

**Advance Power Electronics and Control**  
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**Lecture – 39**  
**Non Linear Control in Power Electronics**

Welcome to our NPTEL courses on advanced power electronics and control, previous class we have discussed about the linear control, today we have discuss few aspect of the non-linear control; non-linear control itself is a very vast subject, so actually one of the method of the non-linear control that is sliding mode control, so you know actually what is non-linear and thus actually linear; go for the linear control, it is rest merely an approximations.

So, thus of course actually, we required to leave lot of juice behind if you consider only the linear control.

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### Sliding Mode Control of Converters

- As a consequence of the discontinuous control action, indispensable for efficiency reasons, state trajectories move back and forth around a certain average surface in the state space, and the state variables present some ripple.
- To avoid the effects of this ripple in the modeling and to apply linear control methodologies to time-variant systems, average values of state variables and state-space averaged models or circuits were presented.

However, a nonlinear approach to the modeling and control problem, taking advantage of the inherent ripple and variable structure behavior of switching power converters, instead of just trying to live with them, would be desirable, especially if enhanced performances could be attained.

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And for this reason, as a consequence of the discontinuous control action, indispensable for the efficiency and the reasons, state trajectory move back and forth in certain average surface of the state space, so we have discussed about the state space, but there is a some limitations of the state space, we have not considers, there is actually due to the discontinuous control action because you know actually with the digital control action, you know or the switching; your switching on and off, so thus there is a discontinuity in a control action which we have seen.

And indispensable for the efficiency reasons because you know we want basically, this term required to be understood very well because when the switch is on, then actually it is in a short circuit condition, the losses across the switch is 0 and when the switch is off, then it is open circuited, no current flows, power losses across the switch is again 0, for this reason, we are saying that indispensable for the efficiency reasons.

So, it required to increase the efficiency of the overall system, so state trajectory moved back and forth around the certain average surface because you want to maintain the capacitor voltage at a certain this level or the inductor current, it is damping on and actually down and it can have an average value, so that is basically a certain average surface in the state space and the state variable represent with the ripples.

So, the average value above it, there will be a some ripple that is a way of actually representing a state space and what we do to here; to avoid the effect of this ripple in the modelling and to apply linear control methods which we have discussed in our previous class that is time variant linear control methodologies in time variant system, we propose the average modelling, so average values of the state space variables and the state space average models are been represented.

But of course you can understand that it is an approximations, it is not at all a proper modelling, however, a non-linear approach a modelling and the control problem, it takes the advantage of the inherent ripple and the variable structure behaviour of a switching of the power electronics converter and for this we have chosen to one of the non-linear control that is sliding mode control and it has got a huge application in the powers electronics.

Instead of just trying to actually leave out them, so we just omit them as a ripple or we take the average modelling, instead of that, we can take this uncertainty and thus your model becomes more close to the real world and would be desirable specially if the enhanced performance could be attained.

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## Sliding Mode Control of Converters (Cont...)

In this approach, switching power converters topologies, as discrete nonlinear time-variant systems, are controlled to switch from one dynamics to another when needed.

If this switching occurs at a very high frequency (theoretically infinite), the state dynamics can be enforced to slide along a certain prescribed state-space trajectory.

The converter is said to be in sliding mode, the allowed deviations from the trajectory (the ripple) imposing the practical switching frequency.

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So, in this approach, the switching power converter topologies as discrete non-linear time variant system are controlled to the switch from one dynamics to the another dynamics whenever needed, so switch over to the one dynamics state to the another dynamic state, we will say that, if switching occurs at very fast frequency, theoretically, it is at infinite frequency but we have practical limitations of the switching.

Switching can takes place in a finite frequencies that is the switching frequency of this actually the switches, 10 kilo watts, 1 kilo watt whatever may be, the state dynamics can be enforced to slide along the certain prescribed state space trajectory, we have to force them to go into the specific way and the converter said to be in a sliding mode, when it is actually fall in this trajectory and they allowed to deviation from the trajectory as a ripple.

And imposing; imposed by the practical switching frequency, your switching may be a 10 kilo watts, so there will be a little variation from this trajectory but ultimately, you will go to the that switching state.

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## Principles of Sliding Mode Control

Consider the state-space switched model of a switching converter subsystem, and input-output linearization or another technique, to obtain from state-space equations, one Equation for each controllable subsystem output  $y = x$ . In the controllability canonical form we can write-

Recall (Canonical Form)

$$\frac{d}{dt} [x_h, \dots, x_{j-1}, x_j]^T = [x_{h+1}, \dots, x_j, -f_h(x) - p_h(t) + b_h(x)u_h(t)]^T$$

$$\dot{x} = Aq + Bu; \quad A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_1 & -a_2 & -a_3 & \dots & -a_n \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$y = Cq + Du; \quad C = [(b_1 - a_1 b_2) \quad (b_1 - a_2 b_2) \quad \dots \quad (b_1 - a_{h-1} b_2) \quad (b_1 - a_h b_2)] \quad D = b_1$$

where  $x = [x_h, \dots, x_{j-1}, x_j]^T$  is the subsystem state vector,  $f_h(x)$  and  $b_h(x)$  are functions of  $x$ ,  $p_h(t)$  represents the external disturbances, and  $u_h(t)$  is the control input. In this special form of state-space modeling, the state variables are chosen so that the  $x_{i+1}$  variable ( $i \in \{h, \dots, j-1\}$ ) is the time derivative of  $x_i$ , that is  $x = [x_h, \dot{x}_h, \ddot{x}_h, \dots, x_h^{(m)}]^T$ , where  $m = j - h$ .

