Analog Electronic Circuits
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Lecture - 02
Small Signal Analysis of Nonlinear Networks

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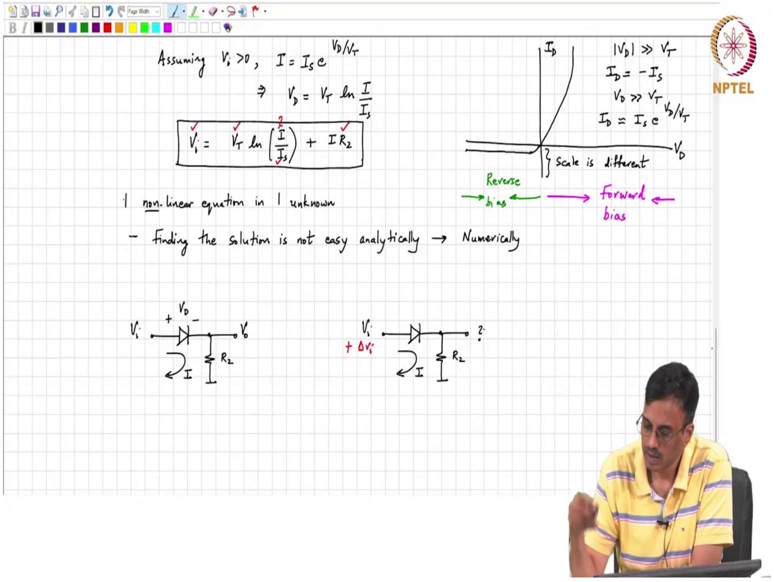

Alright, what is the simplest way of or rather I would say what are the dumbest way of doing this? You know most straightforward way of doing this is to write Kirchhoff's law again.

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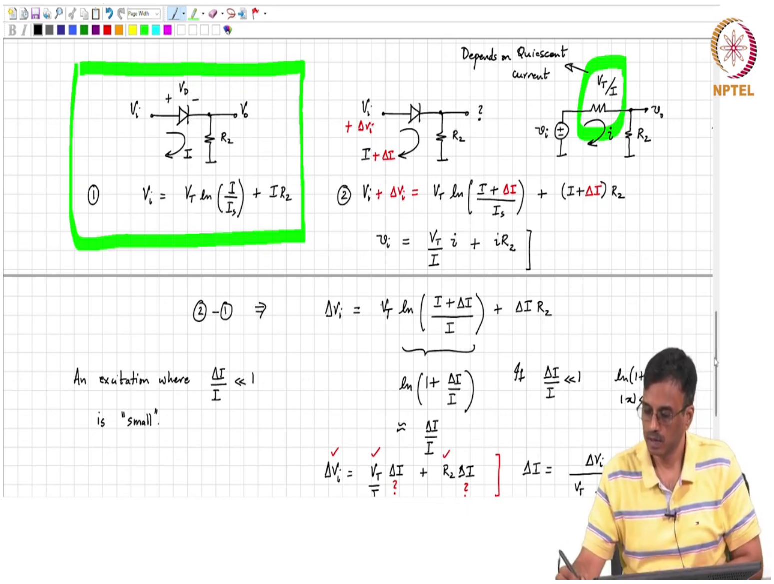

So, now what happens? In the earlier case remember that V_i was nothing but,

$$V_i = V_T \ln ln \left(\frac{I}{I_s}\right) + I R_2$$

In the circuit on the right what do we see? V_i. We do not know what the current will be. So, the current is I it is going to change from I+ΔI. And so therefore, $V_i + \Delta V_i$ must be equal to?

Student: Diode voltage.

The diode voltage + the drop across the resistor what is the diode voltage?

$$V_i + \Delta V_i = V_T \ln ln \left(\frac{I+\Delta I}{I_s}\right) + (I + \Delta I) R_2$$

Correct. And what are the most straightforward ways that the way to do it? It is to say well we solve this numerically, so now we have changed the input voltage.

Well, you go to the computer again and solve for this. So, this new ΔI and then for this new I which is different from the earlier current, and find the new output voltage. Does that make sense? Now, this will work for an arbitrary change in the input voltage, correct. But now, if I told you that ΔV is small, what is the first question that you should ask me?

