

**Analog Electronic Circuits**  
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**Lecture - 06**  
**Graphical Representation of the Y-Matrix**

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NPTEL

$$I_1 = f(V_1, V_2) \quad I_1 + i_1 = f(V_1 + v_1, V_2 + v_2)$$

$$I_2 = g(V_1, V_2) \quad I_2 + i_2 = g(V_1 + v_1, V_2 + v_2)$$

$$I_1 + i_1 \approx I_1 + \frac{\partial f}{\partial V_1} v_1 + \frac{\partial f}{\partial V_2} v_2$$

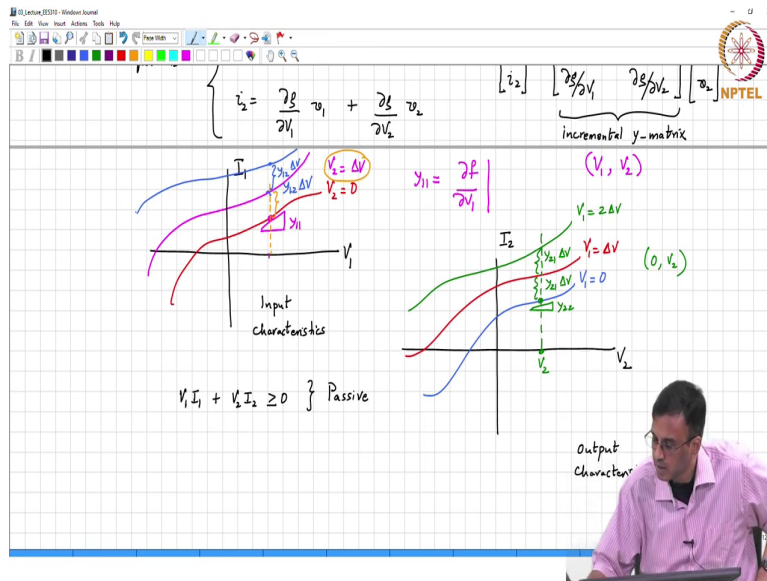
$$I_2 + i_2 \approx I_2 + \frac{\partial g}{\partial V_1} v_1 + \frac{\partial g}{\partial V_2} v_2$$

Linear equations

$$\begin{cases} i_1 = \frac{\partial f}{\partial V_1} v_1 + \frac{\partial f}{\partial V_2} v_2 \\ i_2 = \frac{\partial g}{\partial V_1} v_1 + \frac{\partial g}{\partial V_2} v_2 \end{cases} \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial f}{\partial V_1} & \frac{\partial f}{\partial V_2} \\ \frac{\partial g}{\partial V_1} & \frac{\partial g}{\partial V_2} \end{bmatrix}}_{\text{incremental } y\text{-mat}} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Alright, again a picture is worth 1000 words. So, I just like we did with the linear two port. We would like to do show the performance of the two port in a picture. So, again we cannot draw a 3D surface.

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So, we will draw the input characteristics which are again  $I_1$  versus  $V_1$  for different values of  $V_2$ . So, what comment can you make, how will this look like in general?

Student: It is non-linear.

It will be non-linear. So, something like that. Let us say this is the curve for  $V_2 = 0$ , alright. So, for  $V_2$  some other voltage, let us say for  $V_2 = \Delta v$ . What comment can we make?

You will get another curve will that curve necessarily be parallel to this red line?

Student: No.

No, it will be some other curve like that ok and. So, and what constitutes the operating point? What do we mean by operating point?

So,  $V$  the operating point is,  $(V_1, V_2)$ . In the case of non-linear two terminal element the operating point was simply either the voltage or the current because once you know one you know the other.

Now, you need to know both quantities  $V_1$  and  $V_2$ . So, that is the operating point. So, on this picture for example, let us say this is your operating point. So, what is  $Y_{11}$ . Now what is the incremental  $Y_{11}$  at that operating point.

The slope. Remember that the incremental  $Y_{11}$  is nothing but,

$$Y_{11} = \left. \frac{\partial f}{\partial v_1} \right|$$

What are we doing remember, what is in the linear network, what was  $Y_{11}$ ? In a linear network what is  $Y_{11}$ ? Yes you.

Student: In the output voltage output is shorter.

Huh.

Ok. So, mathematically what is it?

Student:  $I_1/V_1$ .

$I_1/V_1$  when?

Student:  $V_2 = 0$

$V_2 = 0$ . That was what we had in the linear case. Right ok, when the network is non-linear we can only talk about the incremental admittance parameters. So, what is the incremental  $Y_{11}$ ?

Student: Relation between  $V_1$  and  $V_2$ .