Let us take example of the principle of the sliding mode control, we will consider the same state space equations, consider the state space switching model of the switching converter subsystem, input output linearization's or any other technique to obtain the state space equation. One equation of the each controllable subsystems were actually consider,  $y = x$ , we can write in the controllable canonical form.

Generally, if you write in the controllable canonical form, if you have; I hope that most of you have done the control system courses, so this is actually the controllable canonical form and we will write this matrices by the sub transformation into the controllable canonical form and thus we can rewrite this equation and we can have a differentiations of it, so  $\dot{x}$  or  $d/dt$  of actually  $x_h$  to  $x_j$  transpose = basically you can write,  $\dot{x}_h$  to actually  $\phi_h + b_h x + u$ , where  $x$ , this matrix is a subsystem of the state factor.

$f_x$  and  $b_x$  are the function of the  $x$  and here, we had a state transition matrix  $A$  that will be actually changed here, so you will have this as  $A$ , so it represented by the external disturbance and  $u_h t$  is the control import in the special form of the state space modelling, the states variable chosen, so that actually in a digital domain, it is actually its next instant, it may be  $x + i1$  and this variable where it is belong to the;  $i$  belong to a set of  $h$  to  $j - 1$ .

And the time derivative of  $x_i$ , we it exist and thus we can rewrite as actually,  $\dot{x} = h$  dot and so on and where this  $m$  is basically  $j - h$ .

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### Control Law (Sliding Surface)

The required closed-loop dynamics for the subsystem output vector  $y = x$  can be chosen to verify Eq. (2) with selected  $k_i$  values. This is a model reference adaptive control approach to impose a state trajectory that advantageously reduces the system order  $(j - h + 1)$

$$\frac{dx_j}{dt} = - \sum_{i=h}^{j-1} \frac{k_i}{k_j} x_{i+1} \quad (2)$$

Effectively, in a single-input single-output (SISO) subsystem, the order is reduced by unity, applying the restriction Eq. (2). In a multiple-input multiple-output (MIMO) system, in which  $v$ -independent restrictions could be imposed (usually with  $v$  degrees of freedom).

$$\frac{dx_i}{dt} = - \sum_{i=h}^{j-1} \frac{k_i}{k_j} x_{i+1} = - \sum_{i=h}^{j-1} \frac{k_i}{k_j} \frac{dx_i}{dt} \quad (3)$$

Now, we require same way as we have done for the open-loop subsystem, we have a closed loop sub system, like you know,  $G a$ ; this is a with unity feedback, so it become  $G_s / 1 + G_s$  something like that in a linear and here you will see that how we will required to generate a closed loop dynamics for the subsystems of output vector,  $y = x$  and can be chosen from the equation 2 and by selected the variable  $k_i$ ; the  $k_i$  is the  $k$ th instant.

This is the model reference adaptive control and according to its model, it will adopt, this is the part of their adaptive control techniques and the control approach to impose a state trajectory that advantageously reduces a system order that is  $j - h + 1$ , so we shall have a proof you know, actually, we are not going into the detail because of the limitation of the time, so we shall see that what is the application of the power electronics as soon as possible.

Effectively in a single input, signal output we shall consider that is a SISO system in order to reduce by this unity, applying the restriction of the this restriction of equation 2 in the multiple input and the multiple output system which has  $v$ - independent restriction could be imposed and usually have  $v$  degree of freedom or  $n$  degree of freedom, so we can rewrite this equation like this for the MIMO system.

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### Control Law (Sliding Surface) (Cont...)

In fact, as the control action should enforce the state vector  $x$ , to follow the reference vector-

$$x_r = [x_{hr}, \dot{x}_{hr}, \ddot{x}_{hr}, \dots, x_{hr}^{(m)}]^T$$

The tracking error vector will be

$$e = [x_{hr} - x_{h1}, \dots, x_{h(j-1)r} - x_{h(j-1)}, x_{jr} - x_j]^T$$

or  $e = [e_{x_{hr}}, \dots, e_{x_{j-1}}, e_{x_j}]^T$

Thus, equating the sub-expressions for  $dx/dt$  of Eqs. (1) and (2), the necessary control input  $u_h(t)$  is

$$u_h(t) = \frac{p_h(t) + f_h(x) + \frac{dx_j}{dt}}{b_h(x)}$$

$$= \frac{p_h(t) + f_h(x) - \sum_{i=h}^{j-1} \frac{k_i}{k_j} x_{i+1,r} + \sum_{i=h}^{j-1} \frac{k_i}{k_j} e_{x_{i+1}}}{b_h(x)} \quad (4)$$

Now, in fact as the control action should be enforced the state vector  $x$  to follow the reference, thus the desire vector  $x_r$  will be actually,  $x_h$  at dot, triple dot dot dot  $m$  of transpose of it, thus the tracking error will be basically  $x_{hr} - x_{h1}$  and  $x_{j-1,r} - x_{j-1}$ , where these are the reference and where these are the actual; so actual minus reference is definitely the error and for the various system, so automatically you will get an error matrix that is basically  $x_{hr}$  dot dot dot  $x_{hj-1}$  till  $x_j$ .

Thus, the equations of the subsystems for  $dx/dt$ , equation 1 and 2 will actually necessary the control input  $u$ , thus it is control input  $u$  can be actually rewritten in the form of this, please refer to the equation number 1, then only you can correlate,  $+dx/dt/b_h$  and thus by substitution you know of this value you know, you essentially get for this MIMO system, total expression that is 4 and this is your input or the enhanced.

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## Control Law (Sliding Surface) (Cont...)

In most switching power converters,  $u_p(t)$  is discontinuous. Yet, if we assume one or more discontinuity borders dividing the state space into subspaces, the existence and uniqueness of the solution is guaranteed out of the discontinuity borders since in each subspace the input is continuous.

