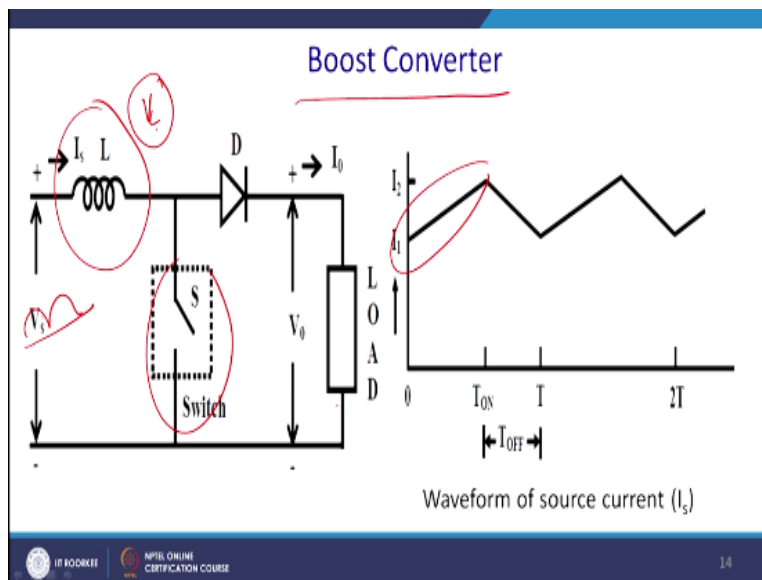


Advance Power Electronics and Control
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Lecture - 20
Non-Isolated DC-DC Converters – II

Welcome to our NPTEL courses on Advanced Power Electronics and Control. We shall continue with the non-isolated DC to DC converter. We are discussing about with the Boost converter, so we continue with that Boost converter.

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So in a Boost converter essentially you will be boosting the voltage and since actually once switch is ON so current input current will ramp on through the switch and when switch is OFF then there will be a stored inductance; there may be a value voltage V_L will appear across it. And this voltage and V_s will basically fit the load and thus you will find that the voltage applied across this load has enhanced.

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Boost Converter cont.

- In this type of converter circuit, inductor is connected in series with supply and switch is in parallel with supply. ✓
- Diode is connected in series with load.
- Here load inductance is assumed to be very small.
- Switch 'S' is ON at $t = 0$. During the interval $0 \leq t \leq T_{ON}$, $V_0 = 0$.
- The current from source i_s flows in the inductance L . In this interval value of current increases linearly with time. So di/dt is positive.
- So equation for circuit becomes-

$$V_s = L \frac{di_s}{dt}$$
$$\frac{di_s}{dt} = V_s / L$$

- 'S' is OFF during the period $T \geq t \geq T_{ON}$. Current through inductor decreases with same direction in circuit.



So this type of converter circuit, inductor is connected in series with a supply and switch is in parallel with the supply. Diode is connected in series with the load to avoid back EMF. Here the load inductance is assumed to be very small. Switch S is ON and $t=0$ during the interval 0 to t_{ON} and output voltage should be equal to 0 because you have shorted switch.

The current on the source I_s , flows in the inductance L . In this interval value of the current increases linearly so since di/dt is positive and the equation is $V_s=L di/dt$, ignoring that actually switch is ON, the drawback that the switch is almost 0 , so you will get V_s/L as ramp of this di/dt . So when switch is OFF during that period actually t is less than T to T_{ON} the current through the inductance decreases with the same direction of the current. This is a DC to DC converter.

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

Boost Converter cont.

- The output voltage is considered to be nearly constant.
- The voltage equation of the circuit can be

$$V_s = V_0 + L \frac{di_s}{dt}$$

$$\text{so } \frac{di_s}{dt} = \frac{V_s - V_0}{L}$$
- As shown in waveform, current varies linearly from I_1 (I_{\min}) to I_2 (I_{\max}) during T_{ON} interval.
- $I_2 - I_1 = I_{\max} - I_{\min} = \left(\frac{V_s - V_0}{L}\right) T_{\text{ON}}$
- Similarly current varies linearly from I_2 (I_{\max}) to I_1 (I_{\min}) during T_{OFF} interval. So $\frac{di_s}{dt}$ can be expressed as

$$I_2 - I_1 = I_{\max} - I_{\min} = \left(\frac{V_0 - V_s}{L}\right) T_{\text{OFF}}$$
- Equating both $\frac{di_s}{dt}$ equations





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So ultimately the $V_s = V_0 + L \frac{di}{dt}$ so here you will find that actually $V_s \cdot dt = V_0 \cdot dt + L \cdot di$. So as shown in the waveform, the current varies linearly I_1 to I_2 that is I_{\min} to I_{\max} during the T_{ON} interval that is actually $I_2 - I_1 = I_{\max} - I_{\min}$ that is essentially $V_s/L \cdot \text{time ON}$ that duty cycle when switch is ON. Similarly, current varies linearly from I_2 to I_1 during the T_{OFF} so di/dt can be expressed as follows, so $I_2 - I_1 = I_{\max} - I_{\min} = V_0 - V_s/L \cdot T_{\text{OFF}}$. So we can equate both the equation $I_2 = I_1$.

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Boost Converter cont.

