Introduction to Smart Grid
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Lecture – 13
Phasor Estimation- I

Good morning to all of you, in this class we will discuss about the Phasor Estimation techniques. One of the phasor estimation techniques is DFT that is the Discrete Fourier Transform and already we have discussed that in case of PMU the main phase estimation technique that is used is the DFT.

So, that is why in this particular class we will discuss about it what is that and how to write it and what are the mathematical formulation and we will also do one assignment if we will take some have sample data; So, how to calculate the phase angle or the magnitude of the voltage current using the DFT.

Now, as per the phasor is concerned how to define a phasor?

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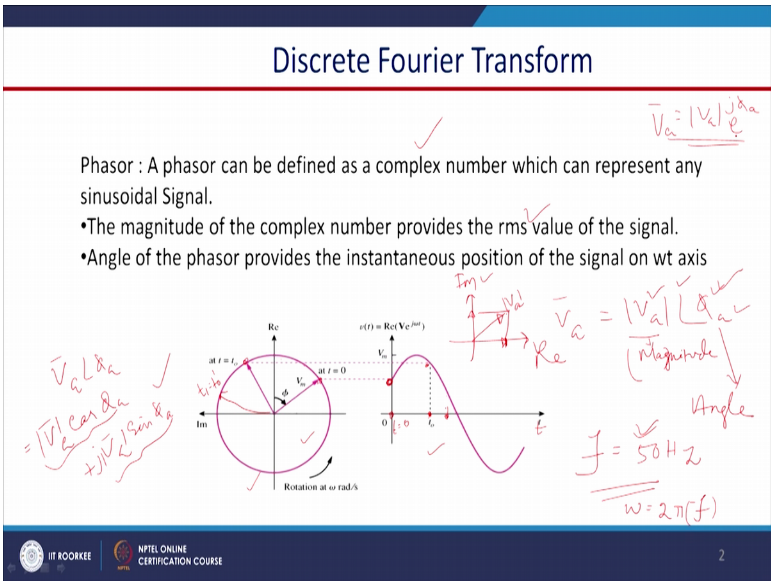

If we will say this V a phasor; so it has a magnitude and also it has angular. So, this two together the magnitude part is the magnitude part and this is our angle part; So, magnitude and angle together forms form the phasor; now one more point we have to also keep in mind when we call about this phasor we will talk about the phasor, the frequency is also important this fundamental frequency if it is 50 hertz. So, at what frequency this particular phasor is calculated that is also important express the magnitude and phase angle both are concerned if we will vary this frequency then their corresponding values are going to be changed.

So, that is why at particular frequency the magnitude and phase angle together called as the phasor. If it this see this particular picture here we have taken one V m 1 phasor position at t is equal to 0 and you can see this is the rotational plane and this is our sinusoidal plane that along x axis we have taken this time excess this is t and here is our magnitude V m. And this is how this t 0 starts from t 0 means it is the starting point of the phasor. Now when this phasor rotates at certain frequency as we have discussed this is angular speed that is omega is equal to 2 pi f to pi stands for 360 degree and this is our frequency that is 50 hertz.

So, this angular frequency this radiant per second; so, it rotates and from t equal to 0 let us say it starts from t equal to 0 here this is a position of t equal to 0 this is the position of the phasor. And when this phasor rotates and it reaches to t equal to t 0 and the corresponding phasor position is here it is t equal to 0; that means, this is the starting point let us this is the another point.

So, it further it moves away let us say t t 1 is equal to t 0 dash so; obviously, it will come to another point in the t axis along the t axis. So, this particular phasor position which is shown here and here together altogether equal; now to represent the position we took also sometimes this real access this is a real axis and this is a imaginary axis. If I want to this decompose this particular phasor, it is a complex number basically; it is a complex number and which has like provides rms value of the signal.

And angle provides instantaneous position of the signal at what angle this signal is rotating at particular time stamp, we can always access it. That is what this real axis will tell the real part of this particular phasor and this imaginary axis will just say what is a imaginary part of the signal that is basically the complex form of representation of the phasor quantities phasors.

Now, if I will just write here this V a phi a. So, how to write in complex form this will be V a cos phi a plus j V a magnitude basically this is magnitude V m magnitude and then sin of phi a. So, this is our the real part and this is our imaginary part. So, along this real axis we can take how much this real component and along this imaginary axis we can take real imaginary component taking the resultant. So, finally, we will get this V a magnitude that is what is the complex form of the phasor and also sometimes we represent in terms of exponential form also in polar form. So, the 3 forms basically we follow to represent the discrete Fourier I mean the phasors.

The first one is this is our polar form this is our rectangular form and this particular signal also can be written this V a phasor also is equal to V a is magnitude e to the power j phi this is our exponential form.

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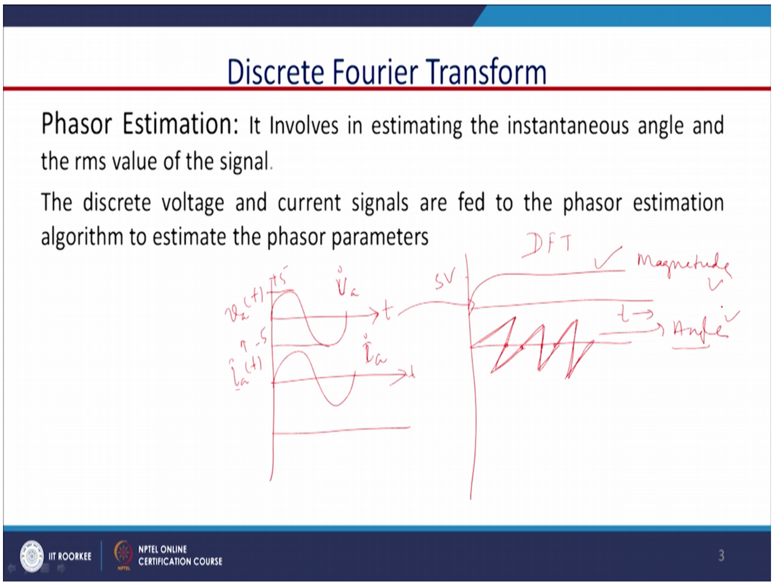

Now, this DFT Discrete Fourier Transform is one of the signal processing techniques which is used to estimate the phases of the voltage and current signal. Let us say we have 3 phase voltages 3 phase currents. So, if we have this one phase voltage and again another phase voltage with 120 degree apart; also we have current phase let us say this is V a and this is I a and this is all instantaneous with respect to time to be to get.

