Advance Power Electronics and Control

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Lecture – 39
Non Linear Control in Power Electronics

Welcome to our NPTEL courses on advanced power electronics and control, previous class we have discussed about the linear control, today we have discuss few aspect of the non-linear control; non-linear control itself is a very vast subject, so actually one of the method of the non-linear control that is sliding mode control, so you know actually what is non-linear and thus actually linear; go for the linear control, it is rest merely an approximations.

So, thus of course actually, we required to leave lot of juice behind if you consider only the linear control.

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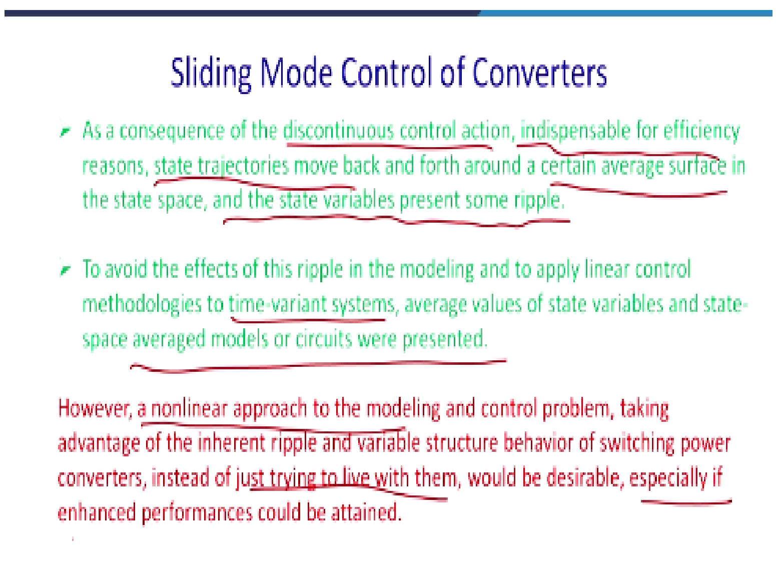

And for this reason, as a consequence of the discontinuous control action, indispensable for the efficiency and the reasons, state trajectory move back and forth in certain average surface of the state space, so we have discussed about the state space, but there is a some limitations of the state space, we have not considers, there is actually due to the discontinuous control action because you know actually with the digital control action, you know or the switching; your switching on and off, so thus there is a discontinuity in a control action which we have seen.

And indispensable for the efficiency reasons because you know we want basically, this term required to be understood very well because when the switch is on, then actually it is in a short circuit condition, the loses across the switch is 0 and which the switches is off, then it is open circuited, no current flows, power losses across the switch is again 0, for this reason, we are saying that indispensable for the efficiency reasons.

So, it required to increase the efficiency of the overall system, so state trajectory moved back and forth around the certain average surface because you want to maintain the capacitor voltage at a certain this level or the inductor current, it is damping on and actually down and it can have an average value, so that is basically a certain average surface in the state space and the state variable represent with the ripples.

So, the average value above it, there will be a some ripple that is a way of actually representing a state space and what we do to here; to avoid the effect of this ripple in the modelling and to apply linear control methods which we have discussed in our previous class that is time variant linear control methodologies in time variant system, we propose the average modelling, so average values of the state space variables and the state space average models are been represented.

But of course you can understood that it is an approximations, it is not at all a proper modelling, however, a non-linear approach a modelling and the control problem, it takes the advantage of the inherent ripple and the variable structure behaviour of a switching of the power electronics converter and for this we have chosen to one of the non-linear control that is sliding mode control and it has got a huge application in the powers electronics.

Instead of just trying to actually leave out them, so we just omit them as a ripple or we take the average modelling, instead of that, we can take this uncertainty and thus your model becomes more close to the real world and would be desirable specially if the enhanced performance could be attained.

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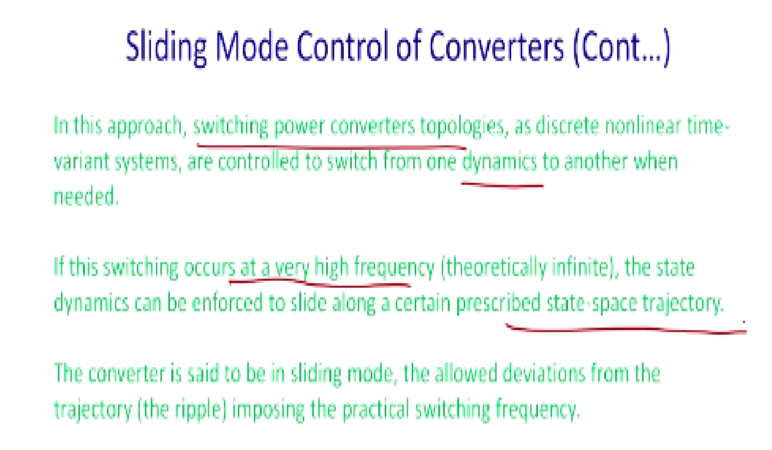

So, in this approach, the switching power converter topologies as discrete non-linear time variant system are controlled to the switch from one dynamics to the another dynamics whenever needed, so switch over to the one dynamics state to the another dynamic state, we will say that, if switching occurs at very fast frequency, theoretically, it is at infinite frequency but we have practical limitations of the switching.

Switching can takes place in a finite frequencies that is the switching frequency of this actually the switches, 10 kilo watts, 1 kilo watt whatever may be, the state dynamics can be enforced to slide along the certain prescribed state space trajectory, we have to force them to go into the specific way and the converter said to be in a sliding mode, when it is actually fall in this trajectory and they allowed to deviation from the trajectory as a ripple.

And imposing; imposed by the practical switching frequency, your switching may be a 10 kilo watts, so there will be a little variation from this trajectory but ultimately, you will go to the that switching state.