Within the sliding-mode control (SMC) theory, assuming a certain dynamic error driven to zero, one auxiliary equation (sliding surface) and the equivalent control input  $u_p(t)$  can be obtained, integrating both sides of Eq. (3) with null initial conditions:

$$k_j x_j \sum_{i=h}^{j-1} k_i x_i = \sum_{i=h}^{j-1} k_i x_i = 0 \quad (5)$$

So, in the most switching power converter, this  $u_p$  basically 1 and 0, basically is discontinuous, yet if we assume one or more discontinuity, border dividing the state space into the sub space, the existence of the uniqueness of the solution is guaranteed that it is a mathematical proof is there, we are not interested to do much mathematics but we just brief it out why this system will work, uniqueness of the solution guaranteed out of the discontinuity border since in each subspace, the input is continuous that is the logic of the actually this  $u_p$  to make the system stable.

Within the sliding mode, the SMC theory assuming that a certain dynamic error to 0, one auxiliary equation of the sliding surface and the equivalent control input  $u_p$  can be obtained by integrating both side of the equation 3, so thus these should be = 0.

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## Control Law (Sliding Surface) (Cont...)

This equation represents the discontinuity surface (hyperplane) and just defines the necessary sliding surface  $S(x_i, t)$  to obtain the prescribed dynamics of Eq. (2):

$$S(x_i, t) = \sum_{i=h}^j k_i x_i = 0 \quad (6)$$

In fact, by taking the first time derivative of  $S(x_i, t)$ ,  $\dot{S}(x_i, t) = 0$ , solving it for  $dx_j/dt$ , and substituting the result in Eq. (4), the dynamics specified by Eq. (2) is obtained.

This means that the control problem is reduced to a first-order problem since it is only necessary to calculate the time derivative of Eq. (6) to obtain the dynamics (2) and the needed control input  $u_h(t)$ .

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The equation presents the discontinuity surface or we say that it is a hyperplane and just defines the necessary sliding surface that is  $x$  of it to obtain as prescribing dynamic equation, so we have to generate a sliding surface and the sliding surface will have the features that  $S$  that is function of  $x_i$  and  $t$  should be summation of  $i_h$  to  $j$   $k_i x_i$  that summation should be  $= 0$ , in fact by taking the first derivative that is  $x$ ,  $\dot{x}$  should be  $= 0$  and that will ensure the stability, solving it for  $dx/dt$  and substituting the results for equation 4, the dynamic specified by the equation 2 is again reobtained.

This means that the control problem is reduced to the first order problem, since it is unnecessary to calculate the one derivative basically, the time derivative of equation 6 to obtain the dynamics that is  $S$  dot and the needed control input  $u_t$ , so this is the basically you required to calculate the basically, what should be the  $u_t$  for this sliding surface and thus it reduces to the first order system.

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## Control Law (Sliding Surface) (Cont...)

- 1) The sliding surface Eq. (6), as the dynamics of the converter subsystem, must be a Routh-Hurwitz polynomial and verify the sliding manifold invariance conditions,  $S(x, t) = 0$  and  $\dot{S}(x, t) = 0$ .
- 2) Consequently, the closed-loop controlled system behaves as a stable system of order  $j - h$ , whose dynamics is imposed by the coefficients  $k_i$ , which can be chosen by pole placement of the poles of the order  $m = j - h$  polynomial.
- 3) Alternatively, certain kinds of polynomials can be advantageously used such as Butterworth, Bessel, Chebyshev, elliptic (or Cauer), binomial, and minimum integral of time absolute error product (ITAE) ✓
- 4) Most useful are Bessel polynomials  $B_e(s)$  shown in Eq. (7), which minimize the system response time  $t_r$ , providing no overshoot, the polynomials  $I_{ITAE}(s)$  shown in Eq. (8), which minimize the ITAE criterion for a system with desired natural oscillating frequency  $\omega_o$ , and binomial polynomials  $B_i(s)$  shown in Eq. (9). For  $m > 1$ , ITAE polynomials give faster responses than binomial polynomials.

The sliding surface as shown in this equation 6, as the dynamics of the converter subsystem must be Routh-Hurwitz polynomial and verify the sliding manifold variance of condition that is  $S(x, t) = 0$  and also  $\dot{S}(x, t) = 0$ , consequently, the closed loop control systems, this something like the (14:31) characteristics, control system behaves as the stable system by order  $j - h$ , whose dynamics is imposed by the coefficient  $k_i$ , which can be chosen by pole placement of the poles of the order that is  $m = j - h$  polynomial.

Alternatively, certain kind of polynomial can be advantageously used such as Butterworth, Bessel, Chebyshev, ellipticals, binomial and minimal integral of time absolute error product, this kind of actually expansions of the actually, we have seen the Taylor series, these are the other method also can be used to describe the function and its first derivative will satisfy this same condition.

Since you have studied in a Bessel functions, we will consider up here the Bessel function here, most useful Bessel polynomial is shown in 7, which minimises the system, response time  $t_r$  providing no overshoot, the polynomials ITAE shows this equation 8 which minimises this actually this ITEA criteria for the system with the desired natural frequency of oscillation  $\omega_o$  and binomial polynomial  $B_i(s)$  shown in equation 9 for system  $m > 1$ , IEA polynomial gives faster response than the binomial polynomials.

