Introduction to Smart Grid
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Lecture – 14
Phasor Estimation- II

Good morning to all of you. In this lecture, today will discuss about the Phasor Estimation and Frequency Estimation techniques. As already we have discussed 1 Phasor Estimation technique that is the Discrete Fourier transform based phasor estimation technique. And in this class, we will discuss about another technique that is the Least square error based phasor estimation technique.

Now, the question comes that if we have already discussed 1 phasor estimation technique; then why you are interested for this particular phasor estimation technique. I will discuss here two-three points that why we are interested for Least Square Error Based Phasor Estimation technique.

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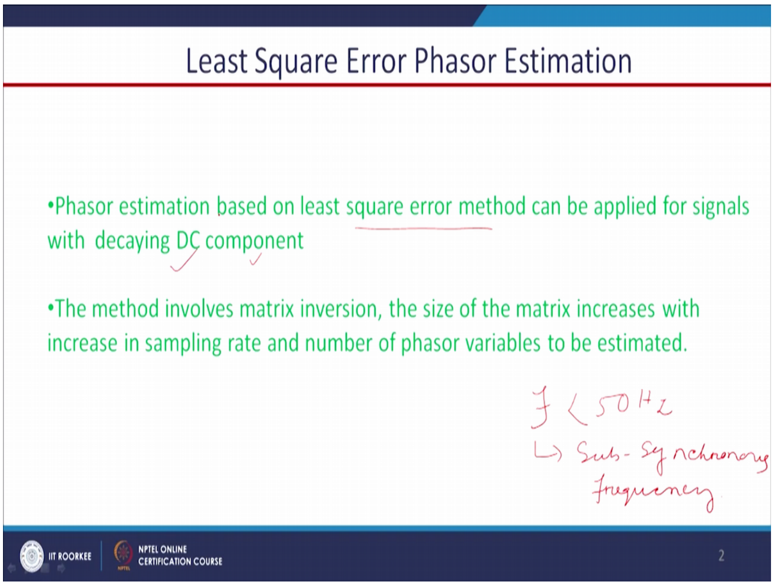

That if you could see that in case of discrete Fourier transform that we are not able to estimate the phases of the synchronous frequency components of the signal. What is the meaning of sub synchronous frequency component of the signal?

The signal which is having frequency below the fundamental component that particular signal is known as sub synchronous frequency component based signal. Now, if some signals having super synchronous frequency component means if the frequency is above the nominal frequency or normal frequency or retired frequency that is 50 hertz; then that particular signal is known as sub synchronous super synchronous based signal.

Now, what happens in case of DFT that we are able to estimate the phasor of the sub synchronous super synchronous and the fundamental component and also the decaying DC component; but the problem here comes the frequency which is less than this 50 hertz that frequency is known as Sub-Synchronous Frequency. That frequency component based signal phasor estimation is difficult in case of DFT, that is the district Fourier transform and also it is difficult to some extent to extract the decaying DC component part in case of DFT.

So, that is why we are interested to help another phasor estimation technique that is known as the least square error based phasor estimation technique where we will be able to estimate the sub synchronous frequency based component. The fundamental component of the signal, the decaying DC component and also the super synchronous frequency component of the signal; that is what we have mentioned here that this phasor estimation based on least square error method can be applied for the signals with decaying DC component. Because in power system may be in smart grid environment, the signal may have the decaying DC component.

So, in that case we can always extract the decaying DC component using this particular phasor estimation based technique. The problem here is see every method has some merits and also demerits. This least error square based technique has the demerits also. What is the main demerit? The first one is it involves the matrix inverse. In the subsequent slides, we will see the mathematics the basically the formulation of this particular technique where you will find this inverse of the matrixes involved, in calculating or for calculating this phasors. And also the size of matrix increases with increase in sampling rate; that is also one disadvantage of this particular. It is not disadvantage.

Yes of course, with a capability of the computing facility of or the capability of the processor of the computer, we can always apply this particular technique for estimating the phasors and also the number of variables to be estimated. So, these are 3 points which are basically the major concern as well as this least square error based phasor estimation technique is concerned. Coming to the mathematical formulation of this particular phasor estimation based technique.

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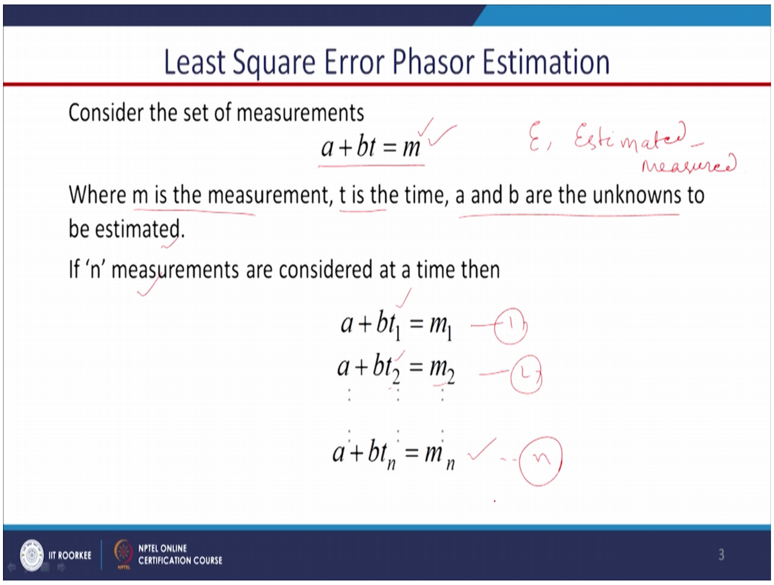