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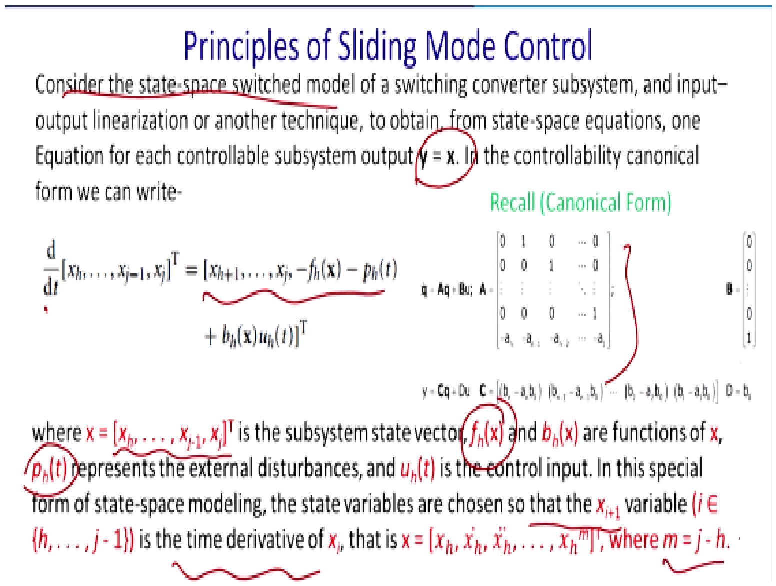

Let us take example of the principle of the sliding mode control, we will consider the same state space equations, consider the state space switching model of the switching converter subsystem, input output linearization's or any other technique to obtain the state space equation. One equation of the each controllable subsystems were actually consider, y = x, we can write in the controllable canonical form.

Generally, if you write in the controllable canonical form, if you have; I hope that most of you have done the control system courses, so this is actually the controllable canonical form and we will write this matrices by the sub transformation into the controllable canonical form and thus we can rewrite this equation and we can have a differentiations of it, so x dot or d/dt of actually xh to xij transpose = basically you can write, xh1 to actually phi h + bhx + ut, where x, this matrix is a subsystem of the state factor.

Fx and bx are the function of the x and here, we had a state transition matrix A that will be actually changed here, so you will have this as A, so it represented by the external disturbance and uht is the control import in the special form of the state space modelling, the states variable chosen, so that actually in a digital domain, it is actually its next instant, it may be x + i1 and this variable where it is belong to the; i belong to a set of h to j -1.

And the time derivative of xi, we it exist and thus we can rewrite as actually, x = h dot and so on and where this m is basically j – h.

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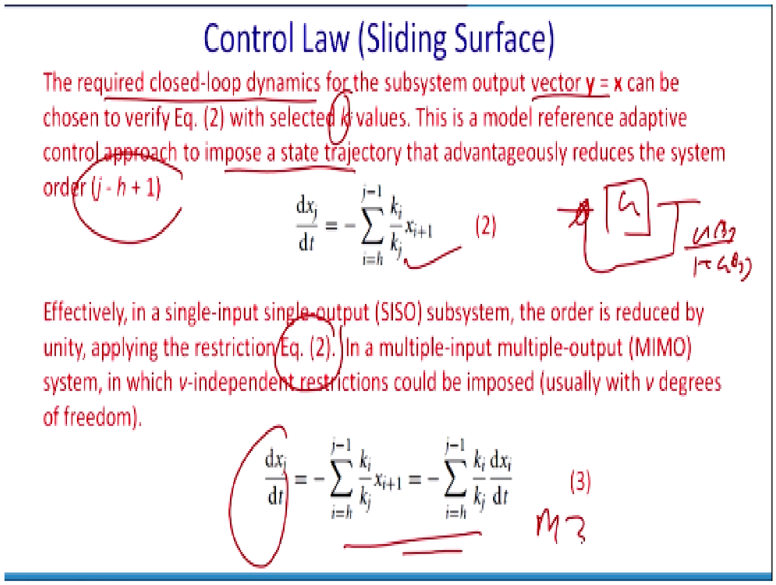

Now, we require same way as we have done for the open-loop subsystem, we have a closed loop sub system, like you know, G a; this is a with unity feedback, so it become Gs/ 1 + Gs something like that in a linear and here you will see that how we will required to generate a closed loop dynamics for the subsystems of output vector, y = x and can be chosen from the equation 2 and by selected the variable ki; the ki is the kth instant.

This is the model reference adaptive control and according to its model, it will adopt, this is the part of their adaptive control techniques and the control approach to impose a state trajectory that advantageously reduces a system order that is j – h + 1, so we shall have a proof you know, actually, we are not going into the detail because of the limitation of the time, so we shall see that what is the application of the power electronics as soon as possible.

Effectively in a single input, signal output we shall consider that is a SISO system in order to reduce by this unity, applying the restriction of the this restriction of equation 2 in the multiple input and the multiple output system which has v- independent restriction could be imposed and usually have v degree of freedom or n degree of freedom, so we can rewrite this equation like this for the MIMO system.

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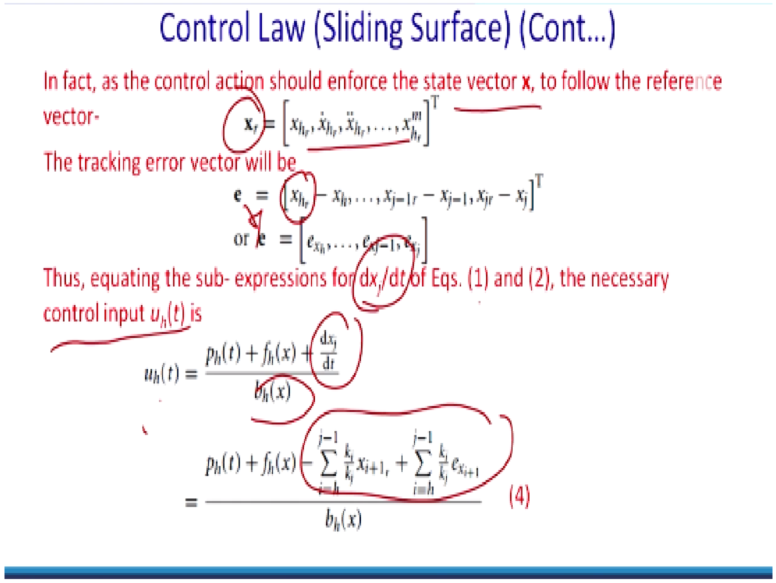

Now, in fact as the control action should be enforced the state vector x to follow the reference, thus the desire vector xr will be actually, xh at dot, triple dot dot dot m of transpose of it, thus the tracking error will be basically hr - h and x1 – x, where these are the reference and where these are the actual; so actual minus reference is definitely the error and for the various system, so automatically you will got an error matrix that is basically xh dot dot xh j – 1 till xj.