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## Control Law (Sliding Surface) (Cont...)

$$\begin{aligned}
 B_E(s)_m &= \begin{cases} m=0 \Rightarrow B_E(s) = 1 \\ m=1 \Rightarrow B_E(s) = s + 1 \\ m=2 \Rightarrow B_E(s) = \frac{(s_r)^2 + 3s_r + 3}{3} \\ m=3 \Rightarrow B_E(s) = \frac{(s_r)^2 + 3.678s_r + 6.459)(s_r + 2.322)}{15} \\ \quad = \frac{(s_r)^3 + 6s_r^2 + 13s_r + 15}{15} \\ m=4 \Rightarrow B_E(s) = \frac{(s_r)^4 + 10(s_r)^3 + 45(s_r)^2 + 105(s_r) + 105}{105} \\ \dots \end{cases} \quad \text{Bessel polynomials Eq. (7)} \\
 I_{TAE}(s)_m &= \begin{cases} m=0 \Rightarrow I_{TAE}(s) = 1 \\ m=1 \Rightarrow I_{TAE}(s) = s + \omega_0 \\ m=2 \Rightarrow I_{TAE}(s) = s^2 + 1.4\omega_0s + \omega_0^2 \\ m=3 \Rightarrow I_{TAE}(s) = s^3 + 1.75\omega_0s^2 + 2.15\omega_0^2s + \omega_0^3 \\ m=4 \Rightarrow I_{TAE}(s) = s^4 + 2.1\omega_0s^3 + 3.4\omega_0^2s^2 \\ \quad + 2.7\omega_0^3s + \omega_0^4 \\ \dots \end{cases} \quad \text{ITAE polynomials Eq. (8)} \\
 B_I(s)_m &= (s + \omega_0)^m \\
 B_I(s)_m &= \begin{cases} m=0 \Rightarrow B_I(s) = 1 \\ m=1 \Rightarrow B_I(s) = s + \omega_0 \\ m=2 \Rightarrow B_I(s) = s^2 + 2\omega_0s + \omega_0^2 \\ m=3 \Rightarrow B_I(s) = s^3 + 3\omega_0s^2 + 3\omega_0^2s + \omega_0^3 \\ m=4 \Rightarrow B_I(s) = s^4 + 4\omega_0s^3 + 6\omega_0^2s^2 + 4\omega_0^3s + \omega_0^4 \\ \dots \end{cases} \quad \text{Binomial polynomials Eq. (9)}
 \end{aligned}$$

So, there are few take as; these are all Bessel polynomials you know if  $m = 0$ , we will prefer basically ITEA and that and will be value will be 1 and if it is  $m = 1$ , it will be a natural frequency of oscillation + in (()) (16:33), so you are mapping basically the this polynomial in a typical Laplace format, so same way actually we had a Bessel functions, if  $m = 0$  that leads to , if  $m = 1$ , that leads to  $str + 1$ , same way actually this is for the ITEA polynomial, this is a binomial polynomial.

And this is basically with the you can chose a different aspect, if you are comfortable in the frequency domain analysis, then you can go for the binomial polynomial because it gives you in terms of the essentially, the natural frequency of oscillations, same way this one is also ITEA is been prefer and this is basically, the Bessel polynomial, so these are 3, 4 technique to generate the surface and which one is more suitable for your generally it comes with experience so, either of it can be chosen.

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## Closed-Loop Control Input-Output Decoupled Form

For closed-loop control applications, instead of the state variables  $x_i$ , it is worthy to consider, as new state variables, the errors  $e_{i,r}$  components of the error vector

$$\mathbf{e} = [e_{x_1}, \dot{e}_{x_1}, \ddot{e}_{x_1}, \dots, e_{x_m}]^T$$

of the state-space variables  $x_i$ , relative to a given reference  $x_{i,r}$ , Eq. (10). The new controllability canonical model of the system is

$$\frac{d}{dt} [e_{x_1}, \dots, e_{x_{j-1}}, e_{x_j}]^T = [e_{x_{h+1}}, \dots, e_{x_j}, -f_e(\mathbf{e}) + p_e(t)$$

$$- b_e(\mathbf{e})u(t)]^T \quad e_{x_i} = x_{i,r} - x_i \quad \text{with } i = h, \dots, j$$

where  $f_e(\mathbf{e})$ ,  $p_e(t)$ , and  $b_e(\mathbf{e})$  are functions of the error vector  $\mathbf{e}$ . (10)

Finally the Routh-Hurwitz polynomial for the new sliding surface  $S(e_{i,r}, t)$  is

$$S(e_{i,r}, t) = \sum_{i=h}^j k_i e_{x_i} = 0 \quad (11)$$

So, the closed loop control applications, instead of the state variable  $x_i$  it is worthy to consider a new state variable, the error  $e_{x_i}$ , component of the error vector, so it will be actually  $e_{x_i}(t)$  of the state space variables  $x_i$  related to the given reference  $x_{i,r}$ , the equation, the new controllability canonical model is obtained and thus actually,  $d/dt$  of this one, it leads to this equation where the error  $e_{x_i} = x_{i,r} - x_i$  and so on.

So, where  $f_e$  and  $p_e$  and  $b_e$  are of the function of error effect  $\mathbf{e}$ , from this the Routh-Hurwitz criteria which we have studied in the linear control model that is also applicable finally, the Routh-Hurwitz polynomial for the new sliding surface will be in this and we should condition that  $\dot{x} = \text{error vector} \cdot t$  should actually minimise a error 0 for this is in this value is 0, now, stability, how we will ensure that it will give the stability?

We have stability conditions with a rank of the matrix in the linear form, so what should be the equivalent here.

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## Stability

Existence condition: The existence of the operation in sliding mode implies  $S(e_x, t) = 0$ .

Also, to stay in this regime, the control system should guarantee  $\dot{S}(e_x, t) = 0$ .

