have seen this multiple times that $\varphi_{REF} - \frac{\varphi_{OUT}}{N}$ that is what the PLL will ensure. We treat the noise which is added by reference or PFD or charge-pump, φ_{OUT} , divider or resistor. These are small perturbations in the steady state of the PLL which means, let me just write, noise is like a small perturbation in steady state of PLL which implies that we can use small signal model of PLL for finding output phase noise or you can say for noise analysis. So, there are a few things which we need to find.

So, how this reference noise gets converted to the output noise. So, before we look at that, I will identify that at each point, whatever variable you had before, what we have done is we have added the noise in that particular variable only. For example, the charge-pump output is a current. So, the charge-pump noise is also taken in the form of current only. So, I will call this as $i_{cp,n}$. So, current at the output of the charge-pump, the noise is added in current domain.

At the output of the PFD, the noise is added in voltage domain which you have, the way we have noted it. At the reference, the noise is added in phase domain because the phase frequency detector is finding the phase error. So, just look at the small signal model of this PLL. So, what the PFD is doing, it is finding the phase error between the input and the feedback and that is multiplied by K_{PD} which is the gain.

So, when you are subtracting the reference with the feedback, it is in phase domain. So, the noise which is added by the reference which we have is in phase domain. So here, this is phase error, this phase error gets converted from phase to voltage by using K_{PD} . So, in using this K_{PD} , the PFD adds in voltage domain. At the charge-pump output, it adds in current domain. This is V_{ctrl} .

So, your voltage noise here adds to the control voltage noise in voltage domain. At the output, we are looking at the output phase. So, the noise from the VCO adds to the output phase in phase domain. At the divider, the output of the divider is taken as φ_{div} . So, the noise from the divider adds in the phase domain.

So, the important part here is that your reference noise, we have to find this also independently that how much noise the input adds independently, PFD adds independently and those noise sources we have to add here. So, reference noise is in phase domain, charge-pump noise is in current domain, resistor noise is in voltage domain because it adds at the node where it changes the voltage, you can use current also but finally you have to look at the control voltage and VCO's noise is in phase domain and your divider noise is in phase domain.

So, we found all these noise sources. Now, after finding all these noise sources, what we need to do is we need to find how these noise sources get converted to the output phase noise by the small signal analysis. So, to do that, I will replace these blocks with their linear models. This is small signal model which is I_{CP} here and this is $\frac{K_{VCO}}{s}$, K_{PD} of the PFD block is $\frac{1}{2\pi}$.

So, we define the term called as noise transfer function. So, what is this noise transfer function? Noise transfer function tells us that how the noise from one source gets converted to the output. So, this is NTF noise transfer function. So, let us say, we want to find, for a simple example, how the reference noise, this particular noise, gets converted to the output noise. So, NTF_{REF} is what you need to define here which means how the reference noise gets added up to the output. So, it is defined as follows:

$$NTF_{REF} = \frac{\varphi_{OUT}(s)}{\varphi_{REF}(s)}$$

Similarly, for the charge-pump noise, we need to define NTF_{CP} . This is the charge-pump current and noise current which is added here. So, NTF_{CP} is defined as follows:

$$NTF_{CP} = \frac{\varphi_{OUT}(s)}{i_{cp,n}(s)}$$

This is how we define the transfer function. Before we go ahead and define all the transfer functions, it will be easy for us if we first define the loop gain and then we work out these transfer functions in terms of the loop gain because all these transfer functions which you are seeing here are for the same loop, just that the point of addition is different, but the point of observation here is same.

So, the loop gain of the PLL loop is given by,

$$LG(s) = \frac{I_{CP}}{2\pi N} \frac{K_{VCO}}{s^2(C_1 + C_2)} \frac{1 + sRC_1}{1 + \frac{sRC_1C_2}{C_1 + C_2}}$$

$$LG(s) = \frac{I_{CP}}{2\pi N} \frac{K_{VCO}}{s^2(C_1 + C_2)} \frac{1 + s/\omega_z}{1 + s/\omega_{p3}}$$

If you just want to recall how we got this, we are looking at the loop gain. So, you can break the loop, find how you get the loop gain multiplication $\frac{1}{2\pi} I_{CP}$, then I_{CP} goes through this loop filter because of which you get $R C_1 C_2$, that gets multiplied by K_{VCO} , it comes in the feedback and then it gets divided by N . So, the divider plays the role of phase output by N and that is what you get back. So, this is the loop gain.

