Modern Computer Vision

Prof. A.N. Rajagopalan

Department of Electrical Engineering

IIT Madras

Lecture-37

So, what is called frequency domain filtering? So all that we saw right now was actually in the spatial domain right and of course you know you can get a you can actually you know except for the you see right a median filter which is non-linear. The usual if you had a convolution operation and all right we understand that you know one can do a frequency domain kind of filtering and the way right I mean and in frequency domain you have to take and you know why a frequency domain it is very clear sometimes right it is easier to you know interpret things in the Fourier domain. For example when I asked you how would you arrive at a high pass filter right you went to the all pass filtering right that is what occurred to you first right and then you sort of right mapped it back to get inverse and that will be a delta and therefore a delta - this. So, you see that there is already another way to you know look at things which is easier to interpret because and similarly that day when I asked you about log right if I just gave you the spatial domain thing right would you have been able to tell us to what might have might be it is this one when we said it is a band pass filter definitely we just gave you that expression right it will not be obvious to you that you know this whole intuitively maybe yes but then to get a real insight into why it should be a band pass filter would have been would be possible only by going to kind of a Fourier domain right where and there are various other reasons right why you want to go. So, it could be like an alternative explanation or you know kind of interpretation alternative interpretation which is more which is or an easier interpretation or it could be that it could be that right I mean you have a computation faster computation like for example, you know that a convolution right if you apply you know if you do it in a Fourier domain it is much faster right because you have the fast Fourier transform whereas in spatial domain I mean it is exactly the equivalent but then you can do it much faster. So, faster computations and so on.

So, there are a bunch of reasons why you can go to a Fourier domain and it is completely 1 to 1 right 1 to I mean it is completely sort of right invertible. So, whatever interpretations you have in the Fourier domain you can always come back and write and sort of you know interpret things in the you know in time domain or in the spatial domain. So, the so there are of course, you know frequency domain does not necessarily mean a DFT alone ok there are other kinds of transforms and all there is a whole class of transforms what are called you know unitary transforms DFT is one of them and for a DFT right if I gave you an image f of m n right then DFT is simple. So, what you do is you write it as f of k, l and then you can do like m n going from ok right depending upon the upon the size whatever it is a finite size some 0 to n - 1 or something and then you will have like f of m, n and then e raise to power - j 2 pi by ok now let me write this.

So, the image size is like 0 to let us say n - 1 and then 2 pi by n then k m + n l and where k and l also go from 0 to n - 1 ok. Now, there is a lot of theory behind why this is so and so and so on right which we will not kind of talk about in this course. But the point is in a straightforward way right this is what it is and this is a straightforward extension of what you have what you have for 1 D right and sometimes right you might scale this by 1 by n square right it depends sometimes you have just a 1 by n there there is a reason for that ok. I do not know what MATLAB implements that is MATLAB do 1 by n square honey root what does it do MATLAB uses it as 1 by n square or 1 by n it is like unitary or you get some and this and again at the same way that you interpret a 1 D filter right you have like low frequency then high frequencies and so on right a DFT just in the same way right you know a 2 D right a DFT is also is also somewhat similar except that right you have to kind of remember that you know that when you when you take a kind of a DFT right and then let us say right this is some 1 D signal f of k and I gave you a sequence f of n right then then then when you reach it by 2 that is when that is when you hit the highest frequency right 0 is like the lowest frequency n - 1 is again heading to the lower frequency in between is where is where you have the highest frequency right. And therefore, what happens is in a 1 D it is ok, but then when you plot it on a kind of 2 D right then this interpretation becomes a little difficult because we are typically accustomed to seeing the center at the center of the of the plane and then we we we would like to believe that as you go out right you get higher and higher frequencies right whereas if you directly plot f of k in this form using MATLAB or something you do not get this feel I mean you see that right I mean because of the fact that the right I mean you know n by 2 is where is where is where right I mean you will have the highest frequencies and so on.

And therefore, what is normally done is there is something called frequency centering ok that people do it is a very simple operation just to swap such that such that right you get the low frequencies in the center and then as you go out right you will get the you will get the you will get the you will get this higher and higher frequencies it is actually simple ok. I mean just a matter of it is a it is a matter of right visualization that is all it has it has nothing to do with anything else just a matter of visualizing do you want to visualize it like this or for example, right I mean you know so it is like saying that saying that right domain you know if I if I wanted to if I wanted to especially you know in a kind of a 2 D domain it becomes more important than a 1 D right I mean you know you do not really bother so much by the way let me ask you just you know a quick question which is the which is the fastest what do you call varying signal in 1 D am I right which is the which is the fastest varying signal 1 D a discrete time a discrete time of course think about it right by the time let me just see - 1 power n yeah equivalently equivalently a cos cos for cos pi n right. So, so I mean so otherwise right you can have you can have something that kind of goes and then changes right where is this like you know alternate numbers like 1 - 1 1 - 1 I mean you cannot have something faster than that right ok. So so the other thing right as I said was was is was is interpretation part which we have already seen right so so when we wanted to do that do that log filter right we we saw I mean how to how to kind of you know get that expression. So, I mean if you go to a go to a continuous domain right you saw that you know that gave you something like so when you have log so you saw that right this you have log right then you saw that right this gives out.