Therefore, the semiconductor switching law must ensure the stability condition for the system in sliding mode, written as-  $S(e_x, t)\dot{S}(e_x, t) < 0$  (12)

The fulfillment of this inequality ensures the convergence of the system state trajectories to the sliding surface  $S(e_x, t) = 0$  since

- if  $S(e_x, t) > 0$  and  $\dot{S}(e_x, t) < 0$ , then  $S(e_x, t)$  will decrease to zero,

- if  $S(e_x, t) < 0$  and  $\dot{S}(e_x, t) > 0$ , then  $S(e_x, t)$  will increase toward zero.

Hence, if Eq. (12) is verified, then  $S(e_x, t)$  will converge to zero. This condition (12) is the manifold  $S(e_x, t)$  invariance condition or the sliding-mode existence condition.

Existence condition; existence of the operation in the sliding mode implies that is S the surface, the error and the time should be = 0, so to stay in this regime, the control system should generate a start e of t = 0, therefore the semiconductor switching law must ensure the stability condition of the sliding mode, it is written as S, ex dot should be < 0, this is a stability condition, we have to show that system is stable.

Fulfilment of this inequality ensures the convergence of the state system trajectory to the sliding surface, that is S e to the x, error and the t vector = 0, since if S is > 0 and S dot is < 0, then it will decrease to 0 and if again, S < 0 and S dot is > 0, then also S dot will increase towards 0, hence equation 12 is verified, then x xdot t converges to 0, this condition 12 is manifold of the variance conditions or the sliding mode existence condition or sliding mode stability condition.

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## Stability (Cont...)

The equivalent average control input  $U_{eq}(t)$  that must be applied to the system in order that the system state slides along the surface Eq. (11), is given by

$$U_{eq}(t) = \frac{k_h \frac{de_{x_h}}{dt} + k_{h+1} \frac{de_{x_{h+1}}}{dt} + \dots + k_{j-1} \frac{de_{x_{j-1}}}{dt} + k_j (-f_c(\mathbf{e}) + p_c(t))}{k_j b_c(\mathbf{e})}$$

This control input  $U_{eq}(t)$  ensures the converter subsystem operation in the sliding mode.

Now, the equivalent average control you may have a multi input system, multi output system, u of t that must be applied to the system in order that systems states sliding along the surface that is u of t is given by this equations and the control input u of t ensures that the converter subsystems actually operates in a sliding mode and ensures the stability of the system, so this is the one way, this is the actually the sliding mode.

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## Reaching condition

The fulfillment of  $S(e_{x_{iv}}, t) \dot{S}(e_{x_{iv}}, t) < 0$ , as  $S(e_{x_{iv}}, t) \dot{S}(e_{x_{iv}}, t) = (1/2) \dot{S}^2(e_{x_{iv}}, t)$ , implies that the distance between the system state and the sliding surface will tend to zero since  $S^2(e_{x_{iv}}, t)$  can be considered a measure for this distance. This means that the system will reach sliding mode.

In addition, from Eq. (10), it can be written as follows:

$$\frac{de_{x_i}}{dt} = -f_c(\mathbf{e}) + p_c(t) + b_c(\mathbf{e}) u_i(t)$$

From Eq. (11) the sliding surface will be written as

$$S(e_{x_i}, t) = \sum_{i=h}^j k_i e_{x_i} = k_h e_{x_h} + k_{h+1} \frac{de_{x_h}}{dt} + k_{h+2} \frac{d^2 e_{x_h}}{dt^2} + \dots + k_j \frac{d^m e_{x_h}}{dt^m}$$

Now, what is reaching condition; the fulfillment of this basically,  $S \times \dot{S} < 0$  as  $S \times \dot{S} =$ ; let us consider that  $1/2 * S \dot{S}$ , so implies that these distance between the subsystems states and the sliding will tend to 0, since  $S^2 \times \dot{S}$  can be considered as a measure for this distance, so this is the distance, this means that the system will reach to the sliding mode.

And in addition from the equation number 10, it can be written as follows that this value that is differentiation of the error is basically,  $-fe + pe - be$  and with that this is the input actually controllable inputs and it will ensure that from this equation 11, this can be rewritten  $S$  as the surface of the  $\dot{x}_i$  that is actually  $\dot{x}_h$  and so on.

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### Reaching condition (Cont...)

- Hence, the system will reach sliding mode, staying there if  $U = U_{eq}(t)$ . This same reasoning can be made for  $S(e_{x_i}, t) < 0$ ; it is now being necessary to impose  $-b_e(e) - u_f(t) = +U$ , with  $U$  high enough to guarantee  $de_{x_i}/dt > 0$ .
- To ensure that the system always reaches sliding mode operation, it is necessary to calculate the maximum value of  $U_{eq}(t)$ ,  $U_{eqmax}$  and also impose the reaching condition:  $U > U_{eqmax}$
- This means that the power supply voltage values  $U$  should be chosen high enough to additionally account for the maximum effects of the perturbations.

So, hence a subsystem will reach sliding mode staying there, if  $U = U_{eq}$  equivalent, this same reason can be made for  $S$  function of  $e$  and  $t < 0$ , it is now being necessary to impose that is  $-b - u_f$  of  $t$  as  $+u$  with  $U$  high enough to guarantee that this actually, condition leads to this basically,  $dx/dt$  is  $> 0$  to ensure that the system is always reaches the sliding mode operation, it is necessary to calculate the minimum value of the  $eq_t$  and  $eq_{max}$  and also impose the reaching condition,  $U > eq_{max}$ .

This means that the power supply voltage values  $U$  should be chosen high enough to additionally to account the maximum effect to the perturbation.