Let us start this a plus b t is equal to m, where this m is the measurement and t is the time and a b are the unknowns to be estimated. Now as the quickly one question comes to my mind that the technique is known as least square error. So, we are basically trying to calculate some error and we also trying to basically minimize that particular error. So, what is that error basically? The error is this error is basically nothing, the estimated value the estimated value minus the measured value because in power system or smart grid environment, we are estimating some parameters or some quantities or some signals and also we have measured signals the difference between these two should be minimum.

So, whatever signal we are measuring so that particular signals should match to our estimated signal that is what the difference and that difference is known as error. So, in this phasor estimation based technique, we are trying to minimize this particular error. So, in this process by minimizing this error will be able to find out the unknowns or unknown variables that is what the main goal of this particular technique.

So, coming to this mathematical formulation of this particular phasor estimation based technique, we have already defined here 1 equation, In this particular equation, we have this measurement on the right side and the left side we have the estimation and from this formulation we will see how the unknowns are basically calculated using the least square error based technique.

Let us say we have n number of measurements in our power system and this is the first equation this a plus b t 1 is equal to m 1; first equation. This is our second equation. This is a plus b t 2 is equal to m 2 and this is this t 1 is time t 1 and this t 2 is at time t 2; the measurement which is basically done or which is in our hand that m 2 is at time t and similarly coming to nth time that is tn time. So, the last equation that is our nth equation that is a plus b t n is equal to m n.

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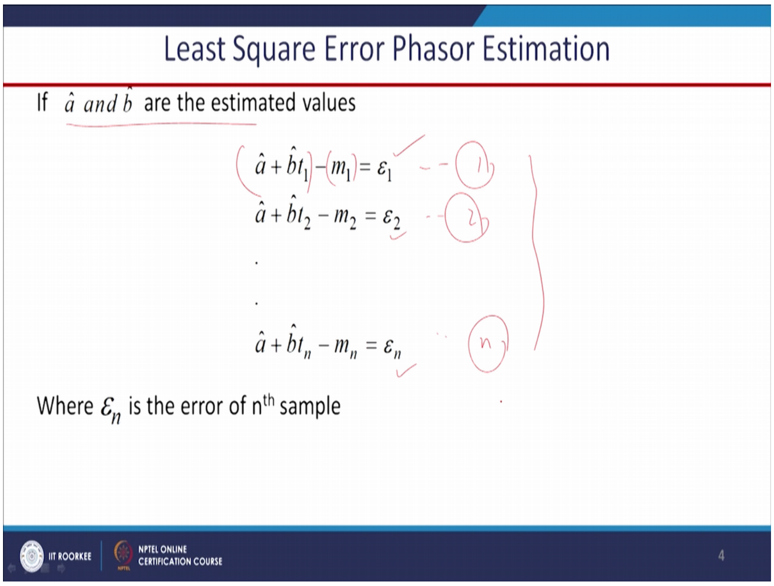

So, if will see we have this a cap and b cap are the estimated values because we are here to estimate our unknowns a and b are unknowns unknown variables. So, we are here to estimate, the unknown variables from the known quantities or known signals that is why this a cap and b cap stands basically the estimated values of the going to we are going to estimate the parameters or variables. So, this a cap and b cap as the estimated values.

Now this a cap plus b cap this t 1 minus m is equal to E 1; already we have discussed this is basically my estimated thing and this is my measured values for difference between these two basically leads to the error and that is why this is my first equation. And similarly, we have second where we have this a cap plus b cap t 2 minus m 2 is equal to epsilon 2. So, on and we have nth equation and nth equation, we have a cap plus b cap t n minus m n is equal to epsilon n. So, in this process we have n number of equations and further, we will just frame one matrix equation from where we can always calculate the unknown variables.

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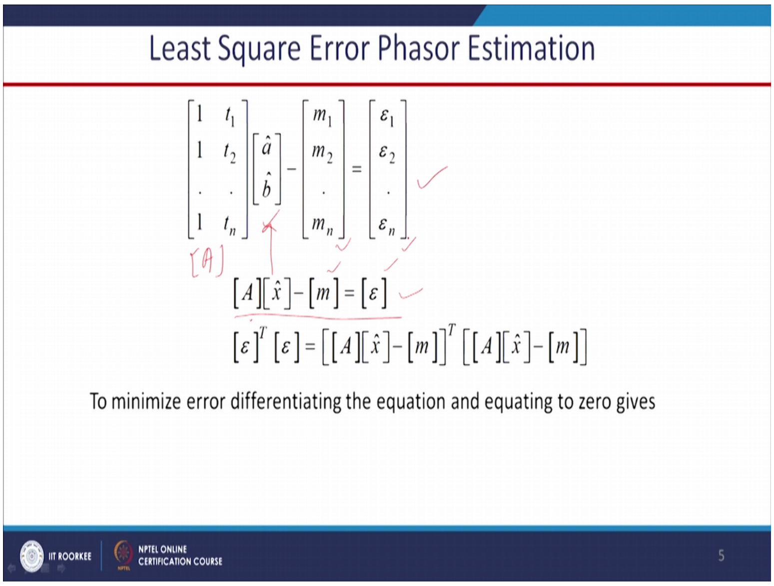