So, so the so the so the Fourier right in sort of a continuous domain looks like - omega square + nu square into e raise to power - omega square + nu square by 2 into you know sigma square. So, right this is a Gaussian with sigma and then and then you are applying applying applying a Laplacian on that right this we saw is actually a band pass filter and the the interpretation became easy because we went to the Fourier domain right otherwise this would not not have been that easy. Now when you do when you do a discrete rate you have to be very careful I mean I hope you people have done that in 1D. So, the the omega rate when you write it I mean you have to be careful ok I am not going to talk about that when you implement it you will know that. If you have not done that before you will realize that there is something that you have to take care of when you do a do a discrete equivalent of that.

Ok now right based upon this let me just show you a few more examples as to how you know how this how this what is this yeah I mean how these how these filtered examples that would look like. So, let us just write quickly run run through them. So, for example so for example right so what ok now the the way the way right the way to kind of write it is like this you have you have a you have an image f of f of m comma n and then right you know you take you take a DFT of that a 2D DFT which gives you f of k comma l and then right this you modify with some with some mask that is what you call that that that frequency domain mask right it could be whatever it could be whatever filter you have right. So, we call this as some kind of a mask and then out of this will come and come and f tilde kl because this is simply a multiplication. So, that mask could be a band pass filter it could be a high pass filter it could be a low pass filter whatever it is right.

So, you directly multiply in the Fourier domain and then then you do an IDFT which is again you know 2D IDFT and then out comes and let us say f tilde m comma n right which is like which is like a modified version of f mn right this is some is a filtered version. And if you try to try to see that here right in these slides right you will be able to see. So, here it is right. So, I was trying to show that here. So, here you see right.

So, if you if you if you see if you see this. So, in the Fourier domain data when you have you have a filtering operation wait a minute I think I will show you the images Gaussian this is. So, here right. So, what do you think what do you think should come should come as this image if you see the if you see the filter on the right filter shown in the shown in the frequency domain right. So, something like this is what it is like a is like a low pass filter therefore, what you should be getting is actually a blurred picture something like this is actually a band pass filter oops what happened right this is like a band pass right I mean in the center you are blocking off some low frequencies then outside band outside the band you are again blocking off in between you are allowing right.

So, it is like a band pass therefore, you see that you get something like edges and then a combination right it is like some it is like a band pass filtering operation then here. So, what do you expect to see this is what it is like a high pass filter right and therefore, right you will get some edges. So, again all this if you if you low pass filter it you should see a blurred picture right and if you did a high pass filter then you should get you know something like you see edges if you did a band pass filtering right you end up with something like that then this is another kind of band pass filter. So, you can have right various examples where oh I think I opened multiple number of them. So, you can have you know different different examples to get some insight about what is happening.

Now, this thing right is actually this idea has been also used in one you know is one in one is interesting kind of an application which I will talk about which is called hybrid imaging. I will just show you that is the picture and then right and then you can tell me what you what you what you can make of that. Let me just go next this noise rate I have already told you how this works this averaging and all right this is that salt pepper case these are the slides are all there. So, it is easy for you to just go through them. Let me just come to this this is a median filter again.

Now, there is there is something right something the way right the way we see things I mean something which is close by and something which is farther off. So, if I showed you this image right what would you see right I mean. So, you see a lady I mean are you able to see that I mean right. So, out in the center it is there a lady and this is actually a this is actually an artistic picture by this guy I think what is his name Salvador Delhi I believe he is some Spanish artist or something right. Now, what else do you see you see that you see that I mean you know you see that there is a lady standing there may be looking out somewhere, but then there is there something else which is which you see in that in that in that image right what is it.

You can see Abraham Lincoln right but actually right what you can do is if you go a little farther off right and watch this right. So, if I just go farther off now what do you see don't you see Abraham Lincoln right. Now, now right this is this is something right because of the way we we see things. So, if you show something to me very closely right I will see those fine details and all and maybe I will see something else, but then when I go away right there is an averaging that goes on right as you go farther as you draw farther and farther away right you you kind of your eyes tend to tend to average things and therefore, when you average it you see a kind of a kind of a low pass filtering effect ok. And and hybrid imaging kind of takes use of use of these you know makes use of both of these notions that we have you know when we see things right you know depending upon how close or how far we are and that is why it is called a hybrid imaging.