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## Switching Law

- From the foregoing considerations, supposing a system with two possible structures, the semiconductor switching strategy must ensure  $S(e_{x_i}, t)\dot{S}(e_{x_i}, t) < 0$ . Therefore, if  $S(e_{x_i}, t) > 0$ , then  $\dot{S}(e_{x_i}, t) < 0$ , which implies, as seen, -
- $b_e(e)u_h(t) = -U$  (the sign of  $b_e(e)$  must be known).
- Also, if  $S(e_{x_i}, t) < 0$ , then  $\dot{S}(e_{x_i}, t) > 0$ , which implies  $-b_e(e)u_h(t) = +U$ .
- This imposes the switching between two structures at infinite frequency.
- Since power semiconductors can switch only at finite frequency, in practice, a small enough error for  $S(e_{x_i}, t)$  must be allowed ( $-\epsilon < S(e_{x_i}, t) < +\epsilon$ ).
- Hence, the switching law between the two possible system structures might be

$$u_h(t) = \begin{cases} U/b_e(e) & \text{for } S(e_{x_i}, t) > +\epsilon \\ -U/b_e(e) & \text{for } S(e_{x_i}, t) < -\epsilon \end{cases}$$

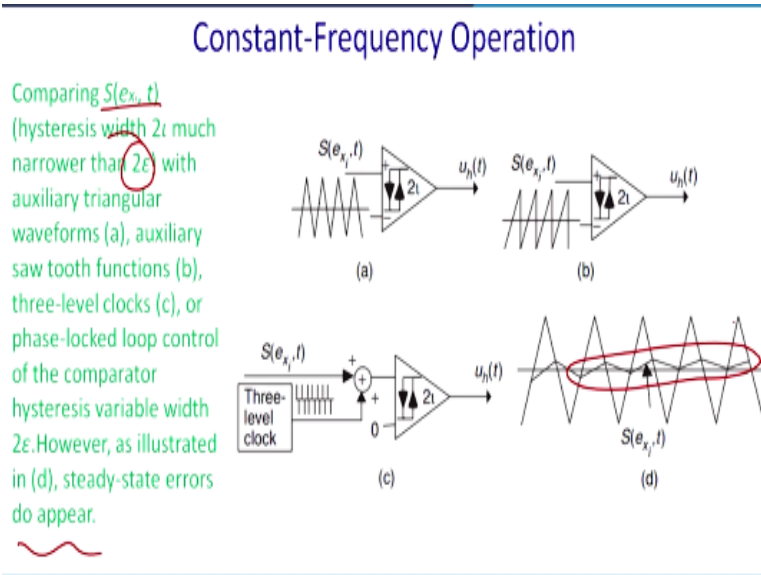
So, what should be the switching law, how will ensure that actually, our converter will be switching from the foregoing consideration supposing, a system will have a 2 possible structures; the semiconductor switching strategies ensure that  $x, \dot{x}$  should be  $< 0$ , therefore if  $S$  is  $> 0$ , then  $\dot{S}$  should be  $< 0$ , which implies as same that is  $b$  of error function of  $Ust$  should be  $-U$ , these sign we must be known them.

Similarly, this means basically, the switch can be off then, similarly if this is  $< 0$  and then  $\dot{S}$  is  $> 0$ , this implies that the switch should be on basically and we should know the basically the sign of  $b_e$  and assuming that the sign of the  $b_e$  is positive, so you require the  $U$  to be positive, if it is the reverse, then that  $U$  sign will change, this imposes that switching between these two structures and infinite frequency will take over this way.

Since power semiconductor can switch only finite frequency in practice a small error for  $S$  will be allowed were basically the  $\epsilon$  is basically will be  $>$  than the sliding surface and  $<$  the positive sliding surface, hence switching law between this 2 possible system structures might be basically  $U/b_e$  for  $x_e$  that is a error  $> 1$ , then we have a positive switching and  $-U/b_e$  for  $S_t$  is  $<$  that error, so we have one, let us take an example, when actually decibels voltage is more, then definitely you will basically switch it off.

And one will be require to build up the decibals voltage which should be switching on, so let us see how does it work practically, so this is also the mathematical part of it and now take a practical aspect of it.

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Then and now comparing this  $S$ , let us have a hysteresis band, this hysteresis band is very narrow of this actually narrower than  $2\epsilon$ , with auxiliary triangle of waveform a, the auxiliary saw tooth function b and the three level clock is c, or the phase locked loop with the comparator any of it will walk, this one, this one, this one and hysteresis variable width  $2\epsilon$ , however as illustrated in the 2D, the steady state error will appear.

Because you know you will switch it on and there will be an error so, ultimately this error will remain and ultimately, this error is going to be accumulated and you will have some amount of the steady state error.

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## Steady State Error Elimination

The steady state error ( $e_{xh}$ ) of the  $x_h$  variable,  $x_{hr} - x_h = \epsilon_1/k_h$ , can be eliminated, increasing the system order by 1. The new state space controllability canonical form, considering the error ( $e_{xi}$ ) between the variables and their references, as the state vector, is

$$\begin{aligned} & \frac{d}{dt} \left[ \int e_{xh} dt, e_{xh}, \dots, e_{x_{j-1}}, e_{x_j} \right]^T \\ &= [e_{xh}, e_{x_{h+1}}, \dots, e_{x_j}, -f_c(e) - p_c(t) - b_c(e)u_h(t)]^T \end{aligned}$$

The new sliding surface  $S(e_{xi}, t)$

$$S(e_{xi}, t) = k_0 \int e_{xh} dt + \sum_{i=h}^j k_i e_{x_i} = 0$$

This sliding surface offers zero-state error, even if  $S(e_{xi}, t) = \epsilon_1$  due to the hardware errors or fixed (or limited) frequency switching.