Now, what I need to find is what is the transfer function for the first one NTF_{REF} . So, NTF_{REF} is you have a change at the input which is your reference and how it comes at the output that is straightforward because the reference noise adds at the input. So, the closed loop transfer function is going to be,

$$NTF_{REF} = \frac{N \cdot LG(s)}{1 + LG(s)}$$

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The slide contains the following content:

- Block Diagram:** A feedback loop with input X , a summing junction, a forward path block $G(s)$, and a feedback path block $H(s)$. The output is Y .
- Transfer Function Derivation:**

$$\frac{Y}{X} = \frac{1}{H(s)} \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{1}{H(s)} \frac{1}{1+L(s)}$$

$$= \frac{G(s)}{1+L(s)}$$
- NTF_{CP} Derivation:**

$$NTF_{CP} = \frac{\left\{ \left(R + \frac{1}{sC_1} \right) \parallel \frac{1}{sC_2} \right\} \frac{K_{VCO}}{s}}{1+L(s)}$$

$$= \left[\frac{\frac{1+sRC_1}{sC_1} \times \frac{1}{sC_2}}{\frac{1+sRC_1}{sC_1} + \frac{1}{sC_2}} \right] \times \frac{K_{VCO}}{s} \times \frac{1}{1+L(s)}$$

$$= \frac{(1+sRC_1)/sC_1C_2}{(C_2+sRC_1C_2)/C_1C_2} \frac{K_{VCO}}{s} \frac{1}{1+L(s)}$$
- Handwritten Equations:**

$$NTF_{CP} = \frac{K_{VCO}}{s^2(C_1+C_2)} \frac{1+sRC_1}{1+\frac{sRC_1C_2}{C_1+C_2}} \times \frac{1}{2\pi N} \times \frac{2\pi N}{I_{CP}}$$

$$= \frac{2\pi N}{I_{CP}} \times \frac{N \cdot L(s)}{1+L(s)}$$
- Speaker:** A man in a blue shirt is visible in the bottom right corner of the slide, likely the presenter.

Reference Noise : Phase domain
 CP Noise : Current domain
 Resistor Noise : Voltage domain
 VCO's Noise : Phase domain
 Divider Noise : ϕ

Noise Transfer Function (NTF) NPTEL
 $- NTF_{REF} = \frac{\phi_{OUT}(s)}{\phi_{REF}(s)}$
 $- NTF_{CP} = \frac{\phi_{OUT}(s)}{i_{CP,N}(s)}$

Loop Gain, $LG = \frac{I_{CP} K_{VCO}}{2\pi N s^2 (C_1 + C_2)} \frac{1+sRC_1}{1+sRC_2}$

$LG(s) = \frac{I_{CP} K_{VCO}}{2\pi N s^2 (1+s\tau_1)} \frac{(1+s/\omega_3)}{(1+s/\omega_2)}$

$NTF_{REF} = \dots$
 $NTF_{CP} = \dots$

In general, you know that for given control loop, let me just define that. For a block diagram like this, if you have a forward path $G(s)$ and you have a negative feedback path with $H(s)$, then the input to output transfer function $\frac{Y}{X}$ is given as,

$$\frac{Y}{X} = \frac{1}{H(s)} \frac{G(s)H(s)}{1 + G(s)H(s)} = \frac{1}{H(s)} \frac{LG}{1 + LG} = \frac{G(s)}{1 + LG}$$

So, this is how it is defined. We are going to use the same thing time and again here. So, what happens to the charge-pump? Charge-pump noise adds in the current domain at this node, so, here you can see this is $i_{cp,n}$. So, $i_{cp,n}$ gets filtered. So, for i_{cp} , we are looking at the output $i_{cp,n}$. The forward path gain includes only this part, this is one way to define. The forward path gain includes only this part which is $\frac{KVCO}{s}$ into the loop filter RC_1 in parallel with other things. So, either you can do it this way or you can also do it that before i_{cp} , you have an extra factor of $\frac{I_{CP}}{2\pi}$, that is the only extra factor.

So, if from the forward path gain, you remove that, then you will get the same transfer function as you get for the reference. So, I will write that you are going to have,

$$NTF_{CP} = \frac{2\pi N \cdot LG(s)}{I_{CP} (1 + LG(s))}$$

How did I get it? Well, noise transfer function for the reference is this, I should just divide the noise transfer function for the reference by the gain of the charge-pump. So, I hope that makes sense.

Now, or if you want to do it directly, well, you can do it brute force also that NTF of charge-pump is forward path gain which is your R in series with $\frac{1}{sC_1}$, in parallel with $\frac{1}{sC_2}$ times $\frac{K_{VCO}}{s}$, this is forward path gain divided by $1 + LG$. So, you have,

$$NTF_{CP} = \frac{\left\{ \left(R + \frac{1}{sC_1} \right) \parallel \frac{1}{sC_2} \right\} \frac{K_{VCO}}{s}}{1 + LG}$$

$$NTF_{CP} = \left[\frac{\frac{1 + sRC_1}{sC_1} \times \frac{1}{sC_2}}{\frac{1 + sRC_1}{sC_1} + \frac{1}{sC_2}} \right] \times \frac{K_{VCO}}{s} \times \frac{1}{1 + LG}$$

$$NTF_{CP} = \frac{(1 + sRC_1) / sC_1 C_2}{(C_2 + sRC_1 C_2 + C_1) / C_1 C_2} \frac{K_{VCO}}{s} \frac{1}{1 + LG}$$

$$NTF_{CP} = \frac{K_{VCO}}{s^2(C_1 + C_2)} \frac{1 + sRC_1}{1 + \frac{sRC_1 C_2}{C_1 + C_2}} \frac{1}{1 + LG}$$