Thus, the equations of the subsystems for dx/dt, equation 1 and 2 will actually necessary the control input u, thus it is control input u can be actually rewritten in the form of this, please refer to the equation number 1, then only you can correlate, +dx/dt/ bh and thus by substitution you know of this value you know, you essentially get for this MIMO system, total expression that is 4 and this is your input or the enhanced.

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Control Law (Sliding Surface) (Cont...)

In most switching power converters $u_h(t)$ is discontinuous. Yet, if we assume one or more discontinuity borders dividing the state space into subspaces, the existence and uniqueness of the solution is guaranteed out of the discontinuity borders since in each subspace the input is continuous.

Within the sliding-mode control (SMC) theory, assuming a certain dynamic error driven to zero, one auxiliary equation (sliding surface) and the equivalent control input $u_h(t)$ can be obtained, integrating both sides of Eq. (3) with null initial conditions:

$$k_j x_j \sum_{i=h}^{j-1} k_i x_i = \sum_{i=h}^{j-1} k_i x_i = 0 \quad (5)$$

So, in the most switching power converter, this ut basically 1 and 0, basically is discontinuous, yet if we assume one or more discontinuity, border dividing the state space into the sub space, the existence of the uniqueness of the solution is guaranteed that it is a mathematical proof is there, we are not interested to do much mathematics but we just brief it out why this system will work, uniqueness of the solution guaranteed out of the discontinuity border since in each subspace, the input is continuous that is the logic of the actually this ut to make the system stable.

Within the sliding mode, the SMC theory assuming that a certain dynamic error to 0, one auxiliary equation of the sliding surface and the equivalent control input uht can be obtained by integrating both side of the equation 3, so thus these should be = 0.

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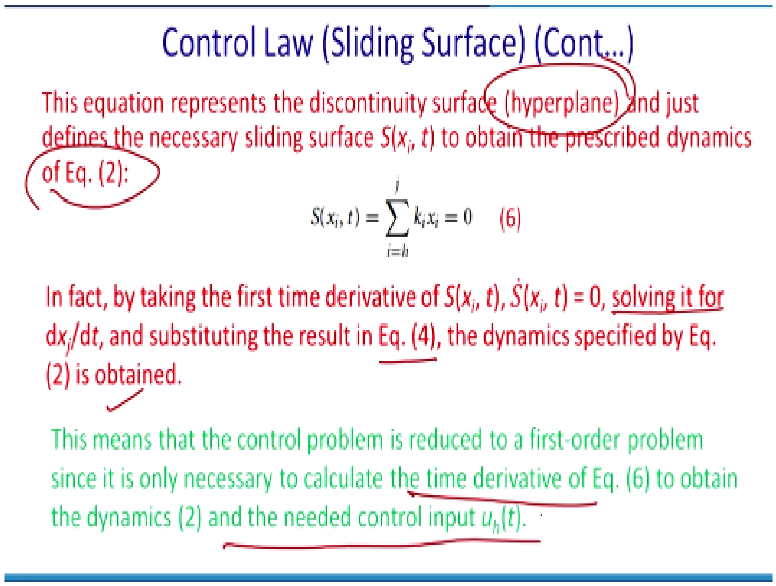

The equation presents the discontinuity surface or we say that it is a hyperplane and just defines the necessary sliding surface that is x of it to obtain as prescribing dynamic equation, so we have to generate a sliding surface and the sliding surface will have the features that S that is function of xi and t should be summation of ih to j ki xi that summation should be = 0, in fact by taking the first derivative that is x, xdot should be = 0 and that will ensure the stability, solving it for dx/dt and substituting the results for equation 4, the dynamic specified by the equation 2 is again reobtained.

This means that the control problem is reduced to the first order problem, since it is unnecessary to calculate the one derivative basically, the time derivative of equation 6 to obtain the dynamics that is S dot and the needed control input ut, so this is the basically you required to calculate the basically, what should be the ut for this sliding surface and thus it reduces to the first order system.

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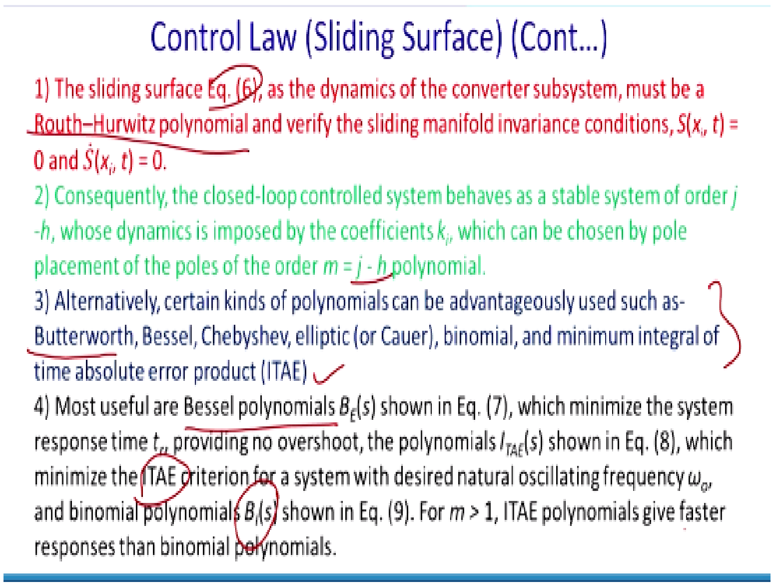

The sliding surface as shown in this equation 6, as the dynamics of the converter subsystem must be Routh-Hurwitz polynomial and verify the sliding manifold variance of condition that is S xi dot t = 0 and also S dot of function x and t should be = 0, consequently, the closed loop control systems, this something like the (()) (14:31) characteristics, control system behaves as the stable system by order j – h, whose dynamics is imposed by the coefficient ki, which can be chosen by pole placement of the poles of the order that is m that is j - h polynomial.

