Modern Computer Vision

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Lecture-43

It is called the Harris Coroner detector, it goes after the people who worked on this problem, HCD if you will, Harris Coroner detector, Harris is somebody's name. This was a paper kind of published actually interesting, it was a conference paper. So to just motivate right what we mean by this right, so of course you know they kind of formulate it in a certain way using gradients and all but just to motivate it why such a thing makes sense. Let us just look at an image and let me draw 3 of them okay. In one case I have what is called uniform plus some noise okay. I am adding noise because you know it does not I do not want it to be wanted to think that you know all images are all intensities are exactly the same maybe there is some amount of noise there.

The second one is actually an edge and the third one is a corner okay. So uniform means what I mean you can think of this as some I plus some N whatever right I mean you know what is 0 mean with some sigma right. This is something like that some Gaussian noise something that is added uniform does not mean uniform noise okay, uniform intensity plus some noise. Edge let us say we have got 0 here I mean we will just take an ideal case okay just to give some insights.

Corner I think I will draw them by a different color, this is a corner right. Now suppose I take a patch there is some patch which I decide some size okay. I take this patch and move around okay in a sense right I kind of move around okay. Suppose I am here at this location let us call that some xi yi I am sitting there and I just move around okay. In terms of the gradients what do you think will happen see for example the appearance of the patch is going to change right as I move around it is not exactly the same right.

So the appearance of the patch rate is going to okay now let us just talk about the appearance okay right. Now let us not even talk about gradients let us just talk about appearance. In the first case how much will be the change in the appearance? Do you expect a lot of change in the appearance or do you feel that maybe there will be just a little bit of change in the appearance? Very little right so we all agree that the appearance will be the change in the appearance is going to be minimal okay because most of it is uniform so except for some noise. What about the next case edge? So let us say I take a patch now I move around yes Abhijayan. When you are okay now but then would this be somewhat going to say direction sensitive the okay so which way would the change be minimal and which way would the change be horizontally right so when you move orthogonal to the edge you will find a lot of variation right horizontally if I pull this patch out like this you will see a lot of variation but if I move along the edge there is going to be hardly any variation if I move along there that means there is some sensitivity here right with respect to which way I move okay which means that the appearance change depends upon how I move and if I move in a particular way there is going to be a lot of change but then if I move in another way which is like along the edge then it looks like there is not going to be much of a change in the appearance.

What about the third guy? I have a patch. So I have the corner there sitting inside that path now I move. Sagnik what will happen now? Any direction if you move there is going to be a change in the appearance right whichever way you move because you are actually sitting at a corner okay so in a sense right you kind of that is what this Harris corner actually tries to capture so at a corner right whichever way you move the appearance I mean appearance of that patch is going to change significantly whichever way you move okay and that is what is you know essence of this and we will try to capture it using whatever math is needed for that right. Of course all this is there in that slide okay that anyway we will upload but you know this too okay so then the idea is this. So let us say that let you know let delta X and delta Y represent shift in X and Y directions shifts in shift in Y X and Y directions respectively directions.

Then the change in appearance of a patch of size W can be found as C of let us call this some C of delta X delta Y which is the change C represent the change in the appearance. So this let us say we compute a summation I of X I Y I minus I of X I plus delta X Y I plus delta Y and X I Y I belonging to this W which is the size of that patch. So what this means is that means is that that right within that window okay if I move by an amount C right delta X delta Y and I want to compare the patch right which I get with respect to that particular motion delta X delta Y and how much is this change in the appearance. So you can of course when we play the standard trick which is to actually take this and expand it in using Taylor's series so you have X I plus delta X make some approximations now because you want something that is computable. So you write this as I of X I Y I plus let us say and all these are scalars now right you just into looking at intensities so delta X again is a scalar just a displacement delta Y is a displacement.

So I am writing a scalar form plus delta X let us say we call this as what is what is in notation I X okay and X I Y I plus delta Y I Y X I Y I and then you will have some higher order terms which we will ignore what happened something. What happened last line I of X I plus delta X yeah correct yeah thanks Y I plus delta Y is equal to this one right X I Y I plus delta X I X plus delta Y okay and plus some terms right higher order terms which we will ignore okay then what we can do is we can actually push that in here and then we will get summation X I Y I then I of X I Y I minus I of X I Y I minus delta X I X of X I Y I minus delta Y I Y of X I Y I. So if we ignore those terms then this is all that we are left with of course these are all this will this approximation will have some effect but that is okay. So I think so what happens these 2 cancel off and you can in a way sense say that right what you are left with is simply delta X I X X I Y I plus delta Y I Y X I Y I the whole square summed over all X I Y I okay. Now if you expand this okay you can expand this one that will give you C of delta X delta Y so you will get summation these are all scalars okay so you get like delta square X I X square X I Y and all this I X and all this gradient right we should have some sort of a mask right to actually do that which we have already seen you know you can I will also talk about that but we are assuming that there is some way to compute the gradient along X and gradient along Y and so on.