So, this is small v a t and this is small i a t; similarly we have like other phase voltages and other phase currents. So, what is DFT? This stands for the abbreviation is DFT the DFT helps in providing the magnitude and phase angle of corresponding voltage and angle. If we will see how it will be if it is 5 volt the signal is of plus minus minus 5 volt voltage signal.

Now we will expect the DFT will provide a magnitude curves like this with respect to time. Now similarly what will be the angle information? The angle will just rotate within 180 degree repeat; so, this is 180 degree 360; 180 degree 360. So, within a cycle this is one complete cycle of the angle like this. So, this magnitude this is the magnitude of the signal ad this is the angle of the signal.

So, this magnitude signal we are interested to extract using this DFT of this particular signal sinusoidal signal; voltage and current which are basically the pause system I am just talking about in terms of like PMU or relaying practice or relaying field. If you will see in the subsequent slides for protection approach in case of for digital relays the phasor of phasors of the voltage and current are very very important. Because when the relays are basically sequence component base relays in some cases we use the first sequence component of the voltage or current or we can use the negative sequence or 0 sequence components of the voltage and current.

So, in that to calculate the sequence components; so we need the phasors of the voltage and current. So, that is why in case of relays also we need some phasor estimation techniques and just way we need in PMS.

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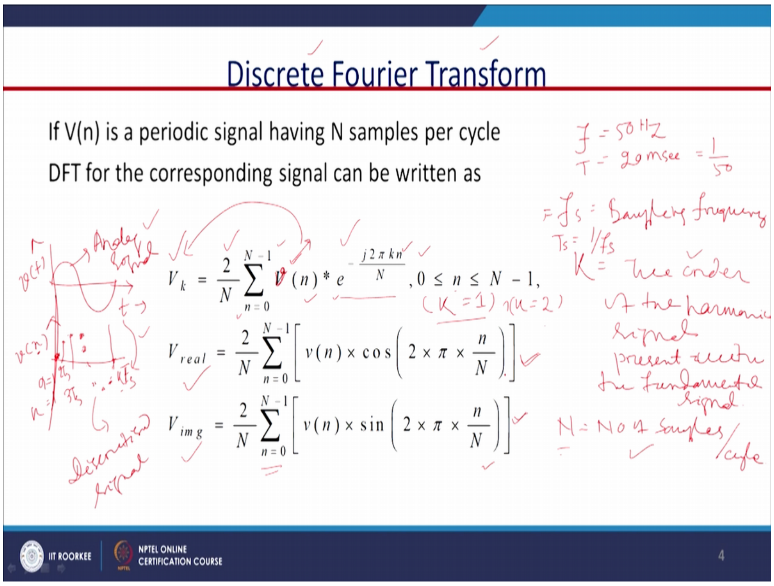

Let us start with this mathematical expression of this DFT this V k is equal to 2 by N summation of n is equal to 0 to N minus 1, V n e to the power minus j 2 pi k small n by capital N. Let us define what are this symbols; what is this k? The k is the it is the order of the harmonic, order of the harmonic signal present with the fundamental one with the fundamental signal. So, as already we have discussed in power network we have fundamental signal also we have harmonic signals.

So, this harmonics are coming due to the presence of electronic devices or non-linear loads also we have. So, due to that we have harmonic signals which are present along with our fundamental signal, but remember for our further processing during this phasor measurement unit or digital relays; we need the fundamental component of the voltage or current signal not the harmonic.

Yes of course, in some cases intentionally we compute the harmonic components of the voltage current signal for our purpose. In some cases some like if you just recall in your B Tech level we studied the second harmonic restrained differential relays or some and second harmonic component is also used for protecting the transformer and some cases also you be need now there in advanced protection strategies; we need some harmonic signals to analyze some protection issues.

So, intentionally we also calculate this frequency harmonic components for our application controller protection purpose. So, here this case transfer the order of the harmonic present in the fundamental signal and this N capital N stands for number of samples per cycle; so we have discussed in our worms technology class third we sample the signal set particular sampling frequency the sampling frequency we stands for f s is equal to our sampling our sampling frequency.

So, if our fundamental frequency is our fundamental frequency is 50 hertz. So, our fundamental time period will be 20 millisecond; it is basically 1 upon 50. So, these are the symbols we have to consider and also here sometimes we write T S the sampling time period that is 1 upon f s. So, these are symbols we use for our Discrete Fourier Transform analysis.

Now, this capital N is the number of samples per cycle and this V n sometimes also in some cases it is written small v n; this small v n is our sample values of the signal. Suppose one signal we have sample that particular sampling frequency if this is my analogue signal and the corresponding samples I will just reconstruct it here. So, this sample values we call it this is our time axis this is my small v t and this signal is basically the analogue signal and if I will just sample down this analogue signals.

So, we will get the digitalized or discretized signals; so, this is known as discretized discretized signal that is in discrete time domain I can write here k t this is at certain time interval we are basically sampling it. So, we call k T S this small t is represented by k T S and along this you know axis we have v small n small v n.

Now, this n stands for the number of samples we have taken for a particular period. If I took like 2 seconds; so for 1 second 1000 samples if I will take sampling frequency as the 1 kilo hertz. So, it will be 2 seconds if I will consider; so there are 200 samples per 2 seconds; so if my sampling frequency is 1 kilo hertz. So, in that sense 20 samples per cycle is a number of samples that is n and this stands for k T s. Because let us say this n is equal to 1 or k is equal to 1 here sometimes also we took it n T S also the position of the sample this n stands for position of the sample when n is equal to 2.

So, here is our n is equal to 1 or k is equal to 1 this is 2 T S and this position is 3 T S and this position is 4 T S 5 T S so on. That means, the point here is that the number of samples positions should be indicated in district domain not in continuous time domain this is a continuous time domain signal and this is a discretized time domain signal that is why the name is Discrete Fourier Transform.

And in Discrete Fourier Transform the name transfer means we are transferring this particular discretized signal to a frequency domain. This is time domain this one is discretized signal in time domain discrete time domain we are converting to a frequency domain signal because we are going to calculate the phasors. Phasors means the frequency domain is must we should know at what frequency we are going to calculate the rms value of the signal and phase angle of the signal; that is why this is how this is our discretized time domain signal and we are converting to this frequency domain signal for calculating the magnitude and phase angle.