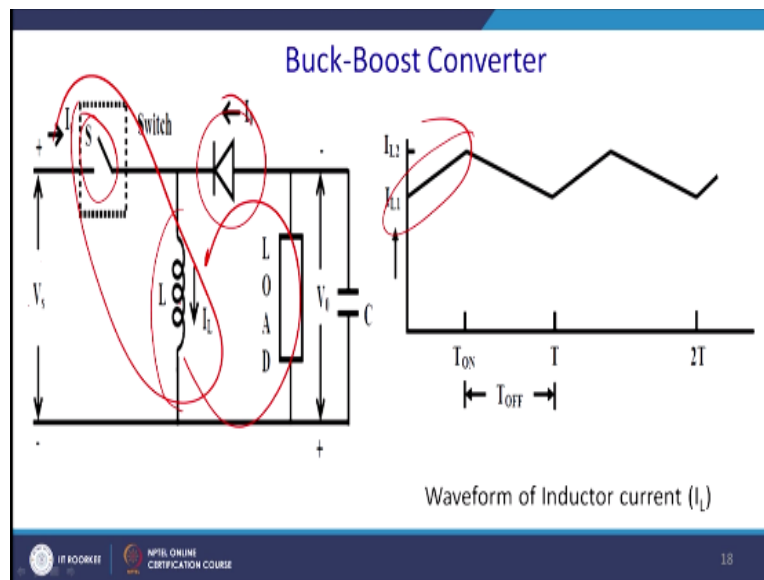
- $V_0 = V_s \left(\frac{1}{1-k}\right)$
 where $k = T_{\text{ON}}/T$
 $T = T_{\text{ON}} + T_{\text{OFF}}$
- Here source current is assumed to be continuous.
- Output voltage is higher than input voltage, so it is called Boost Converter.
- If instead of self commutated device, thyristor is used, it is called Step-Up Chopper.



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And ultimately we can get actually $V_0 = V_s / (1 - k)$ and since $k = T_{\text{ON}}/T$ you can substitute that value actually sometime it is written as $1/(1 - d)$, where d is also same as k . So the source current is assume to be continuous and it is in a senior mode. Output voltage is higher than the higher than

input voltage, for this reason this is called Boost converter. Instead of the commutated device thyristor is used, it is called the Step-Up Chopper. So we can see that when actually it is when we require a regenerative braking the DC motor so we use this kind of application called Step Up Chopper.

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So now let us come to the another topology that is whether you can step up as well as step down. There will be position change, please note that. Change of the position of the switch. So we have to have a we can; so when you have to switch will be in this position in series with the load and inductor will be placed you know, when actually in parallel to the load and thus when it is switch is ON essentially this gives you the Boost topology, because here there is a inductor and there is a switch, and here it is just interchanged.

The another point to note that this actually polarity of the diode. Polarity of the diode is reverse, in Boost topology you will find that polarity is just opposite. So how does it work? You know, you may require to boost the voltage or you may require to buck the voltage depending on the different kind of requirement.

Mostly it is required in case of the variable DC supply and you are going to feed a constant DC voltage, and that kind of applications we may require to Buck-Boost, Buck and Boost the DC voltage. So what happen initially you know, if it is switch ON this is same as your boost

operation and thereafter you know, you will have when it is switch OFF the energy stored into the inductor will feed and ultimately current will flow through like this and then essentially we will have a buck operation.

In recall this actually switch OFF part of the buck you will the same topology. But here this inductor was placed. Here a diode was placed in this position thus this has been interchanged. So you just couple this two and ultimately you come over with this topology called Buck-Boost. Let us see that how you have a principle of operation of the Buck-Boost topology.

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Buck-Boost Converter cont.

- In this converter circuit, inductor is used in parallel with input supply. Diode is connected in series with load.
- Capacitor 'C' is connected in parallel with load.
- Polarity of output voltage is in opposite of input voltage as shown in fig.
- When 'S' is ON, supply current i_s flows through V_s , S, L during interval T_{ON} .
- Current through both source and inductor (i_L) increases. So di_L/dt is positive.
- Polarity of induced voltage is same that of input voltage. So the equation of the circuit can be

$$V_s = L \frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s}{L}$$

- When 'S' is OFF, inductor current tends to decrease with $\frac{di_L}{dt} \rightarrow$ Negative

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In this converter circuit, inductor is used in parallel with the supply, please note that. So in Boost topology it is nothing but a boost topology when the switch is ON but position of the inductor and the other switch has been interchanged. Diode is connected with the series with the load but polarities opposite. Generally, you require a constant DC voltage to hold the DC voltage you will have a capacitor is connected with the parallel to the load which is not shown in the previous period.

Polarity of the voltage is an opposite of the voltage as shown in the figure previous figure. When switch is ON the supply current I_s flow through V_s , I_s L during the interval T_{ON} . The current through both source and the both source and inductor i_L increases. So di/dt is positive. Polarity is induced, polarity of induced voltage is same that of the input voltage. So the equation of the

circuit can be $V_s = L \, di/dt$, so $di/dt = V_s/L$, when S is OFF inductor current tend to decrease with the slope and that is basically then it will be $-di/dt$. So this is the case.

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Buck-Boost Converter cont.

- Polarity of V_0 is opposite of V_s . The current path is through L, parallel combination of load, C and diode during T_{OFF} interval.
- The output voltage remains nearly constant as capacitor is connected across load. So equation of circuit becomes

$$L \frac{di_L}{dt} = V_0$$

$$\frac{di_L}{dt} = \frac{V_0}{L}$$
- Inductor current varies linearly from i_{L1} to i_{L2} during T_{ON} time interval.
- i_{L1} is considered as (i_{Lmin}) and i_{L2} is considered as (i_{Lmax}) .
- So $\frac{di_L}{dt}$ expression can be written as

$$i_{L2} - i_{L1} = \left(\frac{V_s}{L}\right) T_{ON}$$



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Essentially, let us see. The polarity of DC voltage is opposite of the V_s and the current path is through the inductor L, parallel combination of load, C and the diode during the Turn ON interval. The output voltage remains nearly constant as a capacitor is connected across the load will hold the voltage and so the equation becomes $L \, di/dt = V_0$ and thus $di/dt = V_0/L$, inductor current varies linearly with i_{L1} to i_{L2} during the T_{ON} interval; i_{L1} is considered as i_{Lmin} and i_{L2} is considered as i_{Lmax} same in case of the Boost converter. So di/dt expression can be rewritten as $i_{L2} - i_{L1} = V_s/L * T_{ON}$.