Small in relation to what? Correct. But at this point we will revisit this question later, but at this point we will assume that it is small in some sense ok. Or in the right sense. What that sense is will come back to later. So, if that change in the input voltage is very small, then intuitively what do we recognize? We have already solved this problem where the input is so close to our current input ok.

Now, the question is does it make sense for me to go and do all that hard work all over again or can I exploit the fact that I have solved a problem, which is this close to this problem. Perhaps I can exploit the fact that I have done a lot of hard work already. Can I use the results of that to solve my new problem? That makes sense. Now, so, how do we figure this out? Well, if you look at this equation let us call that equation 1 and if I call this equation 2, then what is 2 – 1 yield?

2 is valid, 1 is valid. So, what does the difference between the two yield? On the left-hand side, you have ΔV_i and this is nothing but

$$\Delta V_i = V_T \ln ln \left(\frac{I+\Delta I}{I}\right) + \Delta I\, R_2$$

Quick question is this valid for all ΔV_i or is it only valid for you know small ΔV_i?

There is nothing wrong with this equation this is valid for all ΔV_i because both equations 1 and 2 are valid for all values of excitation. And I have simply subtracted the 2 equations. Now, if ΔV_i is very small, so, in other words if the exciting input has changed just a little bit, right? Where again we will come back to what this just a little bit means going forward, if this ΔV_i has changed by just a little bit what comment can we make about the change in the loop current?

The change in the loop current will also be?

Student: Very small.

Very small, right?

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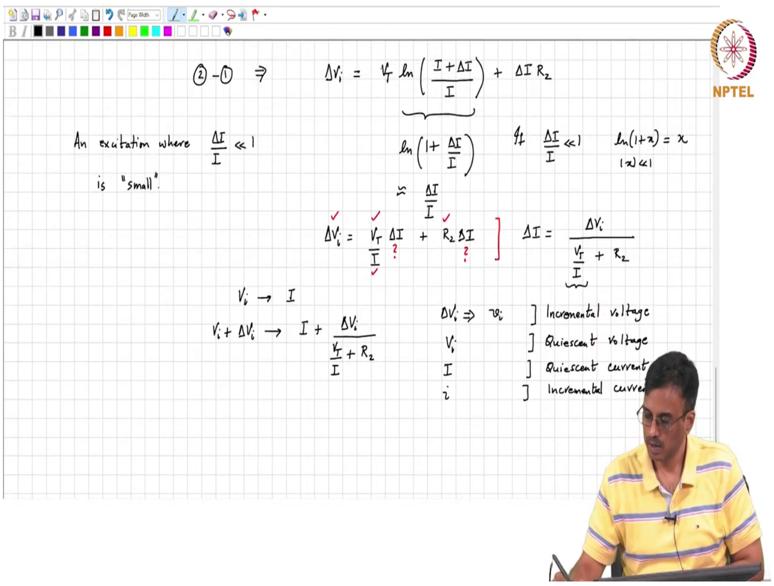

So under those circumstances, remember that this is

$$\left(1 + \frac{\Delta I}{I}\right)\ ;\ if\ \frac{\Delta I}{I} \ll 1$$

what comment can you make about this guy? for $|x| \ll 1$, what is $Ln(1 + x) =$?

I mean $\ln(1 + x) = 0$ for $x = 0$, right? but for small x where x is very small compared to 1, what is it?

Student: 1, x.

1 or x, people?

Student: 1.

2 years of COVID has wiped out your memory also.

Student: Is equal to x.

Is equal to x. Yes or no?

Hmm.

Student: Yes.

Alright. So, ln $ln\left(1 + \frac{\Delta I}{I}\right)$ is what?

Student: $\frac{\Delta I}{I}$.

$\frac{\Delta I}{I}$I ok. So, now, what is this equation reduced to? So, 2 - 1 gives you,

$$\Delta V_i = \frac{V_T}{I}\Delta I + \Delta I\, R_2$$

Now, tell me what is known and what is unknown?

Let us start from the left hand side.

Student: ΔV_i is known.

ΔV_i is known.

Student: V_T known

Is V_T known?

Student: Yes sir.

Yes. Is capital I known?

Student: Yes sir.

Where?

Student: Already from the last class.

Well, yeah. We had already solved that problem by working hard. So that is known. What about ΔI?

Student: Unknown.