Plus let us say delta square Y I Y square X I Y I plus 2 what is this delta X delta Y I X I X I Y I into I Y X I Y and it will get. Now can we can we write this in a more compact form and that this looks like looks a little messy can we write in terms of a matrix vector form or something this is after all you know a quadratic right so this is equal to so we can write this as delta X delta Y and then a matrix what will be the size of this guy again delta X delta Y. So what will be the entries of this I X square X I Y I then I X I Y I will just I will drop that X I Y I it is there I Y I X which is the same as I X I because these are all scalars okay so this is all like gradient at a point okay so that is why it is okay to interchange the order and then you have like I Y square okay this is the same as what you had what what you had here right this is easy to check verify. So now this delta X delta Y right has it is independent of the I right because it is simply a displacement therefore we can actually pull that out and then what we will have is delta X delta Y yeah exactly so summation over so now this I will put this as put this inside sorry did you okay X I Y I then double summation I X X I Y I I Y X I Y I then double sum then again I X X I Y I I Y I Y I and double sum I Y square X I Y I and then delta X delta Y right and this matrix right let us call this as some R okay this is called this as some matrix R okay. Now okay now what we can do is you know so this I X that we can have let us say horizontal gradient we can have a horizontal gradient operator for this and I Y right so horizontal gradient kernel or whatever or mask right something you will need so which is may be of the kind right a simplest that you can think about is maybe minus 1 0 1 all 0s elsewhere this is the simplest that you can think of or you can have right because they actually introduce a Gaussian average so therefore that is why they do not have less minus 1 minus 1 minus 1 that kind they do actually a Gaussian average.

Similarly I Y right you can also have a different vertical gradient and right that is all you need I mean because we just have I X I Y I X square which is simply squaring that number and I Y square which is squaring the Y gradient right. So what this so what okay now I so

I think there is just this other point to mitigate the effect of noise to mitigate the effect of noise the patch is first smooth with a Gaussian or the patch is first that means the patch of size W cross W that you have right is first smoothed with a Gaussian with a Gaussian. So what this means is that is that when you compute your I X right if you had a continuous case then you are doing something like dou by dou X of G of X, Y, some sigma S I call that as some smoothing okay capital S smoothing applied on I at X I Y I. So it is like saying that you know you have a Gaussian sitting on it and then you are doing some kind of smoothing I mean instead of weighting everything uniformly you are having a non-uniform kind of a weightage coming from the smoothness through a Gaussian then in addition in addition similarly for I Y okay so for I Y also you have something similar then also to give more weightage to the point in the middle of the path then there is one more Gaussian so there will be some hyper parameters okay that come in because of all this also to give to give more weightage weightage to the point in the middle of the patch middle of the patch. So what they do is you know so they use another Gaussian use another Gaussian which does the following anyway this I think you know this is not so relevant but I am just writing it because it is there so it is like minus X minus X I square plus Y minus Y I so minus of Y minus Y I square by some divided by some 2 let us say sigma square W or whatever this is not the same as the other one this is a different sigma and now so ideally so it is like saying that when you compute R right so you have something like if you call this as some say W of X I Y I right if this is the weight that you have then it is like equivalent to saying that you are computing your R as some W of X I Y I into I X square at X I Y I I mean okay now these are just some smoothing operations okay this is nothing to get overly worried about this just some smoothing right that they are doing just to make sure that it is a Gaussian because then you can give importance to something in the middle and you know okay what is what is more okay now typically there are some hyper parameters okay sigma W is usually 1 and sigma S is 2 I mean these are all hyper parameters okay of this method but what is more interesting is this is the following.