Alternatively, certain kind of polynomial can be advantageously used such as Butterworth, Bessel, Chebyshev, ellipticals, binomial and minimal integral of time absolute error product, this kind of actually expansions of the actually, we have seen the Taylor series, these are the other method also can be used to describe the function and its first derivative will satisfy this same condition.

Since you have studied in a Bessel functions, we will consider up here the Bessel function here, most useful Bessel polynomial is shown in 7, which minimises the system, response time tr providing no overshoot, the polynomials ITAE shows this equation 8 which minimises this actually this ITEA criteria for the system with the desired natural frequency of oscillation omega 0 and binomial polynomial Bis shown in equation 9 for system m > 1, IEA polynomial gives faster response than the binomial polynomials.

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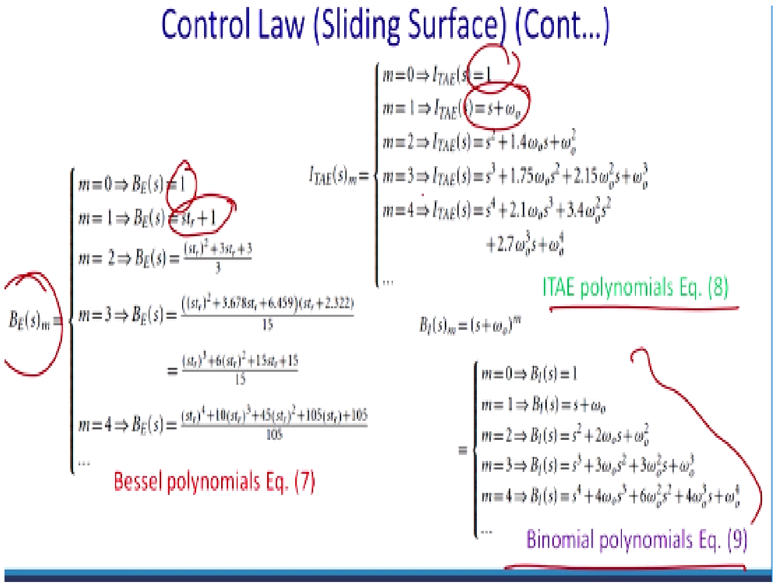

So, there are few take as; these are all Bessel polynomials you know if m = 0, we will prefer basically ITEA and that and will be value will be 1 and if it is m =1, it will be a natural frequency of oscillation + in (()) (16:33), so you are mapping basically the this polynomial in a typical Laplace format, so same way actually we had a Bessel functions, if m = 0 that leads to , if m = 1, that leads to str + 1, same way actually this is for the ITEA polynomial, this is a binomial polynomial.

And this is basically with the you can chose a different aspect, if you are comfortable in the frequency domain analysis, then you can go for the binomial polynomial because it gives you in terms of the essentially, the natural frequency of oscillations, same way this one is also ITEA is been prefer and this is basically, the Bessel polynomial, so these are 3, 4 technique to generate the surface and which one is more suitable for your generally it comes with experience so, either of it can be chosen.

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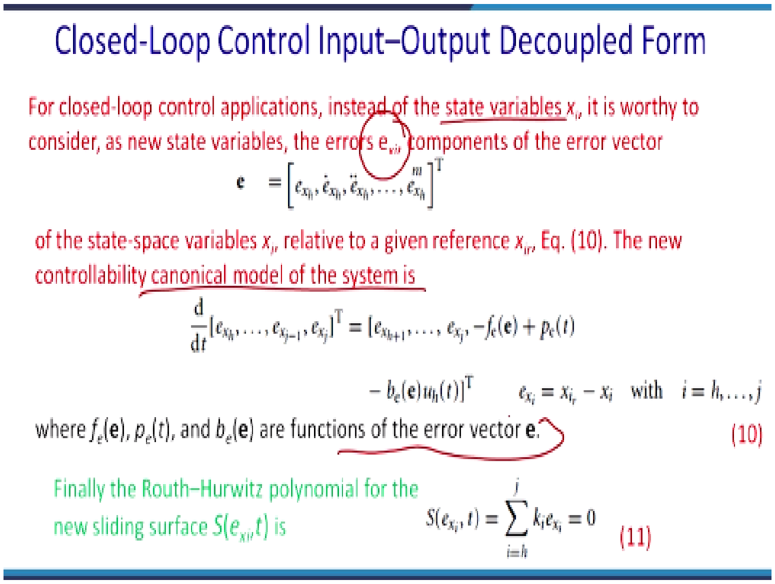

So, the closed loop control applications, instead of the state valuable xi is worthy to consider a new state variable, the error exi, component of the error vector, so it will be actually exh t of the state space variables xi related to the given reference xir, the equation, the new controllability canonical model is obtained and thus actually, d/dt of this one, it leads to this equation where the error xi = xir - xi and so on.

So, where fe and pe and be are of the function of error effect e, from this the Routh-Hurwitz criteria which we have studied in the linear control model that is also applicable finally, the Routh-Hurwitz polynomial for the new sliding surface will be in this and we should condition that x = error vector dot t should actually minimise a error 0 for this is in this value is 0, now, stability, how we will ensure that it will give the stability?

We have stability conditions with a rank of the matrix in the linear form, so what should be the equivalent here.