Now, if you see that another term here it is as I said that small n that is our number of the position of the sample in a particular string. And this lies this sense small n lies between 0 and minus 1 for this case because we are discussing a full cycle discrete Fourier transform, the different types of DFT, the full cycle DFT, half cycle DFT, cosine DFT, sin DFT.

So, in that case we are first discussing the full cycle discrete Fourier transform; now we will decompose this above factor this equation we have real part and also we have imaginary part. This 2 by N summation of n equal to 0 to N minus 1 v n this is small v n into this sin 2 pi small n by capital N this is one expression and this is our imaginary part expression, this is our real part expression.

Now, in this case this k is considered as 1 why when this k is equal to 1 it stands for the fundamental frequency component, when k is equal to 2. So, it will just provide us the phrasor value of the second harmonic component of the signal if k is equal to 3; so it will provide the third harmonic component of the signal; the phasor value of the third harmonic component of the signal; that means so on.

So, this we can vary this k according to our requirement, but for this case we are considering the fundamental component of the signal.

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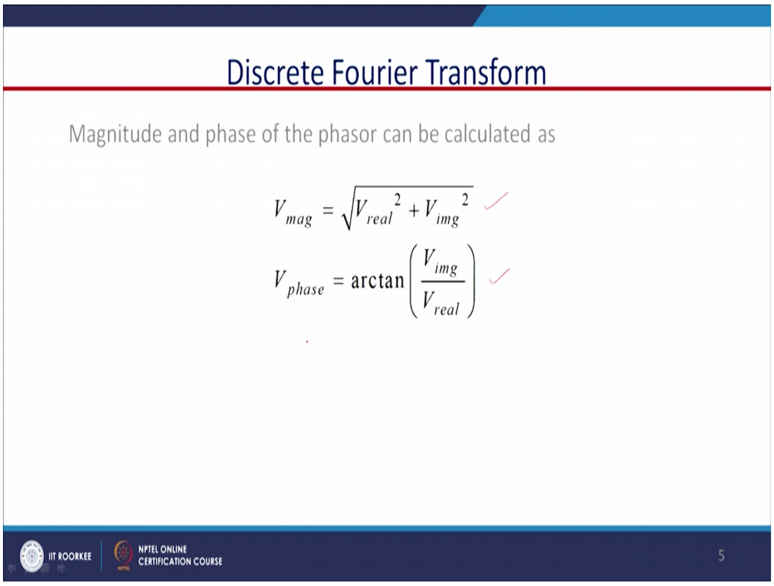

After knowing this imaginary part and real part; so, real and imaginary square if we take. So, we will get the magnitude and the tan inverse of this imaginary to real part. So, you will get the phase angle of the corresponding voltage or let us say current if you want. So, we have to also decompose the current in that manner; we will have the real part imaginary part and then square in together will get the magnitude and also phase angle. This is one table you can take it as an assignment that how to calculate this particular signal.

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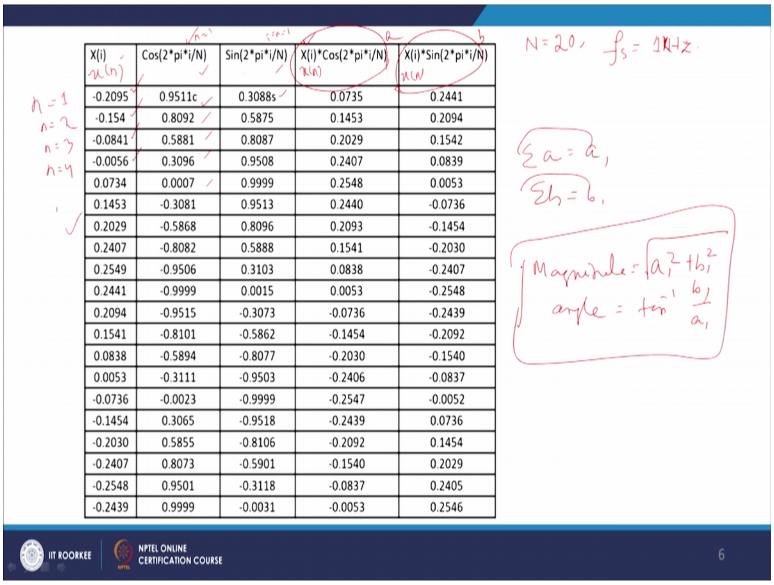

X(i)	Cos(2*pi*i/N)	Sin(2*pi*i/N)	X(i)*Cos(2*pi*i/N)	X(i)*Sin(2*pi*i/N)
-0.2095	0.9511c	0.3088s	0.0735	0.2441
-0.154	0.8092	0.5875	0.1453	0.2094
-0.0841	0.5881	0.8087	0.2029	0.1542
-0.0056	0.3096	0.9508	0.2407	0.0839
0.0734	0.0007	0.9999	0.2548	0.0053
0.1453	-0.3081	0.9513	0.2440	-0.0736
0.2029	-0.5868	0.8096	0.2093	-0.1454
0.2407	-0.8082	0.5888	0.1541	-0.2030
0.2549	-0.9506	0.3103	0.0838	-0.2407
0.2441	-0.9999	0.0015	0.0053	-0.2548
0.2094	-0.9515	-0.3073	-0.0736	-0.2439
0.1541	-0.8101	-0.5862	-0.1454	-0.2092
0.0838	-0.5894	-0.8077	-0.2030	-0.1540
0.0053	-0.3111	-0.9503	-0.2406	-0.0837
-0.0736	-0.0023	-0.9999	-0.2547	-0.0052
-0.1454	0.3065	-0.9518	-0.2439	0.0736
-0.2030	0.5855	-0.8106	-0.2092	0.1454
-0.2407	0.8073	-0.5901	-0.1540	0.2029
-0.2548	0.9501	-0.3118	-0.0837	0.2405
-0.2439	0.9999	-0.0031	-0.0053	0.2546

That this X i sometimes we can also write it X i or this is small x n or this discritized signal n basically n stands here the sample position 1, 2, this is equal to 3 n is equal to 4 and so on, we have number of sample positions this is the first sample, this is second sample, this is third sample, 4 sample and so on.

