# Modern Computer Vision

**Prof. A.N. Rajagopalan**

**Department of Electrical Engineering**

**IIT Madras**

**Lecture-47**

So, let us now go to a normalized, what is this normalized Laplacian of the Gaussian. So, that is normalized log. Now I will just take, so what we will do is we will actually analyze it by taking a step signal. Let us say that this is the, I do not know what I did. So let us say that this is how it is and this is my intensity, I of X versus X in 1D and here it is 0 and whatever and then here it becomes 1 and this is some X0, where there is actually a transition from 0 to 1. So, now we are kind of, we will first see how it actually reacts to an edge.

So our I of X, right in terms of a unit step will now be what, U of X – X0, right that you all know, X – X0, right is what your I of X is. Now let us convolve it, convolve it with a log filter. What does that mean? That means that let me take I of X, let me convolve with, let me call if I call my, this one a Gaussian, let us say I call this as G σ of X, which is 1 by √ 2 π σ e raise to power – X square by 2 σ square and this is my G σ. Then a log will be like d square G σ of X by d X square, right this is what I want, a double derivative of the Gaussian, right where G σ is that.

So, this is nothing but U of X – X0 convolved with d square G *σ* of X by d X square. Then you can play the trick that this is the same as d by d X of U of X – X0 convolved with d σ X by d X, this is okay, right. Now what is this? It is beta of X – X0 convolved with d G σ X by d X and we know that actually convolve something with actually a delta, you get back the same signal in this case it shifted, right. So, what should we get as output d G σ X – X0 d X, right that is what we will get. Now what is, so what is our, so okay let us first find out what is our d G σ X by d X, okay.

So you have your G X of σ, no G σ of X, G σ of X as 1 by √ 2 π σ e raised to power – X square by 2 σ square. Let us do d σ, we need with respect to d X. There is also another sort of a, you know, derivation that is more interesting that you will see later when you actually differentiate with respect to σ, okay. But in this case this is all with respect to X. So you get 1 by √ 2 π σ then with respect to X, right.

So you have like e raised to power - X square by 2 σ square and then what will you get - 2 X into 1 by 2 σ square. So then you have this 2 and 2 will cancel and you will get - what is it 1 by √ 2 π σ cube into X into e raised to power - X square by 2 σ square. So if you go, so therefore, right what we need is d G σ of X - X o, right is what we got in the earlier one, right. This is what we got, right. This is the, this is a response, right.

This is the response of log to a step edge and that is the thing on the left. So then this becomes - 1 by √ 2 π σ cube. The limit of Gaussian σ tending to 0 becomes what? What is it become? Impulse yeah, that you all know, right. e raised to power - X - X o the whole square by 2 σ square, right. That is what we are d σ will look like - 1 by √ 2 π σ cube X - X o yeah e raised to power - X square.

So let us call this as equation 1. So one of the things, right that we should be able to see is that, so as an aside, right, as an aside we can actually ascertain one thing. What is that? This is, this is what is the, is this a response, right to the edge as a function of X. What is the first thing that actually emerges out of this equation? That yeah, right that is has a 0 response that X = X o, which is what we expect, right. That is where the 0 crossing is actually, right.

So right aside clearly the I mean response is 0 at X = X o, which is where the, which is where the edges. So we know that log will give a 0 crossing where the edges, right. So where the edges, this is ok. This is something that we knew all along, but you know earlier we never showed it formally. We knew that you know plotted that is what happens and yeah this is, this also emerges.

Tell me something, right. So what will, what will be the area under the log function? This Mexican hat, right that you have, no. We saw that, right where is that? No, no where is that Mexican hat? I thought I showed it somewhere. I think I, here see this guy.

Yeah, right. So what do you think, what do you think is the area under this, under this signal? Is it 0 or it has to be 0 or what is it? Why are you doubtful of it? I think, I think we said something about this one Mexican hat, right. What kind of a filter did we say it is? A bandpass filter. Can a bandpass filter have a non-zero, non-zero strength at $\Omega = 0$? Can't, no. It should be 0, right. Then only it is a bandpass filter.

If it allows 0, how can it be a bandpass filter? Then what was, what is the frequency response of this guy? - $\Omega$ square, - of $\Omega$ square + $\nu$ square into e raised to power. That is what we saw, no, when we did the frequency response. Put $\Omega = 0$, $\nu = 0$, it has to be 0, right. So in fact it is actually a wavelet, right. I do not know how many of you have done wavelets and also, right.

So it is actually, it is, you know, you can change the scale and all and then, you know, then it becomes actually a wavelet. So it has, it has various, you know. So this log is actually very kind of interesting, interesting kind of a beast. Okay, so clearly the response is 0 and then, okay, now, right, this is, okay, now what we, what we want to know is the peak.