Now, what is missing from this particular part? Well, what is missing is only $\frac{I_{CP}}{2\pi N}$. So, I am going to multiply and divide by $\frac{I_{CP}}{2\pi N}$. Now, this becomes your loop gain. So, what I see here is,

$$NTF_{CP} = \frac{K_{VCO}}{s^2(C_1 + C_2)} \frac{1 + sRC_1}{1 + \frac{sRC_1 C_2}{C_1 + C_2}} \frac{1}{1 + LG} \times \frac{I_{CP}}{2\pi N} \times \frac{2\pi N}{I_{CP}}$$

$$NTF_{CP} = \frac{2\pi}{I_{CP}} \times \frac{N \cdot LG}{1 + LG}$$

This is what we directly wrote previously. So, you see NTF_{CP} .

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The slide contains the following content:

- NTF Derivation:**

$$NTF_{CP} = \frac{\left\{ \left(R + \frac{1}{sC_1} \right) \parallel \frac{1}{sC_2} \right\} \frac{K_{VCO}}{s}}{1 + LG}$$

$$= \left[\frac{\frac{1 + sRC_1}{sC_1} \times \frac{1}{sC_2}}{\frac{1 + sRC_1}{sC_1} + \frac{1}{sC_2}} \right] \times \frac{K_{VCO}}{s} \times \frac{1}{1 + LG}$$

$$= \frac{(1 + sRC_1) / sC_1 C_2}{(C_2 + sRC_1 C_2 + C_1) / C_1 C_2} \frac{K_{VCO}}{s} \frac{1}{1 + LG}$$
- Loop Gain Derivation:**

$$NTF_{PPD} = 2\pi N \frac{N \cdot LG}{1 + LG} = \frac{\phi_{ovr}}{V_{ref}}$$

$$NTF_{DIV} = NTF_{REF} = \frac{N \cdot LG}{1 + LG}$$

$$NTF_R = \frac{\phi_{ovr}}{V_A^R}$$
- Circuit Diagram:** A circuit diagram of a charge pump with a resistor R and two capacitors C1 and C2. The input is Vref and the output is Vcp1(s). The current through R is Icp.
- Transfer Functions:**

$$V_{cp1}(s) = \frac{1/sC_2}{1/sC_2 + 1/sC_1 + R} = \frac{1/sC_2}{C_1 + sRC_1 C_2 + C_2} = \frac{1/sC_2}{C_1 + sRC_1 C_2 + C_2}$$

$$V_{cp2}(s) = \frac{C_1}{C_1 + sRC_1 C_2 + C_2} = \frac{C_1}{C_1 + sRC_1 C_2 + C_2}$$

Reference Noise : Phase domain
 CP Noise : Current domain
 Resistor Noise : Voltage domain
 Vco's Noise : Phase domain
 Divider Noise : 0

$$- NTF_{CP} = \frac{\phi_{OUT}(s)}{V_{REF}(s)}$$

$$\text{Loop Gain, } LG = \frac{I_{CP} \cdot K_{VCO}}{2\pi N \cdot s^2 (C_1 + C_2)} \frac{1 + sRC_1}{1 + sRC_2} \frac{C_1 + C_2}{C_1 + C_2}$$

$$LG(s) = \frac{I_{CP} \cdot K_{VCO}}{2\pi N \cdot s^2 (C_1 + C_2)} \frac{(1 + s/\omega_{P1})}{(1 + s/\omega_{P2})}$$

$$NTF_{REF} = \frac{N \cdot LG(s)}{(1 + LG(s))} \checkmark$$

$$NTF_{CP} = \frac{2\pi \cdot N \cdot LG(s)}{I_{CP} (1 + LG(s))} \checkmark$$

$$NTF_{CP} = \frac{K_{VCO}}{s^2 (C_1 + C_2)} \frac{1 + sRC_1}{1 + sRC_2} \frac{I_{CP}}{2\pi N} \frac{1}{1 + LG} \times \frac{2\pi N}{I_{CP}}$$

Now, the other one is NTF_{PFD} , well, NTF_{PFD} will have the same transfer function like charge-pump, so I will write that also. NTF noise transfer function for PFD noise is given by,

$$NTF_{PFD} = 2\pi \times \frac{N \cdot LG}{1 + LG} = \frac{\phi_{OUT}}{V_{PFD}}$$

NTF_{Div} which is also coming at the same point which is at the input, so, this is also going to be the same as NTF_{REF} which is given as,

$$NTF_{Div} = NTF_{REF} = \frac{N \cdot LG}{1 + LG}$$

Then you have for the resistance noise which is given by,

$$NTF_R = \frac{\phi_{OUT}}{V_n^R}$$

Now, if you look at this resistance, this is little cumbersome in the calculation. So, you have V_n^R here. So, what we do here is we find out that how V_n^R is first changing the control voltage. So, to do that, we know that,

$$\frac{V_{ctrl}(s)}{V_n^R(s)} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_2} + \frac{1}{sC_1} + R} = \frac{\frac{1}{sC_2}}{\frac{C_1 + C_2 + sRC_1C_2}{sC_1C_2}}$$

So, we get,

$$\frac{V_{ctrl}(s)}{V_n^R(s)} = \frac{C_1}{C_1 + C_2 + sRC_1C_2} = \frac{\frac{C_1}{C_1 + C_2}}{1 + \frac{sRC_1C_2}{C_1 + C_2}}$$

So, from V_n^R to V_{ctrl} , we get this, then what we need to do is we need to find from V_{ctrl} , how do we get φ_{OUT} .