Now, if you could remember in previous part the real part is cos 2 pi n by capital N keeping this k is equal to 1. If you put this; so this will be our these are the corresponding magnitudes and similarly this is our imaginary part sin of 2 pi n divided N. So, when you will calculate this I or N we can replace it whatever you like this n equal to 1. So, this is the corresponding value when this I is equal to or n is equal to 1 this is the corresponding value and here you should put N is equal to 20 because the sampling frequency we have mentioned here 1 kilo hertz this is 1 kilo hertz so on.

So, these are after getting this cosine factor sin factor then we will multiply the corresponding x n value with the sin in cosine fact. And finally, we will take if I will just denote this total factor as a and this total factor as b. So, this summation of a let us a is equal to a 1 summation of b is equal to n b 1 or in either way you will write. Now this magnitude is equal to is equal to a 1 square plus b 1 square root over and angle part of this signal is equal to tan inverse of b 1 upon a 1. So, this is one kind of assignment you can take it and you can calculate the corresponding magnitude and angle.

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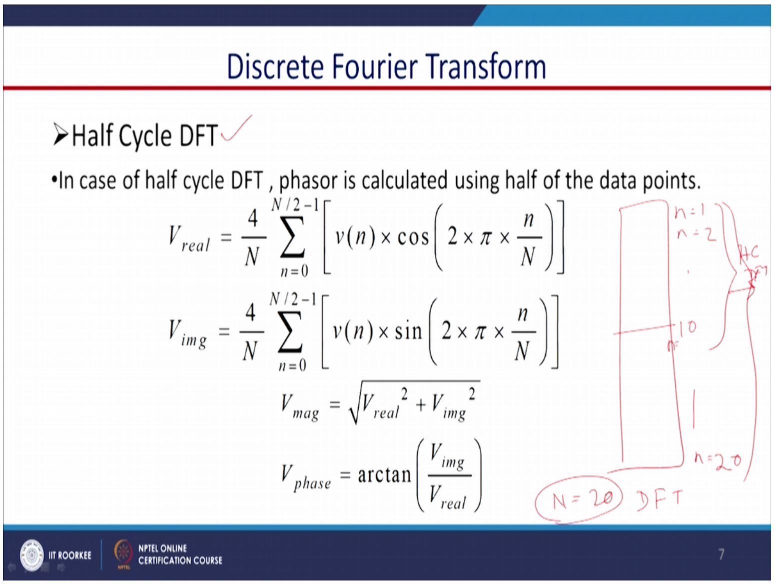

Here we will go for half cycle DFT; in case of half cycle DFT, we basically go for half of the sample values let us say in its sample string we have 20 samples n stands for 1, n stands for 2 up to n is equal to 20.

So, if you will take total 20 number of samples that is N capital N is equal to 20 and if you consider the whole I mean samples 20 number of samples and we calculate the DFT then the DFT known as full cycle DFT. if I will take only 10 samples up to 10 when n is equal to 10 then in that case the DFT is known as half cycle half cycle DFT; that is what is mentioned here.

In that case only the difference will be in case of number of samples, if you just go back to the previous expression of the full cycle DFT here we wrote N minus 1 in this case; the number of cycle number of sample stands starts from n is equal to 0 to N minus 1, but here in this case discuss the number of samples starts from n equal to 0 to N by 2 minus 1.

So, we are taking here half of the samples if 20 is the figure for number of samples for cycle. So, here in this case we will take 10 samples for cycle a half cycle not for cycle the half cycle 10 samples for half cycle. So, half cycle data or samples will take; now (Refer Time: 21:46) other thing remains remain constant other thing remain constant that is v n cos 2 pi small n by capital N and this is our real part and this is our imaginary part.

Here is a v n sin 2 pi small n by capital N; this is our imaginary part now here there is no change in n does not mean that here we will take n by 2 in place of N. No here this N stands for number of samples for cycles; so it is not necessary it is not required to take this N here. If we will take this magnitude in phase formula, it is same as previously we did for the DFT you can see here in case of DFT here we have taken the same.

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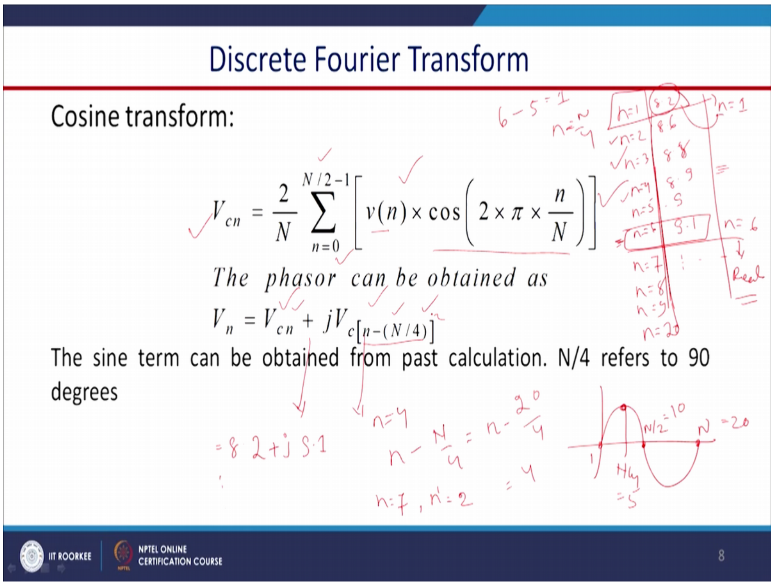

Now, coming to this another part this is a cosine transform of the DFT that is the cosine part of the signal that is 2 by; N stands from 0 to n by 2 minus 1. And here we have v n small v n and cos 2 pi small n by capital N, now this is a real part of this particular signal. What about the imaginary part? If I do not know the imaginary part then how to calculate it is in this way if this is real part and this will be imaginary part what I will do here if I will put n minus capital N by 4.

So, I will get the imaginary part of the signal that is a sine part of the signal because the difference between this if I know cos theta then how to know the sine theta? Then just making minus of minus of N by 4 points one quarter signal or minus 90 degree sine of 90 minus theta is equal to cos theta.

So, in that case let us say I have one string of data n is equal to 1, n is equal to 2, n is equal to 3; let us some values we have 8.2 or 8.6, 8.3 8 n is equal to 4, n is equal to 5 some values arbitrary values I am just writing n is equal to 6 let say this is the 9 or 9.1.