Okay. The peak can be found as follows by equating, by equating d by dx of u of x - x o convolved with d square g σ x by dx square = 0. So we want to find out where exactly, right, does the maximum occur, okay, which then means that, I mean, if you had a, if you had a function whose maxima or minima you wanted to find out, you will take a derivative, equate it to 0 to find out what exit happens, right. Now we have, so, so we know that, right, all of this is equivalent to this, right and therefore, if you do kind of d by dx of equation 1, which is actually d by dx of equation 1, of equation 1. So if we do that, right, then what will happen? So we can pull out - of 1 by √ 2 π σ cube and then maybe I should do x - x o and then we are doing d by dx, right. So e raise to - x - x o the whole square by 2 σ square into - 2 x - x o by 2 σ square + e raise to power - x - x o the whole square by 2 σ square.

So we can actually pull out this - 1 by √ 2 π is common anyway σ cube and then this 2 and 2 will cancel off and then we can pull out e raise to power - x - x o the whole square by 2 σ square and then we can say 1 for this -, there is a - sign here, right, x - x o by σ square, right, not I mean 2 just by σ square, yeah, this is what it will be and this we want to equate to 0, right, in order to get the x at which there is a peak or there are probably multiple peaks we do not know, but because this is square, right, so expect multiple peaks. So, okay, so then this means that 1 - x - x o whole square by, yeah, okay, yeah, so I think this is okay. So 1 -, so that means that x - x o the whole square that is = σ square, right, from there. So if you just send it to the other side or we can say x - x o is = + or - σ or we can say x = x o + - σ because so the peaks occur at, you know, at σ to the left and σ to the right, I mean which is what I think in our figures and all right we keep showing, right, when we see that it is a response, right. It is like σ, I mean if you have a stage, I mean are you able to see this, Karasan, so it is like here and here and there is one on the left and then there is one on the right and it is like σ part, okay.

Now, okay, now, right, this is where, this is where the peaks occur, where the peaks occur. But then what exactly is the peak value, right, that if you want to find out then we should go back to that d σ, okay. So what is the peak value then or what are the peak values, right. If you want to know what is the peak value then we should go back to this, where is that, so equation 1, right. So which was, what is that, so which is that 1, so u x - x0 convolved with d square g σ x by dx which turned out to be - 1 by √ 2 π, what did we have, σ cube.

Okay, now in this equation, right, so in this equation, right, so yeah, so we need to find out, we need to go back to this equation 1 and then there, right, we have to substitute for x, correct, x at those extreme. So what do you get then, x is x0 + - σ, right, and then you have a - x0 by the way, so it is x - x0, right. So that is, so this is your x, e raise to power again you have x in its place, let us find out x0 + - σ - x0 the whole square by 2 σ square, okay. And therefore, the peaks, right, if you see, so I get, so if I put x = x0 + σ, then I get the peak value to be, then the peak value is - 1 by √ 2 π σ cube, so x0 will cancel off and then I will get σ, what happens here, e raise to power -, so you get 1 by 2, right, σ square will cancel off, so you will get like - 1 by 2, okay. And if x is x0 - σ, then the peak value is - 1 by √ 2 π σ cube and then you get - σ here, this of course remains - 1 by 2, which that means that, so the peak occurs at, so the peaks occur at, no not occur at, I mean so the peak values are rather, so the peak values are not where they occur of course, where we know, the peak values are, okay, now this σ will, 1 σ will cancel out, right, so we will have like - 1 by √ 2 π σ square, so we can write kind of, so can we write + –  e raise to power – half, right.

Now if you see, right, what do you need, I mean, see now you can clearly see why the strength was falling off, right, when that curve that I showed, no, as you kept increasing your σ, this σ is actually playing kind of a spoilsport now, right, so it is suppressing the strength and therefore, if you wanted something which should ideally be invariant to σ, what you do now do, you would simply multiply this response by σ square, right, if you do that, then this becomes at the peak just stays at – 1 by √, + – 1 by √ 2 π e raise to power – half, which is just a constant, and that will not interfere with our analysis then, okay, that is what is actually a normalized Laplacian. So always they will write σ square, okay, this one delta square f, but we should remember why that σ square came, right, the σ square has come precisely for this reason, okay, so in order to, okay, so which means that, so the strength falls with increasing σ, with increasing σ, so to make the strength invariant, invariant to σ or you do not want it to σ, multiply log by σ square, and this all for a step edge by the way, okay, I mean, right, this is a response at all that we use a normalized log, right, that we can tyπcally use this all coming from analyzing a step edge. So multiply log by actually σ square to get normalized log, okay. Now, if you see, right, if you see what will happen is that when you have, I do not know whether I have a slide for that. So now when you have a blob, you see, so now if I, no, no, not this, see now, see for example, I know say unnormalized guy would have, would have, would have kind of, would have kind of say died off like that, whereas when you have a normalized scale normalized, right, or when I say normalized, I mean scale normalized, so then you see that these peaks, right, they do not, they do not fall off, right.

