

Analog Electronic Circuits
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Lecture - 33
Effect of Finite Output Resistance on Basic Building Blocks – Part 1

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So, now the question is, well it seems like all along we have been working with an ideal transistor model which does not exist anyway. So, we now go back to all our circuits and start figuring out what this finite earlier transistor was an increment I mean was an ideal incremental voltage control current source meaning that the output impedance was infinite. Now, it is not infinite, after all there is some?

Student: Unknown.

Unknown. Remember one thing in life and engineering right nothing which is supposed to be infinite will be infinite ok alright and nothing which is supposed to be 0 will be?

Student: 0.

0 correct in both are consistent, correct? If you had a perfect 0 then $1 / 0$ is infinite. So, you know you have a perfect infinity that is not possible, right. So, nothing in the world is 0

nothing in the world is infinite. So, now we go back to all our circuits that we started off with and then see what happens when you have finite output resistance.

So, fortunately it is pretty straightforward. Let us start with the common source amplifier which was the first amplifier that we talked about. The incremental picture looks like this. This is R_L and ideally the incremental gain was what ideally what was it? $-g_m R_L$. So, the output voltage was $-g_m R_L v_i$. Now, what happens? We have an extra here which is a part of the transistor r_o . So, what comment can you make? The gain is now minus $g_m (R_L // r_o)$. So, earlier in principle what is the maximum incremental gain that you could get? I mean if you had the freedom to change R_L what was the maximum incremental gain that you could get?

Student: Infinite.

Infinite, earlier in principle if you made R_L an open circuit the DC gain that you would get would be?

Student: Infinite.

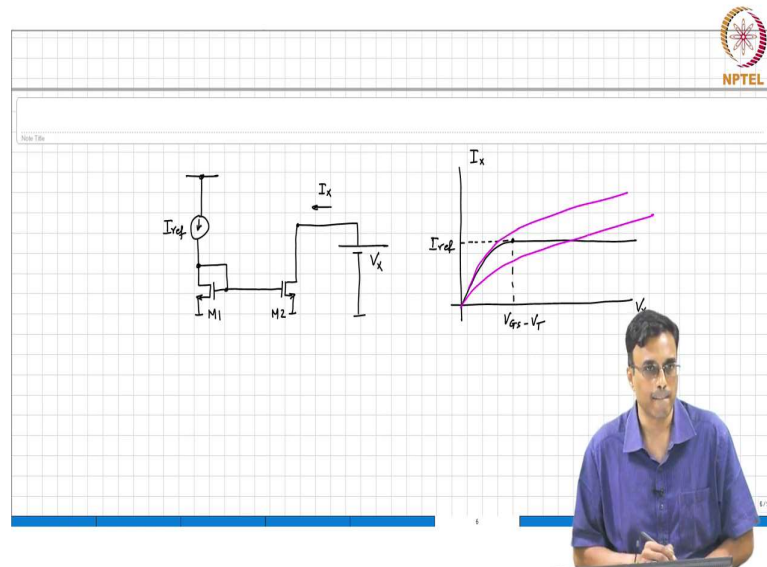
Infinite. Now, what is happening? Even if you make R_L infinity, the maximum gain that you can get is only.

Student: $-g_m r_o$.

So, the maximum magnitude that of the gain you can get is $g_m r_o$ this is often what is called the intrinsic gain of the device, alright. So, it turns out that you know again this value of the intrinsic gain ideally of course, should be infinity, but in practice this $g_m r_o$ depends a lot on the details of the construction of the transistor.

I mean if you choose very short channel lengths the $g_m r_o$ can be as small as you know may be 8 or 10; if you lose choose long lengths for the transistor it can be as high as may be 50, 60 whatever, right. So, you can see that you know the $g_m r_o$ is not a you know is of the order of you know a few tens. That is what you can expect in practice, alright.