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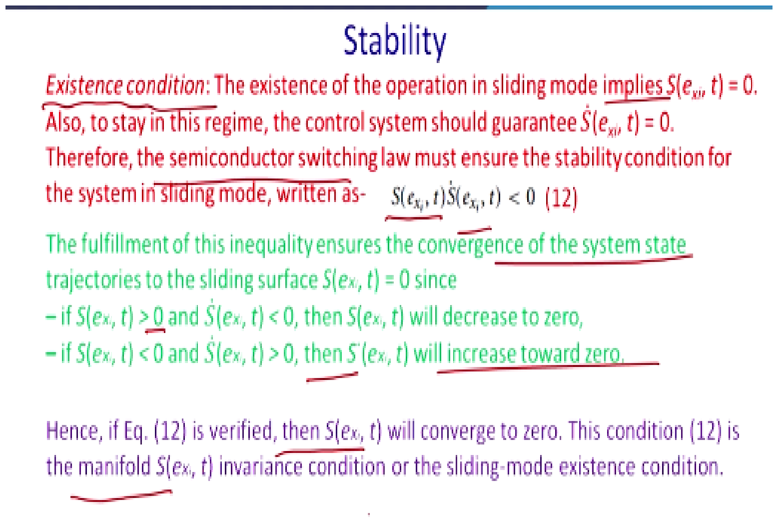

Existence condition; existence of the operation in the sliding mode implies that is S the surface, the error and the time should be = 0, so to stay in this regime, the control system should generate a start e of t = 0, therefore the semiconductor switching law must ensure the stability condition of the sliding mode, it is written as S, ex dot should be < 0, this is a stability condition, we have to show that system is stable.

Fulfilment of this inequality ensures the convergence of the state system trajectory to the sliding surface, that is S e to the x, error and the t vector = 0, since if S is > 0 and S dot is < 0, then it will decrease to 0 and if again, S < 0 and S dot is > 0, then also S dot will increase towards 0, hence equation 12 is verified, then x xdot t converges to 0, this condition 12 is manifold of the variance conditions or the sliding mode existence condition or sliding mode stability condition.

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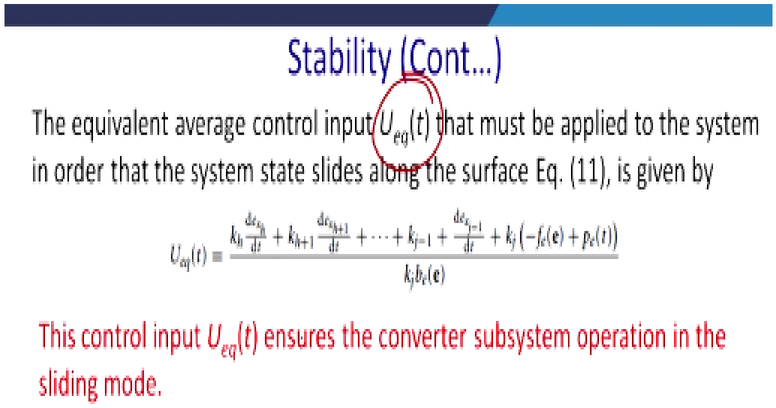

Now, the equivalent average control you may have a multi input system, multi output system, u of t that must be applied to the system in order that systems states sliding along the surface that is u of t is given by this equations and the control input u of t ensures that the converter subsystems actually operates in a sliding mode and ensures the stability of the system, so this is the one way, this is the actually the sliding mode.

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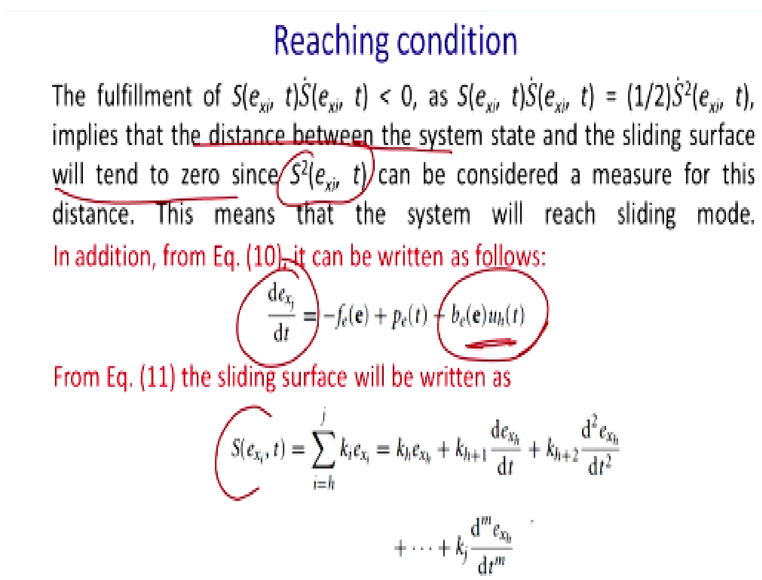

Now, what is reaching condition; the fulfilment of this basically, S x dot is < 0 as S x dot =; let us consider that 1/2 * SS S dot square, so implies that these distance between the subsystems states and the sliding will tend to 0, since S square x dot t can be considered as a measure for this distance, so this is the distance, this means that the system will reach to the sliding mode.

And in addition from the equation number 10, it can be written as follows that this value that is differentiation of the error is basically, -fe + pe - be and with that this is the input actually controllable inputs and it will ensure that from this equation 11, this can be rewritten S as the surface of the kie xi that is actually kh e xh and so on.

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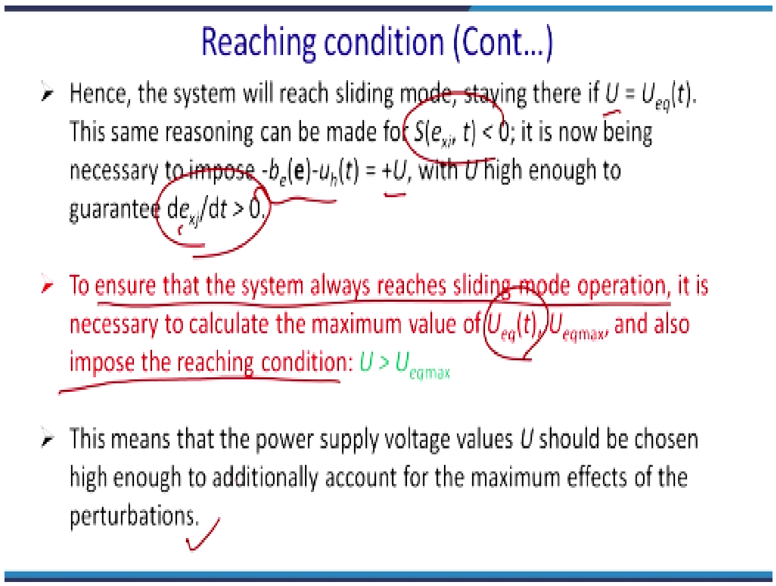

So, hence a subsystem will reach sliding mode staying there, if U = U equivalent, this same reason can be made for S function of e and t < 0, it is now being necessary to impose that is –b – uh of t as +u with U high enough to guarantee that this actually, condition leads to this basically, dx/dt is > 0 to ensure that the system is always reaches the sliding mode operation, it is necessary to calculate the minimum value of the eqt and eq max and also impose the reaching condition, U > eq max.