This is how the previous all the n number of equations look like this is the first matrix, we call it as A matrix; this is A matrix and this is our variable matrix the unknown variable matrix a cap and b cap and this matrix is our measured signal matrix m 1, m 2 up to m n and the last one is our error matrix E 1, E 2 or you can sometimes we can call it as epsilon 1, epsilon 2 up to epsilon n. Now if you just make this matrix, it look like this a cap this A matrix into x cap this x cap is nothing that is basically what the parameters were going to estimate. This x caps stands for the unknown matrix; unknown variable matrix and this is our measured signal matrix and epsilon.

Finally, if you will see that if we will just take the epsilon transpose and multiplying the both the sides, if I will multiply I have just taken after this equation. So, what will do? We will take the epsilon transpose epsilon matrix transpose and you multiply on the both the sides left side and the right side and then, what we will do to minimize the error because our m is in this process to minimize the error to get the parameters that is what our target.

So, to minimize the error so we have to differentiate this particular equation with respect to the error and again, you will equate to 0. This is a process in the metrics that if you want to minimize some parameter something. So, we have to differentiate it and then we have to give it to 0. So, this equation should be taken, I mean should be differentiated with respect to the error and it should be equated to 0 to get the unknown parameters. Finally, after doing this process we have this unknown matrix that is our x cap. So, this x cap stands for whole matrix into the measured signal matrix. So, this matrix is known as the Pseudo inverse of A.

The pseudo inverse of A is a very critical condition that is what already from the beginning I have started that the major element of this particular method is to have this inverse of a particular matrix and if the size of the samples increases, increase then; obviously, it will be difficult to calculate the inverse of the matrix; that is what the major element of this particular technique. After knowing this because you know this A matrix is already known to us and this m matrix is also known to us.

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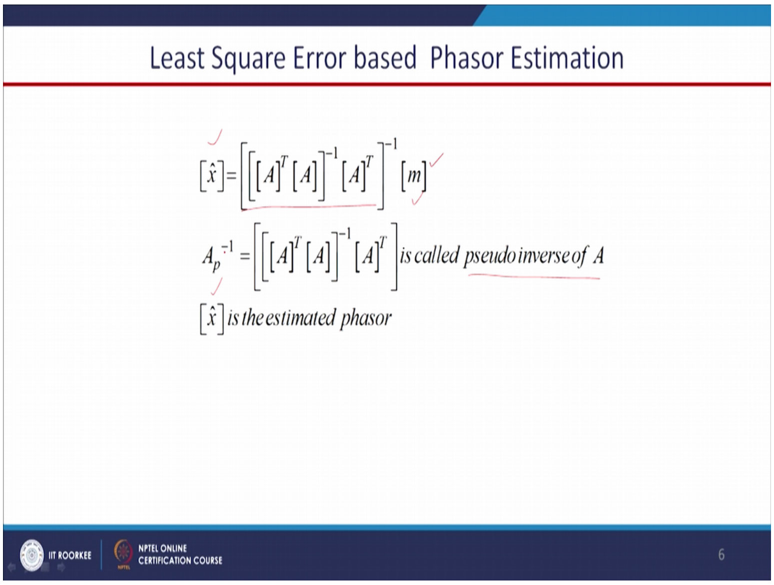

So, by knowing this too; obviously, we can always calculate this x cap that is our the required signal or required parameter or whatever the values we need to calculate. So obviously, we can calculate using this particular equation and this is we and matrix also this in platform it is nowadays using the mat lab, it is very easy to calculate this pseudo inverse just using 1 p pseudo. So, using that particular concept, we can always calculate the inverse of a particular matrix.

Now, we will just apply this concept to our power system signal, where we have voltage signal, we have current signal. So, how we can apply this particular phasor estimation based technique so, that we can calculate the phasor of the fundamental frequency component of the signal or the sub synchronous frequency component of the signal or the phasor of the sub super synchronous phasor of the particular signal or also sometimes decaying DC component which is present in a particular signal.