What should you say, I mean in the linear case, it was the ratio of the input current to input voltage when the output voltage was 0? So, when we talk about incremental quantities, what should we say? Ratio of the incremental  $Y_{11}$  is the ratio of the change in the input current or the incremental current in port 1 to the incremental voltage in applied across port 1 when, the incremental output is 0.

Ok. It is not when the output is shorted ok. What should what would be the absolute voltage at the output port? It would be  $V_2$ .

Student:  $V_2$ .

Right. So, we will establish the operating point by applying a voltage of  $V_1$  at port 1 and  $V_2$  at port 2 to find the incremental  $Y_{11}$  we will make sure that the incremental change in.

Student:  $V_2$ .

In  $V_2$ .

Student: Is 0.

Is 0, right that basically and how do we how do we make sure that the incremental voltage at port 2 is 0?

Student: Constant.

We just keep that keep a voltage source between the terminals of port 2. We know that a voltage source for incremental changes is behaves like a?

Student: Short circuit.

Short circuit ok. And then we will change  $V_1$  by a small amount and.

Student: We will see that it is non-linear.

We will see the we will measure the change in the current in the input port and find the ratio of the change in the input current to the change in the input voltage. Is this clear? Ok, now look at this picture and tell me at the operating, let us assume that this  $\Delta V$  is very small ok. So, look at this picture and tell me how will I find the  $Y_{11}$  at that operating point. What should I do?

Yeah. So, in the in that picture what is  $Y_{11}$ ?

Student: The slope at that point.

The slope at this point is  $Y_{11}$  ok. Now, what other information can I get from these two pictures at that operating point?

Student:  $Y_{12}$ .

Ok. How can I get  $Y_{12}$ ?

Student: I mean we are fetching in  $V_2$ .

Yeah, very good. So, basically what he is suggesting is that at this operating point, we know that  $V_2$  is changed by a small amount. And therefore, what does this quantity, what does that indicate? What does that spacing indicate?

Student:  $\Delta v$ .

$Y_{12}$  into.

Student:  $\Delta v$ .

$\Delta v$  ok. And you can see that this is exactly analogous to what we did in the linear case ok. Now, if I draw another curve for  $V_2 = 2 \Delta v$  what comment can we make?

Where  $\Delta v$  is still very small what comment can we make?

Will the curve be parallel or it will not necessary be parallel. So, you can get another curve like this ok. But what comment if  $\Delta v$  is a small signal indeed qualifies to be a small signal what comment can we make about that difference?

Student: We got the same.

It will be, the same as the one before assuming that  $\Delta v$  is a small signal that will also be  $Y_{12}$  times  $\Delta v$ , ok, but in general the curves will not be parallel. Is this clear? Alright.

So, similarly the output characteristics where we plot  $I_2$  versus  $V_2$  will follow a similar trend. So, these are the input characteristics and these are the output characteristics, alright. And this will be some curve like that right. So, this for example, let us say this is for  $V_1 = 0$  and for some other say  $V_1 = \Delta v$  it will be another curve and say for  $V_1 = 2 \Delta v$ . It is yet another curve and for this operating point  $V_2$ ; what is the operating point of the device?

What is  $V_1$ ? This is the operating point so.

Student:  $0, V_2$ .

$0,$

Student:  $V_2$ .

$V_2$  alright. So, what is  $Y_{22}$  now?

Student: It is like slope.

To the slope around the operating point so that is  $Y_{22}$ . What does that distance represent Revanth?

Student:  $Y_{21}$  into  $\Delta v$ .

$Y_{21}$  into  $\Delta v$  and because the  $\Delta V$  appears to be a small signal that spacing at that operating point will be.

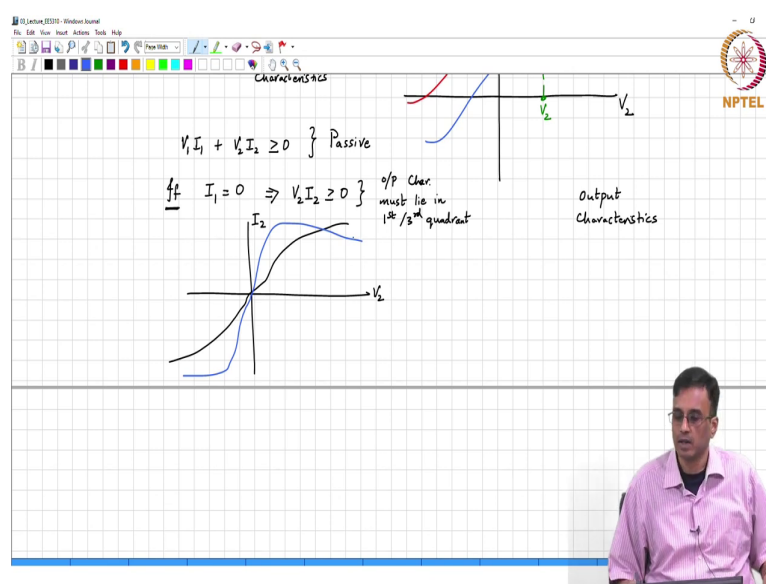
Student: Same.