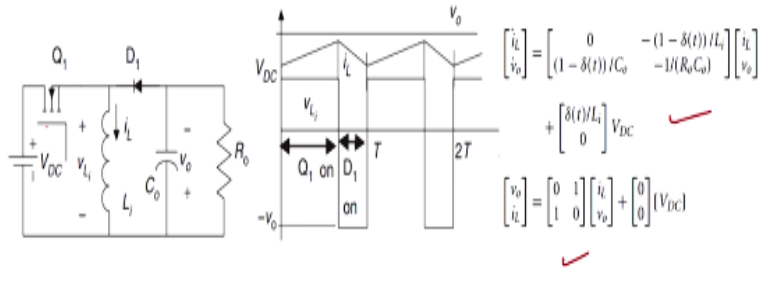
So, steady state error that is basically nothing but something like proportion control error, steady state error of the  $x_h$  variable that is  $x_{hr} - x_h = \epsilon_1/k_h$  can be eliminated by increasing the system order by one, now the new state space controllability of the canonical form considering that error  $e_{xi}$  between the variable and the reference of the state vector is; it will be basically this so, it is something like equivalent to the pi controller and you integrate it ultimately, you get this results.

Now, new sliding surface will be actually  $S = k_0 \int e_{xh} dt + \sum_{i=h}^j k_i e_{x_i}$ , this is due to the integration, so sliding surface will offer 0 state error even if this actually have a some error due to the hardware error or fixed or limited frequency switching, so this can be eliminated by integral based sliding surface.

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## Sliding-Mode Control of the Buck-Boost DC/DC Converter

Consider again the buck-boost converter of previous analysis and assuming the converter output voltage  $v_o$  to be the controlled output. Using the switched state-space model already discussed in linear control section, making  $dv_o/dt = \theta$ , and calculating the first time derivative of  $\theta$ , the controllability canonical model, where  $i_o = v_o/R_o$ , is obtained as:



So, let us take an example first, so this is basically the same circuits and same actually, the matrices which we will have and now this equation, when we try to control this switching by the sliding mode control.

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## Sliding-Mode Control of the Buck-Boost DC/DC Converter (Cont...)

$$\frac{dv_o}{dt} = \theta = \frac{1 - \delta(t)}{C_o} i_L - \frac{i_o}{C_o}$$

This model, written contains two state variables,  $v_o$  and  $\theta$ . Therefore, considering  $e_{v_o} = v_o - v_o^*$ ,  $e_\theta = \theta - \theta^*$ , the control law (sliding surface) is

$$\frac{d\theta}{dt} = -\frac{(1 - \delta(t))^2}{L_j C_o} v_o - \frac{C_o \theta + i_o}{C_o (1 - \delta(t))} \frac{d\delta(t)}{dt} - \frac{1}{C_o} \frac{di_o}{dt} + \frac{\delta(t)(1 - \delta(t))}{C_o L_j} V_{DC} \quad (13)$$

$$S(e_{v_o}, e_\theta) = \sum_{i=h}^2 k_i e_{x_i} = k_1 (v_o - v_o^*) + k_2 \frac{dv_o}{dt} - k_2 \frac{dv_o^*}{dt} = k_1 (v_o - v_o^*) + k_2 \frac{dv_o}{dt} - \frac{k_2}{C_o} (1 - \delta(t)) i_L + \frac{k_2}{C_o} i_o = 0 \quad (14)$$

So, ultimately you have a  $dv/dt$  and all those things, this model written in two states;  $v_0$  and  $\theta$  therefore, considering that actually, this is an error vector that is  $v_0 - v_{0r}$  and the  $\theta - \theta_r$ , we can rewrite and thus we can have a sliding surface that is  $= k_1 (v_0 - v_{0r}) + k_2 \frac{dv_0}{dt} - k_2 \frac{dv_{0r}}{dt} = 0$ , ultimately you know that this should be  $= 0$ ,  $k_1 * (v_0 - v_{0r}) + k_2 * \frac{dv_0}{dt} - k_2 * \frac{dv_{0r}}{dt} = 0$ .

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## Sliding-Mode Control of the Buck-Boost DC/DC Converter (Cont...)

This sliding surface depends on the variable  $\delta(t)$ . Assuming an ideal up-down converter and slow variations, the variable  $\delta(t)$  can be averaged to  $\delta_1 = v_o / (v_o + V_{DC})$ . Substituting this relation in Eq. (14) and rearranging, Eq. (15) is derived:

$$S(e_{x_1}, t) = \frac{C_o k_1}{k_2} \left( \frac{v_o + V_{DC}}{v_o} \right) \times \left( (v_{o_r} - v_o) + \frac{k_2}{k_1} \frac{dv_{o_r}}{dt} + \frac{k_2}{k_1} \frac{1}{C_o} i_o \right) - i_L = 0 \quad (15)$$

This control law shows that the power supply voltage  $V_{DC}$  must be measured, as well as the output voltage  $v_o$  and the currents  $i_o$  and  $i_L$ .

And from this condition relative, we can say that the sliding surface depends on the variable  $\delta(t)$  assuming that ideal up-down converter and a slow variation of the  $\delta(t)$  can be an average that is  $\delta_1 = v_o / (v_o + V_{DC})$ , ultimately will lead to this equation, this control shows the power supply of voltage  $V_{DC}$  must be measured as well as the output voltage  $v_o$  and the current  $i_o$  and  $i_L$ , then only we can generate the sliding surface.

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## Sliding-Mode Control of the Buck-Boost DC/DC Converter (Cont...)