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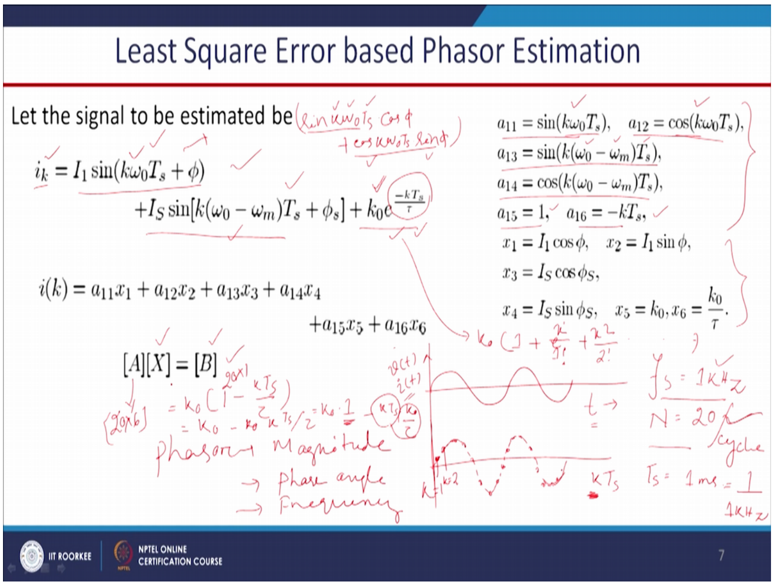

Now, I have just taken here one current signal. This current signal stands like i k this k stands for at kth instant of sampling we have considered the signal i k. We can also write if you could remember in the previous class, we have discussed the sampling of a particular analogue signal.

We are sampling at a particular frequency 1 kilo hertz or 2 kilo hertz 3 kilo hertz or 1 I mean depends or requirement. So, we have discussed 1 kilo hertz something frequent that f s is equal to 1 kilo hertz. In this case, we have this capital N the number of samples per cycle is equal to 20; number of samples per cycle we should write number of samples per cycle that is 20; this N this is per cycle.

Now, that is what if I have some analogue signal in time domain, this is basically in time domain and this is how this small v t stands for our small i t whatever you can designate its possible no problem; we can take the current signal also, we can take the voltage signal. So, here I have taken the current signal. So, let us concentrate on the current signal. This particular signal can be sampled to the discrete domain. The discrete domain means this k is equal to k into T s. This capital Ts is nothing the sampling time interval.

So, at what interval we are just picking up one sample from the analogue signal that is what here it is 1 millisecond because this is basically what 1 upon 1 kilohertz that is why this is 1 millisecond. After 1 millisecond, we will just catch one-one sample from the ADC that is the analogue to the digital converter. Now, so, in this manner; so, we have here the districtized signal, it looks like this. So, here we will get the sample values corresponding to the analogue values and this is and these are the discretized values that is what this k stands for. The k stands for at this is sample position that here the k is equal to 1 in this case k is equal to 2 and so on.

So, in a particular cycle this k stands for k is equal to 1, 2 k is equal to 20 for one particular cycle. As we have discussed that if I will just maintain the sampling frequency as 1 kilohertz. So, in this case my number of samples is equal to 20 samples per cycle that is what I just want to mention here this I k stands for the sample of the signal at kth instant.

So, I will just right here this is equal to I 1 this capital I 1 into sin k omega naught Ts plus phi. The first term of this particular equation is the fundamental signal; the fundamental frequency component of the signal and the second one this second term is I s sin k omega naught minus omega m into Ts. This is very very important term; already we have discussed that this particular phasor estimation based technique allows us to estimate the sub synchronous frequency component of the voltage or current signal and the last one is the decaying DC component that on also we can estimate using this phasor estimation based technique.

Now, if you will see I just want to decompose this three terms. So, that we will just in the process of formulating the list error square technique; where, we are interested to calculate our unknown values, unknown phasors. Already we have discussed phasor means the phasor means we have magnitude, we have magnitude, we have phase angle, we have phase angle. Yes of course, it is linked with one frequency component. The first term is basically the fundamental frequency component where this omega naught stands for the 2 I mean it is 2 phi f naught; this f naught is basically 50 hertz component. Now this first term, if you will see I have just decomposed using this formula of trigonometric equation that is sin a plus B that is sin into cos B plus cos into sin b.

So, in this manner the first one is decomposed and it is written here the known terms. The first known term to us is sin k omega naught Ts and the second one is cos k omega naught Ts. Let me right here, if you will just open it; it will be sin k omega naught Ts into cos phi plus cos k omega naught Ts into sin phi; just I am just opening this the first term, if you will write the trigonometric equation of the sin function sin a plus b.

Now if you will see this one here this sin k omega naught Ts is known, why? Because k is basically the sample position or sample number omega naught is also known to us that is 2 phi f naught and Ts is equal to 1 millisecond. Already we have discussed here that we are taking this sampling frequency as 1 kilo hertz that is why this Ts is also known to us that is 1 millisecond and the second 1 this cos phi is not known. The cos phi is unknown so, we have to estimate this cos phi that is the first variable for us.

Similarly, coming to the second term that cos k omega naught Ts is also known to us, but this sin phi is not known to us. This is the second variable. If you will similarly if you will just keep on decomposing I mean opening this particular also equation the term and also this one. So, finally, we will have 6 variables; 6 unknowns and 6 known quantities this that is what we have designated like a 1 1. The first one known quantity; this a 1 2 that is second one which is also known to us and third one, we name it a 1 3 that is also known to us because omega naught and omega m the fundamental frequency and the sub synchronous frequency component is also both of them are known to us and Ts is also known to us.