So, these are the sample values and so on it goes up to n is equal to 20 because we are discussing about this 20 samples per cycle data. Now the question is if this is my cosine part, so how to make it the sine part? From the same data, I will construct my signal. So, this is like this if I am here let us say n is equal to 4; so how to make the particular sample for this V c n minus N by 4?

So, from where I should start if this is; that means, n is equal to now it is 4 that n minus N by 4 means n small n minus 20 divided by 4 that is 4 if I will come like number of samples I have 20 by 4. So, forth before the fifth point before 4 points I have to move N by 4 means 5th point; here it is if I have the signal here is my N by 4 and here is my N by 2 and here is my N stands. So, number of samples here is N by 4 and here N by 2 and here is; this is 20 number of samples here it 10 and here it is 5 here start from the 1.

Now; that means, I have to starts I have to start from here n is equal to 6 because I have to go back 5 samples. So, here is my n minus N by 4 that is 6 minus 5 that is equal to 1. So, this sample should be taken into consideration if I am starting from here n is equal to 6 and this is for my real part, real part sample value and so for imaginary part my sample positional will be 1 and this 8.2 is the corresponding sample value. If I will just want to write here for n is equal to 6; so, it should be 8.2 plus j 9.1 and so on.

If I will take n is equal to 7; so, my let us say this is for n cn and this c this part will be n dash is equal to, so the 7 minus this 5 it will be 2; that means, it will just keep on sliding this is 7th position.

So, here it will my real imaginary part if it will be 8. So, the in the real imaginary part will be 3 and if it is 9 then it will be 4. So, on so; that means, if I know a string of data it is not necessary to calculate the sine part from the cosine part from the same symbol we can always calculate the real part as well as the imaginary part to find the magnitude and phase angle of the signal.

Now we will go for the Discrete Fourier Transform part this is the DFT we have discussed. In case of DFT we have full cycle DFT, we have half cycle DFT, we have cosine DFT; now we will discuss about the discrete Fourier transform in recursive manner. What is a recursive DFT and why we are interested for this recursive DFT? In case of recursive DFT one advantage is many we will just calculate in windowing manner what is the window? What first we will discuss then we will go for recursive DFT.

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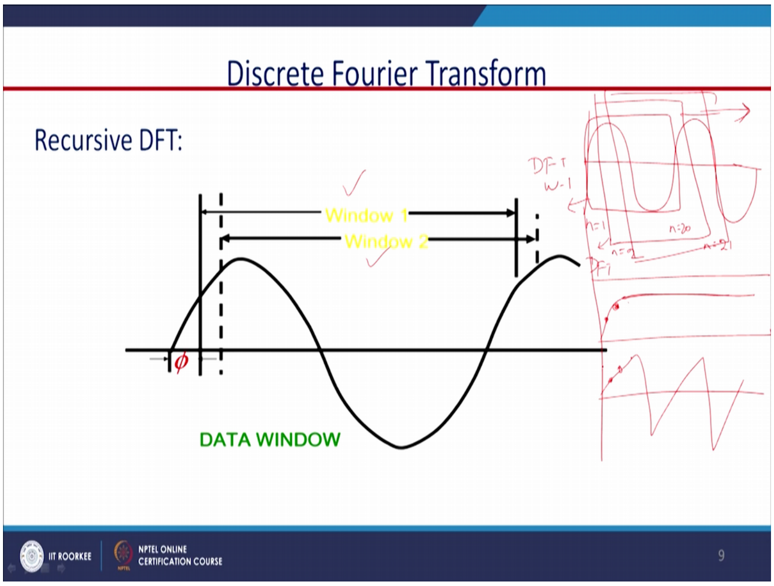

If this is one sinusoidal signal where I have taken 2 cycles and I just want to calculate the corresponding magnitude phase angle recursively; not iteratively it is recursively it will move this is my first window which is having 20 samples per cycle.

It starts the its start from 1 and it ends with 20, now next window will move to the another level this is n is equal to 2 and n is equal to ending one is 21 so on; that means, we this window is moving inside the signal and making taking one, one cycles and 20 samples per cycle and its moves ahead its move ahead. And this kind of moment is known as windowing approach or window approach of the in calculating the phrases of the signal.

Now if it is so, then if for this particular first window we have particular magnitude of the signal and corresponding angle also will be here. Similarly for second window we will have some magnitude point here and the angle point here and so on. So, if you just see then corresponding angle the magnitude will be like this and the corresponding angle goes on like this.

So, we will see this particular picture in more clarity the next PPT or slide now this is our first window this is second window. So, now we will come to the recursive DFT part that with using this recursive DFT we can minimize the time of computation. In DFT if we will just every time we are going to let us say this is a window 1; for this will calculate the DFT and the corresponding magnitude and phase angle will calculate.

Similarly the second window will also we will applied DFT and we will calculate the corresponding magnitude and angle. So, it is time consuming fl every time we are computing the imaginary part the real part and a gain the magnitude and phase angle. So, we are again if you could remember our table. So, this kind of computation is happening for every time. So, this cos theta sine theta part then this X i into cos theta X i into sine theta this computation computations are compulsory.

So, every time for every window we had doing this the computerize doing the same exercise time to time. Now, but what will happened to remove this to avoid this kind of competition for every window. So, we will just do one technique that is known as recursive DFT. In this case only once the DFT will be calculated for the first window; what we will do here? We will just calculate the DFT for the first window and subsequent windows will not we are not going to calculate the DFT anymore.

The first DFT will be utilized to calculate the magnitude and phase angle of the second window or 20 samples for cycle data. And similarly for the next window we will take the help of previous window magnitude and phase angle and then we will proceed and so on. So, this kind of approach is known as recursive DFT, in this manner we can save the computational time which is basically happening in case of Discrete Fourier Transform and where we are calculating the DFT for every window for finding the magnitude and phase angle of the signal.

