Modern Computer Vision

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Lecture-70

Okay, which we have not done, okay. So this is something that we will do next. So estimating, so right a depth by triangulation. Okay, and then this is not the case, not the same as the stereo case right where we had you know baseline and all given and all right. This is like R and T and right everything is sitting there and we want to still be able to triangulate because it makes sense right. I should be able to go back I mean why how does it matter at what inclination both cameras are mutually right.

But then there should be a ray right which should where they should intersect if I do a back projection right. And therefore, okay what does it mainly use right, it uses the fact so right so this is actually a smart idea. So uses the fact that and this is for a sparse set of correspondences right. This we are not doing as I say dense correspondence and also note that we are not doing this because we want to estimate the depth map okay.

Please remember we are not doing this to estimate the depth map at the same way, we are doing this to estimate a camera pose. Once we know the camera pose then we will actually pin it down pin down the dense reconstruction using something called plane sweep stereo and so on which I will talk about later. But this is not to compute the dense depth map and all this is even to know which camera what is the camera configuration, which R T to pick okay. So you should remember that. This is not depth by triangulation does not mean you are trying to get a whole scene depth map and all you may not have that many correspondences.

You have only a sparse set of correspondences that gives you a sense for which camera pose to pick okay. Now it uses the fact that the cross product of 2 vectors in the same direction is 0 right, 2 vectors in the same direction is 0 is 0. So what does it mean? That means that right if I take actually the you know image coordinate which is like x y 1 image coordinate which is like x y 1 that vector okay if I do a cross product with let us say I mean right I am going to write this as okay now no yeah so right this is for the first camera by the way okay for the first camera. So if I do cross product with p x ~ right p x ~ is basically any other point on this on that ray right this is on the same ray and therefore because they are in the same direction this cross product should be 0. So p x ~ this should be 0.

What is the size of p? 3 cross 4. Size of x ~ 4 cross 1. This guy 3 cross 1 and then therefore right if you take this cross product it is another vector which is the 0 vector or we can write this as whatever right x ~ expressed in terms of the cross in terms of matrix form p x ~ right is equal to 0. Now for a corresponding point this is with respect to the let us say one of the cameras right left camera let us say for the right camera well if you want you can write this as p left or something p l or something for the right camera for and for the corresponding point okay this is not for any old point okay because we know the correspondence we know what is the x ~ ‘ to which x ~ is mapping for the corresponding point what can you say? You can say that x ~ ‘ cross p ‘ let us say whatever right or p r x ~ should be equal to 0 right or for that matter x ~ ‘ expressed in matrix form and then p ′ x ~ right is equal to 0. Now if you see right it might actually look like you have got 4 independent equations you know what I wanted to say was you might actually think that there are 6 linearly independent equations actually right because you have got kind of 3 equations coming from the first correspondence and 3 from the second okay but in reality there are only 4 linearly independent equations and I will show you why okay so let us kind of go back to this condition where we had x ~ expressed as a matrix into p what is this p x ~ it is equal to 0 okay now if you write this p as let us say you know let us say p 1 p 2 p 3 where each is a each is a 1 cross 4 row and because p is 3 cross 4 right so let us say p 1 is the first row p 2 is the second row p 3 is the p 3 is the p 3 is the third row then and of course this guy is like x y 1 right x y 1 then if you take actually a cross product right so the way I kind of try to remember is like l 1 l 2 l 3 right if you have then what is the first term here right so it will be like l no it is l 2 l 3 l 1 I think that is the order right I mean you have to have some way to get us to remember this so it is like l 2 l 3 - you see m 3 m 2 that is the first term right so if you guys can tell me then we can start writing here okay so you have like y okay so l 3 is what see p x ~ so the first row for the first entry of the p x ~ is what p 1 transpose x ~ this guy this is actually vector no this is like a 3 cross 1 vector right I am writing p as p 1 transpose p 2 transpose p 3 transpose I am multiplying it with x ~ so the first entry is what p 1 ~ p 1 transpose x ~ is that fine p 1 transpose okay oh yeah okay no well I think p 1 p 1 is already the row fine if you take if you take write p 1 to be the row if you say that p 1 is a row of p 1 then it is simply then it is simply then it is simply p 1 okay so p 1 I think you know the reason why they put a transpose because just to make it explicit that this is a row column multiplication otherwise you can get a little confused right you may think that how come two vectors are getting multiplied to get a scalar I think that is the reason that is the reason why typically a transpose is used here okay so anyway okay so so right so just to make it clear I will write this as p 1 transpose row is p 1 transpose because normally it is better to keep the transpose in your head okay then you will know that that is a number otherwise you know p 1 into something you will be confusing so so each row is like p 1 transpose p 2 transpose p 3 transpose okay so you have like y p 1 transpose x ~ okay or I would not put a bracket and all y into p 1 transpose x ~ then - what is determined so I did l so - m 2 so - m 2 to m 3 right so m 3 is 1 - m 2 wait a minute so what is that I mean can somebody tell me quickly okay anyway I can write this down here I have it so you should be able to this is nothing right this is no great there is no great theory here so okay the next entry is p 1 let us not waste time I will just write it down x ~ - x you guys can just you see verify that this is correct x ~ this is just a cross product right so x p 2 transpose x ~ - y p 1 transpose x ~ okay this is all there is to it okay and this you know is equal to some 0 0 0 right.