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Now, the next thing we started looking at was various biasing techniques. Remember that the first robust biasing technique we looked at was the current mirror. So, this is some I_{ref} . So, ideally regardless of the voltage V_X as long as the transistor M_2 is in saturation. What comment can you make about the current I_X is going to be?

Student: I_{ref} .

I_{ref} . But, there is a slight problem. So, if I plot I_X as a function of V_X what curve should I expect? Ideally, I should expect a constant like that, right. So, this is $V_{GS} - V_T$, and what would this value be? What will that value be?

Student: I_{ref} .

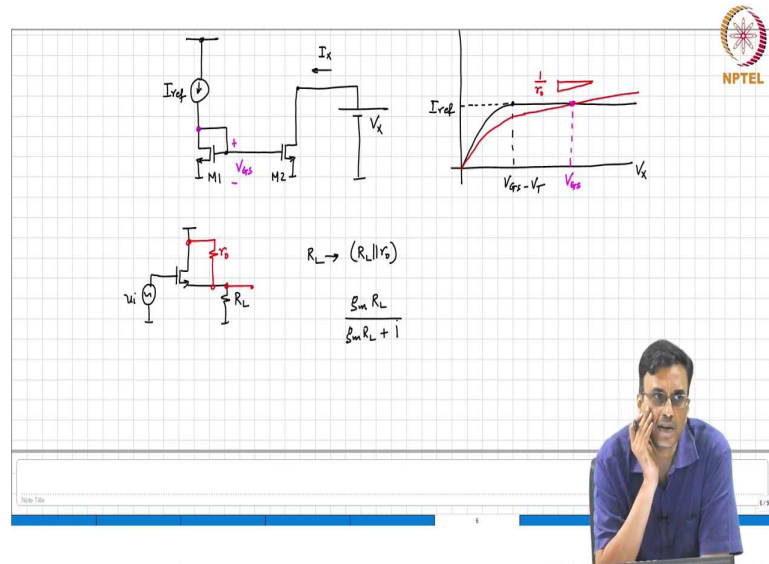
I_{ref} very good, alright. Now, what happens in reality? There will be some slope ok. So, at what value of V_X will the current exactly be equal to I_{ref} ? Ok. Let me rephrase the question you said there is a slope. So, it could be like that, what are we talking about? At what point I am with the question I am asking you is first let us figure out at what point I_X will be equal to I_{ref} .

Student: V_{GS} .

No, why?

So, the claim is that I_X will be exactly equal to I_{ref} when V_X is exactly equal to?

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Student: V_{GS} .

V_{GS} , correct? Why does that make sense? M1 and M2 are identical. Both of them have the same? Both of them have the same gate potential. So, their drain currents will be exactly identical when the drain potentials are?

Student: Same.

Right, because after all the current in the drain is a function of both the gate source voltage and the drain source voltage. So, if the two of them are the gate source voltage is the same as I mean in both cases it is the gate source voltage of course, constrained to be the same. So, if the drain source voltage is exactly the same for both M1 and M2 then you will have exactly the same current, correct.

Now, what comment can you make about this, if you know that if V_X increases beyond V_{GS} do you think the current must increase or decrease?

Student: Increase.

Increase right. So, what will be the slope of the characteristic around V_{GS} around $V_X = V_{GS}$. I hope all of you are able to see this right. It's pretty straightforward. Around the operating

point the change in the current in the transistor due to the gate source voltage is fixed correctly, it is not changing. So, we are interested in the slope with respect to V_X .

So, the slope will be the output conductance of the transistor which is the ratio of these changes in the current drain current due to change in the drain source voltage which is λI_{ref} , ok. So, this is an exaggeration. So, the curve probably looks something like that ok. So, this current source which is ideally supposed to be?

Student: Constant current.

A constant current is now no longer a constant. Does it change?

Student: V_{PX} .

V_{PX} because I mean the output impedance is the incremental output impedance of this current source not infinite, it is finite ok, alright. Now, if you want, we just looked at just a little while ago, ok anyway let us do this so that we will come to this a little later. So, this is something to bear in mind. So, our current mirrors are not that accurate anymore, ok.