ΔI is unknown, R_2 known ΔI is unknown. So, this is also one equation in one unknown. However, what is the big difference between what we did last time and what we are doing now?

Student: Linear equation.

This is a?

Student: Linear equation.

Linear equation in?

Student: In one variable.

In one variable. So, presumably its lot easier to solve than solving a non-linear equation. So, this basically means therefore, that ΔI is nothing but,

$$\Delta I = \frac{\Delta V_i}{\frac{V_T}{I} + R_2}$$

Ok. So, what comment can we make about the total current now? So, earlier when we had excited the circuit with V_i we got some I, now when we excite the circuit with $V_i + \Delta V_i$ you should get, $I + \frac{\Delta V_i}{\frac{V_T}{I} + R_2}$, Ok. What are the dimensions of V_T / I?

It is a its got dimensions of resistance ok. And so therefore, the bottom line therefore, is that if we have a non-linear circuit to find the branch currents and the branch voltages for an arbitrary excitation is fundamentally a difficult thing to do, right? As we have discovered even with our simple diode example, right? Ok.

And you can imagine that even with this simple example we found that we have to solve a non-linear equation in a non-linear in one variable. Now, you can imagine what happens if you have say a network with 10 nodes and you know multiple non-linear elements what will we end up having to solve?

How will you solve it? If you have more complicated non-linear network, how would we go about solving it?

Student: MATLAB.

I mean MATLAB is just a tool, right? So, you know what I going to feed into MATLAB? What did we do here?

Student: KVL KCL.

No what do we do here? What will be plop into MATLAB?

Student: KVL.

Yeah, we basically wrote you know KVL slash KCL equations and dump those equations into a non-linear solver, right? Now, if you have you know 10 nodes and you know a lot of non-linear elements what comment can we make? what will you do?

Student: Write all the equations.

You write all the you know the KCL equations at every node, correct. And now unfortunately because the elements are non-linear what comment can we make? The branch voltage is related to the branch current through a?

Student: Non-linear relationship.

Non-linear relationship. So you will get a system of?

Student: Non-linear equations.

Non-linear equations and these are all coupled correct ok. So, you will now get a system of non-linear coupled equations in the variables and solving even a single non-linear equation is seems to be very difficult. So, you can imagine how it would be to solve a bunch of non-linear coupled equations. So, what do you do? How would we solve it then?

Well, just like we did in the single in the simplest case we will take the system of equations into your favorite mathematical tool and solve the equations numerical. Will make sense people?

Student: Yes.

Right? what we are doing for the simple one we will do for the more complicated one except that now we have more variables and more equations and the equations also because it because there are more variables there are also coupled, right? So, for example, if you put a diode between two nodes the current flowing through the diode is $I_s e^{(V_1 - V_2)/V_T}$. Ok.

So, you can imagine that as you can you know probably imagine the solution to these equations becomes finding it analytically is almost impossible. You have to resort to some kind of numerical technique ok. However, as we have seen in this particular case, of this particular example, if the excitation changes by a small amount from the previous excitation.

What we are seeing is that we do not have to do all that hard work all over again, right? One can actually leverage the work you have done in the for the previous example where found all the voltages and currents for a given excitation, right? And from that it seems a much easier job to find what happens to the branch currents and the node voltages when the excitation is changed by a small amount.

And in this particular example what comment can we make or what qualifies as a small excitation? Look at where we have made the approximation?

Student: $\Delta I/I$.

Yeah. So, basically any excitation where, $\Delta I/I << 1$, is within quotes. So, a small signal a small excitation or a small change in the input is defined as that which results in a change in the loop current where the change is much much smaller than the current time. And that pertains to this particular example, right?

In general each circuit or each non-linear element will have a different definition for what it means for the excitation to be small. In this particular context it so happens that the $\Delta I/I \ll 1$, alright. So, now some jargon this ΔV_i is to avoid having to write capital Δ and capital V, it is often written in small case, right? And this is what is called the small signal or the incremental quantity, ok.

Capital V_i is what is called the quiescent quantity whether it is a voltage or current or whatever, right? In this case because V_i is capital, V_i is a voltage, and it is called the quiescent voltage ok. And the capital I is the quiescent current. And the small I is the incremental current. Make sense people?