Exactly the same or very close to what it was before that is also  $Y_{21} \Delta v$ . Is this clear people? Ok, alright. So, the next thing that I would like to draw your attention to is the notion of what a passive element is? Right, when we looked at a two terminal element we said that for a two terminal element to be passive the product of voltage and current must be always

Student: Greater than or equal to 0.

Greater than or equal to 0 correct. Now what comment can we make now we have a two port what comment can we make?

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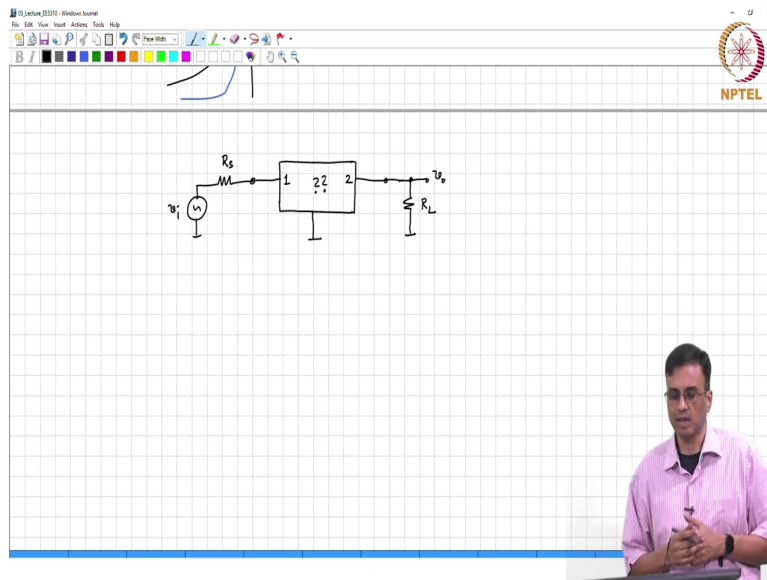


So, basically in the plus  $V_2$   $I_2$  must be greater than or equal to 0 if the device is going to be if the non-linear two port is going to be passive. Is this clear? Ok. Now, under the special condition that  $I_1 = 0$ , correct. If for example, we had a two port where, the input current was 0 right, what comment can we make about passivity therefore?

Then  $V_2 I_2$  must be greater than or equal to 0. So, therefore, what comment can we make about the output characteristics under these circumstances?

The output characteristics must lie in the 1<sup>st</sup> and 3<sup>rd</sup> quadrants. Ok. This is one example, this is another example ok. Anything is possible as long as they lie in the first and third quadrant, ok. This is just an example, is this clear folks? Alright. So, that is all the background we needed to get started on our main quest and namely we want to build an amplifier.

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And what is an amplifier? Well, we have an input source which is the small signal alright. And we have a load  $R_L$  and some voltage across it and we have a box in the middle and for argument's sake the simplest case of the box is that it is a two port. I mean the box is evidently a two port because it got an input port and output port. However, the box in the simplest case is a three terminal two port.

So, there is basically one terminal common between the input and output and we connect it like this to the output, ok. And what we are interested in figuring out is what should be there in this box so that this whole system is an amplifier, what do we understand by an amplifier?

Student: Output power should be greater than Input power.

Output power should be greater than.

Student: Input power.

Input power the fact that the output voltage is greater than the input voltage is irrelevant right because what we need is power gain correct. So, for example, you cannot put a transformer inside the box and then say,

Student: It is amplifying.

It is amplifying even though you get a large voltage the amount of power the moment you put a load resistance the voltage will drop dramatically, so that the total power can only be smaller than what is being sucked out of the source alright, ok. So, the question is what must go into this box to make this an amplifier? First thing is say well let us try this out using a linear network, correct.

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So, option 1, linear passive network, is this a possibility? Linear passive network basically inside what it means is that you have resistors inside, can we get power gain?

Student: No.

No, why?

Student: Power is dissipated.

Well, whatever power is drawn from the source some of it has to be dissipated in the box and therefore, the power coming out has to be smaller than what is being drawn from the source.