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$NTF_R(s) = \frac{\varphi_{OUT}}{V_n^R} = \frac{\varphi_{OUT}}{V_{ctrl}(s)} \times \frac{V_{ctrl}(s)}{V_n^R}$
 $= \frac{\frac{K_{VCO}}{s}}{1 + Lk} \times \frac{C_1}{C_1 + C_2} \times \frac{1}{1 + \frac{sRC_1C_2}{C_1 + C_2}}$
 $= \frac{C_1}{C_1 + C_2} \times \frac{1}{s} \times \frac{K_{VCO}}{1 + Lk}$
 $NTF_{VCO}(s) = \frac{\varphi_{OUT}(s)}{\varphi_{VCO}(s)} = \frac{1}{1 + Lk}$
 $NTF_R(s) = \frac{C_1}{C_1 + C_2} \times \frac{K_{VCO}}{s(1 + s/p_0)}$
 $NTF_{CP}(s) = \frac{2/s}{I_{CP}} \times \frac{N \cdot Lk}{1 + Lk}$
 $NTF_{DIV}(s) = NTF_{REF}(s) = \frac{N \cdot Lk}{1 + Lk}$
 $Lk = \frac{I_{CP} K_{VCO}}{2\pi N} \frac{1 + s/p_0}{s^2(C_1 + C_2)}$
 $NTF_{REF} = \frac{N \cdot Lk}{1 + Lk} = \frac{N}{1 + \frac{1}{Lk}}$
 $\lambda \rightarrow 0 \Rightarrow Lk \rightarrow \infty \Rightarrow NTF_{REF} \rightarrow N$
 $NTF_{VCO} = \frac{1}{1 + Lk}$

Noise Transfer Function (NTF)
 $- NTF_{REF} = \frac{\varphi_{OUT}(s)}{\varphi_{REF}(s)}$
 $- NTF_{CP} = \frac{\varphi_{OUT}(s)}{\varphi_{CP}(s)}$
 $- NTF_{DIV} = \frac{\varphi_{OUT}(s)}{\varphi_{DIV}(s)}$
 $NTF_{REF} = N$
 $NTF_{CP} = \dots$
 $NTF_{DIV} = 0$

Loop Gain, $Lk = \frac{I_{CP} K_{VCO}}{2\pi N} \frac{1 + sRC_1}{s^2(C_1 + C_2)}$
 $Lk(s) = \frac{I_{CP} K_{VCO}}{2\pi N} \frac{1 + sRC_1}{s^2(C_1 + C_2)}$
 $NTF_{REF} = N$
 $NTF_{CP} = \dots$
 $NTF_{DIV} = 0$

Reference Noise : Phase domain
 CP Noise : Current domain
 Resistor Noise : Voltage domain
 VCO's Noise : Phase domain
 Divider Noise : 0

I am finding NTF_R in two steps, one, I need to find $\frac{\varphi_{OUT}}{V_n^R}$, I am finding out like this,

$$NTF_R(s) = \frac{\varphi_{OUT}}{V_n^R} = \frac{\varphi_{OUT}}{V_{ctrl}(s)} \times \frac{V_{ctrl}(s)}{V_n^R}$$

$$NTF_R(s) = \frac{\frac{K_{VCO}}{s}}{1 + LG} \times \frac{C_1}{C_1 + C_2} \times \frac{1}{1 + \frac{sRC_1C_2}{C_1 + C_2}}$$

$$NTF_R(s) = \frac{C_1}{C_1 + C_2} \frac{1}{\left(1 + \frac{s}{\omega_{p3}}\right)} \frac{K_{VCO}}{s} \frac{1}{1 + LG}$$

Now, NTF_{VCO} noise, what we need to find is how the output noise changes in the closed loop with respect to φ_{VCO} . Where are we adding φ_{VCO} ? We are adding φ_{VCO} here. So, φ_{VCO} is added here, the forward path gain is only 1 because you are looking at the forward path, gain is 1 but there is a whole loop behind it. So, we get,

$$NTF_{VCO}(s) = \frac{\varphi_{OUT}(s)}{\varphi_{VCO}(s)} = \frac{1}{1 + LG}$$

So, what we have done so far is we have found the noise transfer function for each noise source from the noise input to its output.