So if you if you look at look at R itself right so the R that you had okay now if you if you actually actually looked at the eigenvectors of R see first of all right R is actually a symmetric matrix right as you saw it is like I X square what is it I X I Y I X I Y and then I Y square right so it is actually a symmetric matrix and you know that every symmetric matrix is actually normal right which basically means that A A transpose is A transpose A right whenever that happens any matrix is what is called unitarily diagonalizable as they say right if it is a normal matrix okay I mean normal does not mean the normal abnormal not that sense normal matrix have you guys encountered a normal matrix? A normal means something like this a matrix is normal if A A transpose equal to A transpose of course for a symmetric matrix it automatically is true but there are matrices right that are not symmetric but actually obey this for example if it is complex right let me just drop A Hermitian is equal to A Hermitian A have you seen a matrix like that DFT sorry no orthogonal will mean A A transpose should be identity that condition is not there it does not have to be identity it is just that A A transpose should be equal to A transpose A I am saying DFT will obey this a DFT is not as symmetric do you know that the DFT matrix is not symmetric right but then it is normal it will satisfy this only thing you should replace transpose with the Hermitian because it has complex entries right so this is like real this is not a real case but it is complex and all that you should be more careful say A Hermitian A is equal to identity in fact for a this one for a DFT it is much more powerful okay so no you call it unitary orthogonal means transpose when Hermitian is involved it becomes unitary okay there is a slight difference between the two okay now the point is this right so when you have so which means that this is a normal matrix and there is actually a theorem right that says that every normal matrix is unitarily diagonalizable or in this case you know unitary because everything we will just take all real numbers so let us not worry about complex entries you know what this means is that you can diagonalize it as P diagonal P transpose okay where this P P transpose is identity okay so all these are actually 2 cross 2 so this guy right so this is actually a diagonal matrix right so this is like lambda 1 0 0 and then you see lambda 2 okay and where are the eigenvectors of R this is called this is an eigenvalue eigenvector decomposition right why do you call it eigenvalue eigenvector decomposition? So which what of the P is it the rows of P or is the columns of P columns of P right so you can show that if I take let us say the ith column of P and if you do R times let us say that P i you can show that that will be equal to lambda i P i right that is why it is an eigenvector so that is why you call it an eigenvector eigenvalue eigenvector decomposition right now the eigenvectors are of R are sitting as eigenvectors of R are the columns of P are the columns of P right now so an eigenvector right what is it what is it really represent an eigenvector you have a covariance matrix right this is R this is like I mean all of all of whatever you expect of a covariance matrix right this guy will actually satisfy now when you have when you have a covariance matrix and you have done this eigenvalue eigen decomposition what do the eigenvectors really tell you they tell what the see variations are right in which directions the variations are from highest to lowest right so for example so for example right I mean you know so if you find that if you find that if you find that right along let us say along say one direction you have the highest variance right how will you capture that that is captured through the eigenvalue right so the eigenvalue should be high for that that is when you say that that is the most dominant sort of a component along which the variations are the highest so it means that so it means that when if you had points right which had a distribution like that then you will say that the spread is maximum along this direction then the probably the next spread is if it is a 2D right then you will say that in the orthogonal direction there is still a little amount of spread right this is what the eigenvectors will capture this is what this is exactly what these eigenvectors of R will also capture and for us right it is all in terms of interpreting this now in terms of what the what it should reflect about my corner now right so what you can show is that the eigenvalues that you get out of this and then the eigenvector it will have an interpretation with respect to the edges that you have and when 2 edges meet that is when you get a corner right therefore for a corner the eigenvalues will have a behavior of a certain kind so you can actually make out what point you are looking at I mean you know it is not just that it will tell you a corner it can tell you whether it is an edge where you are sitting whether it is a flat region where you are sitting or where you are sitting is that a corner right that is the kind of information that this matrix can actually give you which we will see in the next class there are lot of time. Sometimes the corner should be necessarily orthogonal or it can get any other. No no so the thing is right it need not be orthogonal you are saying that what if I had an edge yeah that is also fine that is also fine because there also when you move around right I mean you will see there what will happen is you see in an orthogonal case right you will see when you have something like this which is actually also a corner then what will happen is you know you will get one direction along which the maximum spread is there the other one the interpretation will be hard this interpretation will not be so easy because the spread right the eigenvector will still reflect that there is an orthogonal direction in which the next spread is the highest but if you have like this right then it is much more clean I mean then you know that right along one direction in the orthogonal you can associate with that but corner does not there that is I forgot to tell it does not necessarily mean that they have to meet orthogonally and all it can also be an edge like this corner like that.