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Buck-Boost Converter cont.

➤ Similarly during T_{OFF} interval, inductor current varies from i_{L2} to i_{L1} . So $\frac{di_L}{dt}$ expression can be

$$i_{L2} - i_{L1} = \left(\frac{V_0}{L}\right) T_{OFF}$$

➤ Equating both $\frac{di_L}{dt}$ expressions

$$V_0 = V_s \left(\frac{T_{ON}}{T_{OFF}}\right)$$



$$V_0 = V_s \left(\frac{k}{1-k}\right) = \frac{0.6}{0.4} = \frac{3}{2}$$

Where $k = \frac{T_{ON}}{T}$

$$T = T_{ON} + T_{OFF}$$

$$= \frac{0.8}{0.2} = 4$$

➤ From the output voltage expression, it is observed that for $0.5 < k \leq 1$. Output voltage is lower than input voltage. So it acts as a buck converter.



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So similarly we shall write the equation for the T_{OFF} interval. So inductor current varies from i_{L2} to i_{L1} so $L \frac{di}{dt}$ can be expressed as; sorry di/dt can be expressed as $i_{L2} - i_{L1} = \frac{V_0}{V_L} \cdot T_{OFF}$ and equating both the equations you know $V_0 = V_s \frac{T_{ON}}{T_{OFF}}$ similarly again you can write $V_0 = V_s \frac{k}{1-k}$ where $k = \frac{T_{ON}}{T}$, so you know that $T = T_{ON} + T_{OFF}$, so ultimately you get this expressions as the transfer function.

So $V_0 = V_s \frac{k}{1-k}$ such duty ratio 0.5 so you can find that so output and the input are the same. So for duty ratio less than 0.5 it will operate as a Buck converter and duty ratio have a 0.5 it will acts as a Boost converter. But mind it there is a this ratio actually increases quite non-linearly, because you know actually you can take the value of 0.6, so what will be actually the value of this transfer function?

So it is $0.6/0.4$, so it will be actually $3/2$. If it is 0.8 then it is $0.8/0.2$ it is 4. So it will increase quite drastically. So this is the one of the actually challenge. Increase of the voltage is not propositional with the duty cycle. From the output voltage expression, it can be observed that the value of k , if it is 0.5 to 0 the output voltage is lower than the input voltage so it will acts as a Buck converter.

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Buck-Boost Converter cont.

- From the output voltage expression, it is observed that for $0.5 \geq k > 0$ Output voltage is lower than input voltage. So it acts as a Buck Converter.
- When $0.5 < k \leq 1$, output voltage is higher than input voltage. So it acts as a Boost Converter.
- Therefore for different 'k' value, the converter is considered as Buck-Boost Converter.
- Here inductor current is assumed to be continuous.
- Also it can be termed as Step up/down chopper.

Similarly, if it is above it so output voltage expression is observed that for k more than 0.5 to 1 it will be actually using as a Boost converter. So it is observed that for k less than 0.5 and greater 0 output voltage is lower than the input voltage so it will be a Buck converter. When actually k is less than 0.5 and 0 output is 1; the output voltage is higher than the input voltage so its acts as a Boost converter.

Therefore, the different value of the; different k value of the converter is considered as a for the Buck-Boost converter. Here inductor current is assumed to be continuous also it can be termed as a Step up, step down chopper since it can have a Buck-Boost properties. Now we shall see that how will control less output voltages.

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Control Strategies

➤ In all three types of DC-DC converter, average value of output voltage can be varied.

i) Time-Ratio Control

ii) Current Limit Control

Time-Ratio Control

➤ Here value of duty ratio $k = T_{ON}/T$

➤ It is also classified into two types:

a) Constant Frequency Operation

b) Variable Frequency Operation

Constant Frequency Operation

➤ In this control strategy ON time T_{ON} is varied keeping frequency ($f=1/T$) or whole time period is constant.

➤ This is also called pulse width modulation control.



So all the three types of DC to DC converter, average value of the output voltage can be varied by Time-Ratio Control we will see that, we have already discussed some control strategies in case of the PWM converter. And also the Current-Limit control. So Time-Ratio Control is essentially the control of the duty cycle. And we will assume that if it is not stated current is continuous. So our assumption is quite simple, the load current is continuous.

So we shall discuss later and otherwise we will state that it is a discontinuous mode of operation. Here the value of the duty ratio $k=T_{ON}/T$. And of course this; we have rewrite the expression we have taken the continuous load current. It is also classified into two types; Constant Frequency Operations and the Variable Frequency Operation.