So, it is sort of you know makes use of things that are that are. So, it makes use of the idea that low pass filtering right you know typically has to do with you know being far away and high pass is something like you know being close and and this sort of a dependence on a distance is what is what hybrid imaging exploits and what it does is this. I mean this is this is a simple idea. So, what it does is this right. So, so you have an image I will draw everything in 1D right, but what you what you really do is.

So, so you so you so you take a take a low pass filter right I mean I am just drawing one side of it one side of the frequency spectrum. So, so you so you pick some you see sigma L right you pick some sigma L right for the for the say LPF operation ok. Again the sigma L it all is under your control and for image to image all this can change ok. Nobody knows what is the best value and all, but typically right you will choose a sigma L that means that means right you give an image ok this is the image on which you want to. So, so you have actually 2 images ok.

I mean like these days people talk about style transfer and all right I mean you can think of think of this as some you know a traditional way of trying to do some kind of a blending right. So, so you have a low pass. So, you send this guy right. So, let us say I1 right you send it through this low pass filter and the sigma L is up to you ok. And this another guy is actually so if you do write 1 - this right that will be in the Fourier domain right that will be that will be some kind of an high pass filter.

But then what you can do is you can pick either the same sigma L here or you can pick something else ok you can pick something like sigma H ok. Some other sigma H right I mean you can actually pick I mean and then what you do is so for example, I mean it does not have to be exactly you know sigma H does not have to be exactly equal to sigma L and all. But they say that it is ok even if you take that it works reasonably well and then what you do is you actually fuse the 2 that is the hybrid part. So, you simply pass this guy through I1 pass I1 through the through the through the HPF pass I2 through the sorry LPF I1 and then HPF I2 and then and then and then you simply add them in the spectral domain and you take the inverse right. So, it is all kind of you know everything is actually linear operation.

So, the Fourier everything everything goes through and therefore, when you do that ok then I will show you an example as to what what effect right this can bring in and this you can do for fun. In fact, in fact I think you know we have I think you know we have some examples for you ok that you know for you to try on and ok. Now, see see this right. So, there is an elephant right and if you low pass filter it right with some whatever sigma L right then you get the blurred elephant and then you have at you have actually a leopard right and that if you do some sigma H right with some high pass filter it you get you get you know get this image right in the middle ok what you have here. And then if you fuse them that is this hybrid imaging then what you see is this now right depending upon from where you watch this right you know it will it will you know it will have a different appearance ok.

If you go farther off right it will look like a look like an elephant, but then with a with a it will look like a spotted elephant ok. Whereas, when you watch it from close right it will look like actually a leopard ok and with some with some smear right in the front and back which you cannot take for a trunk. But if you go farther off right and see this it will actually look like a look like a spotted elephant ok. So, I think this is called hybrid imaging these are all small small tricks right that let us say people play and you know when you kind of get some interesting kind of results. Then the other thing right that I wanted to talk about was actually sampling ok just as in 1D right you can sample right.

The next one that I wanted to talk about was sampling and and for us right sampling I mean right you might ask why you want to sample and so on, but especially right a sub sampling. So, you so you want to go from go down from you see one size to another right. So, you want to get a go from a big size to a smaller size and this you normally do because you know there is a computational reason for it. See for example, right I mean you know people have shown that shown that you know if you wanted to would wanted to do some alignment of 2 images or something right. Then instead of directly working in the highest highest sort of a resolution that you have you should actually go down and then what you can do is you know when you are down and then the sizes are very small there it is there it is easy to easy to compute compute the alignment and you get a good initial estimate to start with and then you can get a see build back ok.

And this turns out to be more robust turns out to be faster also ok and because because you have these initial estimates that are very good by the time you come to the highest resolution. Whereas if you start with the high resolution you start with a very bad initial estimate and then you know and then it takes much longer ok. So, there are there are various reasons why you want to do this kind of a kind of a subs you know subsampling that is called you know a pyramid structure ok. So, what you will have is something like this and then you go down then you go down then you go down I mean you know you can go down right up to right up to a single pixel if you wish right this is called this is called you know this one a pyramid. And in order to get and this we will see ok when we do the scale space representation right at the time we will see what is what is the implication of having a pyramid like this.