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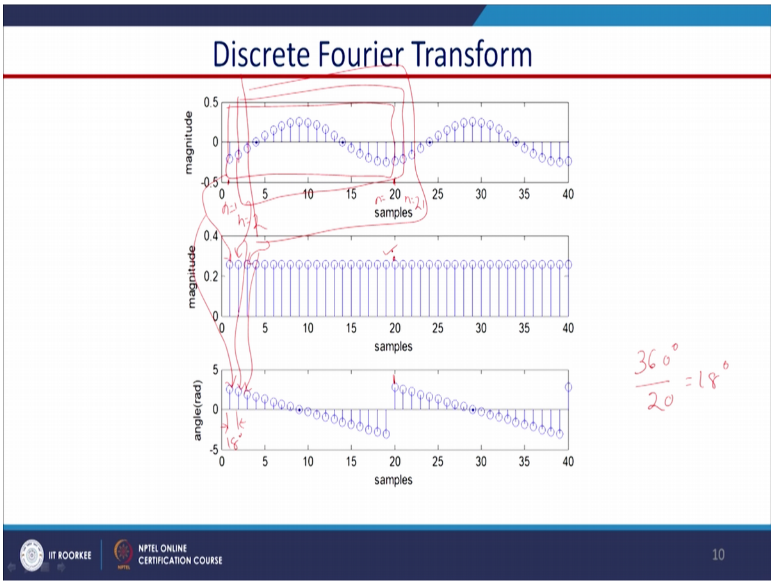

Now, this is how this samples of a 20 samples per cycle here 20 samples.

Here it starts from 1 and it ends with 20 and this is the magnitude part this is the angle part you can see here how this angle varying. This is here one cycle here at this point and here another cycle angle varies like this. If you just calculate the angle between 2 samples at this point it will be 18 degree because we have 20 samples angle is basically 360 degree; 360 degree by 20 it is going to be 18 degree. So, between 2 samples the angle difference is 18 degree that is like this.

So, for every if we will take a window kind of prose this is my first window up to 20th; first window and this will be our second window, where it will start from n is equal to this is n is equal to 1 2, n is equal to 20.

And next window will start n is equal to 2 to n is equal to 21 and similarly and the third window will be like this and so on. This window will keep on progressing going ahead proceeding ahead to find the corresponding magnitude and phase angle. For this first window for the first window this is the magnitude and this is the corresponding angle. For the second window these are the corresponding magnitude and this is the corresponding angle and for third window; this is the third window this is the magnitude and this the corresponding angle and so on.

So, when this window ends within a cycle; so, this is the end of this particular angle this is the end of corresponding magnitude.

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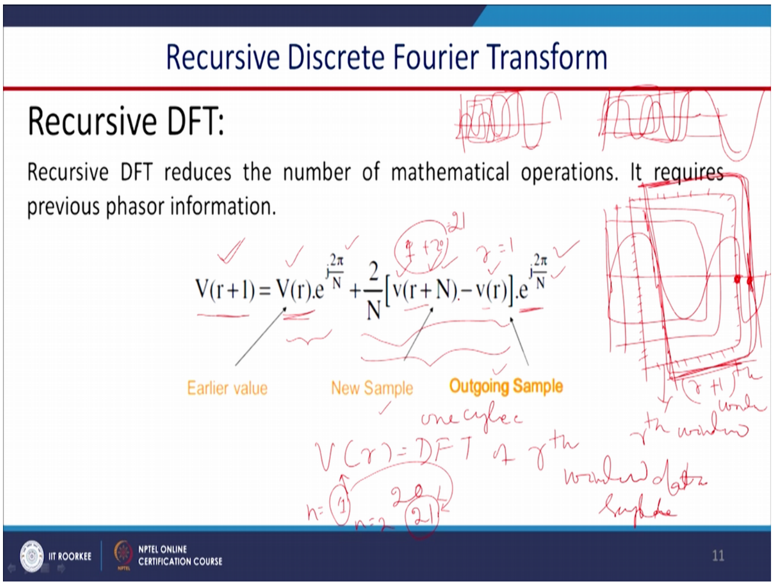

Now, we will come to this mathematical expression of this recursive DFT or recursive Discrete Fourier Transform that here this V the capital V r plus 1 is equal to capital V r into this e to the power j 2 pi by N plus 2 by N small v. So, this is small v small r plus capital N minus small v r into e to the power j 2 pi by N. Now I want to explain this particular expression clearly; so that will understand. So, what is the basic difference between this recursive DFT and the only DFT? Now if you could see what is this capital V r plus 1 if I have a signal has already we have discussed in the previous slide that this particular signal is moving ahead and this is my first window and this is my second window.

So, this arbitrarily let us say I can start from any window my for calculating my phasors. Let us see this is my rth window that is what the meaning of this r small r if I will take this is the say last next window. So, this will be r plus window now this for this particular window r plus 1. So, that will be this will be my phasor V r plus 1 now how to express this is V r is the previous window DFT value; so what is this V r?

The V r is the DFT one cycle DFT one cycle DFT of rth window data sample data sample. This data may be we have voltage data or we may have current data; now this rth window data sample DFT is V r this is capital V r into this j 2 pi by r why this j 2 pi by

N? Between the 2 windows the difference between this angle difference is how much? 2 pi by r n that is 18 degree.

If you will take n is a 20; 20 samples per cycle and what about the second term? We have 2 terms the first is basically the DFT of the first window and the second term is what? The difference between the new sample and the outgoing sample this is the magnitude part of this particular sample and this is the angle part we have to multiply the angle also.

Now, this what is this new sample in this case? If you consider this is my the window just I will make it in a different shape you can understand easily that this dash line is my rth window and the solid one, this solid one is the corresponding new window; we are moving from old window to new window. So, of course when you will move from one window to first one window to another window so within conjugative window we have one old sample is going to be rejected and new sample is going to be entered. So, in this case if you will see that this is how this window ends here and this is the my this was my last sample and this is a new sample which is entered inside; if will tell like this 20 first n is equal to 1 to 20 and next n is equal to 2 to 21.

Now, this one sample the first sample is discarded and the 21th sample is enter to the phase window that is what the meaning of new sample and outgoing sample and the name is like this; the new sample I can write V r plus n and this is my outgoing sample ok. So, if this r is equal to 1 then if N is equal to 20; so this 1 plus 20 that is basically 21 is the my new sample and one is the old sample.

So, old sample is discarded and the new sample is entered to the window the difference between these 2 magnitude sample values in magnitude. So, this difference between these two basically is the difference I mean the what gap exist between the two samples new sample and the old sample. And multiplying this angle we will get the phasor information.