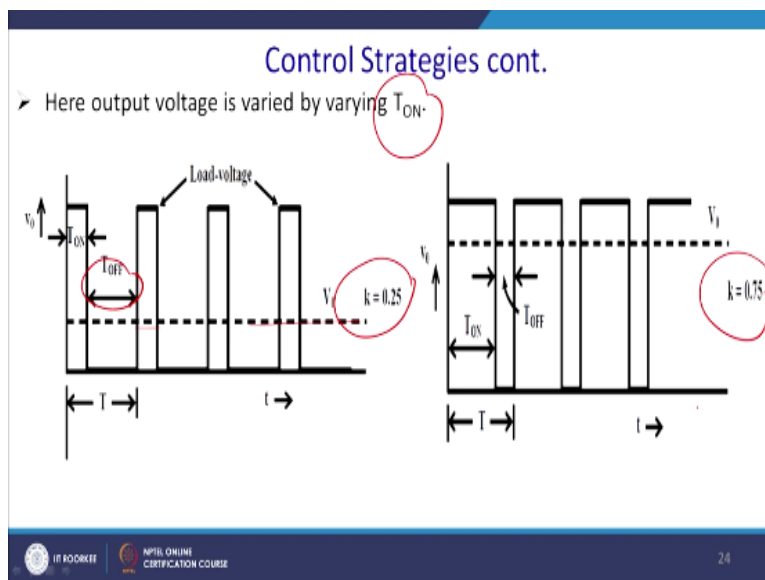
So in the constant frequency operation, frequency of this converter remains constant, so you can you have ease of designing the circuit. What happen you know you can choose a particular switch because you know that actually, if you have a higher switching frequency then you require to use MOSFET but it has to require reduce the power rating. So choosing a switch thereafter choosing the frequency all those issues will comes.

And if you know then actually the constant frequency is constant then design will be easier for choosing the switch as well as design of the inductor and capacitor. But you will pay penalty somewhere else we will see that actually the stability in other issues. Another is the Variable

Frequency Operation. So $(\)$ (15:10) is the controller and other controller the variable frequency operations there you will change the duty cycles that will change the frequency and you may have a constant T_{ON} also.

So there you can have change the frequency and we will have a actually variable frequency operation. In Constant Frequency Operation, in this control that is constant frequency operation strategy T_{ON} is varying keeping the frequency over a frequency kept actually a $1/T$ constant or whole time period is constant. And this is also called pulse width modulation control.

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Now what happen, this is the load voltage and you will actually; this is the value for $k=0.5$; 0.25 and this much will be the T_{ON} and this much will be the T_{OFF} , so ultimately this will be the average voltage and this is for the variation of the T_{ON} , T_{ON} is varying from 0.75 and you can see that this is the pulses. So this is called constant frequency operation with varying T_{ON} .

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Control Strategies cont.

Variable Frequency Operation

- In this control strategy, the time period 'T' is varied keeping either T_{ON} or T_{OFF} constant.
- It is also called frequency modulation control.
- The output voltage is varied by varying duty ratio $k = T_{ON}/T$

Disadvantage of Variable Frequency Operation

- Here to control the output voltage, frequency has to be varied in wide range, for which filter design will be difficult.
- In this control strategy, to control duty ratio and frequency variation would be wide. In this case there is possibility of interference of some other frequencies.
- Large OFF time of this control technique can lead to discontinuous current, which is undesirable.
- Therefore "constant frequency control strategy" is more desirable.

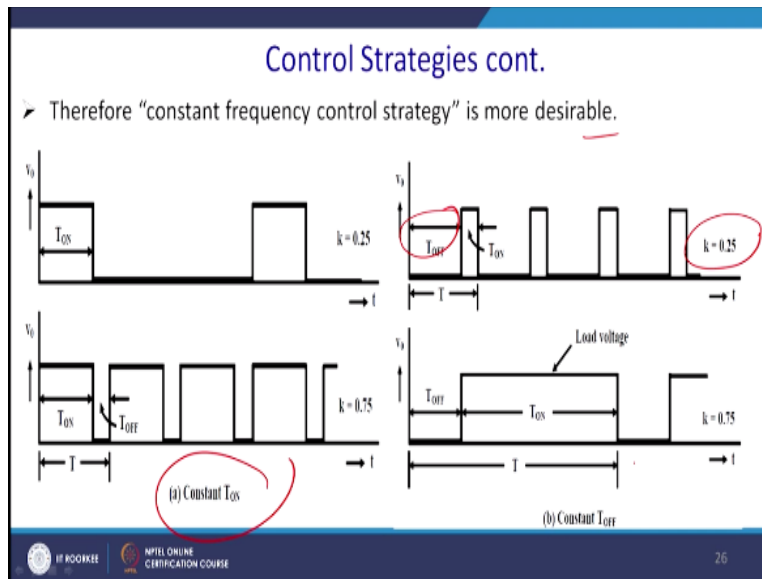


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Now let us come to the Variable Frequency Operation. In this control strategy the time period T is varied either keeping T ON or the T OFF constant. And it is called also the frequency modulation control. The output voltage is varied by varying the duty ratio T ON/T or T OFF/T whichever is a constant. So there are few disadvantage of variable frequency operation because I told you know, we require to design the converter for the particular frequency depending on the switch and the inductor and the other devices.

So that becomes a challenge. Here control the output voltage, here to control the output voltage frequency has to be varied in the wide range, and for which the filter design will be difficult. For this control strategy to control the duty ratio frequency variation would be wide. In this case, this possibility of interference of some other frequencies also come into the picture and ultimately you may have a high spectrum and it may interfere with the other high frequencies component and that may give you the actual issues of the EMI, EMCs.

And large OFF time of this control technique may lead to the discontinuous current, and which is undesirable, current become picky and you have a high EMI/EMC problems and whatever expression we have derived that nothing is application in case of the discontinuous conduction mode, we require to recalculate all the expressions and the duty cycles. And therefore, constant strategy control is more desirable. But sometime we will require to force use a variable frequency operation because of ease of control.