So, in terms of our new jargon we can basically say that if that incremental excitation is, if it qualifies to be a small signal, then after subtracting the equation 2 from equation 1, we basically get V_i. Which is nothing, but $(V_T/I) \times i + i R_2$, ok. And once you have a linear equation, you can also have a circuit, which represents this linear equation alright. And what is that circuit do you think? What is it?

Student: Non-linear element.

So, the source is replaced by the small signal equivalent alright. And what happens to the non-linear element?

Student: Replaced by a resistance.

Replaced by a resistance capital V_T/I. And what comment can you make about the resistance R_2?

Student: Voltage small.

R_2 and what is that voltage?

Student: Small.

Small.

Student: V_o.

V_o. What is the current there?

Student: Small.

And to find small V_o is you know as straightforward as finding the output of Resistor divider, and as you can see the small i is simply nothing but, i= v_i $R_2/(R_2 + V_T/I)$. So, for small signals therefore, we have replaced the non-linear element in this case a diode by,

Student: Linear one.

By a linear one, right? And the what are the properties of the linear element? What is the value of that resistance depending on?

I mean V_T of course, is you know is a given at a given temperature it depends on?

Student: Quiescent current.

The quiescent operating point or the quiescent current ok. And this is often also called the small signal resistance of the diode, ok.

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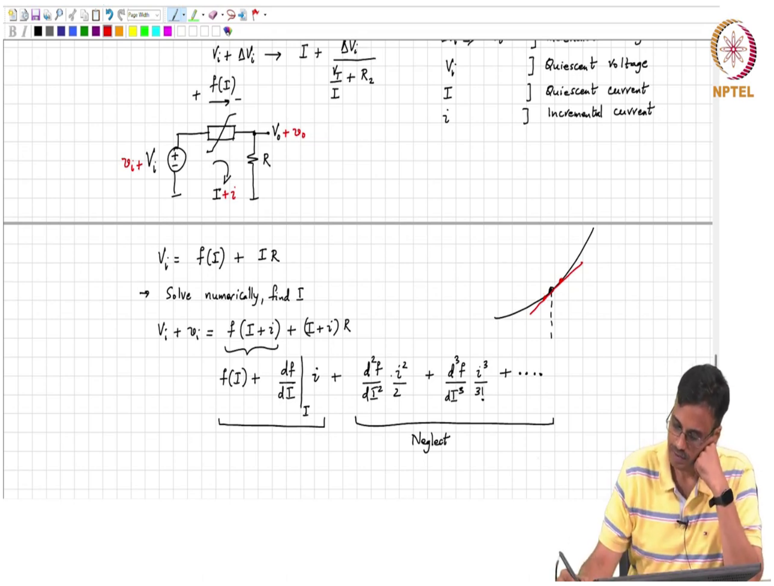

Now, in general therefore, if that non-linear element, let me quickly go through this. Let us generalize. Let us say you had a network with capital V_i some non-linear element and some resistance R and let us say this current or this voltage was some function of the current flowing in the network ok. What comment can we make about the output voltage V_o, how do we find it?

No there is no increment here this just have applied a voltage to a non-linear network how will we find V_o?

Student: First we will write KVL.

KVL what does KVL give yield V_i is nothing but.

That is some function of I + I.

Student: I + I times R.

Times R. And then you solve this numerically, then find I. That make sense? Now if I add a small signal the current changes by a small amount and the output voltage also changes by a small amount. So, what do we say? $V_i + v_i = f(I+i) + I$.

Now, how will you relate f(I+i) in terms of I?

Student: Taylor series.

Use the Taylor series. So, in general this is nothing but f(I)+ df /dI.

where I used notation a little bit. What I mean is when I say df/dI is the derivative of the function with respect to its argument evaluated at I.

Student: I.

Just to make this clear I will write that as ok, and what should be there?

$$V_i + v_i = f(I + i) + (I + i)R$$

$$= f(I) + \frac{df}{dI}\Big|_I i + \frac{d^2f}{dI^2}\Big|_I \frac{i^2}{2!} + \frac{d^3f}{dI^3}\Big|_I \frac{i^3}{3!} + \ldots\ldots\ldots\ldots$$

Does it make sense folks? Ok. Now, if that current change is very small what comment can we make, how can we approximate this f(I + ΔI)?

Yeah, so basically we take only this and neglect.

Student: Higher order terms.