Now the point is okay this now you can show that you can show that right I mean even though it looks like there are 3 equations here given just one correspondence right by the way given this there is one point not even correspondence one point back projected you have got to see 3 equations but you can show that right you can show that you know you can multiply the first row right if you multiply the multiply the first row by let us say - x and you multiply this by you see - y then you can then you can show see that addition of the 2 is actually the last row it is easy to see that right. See for example - x if you do right then you get like x p 2 transpose x ~ which is sitting here and then - x y you will get here but if you multiply - y here then there is - x y and this + x y will knock each other off and this - y will go here and sit as - p - y p 1 transpose x ~ which is the second entry in the third row right. So the third row is simply a linear combination of the first 2 okay therefore you actually have only 2 linearly independent equations but you will also have therefore right effectively only 2 linearly independent equations right - x times first row + - y times second row gives you gives you the third row. And similarly right what will you have you will also have similar set of equations for let us say y whatever right so you will have x ~ ‘ you know for the other point so you will have like y ‘ p 1 ‘ transpose x ~ but x ~ would not change okay - p 2 ‘ transpose x ~ you will have p 1 ‘ transpose x ~ - x ‘ p 3 ‘ transpose x ~ x ‘ p 2 ‘ transpose x ~ - y ‘ p 1 ‘ transpose x ~ wait a minute yeah you will have actually a p right right so you will have a p 1 ‘ p 2 ‘ p 3 ‘ and again the same thing already here also you can get the third one out of the out of the first 2 therefore right effectively given a point correspondence right you have got like 4 equations and their x ~ if you put that as an unknown it has only 4 entries in it right. So what you can do is so you can solve for x ~ as follows right you can do this as y right I am going to rewrite this okay I am going to write this as y p 3 transpose - p 2 transpose p 1 transpose all this is coming from the okay from the way right from the way okay we have actually multiplied things so you are just stacking up now okay this is just by the way the first one is y p 3 transpose okay this is not p 1 okay there is a mistake there y p 3 transpose like I know the p 1 transpose x ~ - x by x p 2 this is also p 3 okay that is why you are getting a y p 3 transpose here - p 2 transpose we got p 1 transpose - what will you get x p 3 transpose it is all coming from here you know from those equations directly then what will you have here y ‘ p 3 ‘ transpose - p 2 ‘ transpose and the last one will be x ‘ there is no vector here okay this is just a number x ‘ p 2 ‘ transpose - y ‘ p 1 ‘ transpose these are all vectors by the way p 1 p 2 these are all vectors.

So I wrote the wrong one it should be this one it should be this guy p 1 ‘ transpose yeah thank you p 1 ‘ transpose - x ‘ p 3 ‘ transpose yeah good for being attentive right so - x ‘ p 3 ‘ transpose right so you got my 4 equations now multiply this with what x ~ right and this should be equal to 0 okay. Now again how will you solve for this right SVD standard right SVD and then look for the one that has the look for that eigenvector which has the smallest eigenvalue right so do you see SVD and once you do this SVD then you have your x ~ now look at the z component and then see right if it is positive or not and this you can do for every camera configuration you got 4 configurations right so this p will change accordingly right as you change your r and t your p will change and therefore this p 1 p 2 p 3 are the only guys that are going to change x ~ and x ~ ‘ would not change that is the correspondence given already the only thing that can change is p and therefore you try for all 4 configurations of p and find out which one actually projects the points both the points in front of projects a point in front of both the cameras. Is this clear? Just a matter of doing triangulation sparse only to get the camera poses okay and the z component of this guy right examine the z component of x ~ for its sign to choose the correct camera configuration right okay we will stop here I think I have exceeded I did not realize that okay so fine so the other question right that you can ask is well you know here I had an essential matrix which means that the intrinsic I knew right and therefore I could come to the essential matrix and I could do this. Now more complicated cases if you did not know the intrinsic at all and all that you have is a fundamental matrix right then can you kind of do something so which is another question that is actually pertinent question to ask right we will see that in the next class which is actually tomorrow and then we will also see what other ambiguities that can arise right the more freedom you give right the more you kind of let this thing go loose right then the more unstructured right can be the depth map.