Then what happens is you keep on increasing, right, there comes a point when it actually hits, hits and hits an extremum and then, and then again, right, again. So, so, so the point is, right, this has to do with the way, with the, with the way, right, with the way, with the way, okay, what do you call, with the way this is a response behaves and, and you get, you get a point, right, where the, where the maximum occurs and then after that, right, again it kind of, and again it kind of begins to fall off. What do you think might be the reason why let us say, of course, it is clear from the math that, that, that there was, that there was a point, right, where of course, you know, where of course, it had a peak, which means that right elsewhere, of course, it did not have a peak, but is there some intuition into why this might be going on? No, I am saying, no, okay, one thing is that we just accept the fact that well, we had an extremum and therefore, therefore, right, at that point, right, we get a peak and then elsewhere we do not have a peak. But see, that is the kind of simplest way to simply, simply say, you know, go with what we had, right, you know, as, as the, as you kind of write as a math, but, but then, right, I am just asking if, is that, is that some other way to kind of interpret why the strength is decreasing beyond a point or for example, why let us say, right, if you had a blob, then then right at some, at some scale, right, it actually, it actually svgly fits in and then at other scales, right, it sort of begins to, begins to drift away from the maximum. This should have something to do with the area, right.

See, I mean, a response is after, after all what, I mean, it is simply a sort of a dot product, right, it is like an inner product, right, right, between, between two functions. So, if you think about it, right, if I, if I were to, if I were to think about it, right, so if I have, okay, no, no, this is, let us go back to, so, so for example, okay, so let us call this scale normalized, okay, that is the actual name, let us also use the same name scale normalized log. So what I am saying is, right, see, if you had, if you had a step like this, right, let us say it is 1 here, it is 0 here, then again goes back to 1, okay. Now, I have, let me draw, right, 3 kinds of Mexican hats, okay, one, I mean, I am not drawing it exactly, but you know, one like this, okay, another, okay, or for example, okay, or for example, a simpler thing to sort of do it is that, say is that I have, I have a Mexican hat at a certain scale, then I am examining this, then let us say I am examining another like this and I am examining another like this. So in each case, the blob width is changing, right, so, so, right, it is like saying that, say, I have a blob of kind of say radius r and this r, right, in some cases, it is very big, in some cases, it is very small.

Now, when you are, okay, now, right, this is your log operator that is, so, each one of these is your edges, why do you think that, think that, I mean, so, suppose I asked you, right, what will be the, for a blob, right, if I asked you, where should it fire, right, if I asked you at what, so, what should be that σ, right, at which it should fire, then first of all, right, we know, right, priority when arriving at the equation, which is very simple to actually derive, how would you, how would you, how would you analyze it, think about it, why, why is that, so, yeah, you are right, so, why is that so, yeah, you are almost there, but I just, σ, no, see, see, what he is saying is this 0 crossing should match this, that is what he is saying, right, what happened, with 0 crossing should match what, this 0 crossing, no, no, I have only one log at some scale, we are trying to see with respect to, with respect to one of them, right, we are saying that, we are saying that we will get, we will get a kind of maximum response, correct. So, a response is like what, when you, when you, when you take, when you take a sort of, you know, a dot product, right, between the two. So, the point is this, right, when you, when you, when this guy comes and multiplies this, what will happen, right, because of the, because of the fact that, because of the fact that from here to here it is all 0, if you multiply that with this negative, this, this negative has no role and, and all that, all that you will end up is, end up is adding up, adding up these areas, right, under whatever is above 0, the positive one. If you had something, something which is very big, right, then basically what will happen is then all of, so for example, so, so what will happen is, right, so, so this portion, so if you try to see, right, there is going to be, going to be a negative portion, right, here, see from, from here to here if you see, right, there is going to be a negative portion of this guy which will actually enter. Can you see that? I mean similarly from here, right, you will have actually, so, so because of the fact that, one mivte, no, no, I think, right, this is the other way around, right, so you have 1 and then, and then you have 0, okay, no, I mean, right, negative will get knocked off because you have a 0, but then the positive area will shrink, right, because of the fact that you have 0 for that big, right, that big a width.

Do you guys see this? This, the negative won't come because you multiply it with 0, no, that won't come, but what will happen is the positive area that you are considering will shrink because, because your 0s are extending beyond the lobe. On, on the, in contrast if I make it very small, right, then what will happen? You will, you will actually get, get this kind of say negative portion into the, into the area calculation. So, the overall response will change. Yeah, which, which negative area, not the one here because that is going to 0, but this one which is on the fringes, right, that will come in, no, because you have, you have a, you have a smaller width, right, therefore those guys will come in, in between the, the middle one won't come because multiplying that with 0. So, this is the best case is when, is when just, just you get the positive areas that are all adding up, right, and, okay, that's, that's one way to actually, right, one way to kind of, you know, think about why it should svgly fit in and when you, when you say should svgly fit, fit in for a log it means that the blob width should actually, should, should, the 0 crossing should match that, right, when that happens you will get, you will get the maximum response, right, and because our whole thing is about blob detection, right, so, so, so then, right, one can even argue that, okay, that I am going to leave it as an exercise, right.