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So what happen, therefore, control frequency control strategies is more desirable. So this is actually constant T ON so you make the T ON constant and you vary the frequency. And accordingly actually the voltage changes and here you vary you make actually T OFF constant and you vary the T ON the duty cycle changes. So these are the two strategies for the variable frequency operation.

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Control Strategies cont.

Current limit Control

- In all three types of DC-DC converters, current is assumed to be continuous.
- In "current limit control" strategy, the switch in DC-DC converter turned ON and OFF so that current is maintained between upper and lower limit.
- When current exceeds maximum limit, 'S' is turned OFF and when current goes down below minimum limit, 'S' is turned ON.
- This type of control is either possible with constant ON time (T_{ON}) or constant frequency (f).
- This type of control is only possible, when load is inductive.
- This type of control strategy reduces ripple of current with increasing switching frequency and switching losses.

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Now we come to the another control strategy that is called a Current limit Control. Please note that you got di/dt which is positive in case of the Boost converter, so current is ramping on. In any case, when your current is ON when switch is ON all those DC to DC converter we have

discussed current ramps on. And current is ramp on through the switch. So we require to actually limit the switch because switch has a limited capability of carrying current.

So that is said to be the current limit control. So you will control, you will allow to current to ramp on and it will actually transfer the energy and till it hits the current limit you will switch off it. So automatically you will what will have you essentially these two methods is deals with the output voltage. It does not take any input of the input current.

While changing this actually these frequencies you know what happen your while changing this actually the T ON time keeping T constant you may actually cross the limit of the current which has been prescribed by the switch we have chosen. So and you may ultimately you may lead to the damage of the switches. But in this case, you know there is no possibility of the damage of the switches because you are basically protecting the devices from the having high (()) (21:29) current and you are controlling the current.

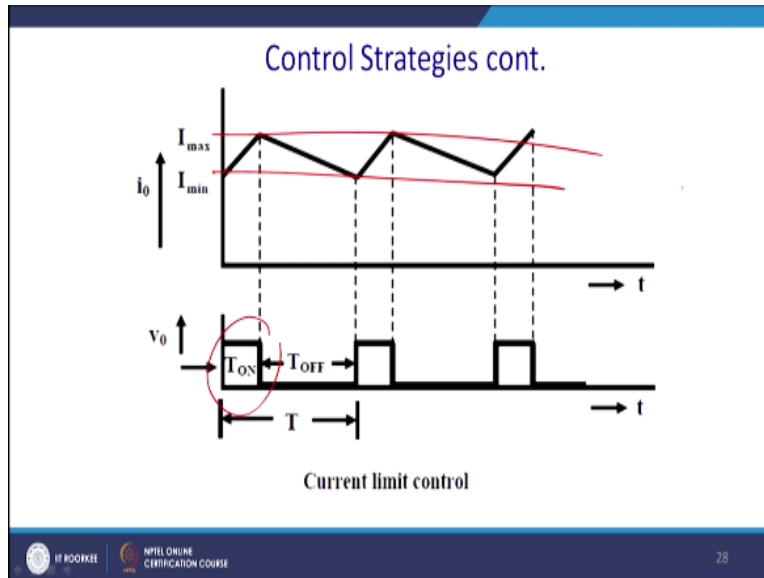
All types of DC to DC converter current is assumed to be continuous that is what our thumb rule. The current limit control strategy, the switch of the DC to DC converter when it is turned ON and turned OFF that maintains the current actually switches between as we have seen, upper and the lower limit of the current, that is basically maintain between the upper and the lower limit. When current exists the maximum limit S then switch will turn ON and when current goes down to the below the minimum limit it will turn ON it S will operate in a band of the hysteresis.

Like you will see AC will turn ON because if you are setting the switch let us say 25 degree centigrade, when it will turn ON let us say it is 25.2 again it will continue to chill down and while the actually your temperature goes below let us say 24 point it will again turn ON. So the bandwidth is basically 4 degree. So it is something like this. This type of control is either possible with the constant T ON or constant frequency.

We can do either of it. So generally, we will prefer constant frequency. So we will have a ease of designing the component as well as choice of the switches. The type of control; this type of control is possible when load is inductive. So please note that when actually you have this issues

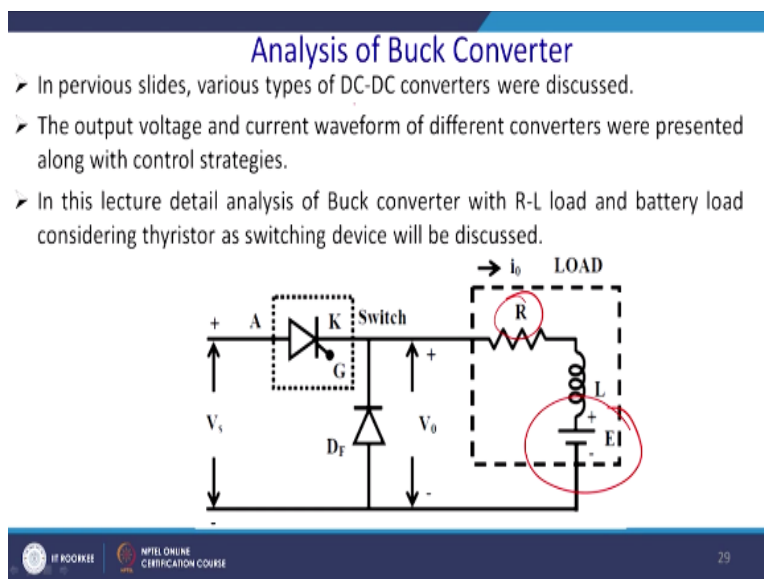
are coming to the picture. And this type of control strategy reduces the ripple current with increasing this switching frequency and the switching losses.