Now, let us guess what these noise transfer functions will look like. So, because we have calculated at many different places, what I will do is I will just summarize at one place.

$$NTF_R(s) = \frac{C_1}{C_1 + C_2} \frac{1}{\left(1 + \frac{s}{\omega_{p3}}\right)} \frac{K_{VCO}}{s} \frac{1}{1 + LG}$$

$$NTF_{CP} = \frac{2\pi}{I_{CP}} \times \frac{N \cdot LG}{1 + LG}$$

$$NTF_{Div} = NTF_{REF} = \frac{N \cdot LG}{1 + LG}$$

Now, to make sense from these transfer functions, the important part which we understand is this loop gain parameter. So, the loop gain, we have seen that also, it is having two poles at zero and then it has one pole at ω_{p3} . So, we have,

$$LG = \frac{I_{CP}}{2\pi N} \frac{K_{VCO}}{s^2(C_1 + C_2)} \frac{1 + s/\omega_z}{1 + s/\omega_{p3}}$$

So, for this, if we are looking at NTF_{REF} , we have,

$$NTF_{REF} = \frac{N \cdot LG}{1 + LG} = \frac{N}{1 + \frac{1}{LG}}$$

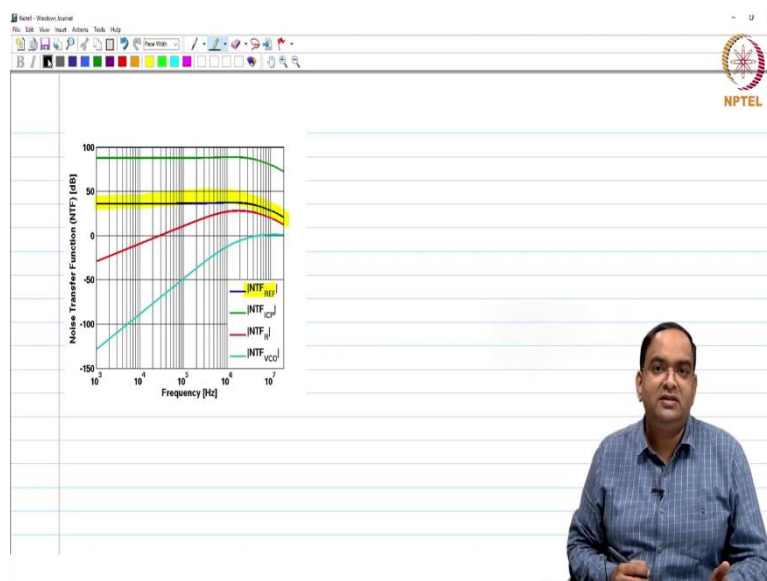
You can very well plot it and understand but you should understand even by looking at the expression. It is $\frac{N}{1+\frac{1}{LG}}$. Loop gain is given to you here. So, what you see here is as $s \rightarrow 0$ in this case, $LG \rightarrow \infty$. This is just handwaving here that as $s \rightarrow 0$, it implies that $LG \rightarrow \infty$. As $LG \rightarrow \infty$, $NTF_{REF} \rightarrow N$ and in the opposite case what you see here is as $s \rightarrow \infty$ at very high frequency, as $s \rightarrow \infty$, $LG \rightarrow 0$, and if $LG \rightarrow 0$, $\frac{1}{LG} \rightarrow \infty$ and $NTF_{REF} \rightarrow 0$.