And similarly for this say a 1 4 is also known a 1 5, a 1 6 here I just want to discuss 1 point that how this a 1 5 is equal to 1 and a 1 6 is equal to minus k Ts. If you will see the last term of this equation that is k naught e to the power minus k Ts by tau that is basically the exponential term; exponential decaying this because it is a decaying DC component of the current signal. I just want to open this how does it look like? Here will do it, this is k naught if you will write e to the power x. So, how the equation look 1 plus e to the power x; I mean that is x e to the power 1 divided by x 1 1 factorial plus x square divided by 2 factorial so on.

So, in this equation what is this x stands for? The x is equal to nothing is our minus k Ts divided by tau. You know for this decaying DC component k naught is the gain and this tau is equal to decaying DC time constant and this T s is our sampling time period and this small k is the kth instant or sample position number. So, in this process we have two unknowns that is basically this k naught and the small tau this one. So, these are the unknowns and the small k and Ts unknown to us.

Now if I will just put this particular equation to this our decaying DC component based part of my current signal, now it will just behave like this. This is equal to I will write here k naught 1 minus k Ts divided by tau omitting the higher orders if will just omit the higher orders. So, our equation will look like this and further if we will go k naught minus k naught small k T s divided by tau.

So, that is why this if I will take basically this k naught as 1 variable, here we will come to this point; then in this process here this is equal to k naught into 1. So, this 1 is known to us; that is why this a 1 5 is equal to 1. And similarly here a 1 6 further minus k Ts whole into k naught divided by tau. So, this is basically known and this unknown and that is why this a 1 6 is equal to minus k Ts; that means, first we have the known values and second we have the unknown values; unknown parameters or unknown quantities.

In this process we have written this i k is equal to a 1 1 x 1 plus a 1 2 x 2 a 1 3 x 3 plus a 1 4 x 4 and so on and coming to this matrix form a into x is equal to b; that means, this x is basically the unknown variable matrix. The matrix of the unknown variables and this B is the measured signal; already we have discuss the mathematics of the list error square base technique that this m basically relates to this B matrix. Already we have discussed that m is measured signal matrix.

So, similarly here the B stands for the measured signal matrix and this a is also known to us that already we have modeled here; this signal is already modeled like this as if that signal this current signal contains the fundamental the sub synchronous frequency component and the decaying DC component. So, that is why this A matrix is also known to us.

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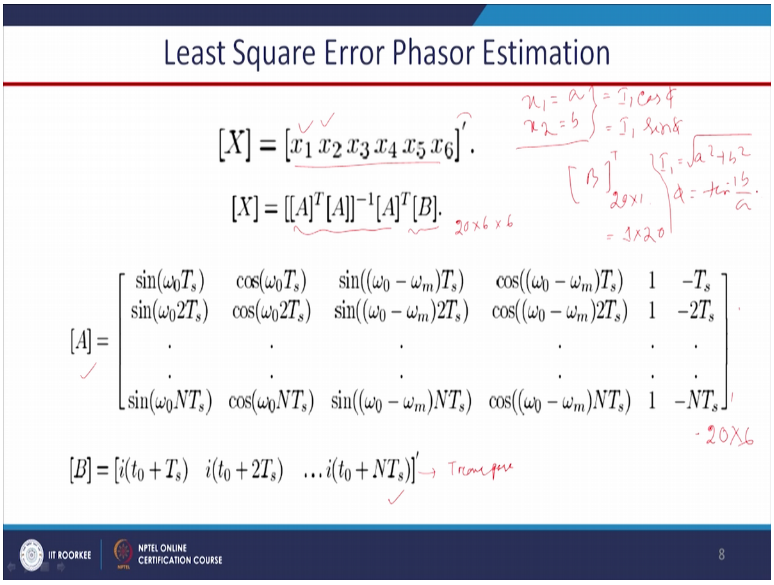

So, now by taking this X the unknown matrix; so, this is our pseudo inverse of A matrix into the measured matrix that is the measured signal matrix that is B. So, at every instant of time we are just taking the samples of the current signals. So, that is matrix is basically the B matrix. If you will see finally, this A matrix is presented here. So, the dimension of this A matrix is how much? We have as I said we have taken here 20 samples per cycle. We are taking here 1 cycle data; 1 cycle sample of the current signal. So, in this case it will go up to 20th row ok. So, in this case we have 20 cross 6; 20 number of rows and 6 number of columns.

So, that is why the size of this same matrix is 20 cross 6. What about this if we will come to here B matrix. This B matrix stands for all the samples we have collected with respective time k is equal to 1 2; k is equal to 20. So, we have all the samples of the current signal; we have measured at different time instant like t stands for 1 I mean some sample 1 millisecond, 2 millisecond, 3 millisecond, up to 20 milliseconds.

So, in this process this B matrix size will be much. So, it will be we have 20 number of rows it will be 20 cross 1 this B matrix and already we have discussed this A matrix size is 20 cross 6 and what about this x matrix size will be? So, this x matrix size if you will see here it will be 20 cross 6 into 6; we have to take the transpose of course. This is how this matrix stands for A matrix 20 cross 6 and this B matrix if I will kept this dot the symbol, it stands for transpose; transpose. The symbol stands for in the matrix formulation on matrix calculation the MATLAB program.