Higher order terms in the Taylor series and the intuition is that well if you have a curve and if you are looking at if you are point about which you are expanding is that then around that operating point ok. So, if you are sufficiently close by you just need the slope. If you go too far away then you need curvature and all that stuff ok. So, now, if so therefore, what you know in general what qualifies as a small signal?

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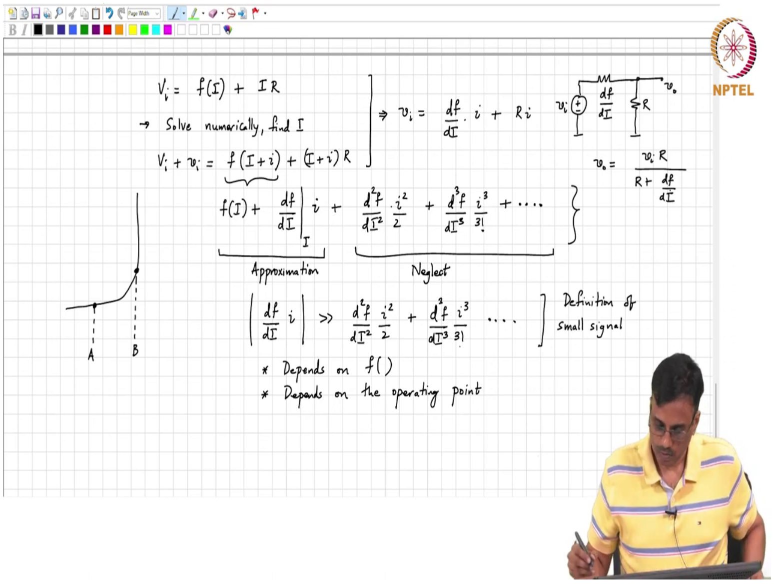

When we will this be a good approximation?

In this Taylor series when is this a good approximation?

So, basically when your approximation is good, right? And this is the complete series is the true thing, right? You are trying to approximate it by just the first two terms. So obviously, the approximation is only valid when df/dI x i in magnitude is much much greater than all the higher order terms.

This is the true definition of what it means to for a signal to be considered small ok. So, this is the definition of what constitutes a small signal and what does this depend on. As you can see from this equation what all does this depend on? It evidently depends on the function what else?

It depends on what?

Student: Summation of.

It depends on f, It also depends on I mean all this is basically saying that we are evaluating the derivatives where?

Student: At the quiescent operating point.

Quiescent operating point, correct. So, it not only depends on the function it or even for a given function, it depends on? On the operating point ok. Does it make sense folks? Right, for instance let us say you had a function like this ok and you are operating this is your operating point 1, operating point A, this is your this is your operating point B, right?

And let us say I have a fixed increment around both these operating points at which of these operating points will the notion of small signal be larger? Which signal will count as a small signal you know or rather there is; obviously, at each operating point there is a limit for what constitutes a small signal. For which of these operating points A and B will that limit be larger?

Student: A.

A, why?

Well A looks like you know a lot more linear and whereas, B it the things curving like this ok. So, clearly you can see that this definition of small signal is not absolute, right? It not only depends on the device, but also depends on?

Student: Operating point.

The operating point for that particular device ok. So, it is very very context dependent. It is not something absolute, right? And a good case in point is the diode, right? You know in the reverse bias region what comment can you make about the operating I mean what constitutes a small signal?

You can see that the curve is virtually flat so, a small you know a large change in the applied voltage will result in a in expected to you know result in you know a linear change in current with that proportionality constant is close to 0 correct. Whereas, in the forward bias region you can see again that you know its exponential. So, to evaluate what constitutes a small signal you have to we have to find the higher order terms of the Taylor series compare it with the first order term and then basically you understand.

So therefore, let me just finish this topic. So, a so what comment can be make when we subtract these two what do we get under the small signal approximation. What do we see? Let us subtract the first equation from the second what do we see?

Student: v_i equals to.

v_i equals.

$$v_i = \frac{df}{dI}.i + Ri$$

And therefore, equivalently if you draw to draw a network, which models the difference between the two equations. This becomes a linear resistor with a value df / dI and resistance R and this is v_o and therefore, v_o is nothing but,

$$v_o = \frac{v_i R}{R + \frac{df}{dI}}$$

Does it make sense people? Ok, alright. So, tomorrow we will continue.

Thank you.