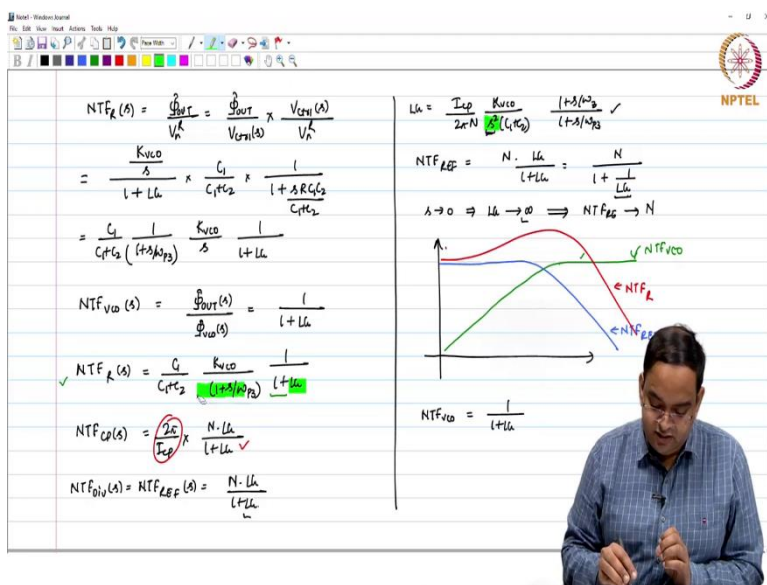
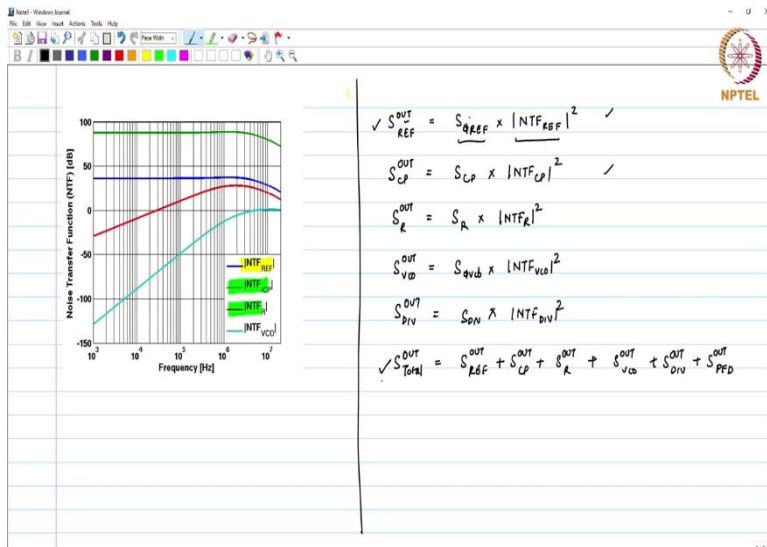
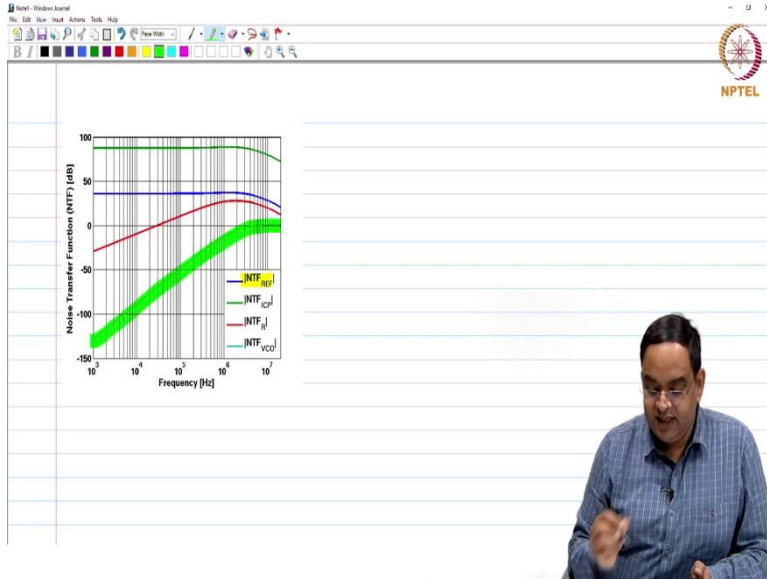
So, what we expect here is that the NTF_{REF} will look something like this, then for all other transits, it should appear like this. Then if you look at the transfer function NTF_{VCO} , $NTF_{VCO}(s) = \frac{1}{1+LG}$. So, as $s \rightarrow 0$, $LG \rightarrow \infty$. As $LG \rightarrow \infty$, $NTF_{VCO} \rightarrow 0$. So, what you will find is that NTF_{VCO} will look like this apparently.

Then, NTF_R , in NTF_R , you are having $\frac{1}{1+LG}$. So, it is like this transfer function getting multiplied by a low pass transfer function which is $\frac{1}{\left(1+\frac{s}{\omega_{p3}}\right)} \frac{K_{VCO}}{s}$, that is what you have here. So, if that is

the case, the high pass transfer function is getting multiplied by low pass transfer function which is a low pass transfer function. It depends on where the poles and zeroes are, maybe something like this. So, overall, what you may find for the resistor transfer function is that it may look like a bandpass transfer function because at high frequency, NTF_{REF} is going to take it to a low pass transfer function effect which is s^2 , that is going to take it down. So, this may be NTF_R and then NTF_{CP} , that is only a gain factor here. NTF_{Div} and NTF_{REF} are the same. NTF_{PFD} is only a gain factor.

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So, let me just plot all these transfer functions for one particular example after choosing certain numbers and show it to you. So, you see this plot. In this case, NTF_{REF} is this blue one. It is plotted like this and it is low pass. Here I am choosing the reference frequency as 40 MHz and my plot is only till $\frac{f_{REF}}{2}$. If you recall our discussion in the previous session, our continuous time or s-domain analysis, the way we are doing is valid only for frequency contents which are for bandwidth which are lesser than $\frac{f_{REF}}{10}$ and it is a sampled system, sampled at f_{REF} , so we are showing only the plots till $\frac{f_{REF}}{2}$. So, till $\frac{f_{REF}}{2}$, what you see here is it is a low pass transfer function.