So, therefore, this is a nonstarter, very clear right. Because there is no source of energy at all anywhere ok alright.

Then I say well this is very clearly a nonstarter. Then what I am going to say is I can see that this cannot satisfy I mean this does not satisfy I mean evidently you cannot get more energy in  $R_L$  from that compared to that drawn from  $v_i$ . Is that clear? Ok.

So, then I will say well, I am going to add up energy since you said that this is not possible because there is no energy source I am going to add energy sources. In the most general case, I will add one battery at the input one battery at the output I mean if you say I am only going to put energy at the input that is easy we can just simply set  $V_2$  to 0 or if you are only interested in putting energy at the output port you can set  $V_1$  to 0 but this is the generic case, alright ok.

Now, what comment can we make for the voltage across  $R_L$ . Can this option 2 linear passive network plus 2 sources of energy. I have added voltage sources, but it could as well be current sources, it does not matter ok. Now we have potentially I mean the power dissipated in  $R_L$  can now be greater than the power dissipated in the power drawn from.

Student: Input source.

From the input source. So do you think we can I mean this is an option that will work.

It depends on what all voltages what can you write the output voltage the voltage across the load resistance can be written as.

Student: linear combination.

A linear combination of  $k_1 v_i + k_2 V_1 + k_3 V_2$  correct just by superposition alright. So, what therefore, in general therefore, what comment can you make about the power dissipated in  $R_L$ , can it be greater than the power drawn from  $v_i$ ?

It can be right, but all that power is basically due to what? It is due to.

Student:  $V_1$  and  $V_2$ .

$V_1$  and  $V_2$  that is not what we are interested in, correct. We want that component the power dissipated due to  $v_i$  to be more than the signal power drawn from  $v_i$ , does it make sense?

Right. The other these components only contribute to? They just contribute to joule heating by simply burning DC power.

Student: Load.

In the load system, is this clear? So, by adding  $V_1$  and  $V_2$  which are external power sources it is indeed true that the power dissipated in  $R_L$  can be greater than the power drawn from  $v_i$ . However, most of that power is wasted power right because it is due to the DC sources  $V_1$  and.

Student:  $V_2$ .

$V_2$  right. The signal component of the power finally, we are interested in making the power corresponding to the input voltage source  $v_i$  right, more than the input power drawn from  $v_i$  ok. That still is very similar to I mean is exactly the same as the initial option 1 where we did not have the 2 DC voltage sources, does it make sense? Ok.

So, basically this also I hope all of you are convinced that this does not work alright. So, what is the next option? This box must be non-linear.

Student: Passive.

And passive. Will that work?

Yes, what do you think?

Student: No.

No, why it would not work why?

Exactly right. So, it does not matter whether that box is active or passive, I mean, even linear or non-linear. It is passive basically means that some energy is going to be consumed. So, even if this box is non-linear, it will you will not be able to get any gain unless there is the only other choice we have is two sources of power.

We still do not know if this will work, but at least this has a this we have to we have to investigate, does that make sense? Ok. So, this therefore, has to be a non-linear two port ok. The moment we talk of a non-linear two port we can only talk of incremental gain correct ok.

So, we want the output the voltage across  $R_L$  to be a linear function of  $v_i$ . So, that basically means  $v_i$  must be a.

Student: Small signal.

Must be a small signal, right.

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And therefore. So, if you want to we want the voltage across  $R_L$  to be a linear function of  $v_i$  so, we have to find the incremental gain.

So, how will we find the incremental gain between  $v_i$  and  $v_o$ ? If you want to find incremental gain you replace every non-linear element in this network by its.

Student: Incremental model.

Incremental model and how will that look like? What happens to  $V_1$ ?

Student: Short circuit.

Short circuit so, basically what we end up therefore, on the input side is  $v_i$ ,  $R_s$ , what do we replace the two port with? It is a non-linear two port so, around the operator, what is the operating point of the two port?

Student:  $V_1$   $V_2$ .

It is not really  $V_1$  and  $V_2$ . It is what comment can you make about the drop across  $R_L$  and  $R_S$ .

Student:  $v_i - i$ .

To find the operating point what will you do to the small signals?

To find this small operating point what will you do?

Student: disable.

You disable all the.

Student: Small signals.

Small signals. So, when you do that  $v_i$  will become 0. There will be some current here. There will be some current there. So, what is the operating point of the non-linear two port?

Student:  $V_1 -$

$V_1 - I_1 R_S$  and  $V_2 - I_2 R_S$ . So, there is some operating point, around the operating point what should you do?

You replace this stuff by its.

Student: Incremental model.

Incremental model, ok. So, we will do it tomorrow.