If you want the current mirror to be accurate you would have to absolutely make sure that the drain source potentials are also matched, alright. Then what else? Then we did the common drain amplifier. This is v_i , earlier we had R_L and now with the new model for the transistor what is the extra element we have?

Student: r_o .

We have r_o here and therefore, what comment can we make about the incremental output voltage? So, all that you have to do is replace R_L in earlier whenever you saw R_L now you replace it with $R_L // r_o$. So, the incremental gain remember was earlier what was the incremental gain with finite g_m ?

Student: $g_m R_L$.

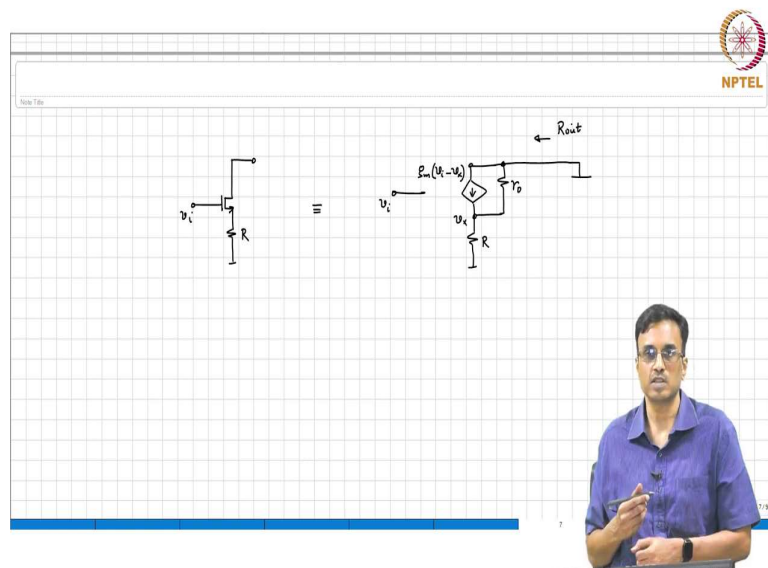
$g_m R_L / (g_m R_L + 1)$. So, if $g_m R_L$ is much larger than 1. What comment can you make about $g_m R_L // r_o$ remember $g_m r_o$ is a large number. So $g_m R_L$ only so, as long as r_o is very large $g_m R_L$ is large and as long as r_o is larger than much larger than R_L right you will find that $g_m R_L$ was a large number, it remains a large number. So, the closed loop I mean the incremental gain

remains largely the same ok. So, it is benign. So, if earlier if R_L was infinite what comment can you make about the gain? Earlier if R_L was infinite what was the gain.

Student: 1.

1, right. Now, even if R_L is infinite what will be the gain $g_m R_o / (1 + g_m R_o)$. So, if $g_m R_o$ is 50 then you will get 50 by 50 ok pretty straight forward, alright. The next thing is the voltage control current source.

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This is v_i , ok earlier what was the output resistance?


Student: Infinite.

Infinite, what is the input resistance earlier?

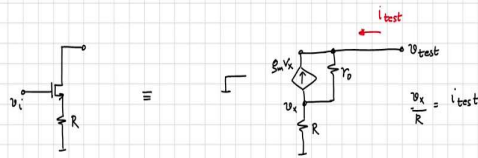
Student: Infinite.

This is R , now we have an extra here, this is r_o , let us call this v_x and this is nothing but $g_m (v_i - v_x)$, alright and this is the output port. So, the first question to ask is what is the output resistance. To find the output resistance what will we do?

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Node Title



$$\frac{v_x}{R} + g_m v_x + \frac{v_x - v_{test}}{r_o} = 0$$

→



You set v_i to 0. So, this becomes $g_m - v_x$ which is equivalent to saying this is $g_m V_x$, this is v_{test} . The question is what is i_{test} ok? Before we do the math I should i_{test} be positive I mean should i_{test} be 0 or should it be still 0 I mean should it be 0 or non-zero?