Now if I will do this. So, initial this B matrix if the B matrix size was basically 20 cross 1. So, it will just it will be opposite it will it will have 20 number of columns and 1 row. So, it will just transpose if it will take it will be 1 cross 20; multiplication purpose we are just doing this. For multiplication purpose, we generally took this transpose for easier calculation. Now, finally, we have this x matrix; this x matrix if you will see here we have 6 number of variables and the size of matrix if you will not take the transpose, it will be how much? It is just like a column matrix where we have 1 I mean 6 number of rows and 1 column. Now if you will take the transpose it will be just opposite to us and opposite to the previous number.

Now, this is all about the matrix formulation of this particular list error square based technique where we have applied the technique to estimate the fundamental components or we are going to calculate the sub synchronous super synchronous frequency component of the particular signal or we are going to calculate the decaying DC component of the signal. Now, you see how we are going to calculate this x 1 and x 2? After multiplying this A inverse into B. So, we will have this x 1 value, x 2 value, x 3, x 4, x 5 and x 6. So, on now after getting this, if I know this x 1 value let us say something about let us say and this x 2 is equal to b.

So, what about this your fundamental components; Now, this is the basically the fundamental frequency component phasor values we are going to calculate. If you could remember that this x 1 is basically I 1 cos phi 1 and this x 2 is equal to I 1 sin phi; that means, we now calculated we have estimated the fundamental frequency real part and the imaginary part this I 1 cos phi 1 is the real part and this x 2 that is I 1 sin phi is the imaginary part.

So, if you know this two components it is very easy to calculate the magnitude part as well as the phase angle part. So, if you could see this equation this I 1 is my fundamental frequency component magnitude and this phi is the basically the fundamental frequency phase angle. So, that is how we will calculate this is basically equal to I 1 cos phi and this is equal to I 1 sin phi and now what about this magnitude? This I 1 is nothing is equal to a square plus b square root over and this phi is nothing tan inverse of b upon a.

So, it is very easy to calculate all the frequency components, I mean using this particular technique and similarly also we can go for this x 5 x 6 where the decaying DC components are present. If you could see here this x 5 is nothing k naught if you got this x phi; that means, we know the basically the constant of the decaying DC gain of the decaying DC component and after knowing this k naught. So, if you will put in here so; obviously, we will get time decaying DC component time that is the tau.

So, it is very easy already we have discussed how to calculate the frequency the phasor estimations of different frequency components of particular signal. This is how we have demonstrated 1 signal by taking 20 samples this is how this A p looks like.

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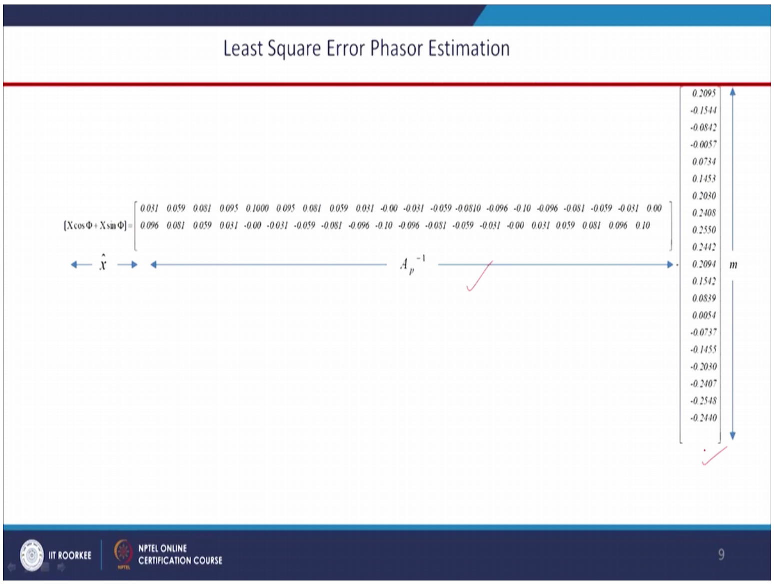

And this is how this measured signal looks like. So, after that we can always calculate this x cap; this x cap is basically the estimated parameters or estimated phasors.

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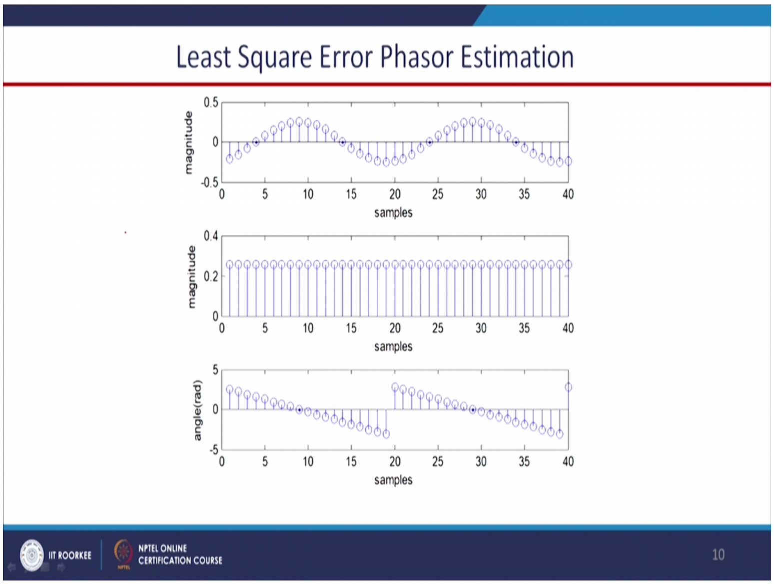

And this is one demonstration where we have taken the current signal and the corresponding angle; this is the magnitude and this is the angle of this particular signal.