Similarly, you look at NTF_{VCO} . So, NTF_{VCO} is this. So, for this NTF_{VCO} , what you see here is this is a high pass transfer function, it increases and then it becomes flat. NTF_R , so, NTF_R , we were looking at that. It should be a kind of a bandpass transfer function, it depends on which has the higher order. So, let us look at it, this term is having second order whereas loop gain term is having third order. So, what is going to happen here is that in loop gain as you see, it is a second order at $s = 0$, that is loop gain close to $s = 0$ whereas this particular at $s = 0$, you are only going to get an order from the other term. So, this is going to dominate, so in place of the red curve which is starting from here, it may start from here.

So, that is what you see for the NTF_R . NTF_{CP} , it is a low pass transfer function just multiplied by K . So, without even plotting it, just looking at the expression, you can make out that which is going to be a high pass, which is going to be a low pass and other stuff. So, what you can conclude from here is that all these noise sources are getting filtered by different noise transfer

functions. When different noise sources are getting filtered by different noise transfer functions, what we need to do is, we need to find the output noise.

So, the output noise due to the reference noise is going to be the noise spectral density of the reference noise source multiplied by the noise transfer function of the reference. So, this was the noise power spectral density, noise transfer function was not in the power domain, it is like either voltage to voltage, phase to phase or current to voltage. So, when you are multiplying these transfer functions, this will be square here. So, the noise power spectral density gets multiplied by the noise transfer function to give you the output noise due to reference. So, we have,

$$S_{REF}^{\phi OUT} = S_{\phi REF} \times |NTF_{REF}|^2$$

Similarly, the noise at the output due to charge-pump is given by,

$$S_{CP}^{\phi OUT} = S_{CP} \times |NTF_{CP}|^2$$

The noise at the output due to resistance is going to be,

$$S_R^{\phi OUT} = S_R \times |NTF_R|^2$$

$S_{VCO}^{\phi OUT}$ that is VCO phase noise is given by,

$$S_{VCO}^{\phi OUT} = S_{\phi VCO} \times |NTF_{VCO}|^2$$

So, what we have done is we found the noise sources, we found the noise transfer function for each of these sources and after finding the noise transfer functions, we calculated the noise at the output due to the different noise sources.

Similarly, you will have for the divider. So, we have,

$$S_{Div}^{\phi OUT} = S_{Div} \times |NTF_{Div}|^2$$

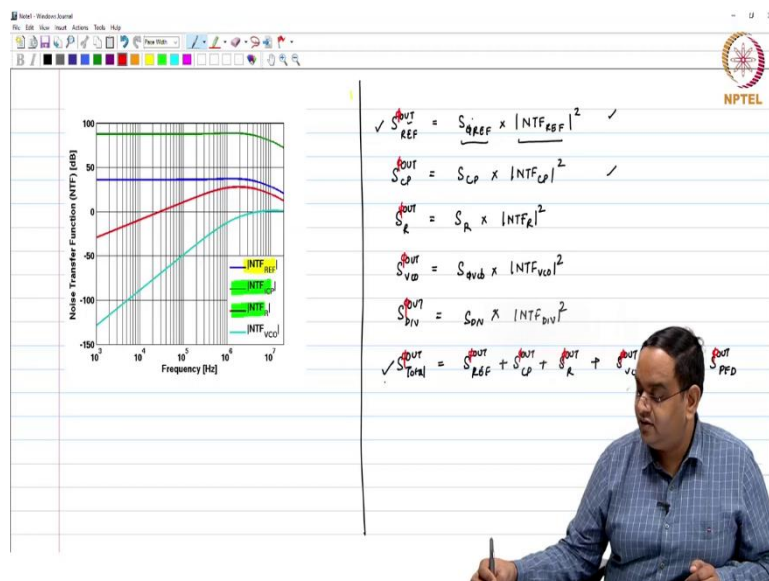
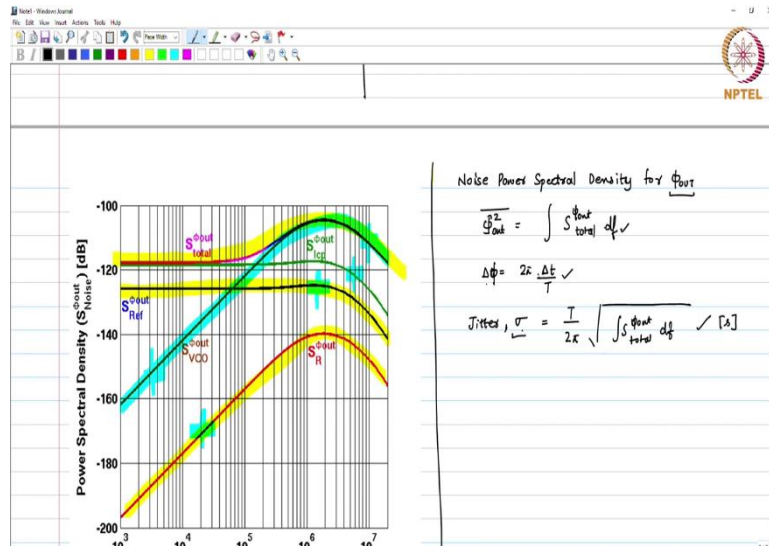
All these 'S' are actually the output noise power spectral density due to different sources. So, to calculate the total noise, $S_{Total}^{\phi OUT}$, all these noise sources are independent of each other, their noise spectral densities are going to add up. So, we get,

$$S_{Total}^{\phi OUT} = S_{REF}^{\phi OUT} + S_{CP}^{\phi OUT} + S_R^{\phi OUT} + S_{VCO}^{\phi OUT} + S_{Div}^{\phi OUT} + S_{PFD}^{\phi OUT}$$

So, we can get the final output noise power spectral density due to different noise sources or due to the noise contribution from different blocks and we can find the total output noise

spectral density. Let me show you one of the simulations which we have done to find the total output noise.