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So ultimately you will have a i_L and the I_{max} . You will be restricting it to the I_{min} and I_{max} and ultimately you will have the switches that is I_{max} and I_{min} so T_{ON} will be there when this ramp on; once actually it is sitted to I_{max} come down and it will be following like continuously it will follow.

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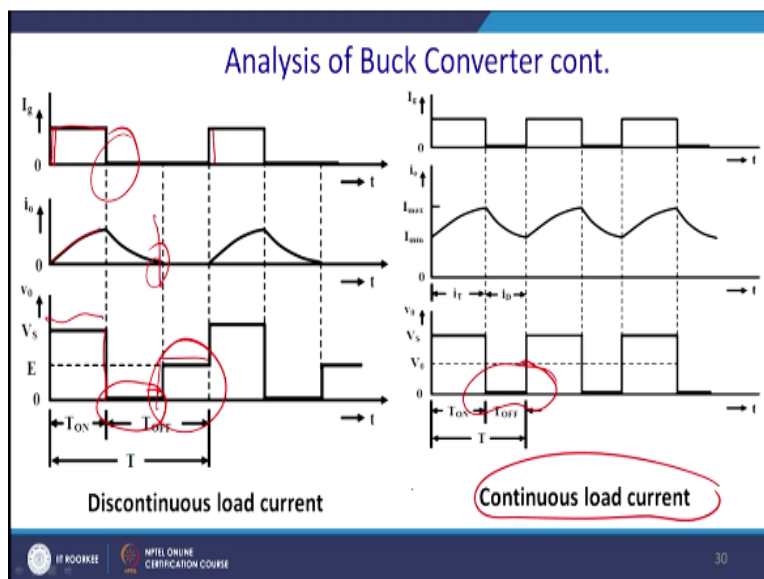
Now while discussing this control strategy we shall apply this into the Buck converter. And it is quite relevant you know we have R-L load it may be so then the Buck converter or the actually

the Buck chopper its feeding a DC motor separately excited DC motor so where this is basically R can be the armature resistance and this is the inductance part of it, it is a feed part of it that is giving to the LA and you got a back Emf because motor is working and gives you a constant voltage, so we can have something like this.

Or you may be searching the battery. So that kind of application also we can model as a R-L load. So we have already discussed various type of DC to DC converter now let us discuss the Buck converter. The output voltage and current of the DC to DC converter and these strategies already been discussed.

So in this lecture detail analysis of the Buck control converter with R-L load, R-L and the bank Emf or the battery is been considered and Thyristor as a switching device has been discussed. So and this since it is a Thyristor actually we require to have a force commutation otherwise it will continue to flow.

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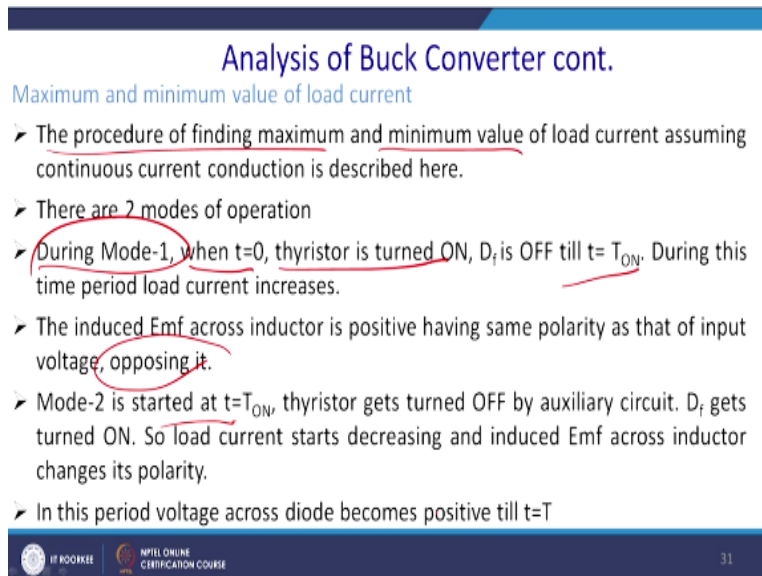
So you give a get pulse and if it is actually and it will trigger it ON thereafter, you may off basically the you may off basically the get current. And current will ramp on. But if it is instead of this Thyristor if it is other devices like IGBT you require to actually give a continuously get voltage or get current.

So current will flow like this and accordingly actually the load will get the voltage V_s and where this dotted line corresponds to the value of the back Emf or the battery voltage. So here you have triggered it off by forced commutations or if it is IGBT then by actually the switches by withdrawing the gate pulses.

Then what will happen then voltage; load voltage will come down and ultimately since you got a freewheeling diode ultimately current will take this shape and no voltage will be appearing to it. And if it is actually commutated; it is actually discontinuous mode of conduction and current become 0 at this point then again we have a motor inertia that will be in second and this phenomenon are in microsecond; sorry this phenomenon is in millisecond or this is again the voltage will be build up to the value of E .

Again, this thyristor is triggered and ultimately current will flow like this. Please note that this is basically the current to the thyristor. So if you have a; if the load current is continuous then actually back Emf will be not manifested in the load or the battery voltage will not be manifested, you got this kind of waveform in a continuous load current and this will be the discontinuous load current.