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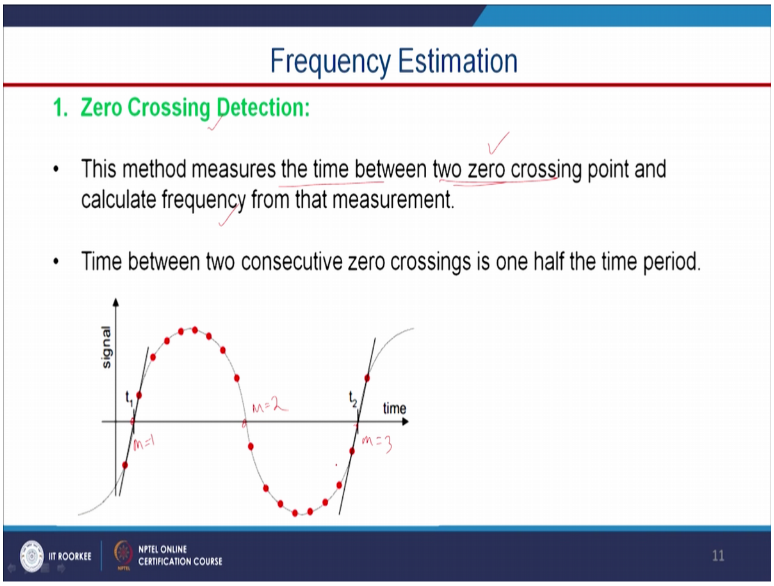

Now, the second part will come here that how to estimate the frequency that already we have discussed in our previous lectures, that the frequency estimation is also very very important in power system or even you can say in your smart grid environment where we have of course, every time we are going to basically have this frequency measurement or frequency estimation. If you could remember in our in case of our wide area measurement based technique where the frequency estimation is one of the important technique you know and for this islanding detection also we need this frequency estimation based technique.

So, that is why we will discuss two techniques where we will see how the frequency can be estimated of a particular current signal or even for a voltage signal. The first technique is the zero processing based technique; it is very easy one, but the technique is erroneous when the signal will have some basically noise. The noise will create problem.

This method measures the time within two zero crossing; this is important. This particular method measures the time between two zero crossing point and calculate the frequency from the measured value. This is the basically the concept of this particular technique. First of all we have to detect the consecutive zero crossing time and after getting the time period, we can always calculate the frequency that this is how the equation looks like.

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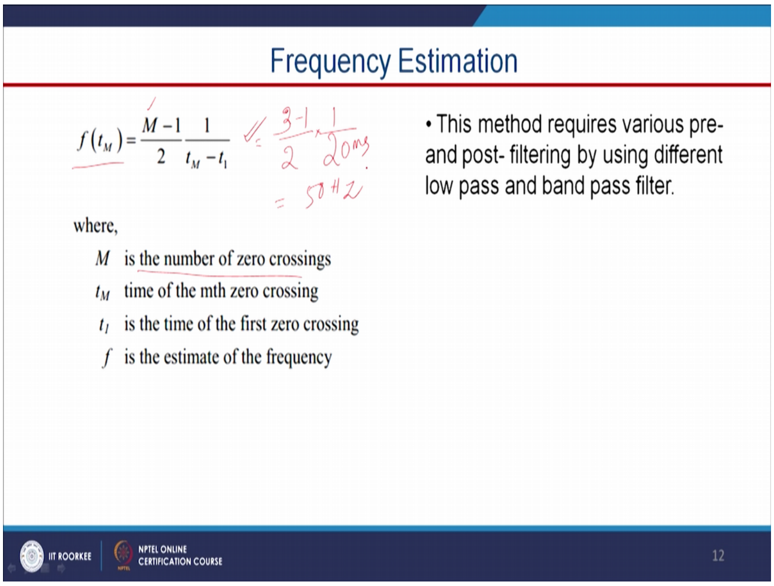

This f of tM because this frequency is dependent on time factor we have to basically the measure the time of zero crossing; at what time the first zero crossing has occurred and next what is the time where the next zero processing has occurred so, that is so on. So, we have to basically the measure the time that is why I have written here f of t M the frequency is a function of time. What is this M stands for this? M is the number of zero crossings. If you could see here previous equation I mean the diagram signal see here the equation is M minus 1 by 2 whole into 1 divided by t M minus t 1. In the previous equation, if you see this signal we have started here; that means, M is equal to 1 this is my first zero crossing.