Student: Non-zero.

Non-zero. Clearly now you know there is some connection between the drain and the source, right. So, alright. So, what is the unknown that we need to find?

Student: v_x .

v_x ok. Once we find v_x we will be able to find the i_{test} as simply being equal to.

Easy way?

Student: v_x/R .

i_{test} remember that this is still a Gaussian surface type thing right whatever current is pulled out.

Will be pulled out and will be coming out from the other side. So, v_x / R must be equal to i_{test} . The question is what is v_x . So, you write KCL at the only node that is there and what is that $g_m v_x$ plus i_{test} sorry, we write KCL. So, $v_x / R + g_m v_x + (v_x - v_{test})/r_o$.

$r_o = 0$ which means that actually this certainly has something to do, but as I also like to illustrate a point here. So, in general when you are trying to find the output resistance looking into a port you can do it in two ways, right you either put a voltage measure the?

Student: Current.

Current or you can put a?

Student: Current.

Current and measured?

Student: Voltage.

Voltage. Eventually you should get the?

Student: Same.

Same answer whatever you do, but in you know, but in say I mean there will be cases where it is convenient to do one over the other, right and this particular case happens to be one such case, right. What do we notice here? A whole bunch of arms are in series correct that this character here is in series with R.

So, do you think it is easier to apply a voltage measure the current or apply a push current to measure the voltage? When things are in series it makes a lot more sense to push?

Student: Current.

Current. So, we will do that, we will push i_{test} .

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The slide contains the following handwritten equations and notes:

$$R_{out} = g_m R r_o + R + r_o$$

$$i_{test} (1 + g_m R) r_o + i_{test} R = v_{test}$$

$$\approx g_m R r_o + r_o \quad (g_m R \gg 1)$$

$$\approx g_m R r_o$$

So, what is the voltage v_x ? Well, what is the current flowing through R?

Student: i_{test} .

i_{test} . So, the voltage v_x is $i_{test} R$. So, what is the current flow, what is the output voltage v_{test} g_m v_x therefore, is nothing but g_m times.

Student: (Refer Time: 18:13).

$R i_{test}$. So, the current flowing through R_o is?

$i_{test} (1 + g_m) r_o + i_{test} R = v_{test}$, correct. So, the output resistance is nothing but, $g_m R r_o + R + r_o$.

Now, if you want to approximate it remember in a $g_m R_o$ is much much larger than 1, ok. So, comparing these two terms? Which can we neglect between these two terms? $g_m R_o$ is much much larger than 1?

Student: r_o .

You can neglect the R, ok. So, this becomes $g_m R r_o + r_o$. Now, in a good in a good voltage controlled current source what comment can we make about $g_m R$? If you want to build a good voltage controlled current source, what comment can we make about $g_m R$? So, $g_m R$, if

you build a good current source, a voltage controlled current source $g_m R$ is much larger than 1. So, which can you neglect here now?

Student: r_o .

$g_m r_o$, alright. Is that clear? So, what comment can you make about the output resistance therefore? A sanity check first of all sanity check, this is the exact expression what all sanity check can we run?

Student: g_m tends to infinity.

Student: r_o .

r_o tends to infinity ok alright. g_m tends to if R tends to 0? What should you get? You should get a small R_o that is indeed correct, alright and small r_o tends to infinity.

Student: infinite.

You must get infinite again to correct all those cases we have seen earlier. So, if $g_m R$ and $g_m R_o$ are very large, the output resistance is simply $g_m R_o R$ or $r_o g_m R$, ok. You can see therefore, that the output resistance has been enhanced by this, I mean the resistance looking in the drain is the resistance in the source multiplied by the $g_m R_o$ of this transistor, alright.

So, basically the bottom line therefore, is that the output resistance looking in here is approximately $g_m R r_o$ right, but the true thing is basically to this you need to add both R and r_o ok, alright.

With that we will stop. We will continue in the next class.