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Analysis of Buck Converter cont.

Maximum and minimum value of load current

- The procedure of finding maximum and minimum value of load current assuming continuous current conduction is described here.
- There are 2 modes of operation
- During Mode-1, when $t=0$, thyristor is turned ON, D_f is OFF till $t=T_{ON}$. During this time period load current increases.
- The induced Emf across inductor is positive having same polarity as that of input voltage, opposing it.
- Mode-2 is started at $t=T_{ON}$, thyristor gets turned OFF by auxiliary circuit. D_f gets turned ON. So load current starts decreasing and induced Emf across inductor changes its polarity.
- In this period voltage across diode becomes positive till $t=T$

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So let us see what we have observed. The procedure of finding the maximum and the minimum value of the load current assuming the continuous conduction mode is required to be discussed

here and we will find out the equations based on the continuous conduction mode or critical conduction mode.

There are two modes of operation during mode 1 when $t=0$ thyristor is ON, Df is OFF till $t=T$ ON. During this period the load current increases. The induced Emf across the inductor is positive having the same polarity as that of the input voltage, opposing it. Mode-2, mode-2 is started by commutations of the thyristors and $t=T$ ON, thyristor get turned OFF by the auxiliary circuits; we have auxiliary commutation circuits.

And Df get turned ON and so the load current is start decreasing and the induced Emf across the inductor changes its polarity and this create voltage across the diode becomes positive till $t=T$ or the complete of the duration. So we shall write the equation for the T ON.

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Analysis of Buck Converter cont.

Mode 1: The equation for the load (output) current in the circuit during this time interval, $0 \leq t \leq T_{ON}$ is,

$$V_s = Ri_0 + L \frac{di_0}{dt} + E \quad \text{or. } V_s - E = Ri_0 + L \frac{di_0}{dt}$$

The current is the load current, same as the source current during this time interval. The values of the load current (i_0) at $t=0$ and $t=T_{ON}$, are I_{min} and I_{max} respectively. The expression for the load current is, $i_0 = Ae^{-t/\tau} + B$

where A and B are constants, and time constant is, $\tau = L/R$.

At $t=0$, $i_0 = A + B = I_{min}$

At $t=\infty$, $i_0 = B = [(V_s - E)/R]$

So, $A = I_{min} - [(V_s - E)/R]$

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When switch is ON, the sequence of the load current during this interval is $V_s = Ri_0 + L di_0/dt +$ back Emf, so you can take back Emf or the battery voltage in this part and you can rewrite the equations. And we can solve this equation ultimately it will be a first order differential equation and the first order any differential equation are the initial value problem, so to find it out the constant A and B.

The current in the load, current is as same as source current during this interval. The values of the load current i_0 at $t=0$ and at $t=T$ ON are I_{min} and I_{max} respectively. So at the beginning it was I_{min} so it will turn it, it will go up to the I_{max} , so ultimately we can find it out $t=0$ such as $A+B=I_{min}$ and at $t=\infty$ that value= I_0 that value equal to B so $B=B R$. And so A become $I_{min}-B E/R$.

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Analysis of Buck Converter cont.

Substituting the values of A & B, the expression for the load current is,

$$i_0 = \left[\frac{(V_s - E)}{R} \right] (1 - e^{-t/\tau}) + I_{min} e^{-t/\tau}$$




At $t = T_{ON}$, $i_0 = I_{max}$. So,

$$I_{max} = \left[\frac{(V_s - E)}{R} \right] (1 - e^{-T_{ON}/\tau}) + I_{min} e^{-T_{ON}/\tau}$$

or, $I_{max} - I_{min} e^{-T_{ON}/\tau} = \left[\frac{(V_s - E)}{R} \right] (1 - e^{-T_{ON}/\tau})$

Mode 2: The equation for the load (output) current in the circuit during this time interval, $0 \leq t \leq T_{OFF}$ is,

$$0 = R i_0 + L \frac{d i_0}{d t} + E \quad \text{or,} \quad -E = R i_0 + L \frac{d i_0}{d t}$$




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So we can substitute this results so the whole equation becomes $I_0 V_s - E/R 1 - t/\tau +$ where τ is the actually T ON. So $I_{min} E$ to the power t/τ . So at T ON, so $I_0 = I_{max}$ so we can substitute that value of the I_{max} here $V_s - E/R 1 - t/$ instead of that τ is a substitute T ON, so you get this equation $I_{max}/I_{min} = V_s - E/R 1 - T$ ON/ τ . So this is the equations for mode-1. Mode-2 just actually it will start from the I_{max} and actually aimed at I_{min} . So same equations but you do not have any input voltage. So $I_0 + L di/dt + E$, so you can rewrite this equation like this.

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Analysis of Buck Converter cont.

The expression for the load current is, $i_0 = Ae^{-t/\tau} + B$.

$$\text{At } t=0, i_0 = A + B = I_{\max}$$

$$\text{At } t=\infty, i_0 = B = -(E/R)$$

$$\text{So, } A = I_{\max} + (E/R)$$

Substituting the values of A & B, the expression for the load current is,

$$i_0 = -\left(\frac{E}{R}\right) \left(1 - e^{-t/\tau}\right) + I_{\max} e^{-t/\tau}$$

$$\text{At } t = T_{\text{OFF}}, i_0 = I_{\min}. \text{ So,}$$

$$I_{\min} = -\left(\frac{E}{R}\right) \left(1 - e^{-T_{\text{OFF}}/\tau}\right) + I_{\max} e^{-T_{\text{OFF}}/\tau}$$

$$\text{or, } I_{\max} e^{-T_{\text{OFF}}/\tau} - I_{\min} = -\left(\frac{E}{R}\right) \left(1 - e^{-T_{\text{OFF}}/\tau}\right)$$



Same way you can have a solutions of the load current. So we can actually get $t=0$ so that value equal to $A+B \cdot I_{\max}$ $t=\infty$ it will be actually $B-E/R$, so while substituting you can get the value actually $-E/R 1 - e^{-t/\tau} + I_{\min} e^{-t/\tau}$. So once you write these equations actually you get $I_{\max} E$ to the power $T_{\text{OFF}}/\tau - I_{\min} = -E/R 1 - e^{-T_{\text{OFF}}/\tau}$.