Now the question comes the between these two samples the information the gap will be 0 throughout unless until any disturbance is going to be inserted. In fact, the first sample is basically similar to the 20th sample of the next window; the first sample is just similar to my 21st sample of the next window; the difference is basically 0, but when any fault occurs when any fault occurs or any disturbance occurs now the signal magnitude increases to another level.

Now, the difference if you see the this window moves on, but; however, when the when the window moves or it reaches within this changed signal part; then it will see some difference between these two samples. The new sample and the old sample difference will be significant, when this window will reach within the disturbance area this is our disturbed area or I can clearly I can so, here this is my signal before the disturbance and this is a signal after the disturbance. Let us this current magnitude has been increased to another level after the disturbance.

Now, this window will keep on moving and when this window moves inside the disturbance area now there is a basically significance change between these 2 samples.

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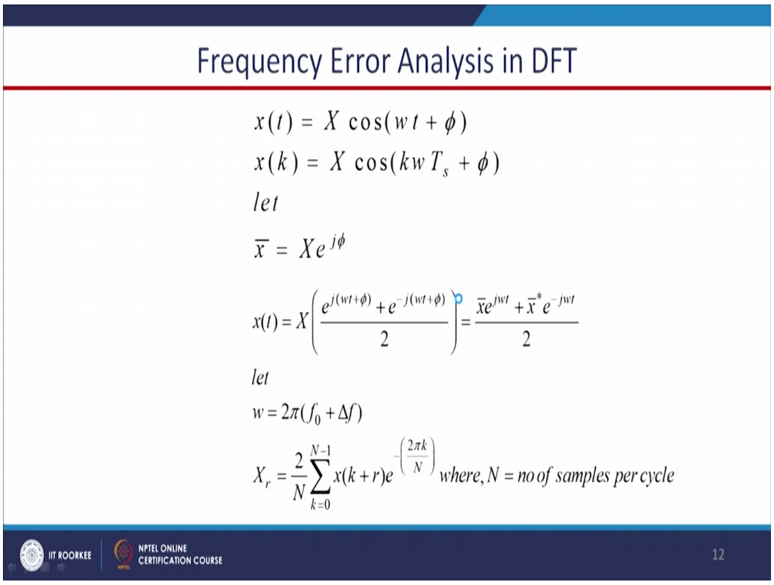

Now, this is another part which is known as frequency error analysis and DFT and before going to that let us just give some remark on this recursive DFT. That in case of recursive DFT we observed that only we have to calculate the DFT once this is r for this V r and rest of the window for rest of the window we are not going to calculate the DFT, time to time taking this second expression we can just add to the DFT value of the previous window so that we can calculate the DFT of next window. So, in this manner the recursive DFT saves the time and also in computational wise it is quick and we can save the time that is our aim of the recursive DFT.

Coming to the error analysis that is frequency error analysis as we are operating in the frequency domain as far as the DFT or recursive DFT are concerned the operation is frequency domain. Now if the frequency of the system varies then the nominal values let us say we have 50 hertz if the frequency varies something like 49.5 or forty 49.8 hertz or sometimes it increases to 50.1 up to 50.2. So, in that case how this magnitude and phase angle which are calculated is in the DFT or recursive DFT they are going to be affected that is what the frequency error analysis.

To have this we have certain steps mathematically and we have to understand it here is the first signal we have taken step 1.

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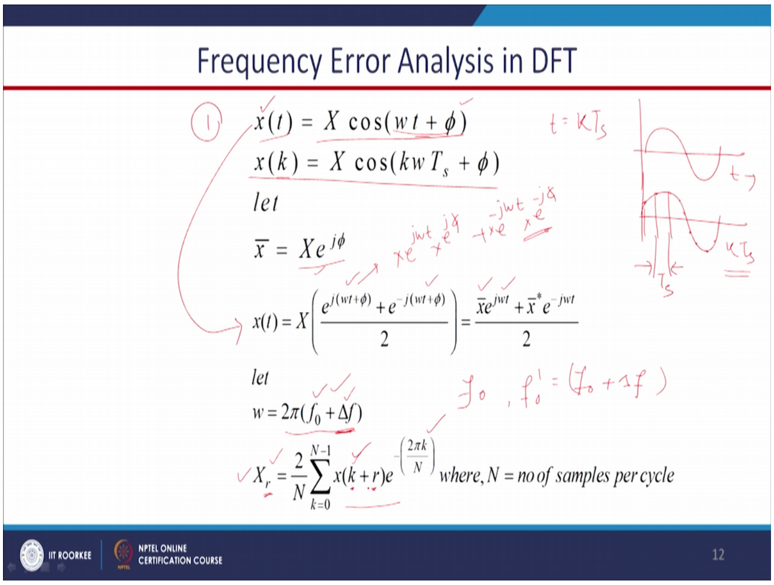

This is step 1 we have taken one instantaneous signal that is xt in time domain where the signal is expressed like this it is x capital X is the magnitude of the signal cos omega t plus phi it is a cosine function let us phi is the phase angle of the signal with respect to certain reference.

And this small xk is the signal in discrete domain that is small x till exist this is in analogue or infinite that is a continuous time domain and the small x k is the signal in discretized timetable. And only the difference is here is in case of t this small t we have to replace k T S that is a difference where already we have discussed that for a signal if it is in time domain t and in discrete domain when we will just convert this small t will be eplaced by k T s; the signals samples are basically sampled (Refer Time: 42:16) within a certain interval.

So, this distance is basically k T S this T S is the sampling time interval within two samples conjugative samples the time gap is basically known as T S now let this particular signal again we will write x e to the power z we will just denote x small x bar is treated as capital X into e to the power j phi.

Now, by rewriting this x t in exponential form like will just decompose like cos theta is equal cos e to the power j theta plus cos e to the power minus a theta divided by 2. So, in that form we will write x this is the theta omega t plus phi. So, first one will be e to the power j omega t plus phi plus e e to the power minus j omega t plus phi. As we have already taken this by decomposing this e to the power j omega t into e to the power j phi plus e to the power minus j omega t into e to the power minus j phi.