Here, M is equal to 2 and here M is equal to 3. So, we have seen 3 zero crossings. So, by putting this M is equal to 2. So, I can always right to this is 3 minus 1 divided by 2 whole into 1 upon this t M minus t 1; starting time and the end time; that means, we have started here in this case the whole time say 1 cycle time period 20 milliseconds. The difference between these two; it should be equal to 20 milliseconds. That is why it will be 20 milliseconds. So, in this case; that means, we are going towards that is our capital I mean f the fundamental frequency that will be 50 hertz. So, this is how we basically calculate the frequency of a particular signal by detecting the zero crossing.

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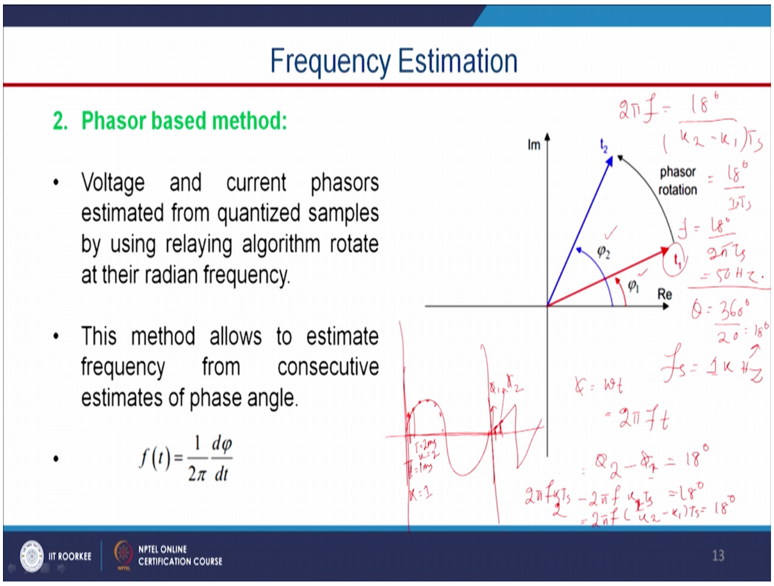

Next we will go for phasor estimation based technique. I mean this is basically from the phasors we are going to calculate the frequency of a particular signal. If we know the phasors basically of the voltage or current signals; then always we can calculate the frequency. Let us say at certain time t 1, we have the phase angle of a particular voltage or current rate phi 1 and the time t 2 the phase angle is basically phi 2; take the difference because you know that our phi is equal to omega t; omega t is how we will write this is 2 phi f into t; If suppose, we have a certain signal at certain time at time t is equal to 1 millisecond.

So, here at this point Ts is equal to 1 millisecond and ours let us k is equal to 1; the sample position. We have samples, we have samples I mean the phasors basically the phasors corresponding samples and corresponding angle basically we have taking here. So, at time t is equal to 1 millisecond and k is equal to 1 and next if you will go. So, t is equal to 2 millisecond and k is equal to 2 if we will take the angle of this particular signal. So, angle looks like this. The first angle at k is equal to 1 and this is a second angle. Thus, we are basically calculating the phase angle and the magnitude of that particular signal using the phasor based technique. So, from the phasors phase angle, we are going to calculate the frequency. The first phase angle is phi 1 and this is phi 2.

Now, what is the difference between these two phi 2 minus phi 1 is equal to 18 degree if we will take the sampling frequency f s is equal to 1 kilo hertz. How it is matter, what comes like this? Because you know the total signal angle theta is equal to 360 degree and how many samples we have 20 samples. So, if we will divide by 20 it comes to be 18 degree between 2 consecutive samples. Yes, remember this sampling frequency affects the phase angle difference. Now this 1 kilo hertz leads eighteen degree phase angle difference between 2 consecutive samples. Now if I know this theta 2 minus theta 1 is equal to 18 degree and what about this theta 2 minus theta 1 that is 2 pi f what about this theta 2? The theta 2 is equal 2 pi f k Ts this is k 1 Ts minus 2 pi f k 2 Ts is equal to 18 degree.

So, this should be 2 this should be 1. Now if we will take command 2 pi f whole into this particular k 2 minus k 1 whole into Ts is equal to 18 degree. Now just to divide now this 2 pi f is equal to 18 degree divided by this k 2 minus k 1. The consecutive signal I mean the positions differences always 1; that means, it is equal to 18 degree divided by 1. Now this frequency is equal to 18 divided by 2 pi. So, here we missed 1 term that is the Ts, here it is this Ts will come here into Ts 1 into Ts. Now it leads to 50 hertz. So, this is a very very important concept where the frequency of the particular signal can be calculated using the phasor based technique. If you know the phase angle of a particular signal; So, consecutive phase angle we have to just track and by taking the difference between these 2 divide by this 2 pi and Ts.

So, we can always estimate the frequency of the signal. So, in this particular lecture, we have discussed very clearly the difference between this DFT and the least error square based phasor estimation based technique, that why we are interested for this phasor estimation technique based on the least error square. Because, here we are the major advantage like we can extract the phase angle or magnitude of the sub synchronous and super synchronous frequency components of the signal. And also we can calculate the decaying DC component of the signal like gain and the time constant. And also we have discussed very clearly the frequency estimation of the signal using two techniques that is zero crossing based technique and also phasor based technique.

Thank you so much.