This means that the power supply voltage values U should be chosen high enough to additionally to account the maximum effect to the perturbation.

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Switching Law

- From the foregoing considerations, supposing a system with two possible structures, the semiconductor switching strategy must ensure $S(e_{xi}, t)\dot{S}(e_{xi}, t) < 0$. Therefore, if $S(e_{xi}, t) > 0$, then $\dot{S}(e_{xi}, t) < 0$, which implies, as seen, $-b_e(\mathbf{e})u_h(t) = -U$ (the sign of $b_e(\mathbf{e})$ must be known).
- Also, if $S(e_{xi}, t) < 0$, then $\dot{S}(e_{xi}, t) > 0$, which implies $-b_e(\mathbf{e})u_h(t) = +U$.
- This imposes the switching between two structures at infinite frequency. Since power semiconductors can switch only at finite frequency, in practice, a small enough error for $S(e_{xi}, t)$ must be allowed $(-\varepsilon < S(e_{xi}, t) < +\varepsilon)$.
- Hence, the switching law between the two possible system structures might be

$$u_h(t) = \begin{cases} U/b_e(\mathbf{e}) \text{ for } S(e_{x_i}, t) > +\varepsilon \\ -U/b_e(\mathbf{e}) \text{ for } S(e_{x_i}, t) < -\varepsilon \end{cases}$$

So, what should be the switching law, how will ensure that actually, our converter will be switching from the foregoing consideration supposing, a system will have a 2 possible structures; the semiconductor switching strategies ensure that x, x dot should be < 0, therefore if S is > 0, then S dot should be < 0, which implies as same that is b of error function of Ust should be –U, these sign we must be known them.

Similarly, this means basically, the switch can be off then, similarly if this is < 0 and then S dot is > 0, this implies that the switch should be on basically and we should know the basically the sign of be and assuming that the sign of the be is positive, so you require the U to be positive, if it is the reverse, then that U sign will change, this imposes that switching between these two structures and infinite frequency will take over this way.

Since power semiconductor can switch only finite frequency in practice a small error for S will be allowed were basically the e is basically will be > than the sliding surface and < the positive sliding surface, hence switching law between this 2 possible system structures might be basically U/b for xe that is a error > 1, then we have a positive switching and –Ub for St is < that error, so we have one, let us take an example, when actually decibels voltage is more, then definitely you will basically switch it off.

And one will be require to build up the decibals voltage which should be switching on, so let us see how does it work practically, so this is also the mathematical part of it and now take a practical aspect of it.

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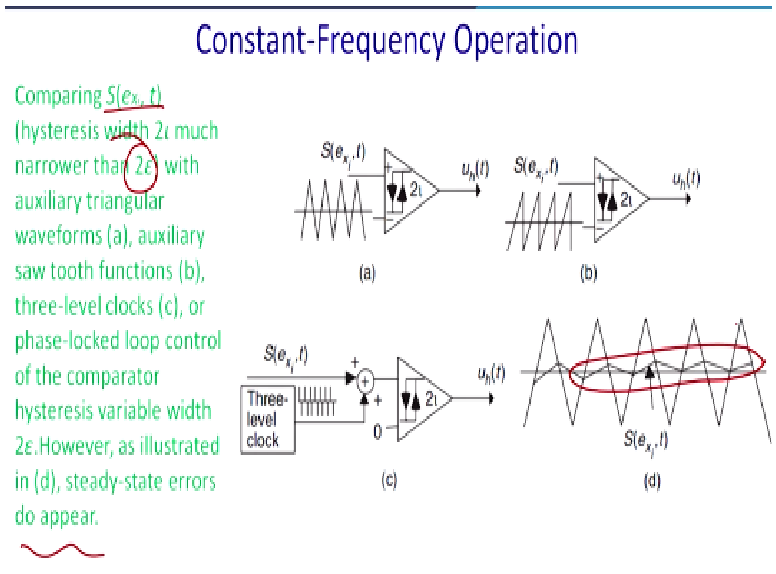

Then and now comparing this S, let us have a hysteresis band, this hysteresis band is very narrow of this actually narrower than 2e, with auxiliary triangle of waveform a, the auxiliary saw tooth function b and the three level clock is c, or the phase locked loop with the comparator any of it will walk, this one, this one, this one and hysteresis variable width 2e, however as illustrated in the 2D, the steady state error will appear.

Because you know you will switch it on and there will be an error so, ultimately this error will remain and ultimately, this error is going to be accumulated and you will have some amount of the steady state error.

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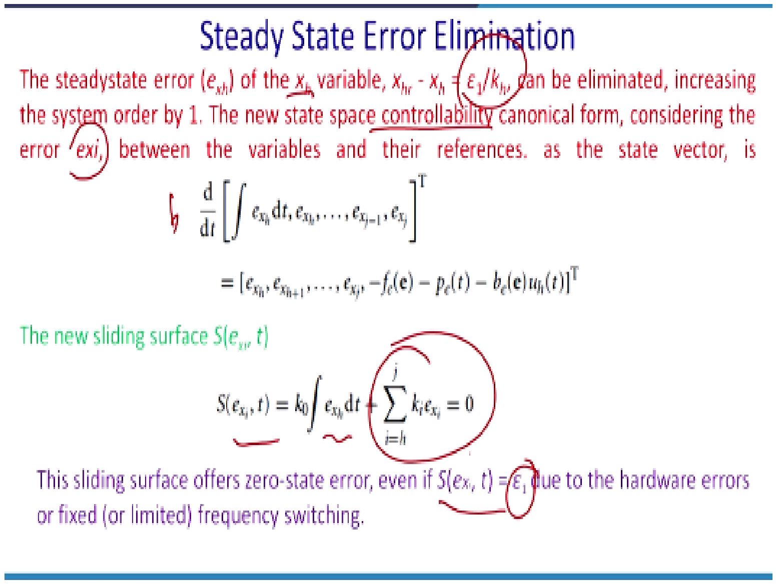

So, steady state error that is basically nothing but something like proportion control error, steady state error of the xh variable that is hr – h = E1/kh can be eliminated by increasing the system order by one, now the new state space controllability of the canonical form considering that error Exi between the variable and the reference of the state vector is; it will be basically this so, it is something like equivalent to the pi controller and you integrate it ultimately, you get this results.