Now, in place of this if we will multiply x with all together x. So, this expression can be written as we have abbreviated here x small x bar the small x bar into rest of the term e to the power j omega t. Now let us say f naught is our nominal frequency and due to the some disturbance we have a frequency deviation, then our nominal frequency that is let us say that is delta f. This delta f is the frequency deviation and in that case how to write it, what is the change in frequency? That will be f naught plus delta f earlier it was f naught this f naught dash is equal to f naught plus minus del f; this frequency deviation when we added or it may be subtracted.

Now, if by considering this omega as with the change frequency scenario this x r can be rewritten this is how our expression was for the previous expression like we have for DFT. If you could remember the DFT of a particular signal 2 by N k is equal to 0 to N minus 1 x of k plus r e to the power minus 2 pi k by N. Now what this r stands for and k stands for already we have discussed capital N is the number of samples per cycle, but here this r stands for what? Window we are looking for rth window r plus 1 or r plus 2 and so on.

Now, what about this k? k is basically number of sample position as if we have taken the previous case n small. So, that this k stands for the sample position and this r small r stands for the window position or window number.

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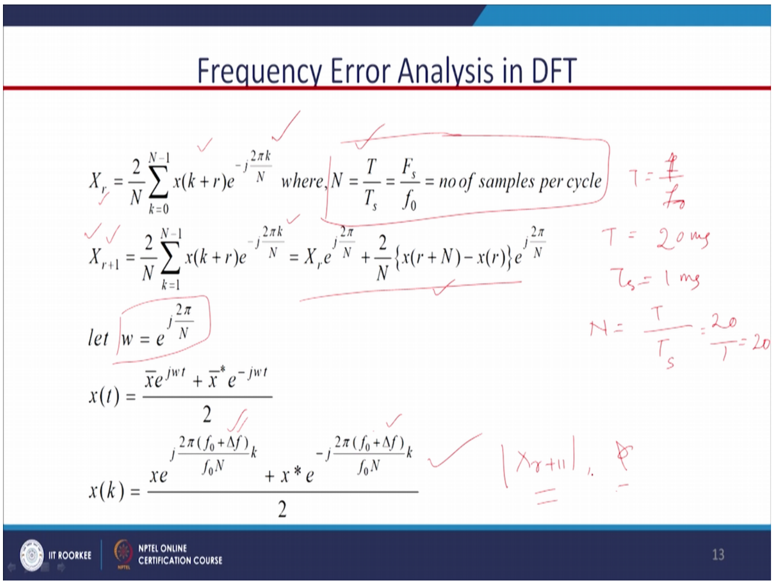

Now by keeping this in mind like keeping this mind; so this X r can be written in this manner where this small this capital N can be written as T divided by T S this is our formula because T is basically 20 milliseconds and T S is equal to 1 milliseconds. So, how to get this number of samples the relationship between this capital T and T S is equal to 20 divided by 1 that is 20 that is how this N, T and T S are related to each other.

Now, in place of capital T also we can put as 1 upon 1 upon f naught. Now putting all this we will just get 2 upon N this x k plus r e to the power minus j 2 phi k by N. And this is how our recursive DFT part I should not discuss about this part because already we have discussed how to take the recursive DFT. This is the first window rth window and this is by r plus oneth window if this is the DFT for my rth window.

So, this will be my DFT using the recursive concept for x plus r r plus oneth window now this is one other symbol we have we will use here omega is equal to w or you can say because omega is our radiant frequency small w e to the power j to pi by n now by putting all this expressions. So, this will my final expression for kth window signal.

Now, you can see here that very clearly that in this expression that this f naught plus delta f is represented here very clearly as per the frequency deviation is concerned. And that is why if previously if you could see here our expression was here that the frequency for this case was f naught only, but now in this case the expression is f naught plus delta f. So, as if this is the case so; obviously, the magnitude of this particular X of plus r 1 this magnitude and also the phase angle are going to be affected because the frequency has been deviated from the nominal values. This is how this is for recursively, if you are running this i stands for 1 recursive window if you will just scored this mathematical equations.

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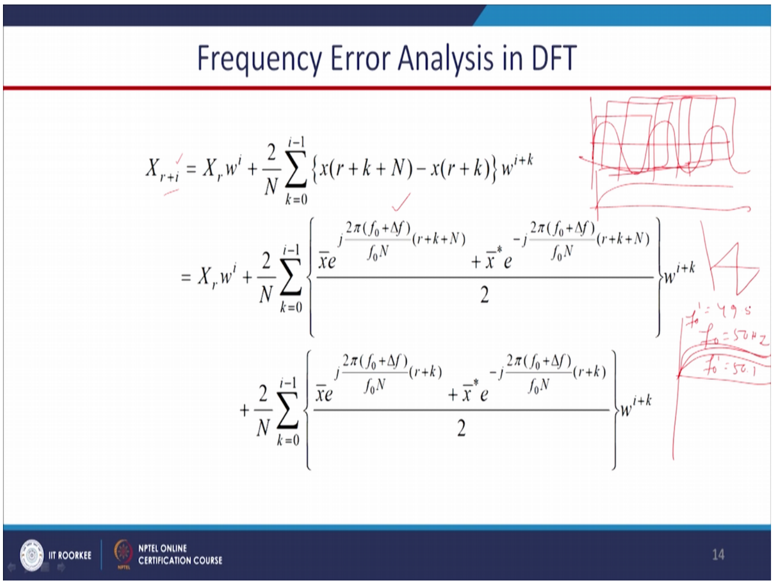

So, slowly we will find the window will move ahead using this i this is basically for low case type.

So, this is my signal if it is so on moving now this window will keep on moving one by one; this is how it will just move ahead. So if you just move ahead then we will find the corresponding magnitude and the corresponding angle also already we have discuss this. Now the question here is due this frequency deviation, you can see here due to this frequency deviation; let us say we got some magnitude with f is f naught is equal to 50 hertz.

Now we will get some deviation in the magnitude part if f naught dash is equal to either 50.1 or with some this f naught dash is equal to 49.5 or 49.8; whatever the frequency deviation.

That means the difference between these 2 magnitudes are basically errors; if my frequency is deviating from 50 hertz to another frequency value that deviation is basically the error in terms of like maybe it is in magnitude part or also in the angle part.

So, this is how today our class we did we have discussed the one of the important phasor estimation techniques; that is the DFT Discrete Fourier Transform.

And again also we have discussed recursive DFT that is recursive Discrete Fourier Transform and also we have analyzed the frequency error part which is going to be happen in our magnitude and phase angle case.

Thank you all.