To obtain the switching law from stability considerations, the time derivative of  $S(e_{x_1}, t)$ , supposing  $(v_o + V_{DC})/v_o$  almost constant, is

$$\dot{S}(e_{x_1}, t) = \frac{C_o k_1}{k_2} \left( \frac{v_o + V_{DC}}{v_o} \right) \times \left( \frac{dv_o}{dt} + \frac{k_2}{k_1} \frac{d^2 v_{o_r}}{dt^2} + \frac{k_2}{k_1} \frac{di_o}{dt} \right) - \frac{di_L}{dt} \quad (16)$$

If  $S(e_{x_1}, t) > 0$ , then  $\dot{S}(e_{x_1}, t) < 0$  must hold. Analyzing Eq. (16), we can conclude that if  $S(e_{x_1}, t) > 0$ ,  $\dot{S}(e_{x_1}, t)$  is negative if, and only if,  $di_o/dt > 0$ . Therefore, for positive errors  $e_{x_1} > 0$ , the current  $i_o$  must be increased, which implies  $\delta(t) = 1$ . Similarly, for  $S(e_{x_1}, t) < 0$ ,  $di_o/dt < 0$  and  $\delta(t) = 0$ . Thus, a switching law is obtained:

$$\delta(t) = \begin{cases} 1 & \text{for } S(e_{x_1}, t) > +\epsilon \\ 0 & \text{for } S(e_{x_1}, t) < -\epsilon \end{cases} \quad (17)$$

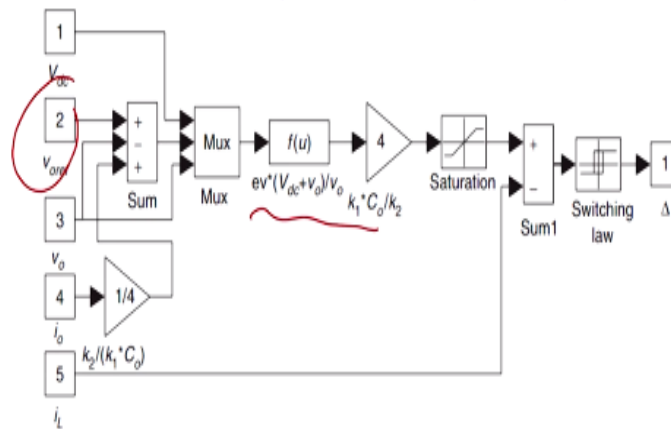
And we will ensure that the conditionality of the sliding surface explained and now, we have to check the conditions of the stability to obtain the switching law, we have to take the derivative of it and to obtain the switching law from the stability consideration, the time derivative will be

basically  $v_0 + V_{DC}/v_0$  and it is almost constant, so we can put it that and thus what we can say is that if the sliding surface is  $> 0$ , then  $S \dot{}$  must be  $< 0$ , must hold.

Analysing this equation 16, we can conclude that if it is  $> 0$ , then  $x \dot{}$  is negative and only if  $di/dt$  is  $> 0$ , then what we can conclude; therefor, for the positive error if  $v$  is  $> 0$ , the current  $i_l$  must be increase then, switch should be on  $< 0$  than zero and thus  $\Delta t$  should be  $= 0$  and thus it should be off and this is the conditions for the switching and we can ensure in case of the sliding mode surface.

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### Sliding-Mode Control of the Buck-Boost DC/DC Converter (Cont...)



Block diagram of the sliding-mode nonlinear controller for the buck-boost converter

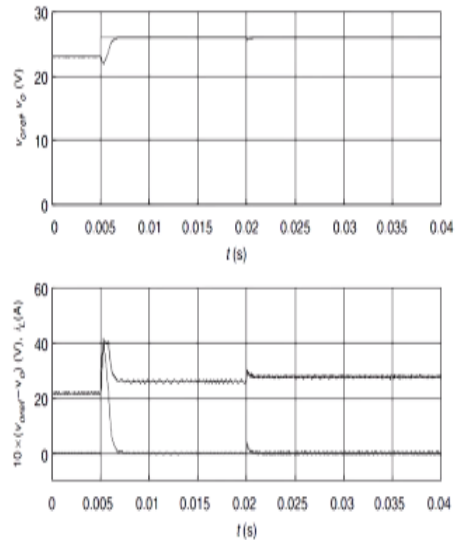
So, this is the overall control cycle, this is a VDC, this is basically  $v_{df}$ , this is  $v_0$ , this is  $i_0$  and  $i_l$ , you will sum it up, there after you will actually put it into the sum of what; you have multiplex, there after you will generate the function, you have multiplicity of 4, there after you have a saturation block, then you have actually subtractor, you will check the condition whether which one is zero, accordingly we will implement.

It can be easily implemented if you wish to have a actually simulation you can easily implement with the Simulink blocks.

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## Sliding-Mode Control of the Buck- Boost DC/DC Converter (Cont...)

Transient responses of the sliding mode controlled buck-boost converter. At  $t = 0.005$  s,  $v_{ref}$  step from 23 to 26 V. At  $t = 0.02$  s,  $v_{ref}$  step from 26 to 23 V. Top graph: step reference  $v_{ref}$  and output voltage  $v_o$ . Bottom graph: trace starting at 20 is  $i$  current; trace starting at zero is  $10 \times (v_{ref} - v_o)$ .



So, these are the results, you know I do not go into much explanation, this slide says that transient response of the sliding mode control of the Buck Boost converter at  $t = 0.005$  and  $v_0$  ref steps from this, then you can see that actually it is rising very fast and top graph reference and bottom graph actually traces out  $i_l$  and the current will be actually tracing back from this actually starting from 0 to 10, so these are the actually the results of the sliding mode control.

Now, we have taken one important aspect of the nonlinear control, there are so many non-linear control and just I show that actually what I; we try to picture it basically that this kind of the sliding mode control or nonlinear control gives you the better stability and the better transient response than the linear control. Thank you for your attention, I take the next class that is basically will be the concluding class and there we will try to sum up the whole courses, thank you for your attention.