But then right if you thought that you know I can just take this image at whatever size let us say n by n cross n and suppose I picked every alternate you know row and column value then I will reduce it to n by 2 by say n by 2 right. So, the next guy will be like n by 2 by n by 2 and then the next guy will be like n by 4 by n by 4 and I can keep going down if I wanted to reduce by a factor of half right all the way or by sorry by a factor of 2 right if I wanted to reduce by a factor of 2. But then what typically happens is if you try doing this right directly just like that then you get then you get actually what is called the aliasing effect. It should happen right in 1D what will happen I mean if I did if I did you know x of let us say right 3T, 2T if I did then what will happen in the what will happen in the Fourier domain. If I did x of 2n right I mean instead of x of n that means I am dropping right every other sample.

If I do that in the in the in the Fourier domain what will happen the spectrum will it expand or will it shrink? It is running faster it would not shrink it will actually expand right and you will have an aliasing effect. Let I mean see so for example so what I mean when it expand right you will get you will get the you will get the higher frequency rate aliasing with the lower rate that is what will happen. So similar thing will happen right even with the images I mean you have done that right in 1D this sub sampling at all you must have done right I mean x of m by n in general right where you where you sort of which is also the reason why you first up sample and then and then you downs up because if you first down sample then then right I mean all of that will get already mixed up. This is no point they are not trying to interpolate and they say up sample right. So you first up sample so that you squeeze and then and then you get a say down sample right.

So so here of course there is no up sampling we are talking about sub sampling which means that you are going down in terms of the size and and therefore right what you what you do is so if you try to do that directly right this is what will happen. So therefore what basically people do is they will always blur it. So which means that which means that you limit the limit the frequency right that this that this image has to some extent. So you may say then I may not kind of losing because I am blurring but it is okay. If you do not want aliasing you actually you actually limit the frequencies that it has so that after that if you sub sample you do not have this aliasing effect.

And therefore most of these scale pyramids that we see right you will have a blurring operation always okay that is that is to simply limit the aliasing effect. So there are some figures right which might actually explain this better to you. So let me just go show that okay. So if you see right okay so right sampling I think you know I can probably okay so for example if this guy if you take right and then then then then this is too big and you start to down sample right this is how it will happen if you simply take you know take every every alternate without actually blurring. And if you try to blow this guy back up to the original size right that is when you will see how bad right it is become okay.

So it is like you know you are like 1 by 8 the size and then if you try to blow it back to the original size you will see what all you have lost and you can see that you lost you know you lost a lot. Of course you may not see the aliasing effect that well here the aliasing effect can come from here see here right do not you see the do not you see that you know that you see picture you know looks very very quite far away from the other one right. And these are these are actually aliasing effects right that you that and then in images it can have a pretty damaging effect on the appearance right you might think that that is how it is actually that is because of the way you sub sampled it. And there is I think one more example also here that that also illustrates this point see something like this you know a checkerboard if you if you sub sample directly right at the end okay you know you see that there is a there is a there is a there is a there is a lot of the aliasing right. Because those are kind of closely packed right and therefore the aliasing effects are higher there.

And therefore right what you what you can do is of course you know you can say that well maybe I know you should have a better sensor and all that you know I should be sampling the scene at a higher rate and all that but instead of that people use what is called what is called you know a pyramid a Gaussian image sort of pyramid where again the Gaussian right that we that we have been talking about all along right is actually used. So, what you do is I mean you know at whatever level you are you first you first blur it with a Gaussian again it we will see later I mean you know what is what is what is and this exact idea is also used to get a get a Laplacian of the Gaussian approximation because you are anyway blurring with a Gaussian. So, if you had 2 different Gaussians blurring an image and if you were to take a take a difference I mean if you blur them the correct way and then take a difference then you can get a log approximation okay and such things are done. But this kind of a you know a pyramid thing right now as far as we are concerned it may be for various reasons could be for computational reasons or it may be a scale space representation you know where you want to see structures in an image in a certain way and which we will talk about it when we do the when we do you know shift and all feature detectors and yeah right. So, basically this is how it looks like and blur and blur and then it is a sub sample.

So, it is like always blur and then sub sample and then and then if you do this right then okay then these are all like 1D examples okay and then right what you see is that what you see is that the aliasing effects are actually much less I mean you can take some examples and show this I am not so sure whether these examples bring that effect out but then right that is that is really the point okay. Then when you expand it aliasing would not happen. No actually no the idea is not like when you expand it would not happen the idea is when you down sample it the aliasing should not happen. Yeah I mean by expanding.

Exactly that is that is the idea. To not have aliasing. Not have aliasing at the at the expense of losing some fine details. So, now right so for example in this case I think so now right yeah this is an example. So, when you blow it back right probably some people may like this better you know again but then you know but for this example it does not seem to matter whether I had the phase the other way where I did not blur and sub sample whereas here I have blurred and sub sample but then like the other when I showed a tower right there it is going to matter a lot because when you blow it back up I mean you will get other patterns which are not there in the original image itself okay. Yeah I think I think I will stop here.