Now, new sliding surface will be actually S k0 kx + this submission, this is due to the integration, so sliding surface will offer 0 state error even if this actually have a some error due to the hardware error or fixed or limited frequency switching, so this can be eliminated by integral based sliding surface.

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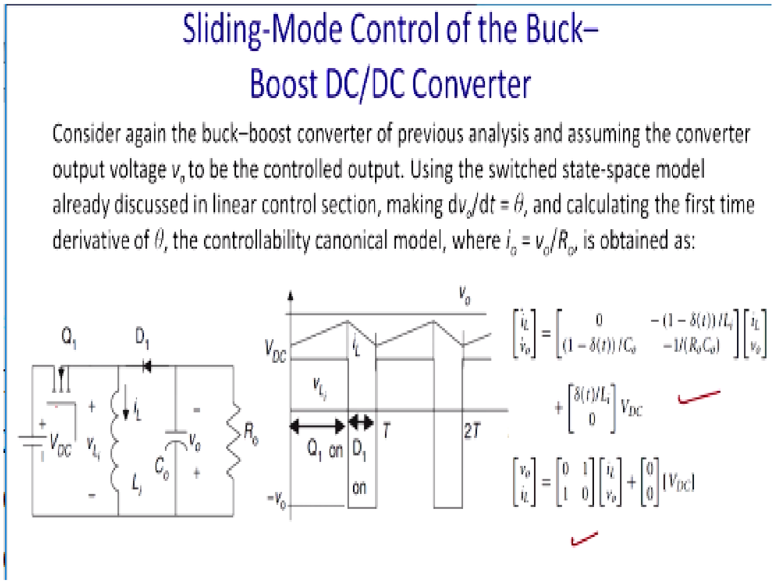

So, let us take an example first, so this is basically the same circuits and same actually, the matrices which we will have and now this equation, when we try to control this switching by the sliding mode control.

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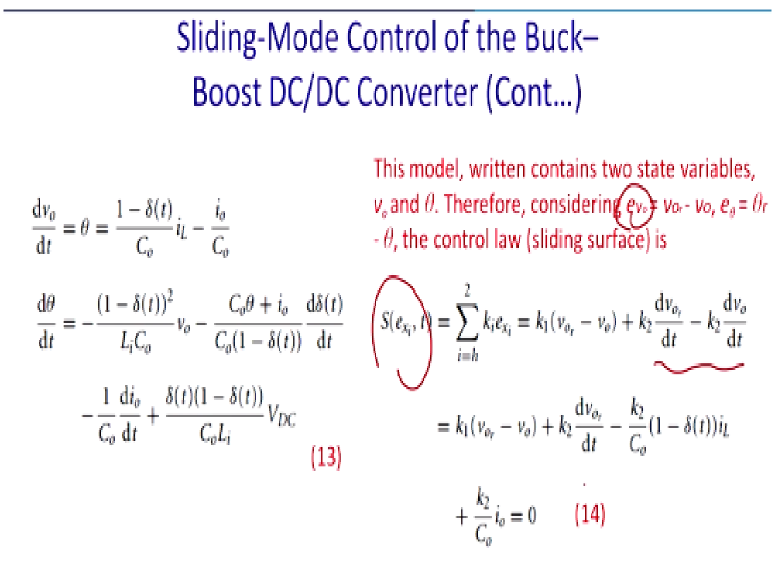

So, ultimately you have a dv/dt and all those things, this model written in two states; v0 and theta therefore, considering that actually, this is an error vector that is v0 – vr0 and the theta r – theta, we can rewrite and thus we can have a sliding surface that is = k1 v0r – v0 + this 2 constant and ultimately you know that this should be = 0, k1 * v0r – v0 k2 * this derivative + switching should be = 0.

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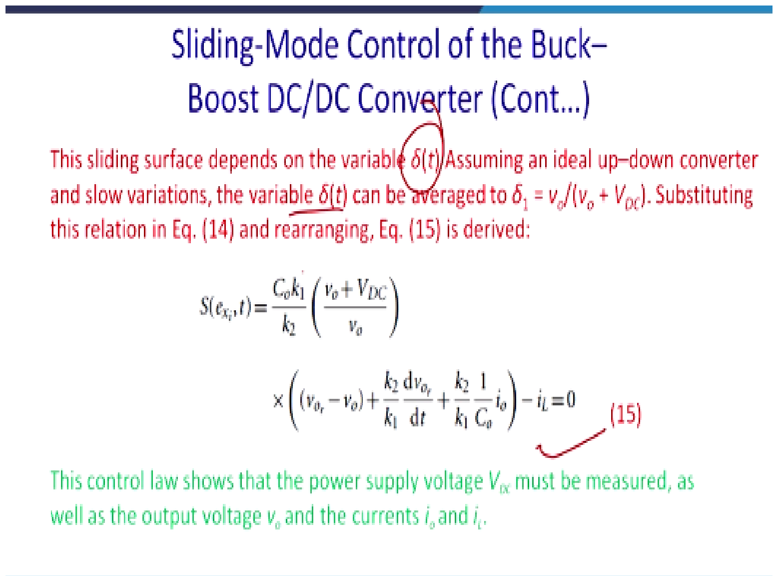

And from this condition relative, we can say that the sliding surface depends on the variable del t assuming that ideal up down convertor and a slow variation of the delta t can be a average that is delta 1 = v0 / this one, ultimately will leads to this equation, this control shows the power supply of voltage VDC must be measured as well as the output voltage v0 and the current i0 and il, then only we can generate the sliding surface.