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Analysis of Buck Converter cont.

This is the second expression obtained for mode 2 between I_{\max} and I_{\min} .

From these two expressions, the currents, I_{\max} and I_{\min} are derived as,

$$I_{\max} = \left(\frac{V_s}{R}\right) \left[\frac{1 - e^{-T_{\text{ON}}/\tau}}{1 - e^{-T/\tau}}\right] - \left(\frac{E}{R}\right) \quad \text{or,} \quad I_{\max} = \left(\frac{V_s}{R}\right) \left[\frac{1 - e^{-RT_{\text{ON}}/L}}{1 - e^{-RT/L}}\right] - \left(\frac{E}{R}\right), \text{ and}$$

$$I_{\min} = \left(\frac{V_s}{R}\right) \left[\frac{e^{T_{\text{ON}}/\tau} - 1}{e^{T/\tau} - 1}\right] - \left(\frac{E}{R}\right) \quad \text{or,} \quad I_{\min} = \left(\frac{V_s}{R}\right) \left[\frac{e^{RT_{\text{ON}}/L} - 1}{e^{RT/L} - 1}\right] - \left(\frac{E}{R}\right)$$



So you just equate. Since I_{\max} and I_{\min} are the boundary condition they are same. So second, this second expression is obtained mode-2 between I_{\max} and I_{\min} so for this two equations you an equate. So $I_{\max} = V_s/R 1 - T_{\text{ON}}/\tau 1 - t/\tau - E/R$ or you have this expression. Similarly, I_{\min} you can have the expression $V_s/R T_{\text{ON}}/\tau 1 - e^{-t/\tau} - 1$ or you can have this

expression. So these are the actually the current varying between I_{max} and I_{min} in case of the Step down chopper with the Thyristors or the Buck converter feed in early on.

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Analysis of Buck Converter cont.

Chopper

Ripple content in the Load Current

As given earlier, the load (output) current varies between the maximum and minimum values (I_{max} and I_{min}). Therefore, the ripple content of the current is,

$$I_{\max} - I_{\min} = \left(\frac{V_s}{R} \right) \left[\frac{(1 - e^{-T_{\text{on}}/\tau})(1 - e^{-(T - T_{\text{on}})/\tau})}{(1 - e^{-T/\tau})} \right] = \left(\frac{V_s}{R} \right) \left[\frac{(1 - e^{-T_{\text{on}}/\tau})(1 - e^{-T_{\text{off}}/\tau})}{(1 - e^{-T/\tau})} \right]$$

The above expression for ripple content is independent of battery voltage or back emf (E). Using the duty ratio $k = T_{\text{ON}}/T$, the expression becomes,

$$I_{\max} - I_{\min} = \left(\frac{V_s}{R} \right) \left[\frac{(1 - e^{-kT/\tau})(1 - e^{-(1-k)T/\tau})}{(1 - e^{-T/\tau})} \right]$$

its per unit value being,

$$\frac{(I_{\max} - I_{\min})}{(V_s/R)} = \left[\frac{(1 - e^{-kT/\tau})(1 - e^{-(1-k)T/\tau})}{(1 - e^{-T/\tau})} \right]$$

The current (V_s/R) is taken as 1.0 pu (100%).

Now let us analyze the Buck Chopper for the concept of ripple current. Because we want a steady state current we have seen that while turn ON actually current goes to the I_{max} that is coming to the I_{min} for the continuous conduction mode and this is the waveform for the critically conduction mode. Now, as we have talked about the load current varies between the maximum and the minimum values.

If I_{max}; I_{min} touches 0 then we will say that it is in a critical conduction mode. Therefore, from this previous analysis we can write that what should be the content of the ripple that mean that actually ΔI that is recall to I_{max}-I_{min}. So you can substitute, you can subtract I_{max}-I_{min} we have got our expressions of I_{max} and I_{min} that is V_s/R 1-e to the power T ON/Tau*1-e to the power t-T ON/Tau/1-e to the power t/Tau.

And if you simplify multiply this equations you know, so essentially you get V_s/R 1-e to the power T ON/Tau*1-e to the power T OFF/Tau/1-e to the power t/Tau. Now, if you can investigate this equations you know, let us put you know k is the basically the duty cycle. So k is the duty cycle and it is T ON/T and the expression becomes actually V_s/R 1-e -kT/Tau 1-kT that is basically T OFF/Tau/T.

So essentially, its per unit ripple value if you say in terms of the percentage it is $I_{max}-I_{min}$ by actually the average value that is nothing but V_s/R it will be basically that will depend on $1-e$ to the power $kT/\tau * 1-1-kT/\tau/1-e$ to the power T/τ . Now accordingly you can see that the value of this $I_{max}-I_{min}$ will actually varying by this equations. So thank you for your attention. In next class we shall continue with the different kind of isolated converter and different kind of chopper in the next class. Thank you.