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$$NTF_R(s) = \frac{\Phi_{OUT}}{V_N} = \frac{\Phi_{OUT}}{V_{REF}(s)} \times \frac{V_{REF}(s)}{V_N}$$

$$= \frac{K_{VCO}}{s} \times \frac{G_1}{C_1 + sC_2} \times \frac{1}{1 + sRC_1C_2}$$

$$= \frac{G_1}{C_1 + sC_2} \times \frac{K_{VCO}}{s} \times \frac{1}{1 + Lk}$$

$$NTF_{VCO}(s) = \frac{\Phi_{OUT}(s)}{\Phi_{VCO}(s)} = \frac{1}{1 + Lk}$$

$$NTF_R(s) = \frac{G_1}{C_1 + sC_2} \times \frac{K_{VCO}}{s} \times \frac{1}{1 + Lk}$$

$$NTF_{CP}(s) = \frac{2K}{I_{CP}} \times \frac{N \cdot Lk}{1 + Lk}$$

$$NTF_{OIN}(s) = NTF_{EBF}(s) = \frac{N \cdot Lk}{1 + Lk}$$

$$Lk = \frac{I_{CP}}{2\pi N} \times \frac{K_{VCO}}{C_1 + sC_2} \times \frac{1 + sRC_1C_2}{1 + sRC_1C_2}$$

$$NTF_{EBF} = \frac{N \cdot Lk}{1 + Lk} = \frac{N}{1 + \frac{1}{Lk}}$$

$$\lambda \rightarrow 0 \Rightarrow Lk \rightarrow \infty \Rightarrow NTF_{EBF} \rightarrow N$$

$$NTF_{VCO} = \frac{1}{1 + Lk}$$

So here, we found the noise spectral density, it actually depends on whether the input noise is flat or not. So, let us look at the resistor one first. So, we know that the resistance noise is $4kTR$ which is flat and the noise transfer function is bandpass in nature, that is what you see the $S_R^{\Phi_{OUT}}$ as.

Now, you look at the reference noise. Reference noise depending on what noise we have, the reference noise will also be low pass if it is flat. Charge-pump noise, the current noise is normally flat as we have talked about. So, though we have not talked specifically about the charge-pump noise but the current element which you have there, that noise is flat and it turns out that the charge-pump noise spectral density is also flat. So, the output noise spectral density of the charge-pump is also low pass.

Then, you get the VCO. VCO is a little tricky part and what happens is that the VCO phase noise is not white or not flat, it has some transfer functions. Well, whatever that noise we have, we multiplied that noise with the noise transfer function and it turns out that this noise appears to have bandpass characteristic here and then we add up all these noise sources whichever you are seeing and the final resultant noise is shown to you in terms of, let me just erase this part so that it becomes little clearer. So, you add all these terms and the final output noise is this.

When you have the final output noise power spectral density for the phase, what do we have effectively? We have the noise power spectral density for ϕ_{OUT} , this is the power spectral density, so, the total noise power for the phase error because this is what we are calculating is due to noise, so that is like the disturbance in the output phase.

So, we can find out the mean square value of output phase error as follows:

$$\overline{\varphi_{OUT}^2} = \int S_{total}^{\varphi_{OUT}} df$$

So, we know that the area under the plot in the frequency domain will give us the power for the signal in time domain.

So, we get this φ_{OUT} , and how is jitter related to φ_{OUT} ? Well, we have,

$$\Delta\varphi = 2\pi \cdot \frac{\Delta t}{T}$$

So, the time domain jitter value σ is defined as,

$$\sigma = \frac{T}{2\pi} \sqrt{\int S_{total}^{\varphi_{OUT}} df}$$

From the above equation, you will get the jitter in seconds.

So now, you see that based on the loop gain transfer function, your noise transfer functions depend and depending on the noise transfer functions and your input noise, noise for different blocks, you get the total output noise which will finally lead to jitter. If you vary your bandwidth, you vary the noise transfer functions, this jitter will vary.

The next thing which we need to look at is that how this jitter varies with bandwidth, right now we have said the fundamental element is resistor or transistor, but we need to see how noise from different blocks like PFD, charge-pump, VCO, etc. translates to the output phase noise. And once we know that, then we need to understand how to choose an optimum bandwidth and look at the trade-offs. Thank you.