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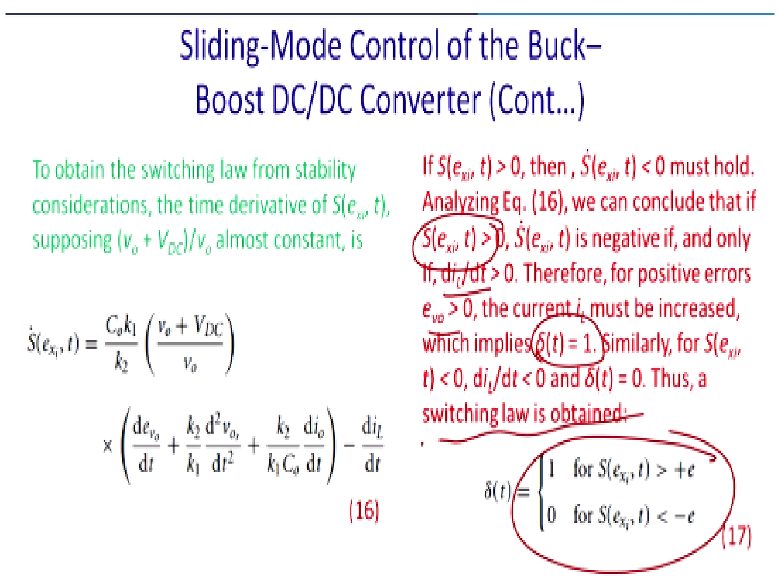

And we will ensure that the conditionality of the sliding surface explained and now, we have to check the conditions of the stability to obtain the switching law, we have to take the derivative of it and to obtain the switching law from the stability consideration, the time derivative will be basically v0 + VDC/ v0 and it is almost constant, so we can put it that and thus what we can say is that if the sliding surface is > 0, then S dot must be < 0, must hold.

Analysing this equation 16, we can conclude that if it is > 0, then x dot is negative and only if di/dt is > 0, then what we can conclude; therefor, for the positive error if v is > 0, the current il must be increase then, switch should be on < 0 than zero and thus delta t should be = 0 and thus it should be off and this is the conditions for the switching and we can ensure in case of the sliding mode surface.

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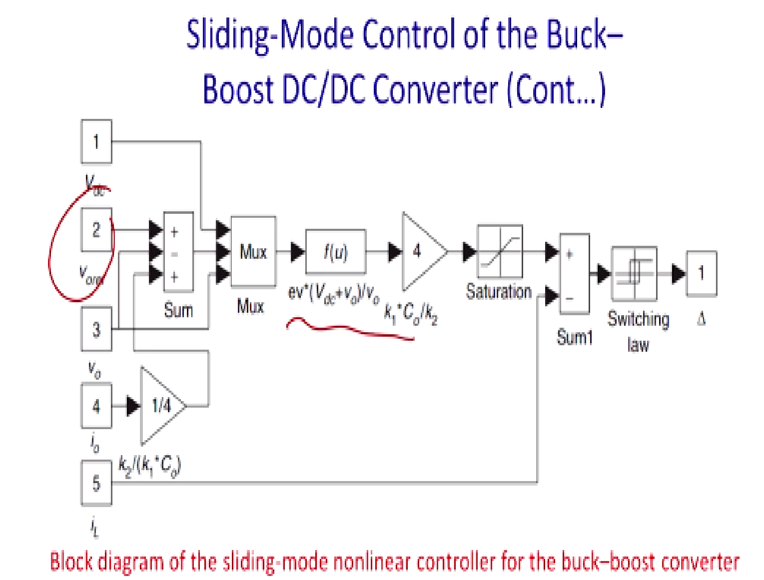

So, this is the overall control cycle, this is a VDC, this is basically vdf, this is v0, this is i0 and il, you will sum it up, there after you will actually put it into the sum of what; you have multiplex, there after you will generate the function, you have multiplicity of 4, there after you have a saturation block, then you have actually subtractor, you will check the condition whether which one is zero, accordingly we will implement.

It can be easily implemented if you wish to have a actually simulation you can easily implement with the Simulink blocks.

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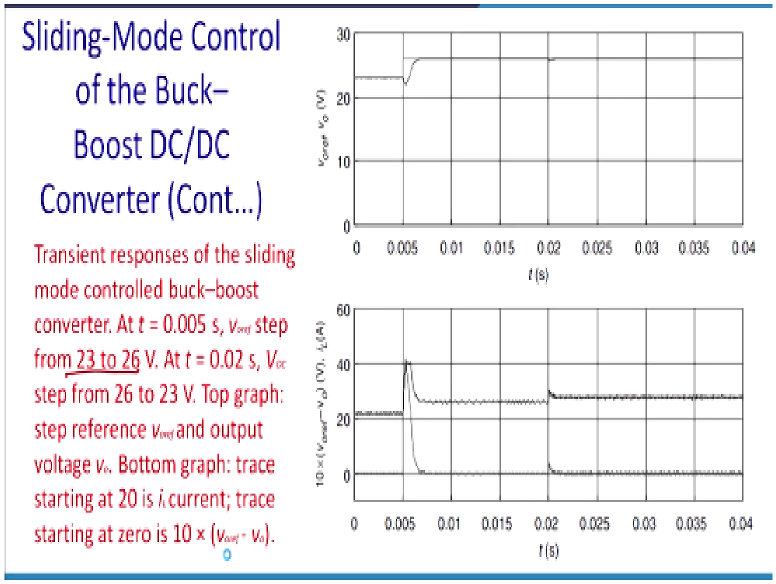

So, these are the results, you know I do not go into much explanation, this slide says that transient response of the sliding mode control of the Buck Boost converter at t = 0.005 and v0 ref steps from this, then you can see that actually it is rising very fast and top graph reference and bottom graph actually traces out il and the current will be actually tracing back from this actually starting from 0 to 10, so these are the actually the results of the sliding mode control.

Now, we have taken one important aspect of the nonlinear control, there are so many non-linear control and just I show that actually what I; we try to picture it basically that this kind of the sliding mode control or nonlinear control gives you the better stability and the better transient response than the linear control. Thank you for your attention, I take the next class that is basically will be the concluding class and there we will try to sum up the whole courses, thank you for your attention.