Microelectronics: Devices to Circuits
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Lecture-40,
S-Domain Analysis, Transfer Function, Poles and Zeros-I

Hello everybody and welcome to the next edition of NPTEL Online Certificate Course on Microelectronics: Devices to Circuits. In our previous module, we had looked into simple multistage amplifier, differential amplifier based operational amplifier and we also saw the methodology for extracting various parameters of the OP-AMP.

In today's lecture we slightly change gears and we try to do what is known as S-Domain Analysis. We will start with S-Domain Analysis today and we will show to you that how do you actually do a frequency response which means that with varying frequency how does the gain of an amplifier changes. We will also look into the concepts of poles and zeros and what is a transfer function.

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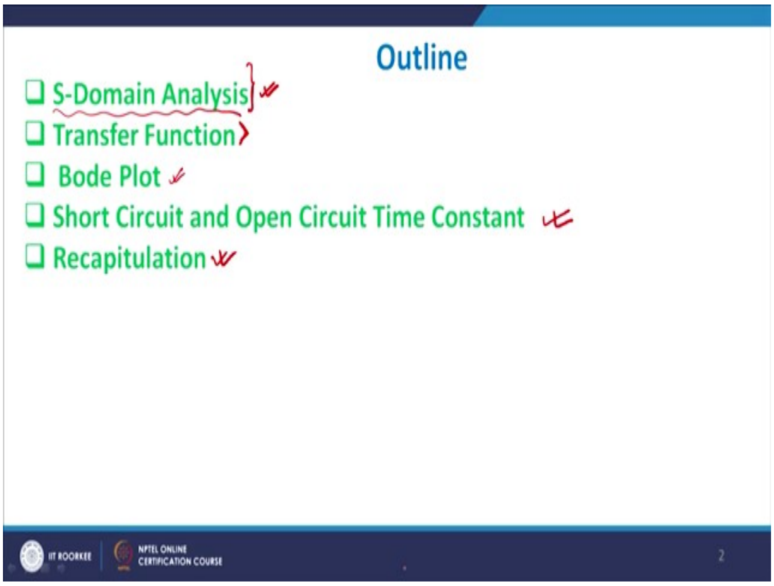

So, the lecture module is named as S-Domain Analysis, Transfer function and Poles and Zeros. So, these 3 we will be covering maybe by this module and then we will move along and we will see as we go ahead. So, the outline of the stock is something like this that we will start with first of all, we will explain to you later on maybe what is S-Domain, though I have written it here at the very first instance but we will be using this later on.

So, we will be using this later on, right. So, we will be using the concept of S-Domain Analysis later on and we will finally come to, we will first initiate our discussion with Transfer Function, explain to you what is the Bode Plot and then look into what is a Short Circuit and Open circuit Time Constant and we recapitulate our work. So, this is the general scheme of things which will be following as far as this course is concerned.

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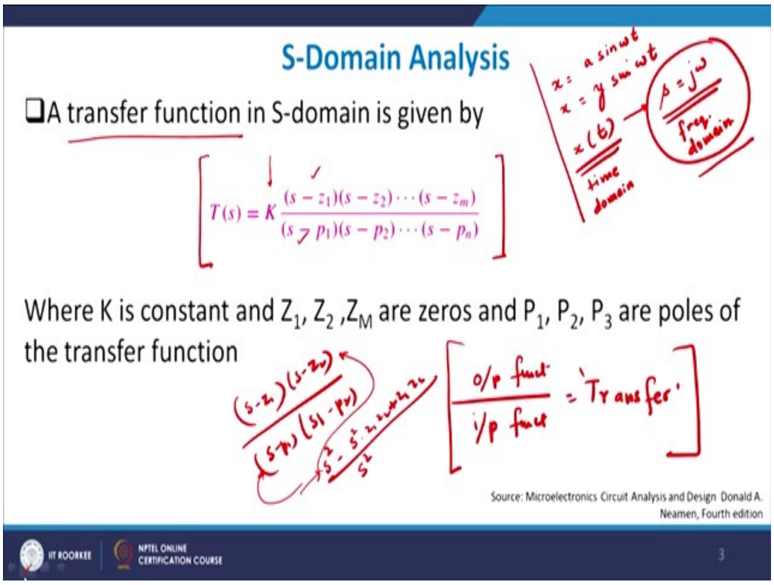

Now, when we say in S-Domain, typically most of the input signals which you will be encountering will be in time domain for example let us say x equals to a sin of omega t or let us say x equals to y sin omega t. Which means that x is basically a function of time t. So, this is in time domain. Now, there was a requirement, we will not underline at this stage not why but to convert this time domain into frequency domain, right.

So, we define s to be equals to j omega and this is what we define as frequency domain. So, this is known as the time domain analysis and we have got here a frequency domain analysis. So, in most of the cases we will be actually looking into frequency domain analysis and then when time permits we will go to time domain analysis. But, at this stage we are not looking into the methodology by which you can convert a time domain into frequency domain and vice versa.

We are assuming that we do already have in S-Domain which is in the frequency domain we do have the certain characteristics available and from there I can find out the values of this thing, various characteristic of the device of circuit. We define a new term here which is known as Transfer Function. Transfer Function as the name suggests is a very simple and basic pack. It is basically output function divided by input.

So, this is known as Transfer Function. As the name suggests, it is transferring from output to input. So, we define this to be an output function divided by input function. We define output function, output can be current voltage, input can also be current voltage. So, therefore, I can have sort of 4 types Transfer Function available to me. Because, there are 2 inputs and there are 2 outputs, so total about to 4.

I can have 4 combinations of transfer function available (to us) to me in a much more detailed manner. But, before we move forward, let me show to you that what is the meaning of poles and zeros before we move forward. I will be keeping to bare minimum understanding so that we can concentrate more on the circuit aspects of it. For understanding more of poles and zeros, I would refer that you go for a basic course of the network theory and synthesis part of the network theory and there this will be discussed in a detailed manner.

Look at this Transfer Function, if you look Ts is given as K times, in the numerator you have s minus Z_1, s minus Z_2, s minus zm divide by s minus p_1, s minus p_2, s minus pn. Now, we all know that the output current or voltage or input current or voltage can be expressed in a linear relationships and this can be there for factorized as given as this and this, right. This K is basically constant, which is basically an s, it does not depend upon the frequency, it's basically an integral constant and it is kept outside here, right.

It is also referred to as gain for most practical purposes. I guess, therefore s minus Z_1, s minus Z_2, so if you for e.g. if this would have been s minus Z_1 and then s minus Z_2 divided by s minus p_1 into s minus p_2, then you open it up, I will get s square minus square Z_1Z_2 plus Z_1Z_2 divided by s square, so on and so forth. So, this is basically a quadrative equation here and I can remove the quadrative equation in form of partial factors and these are the partial products of each one of them, right.

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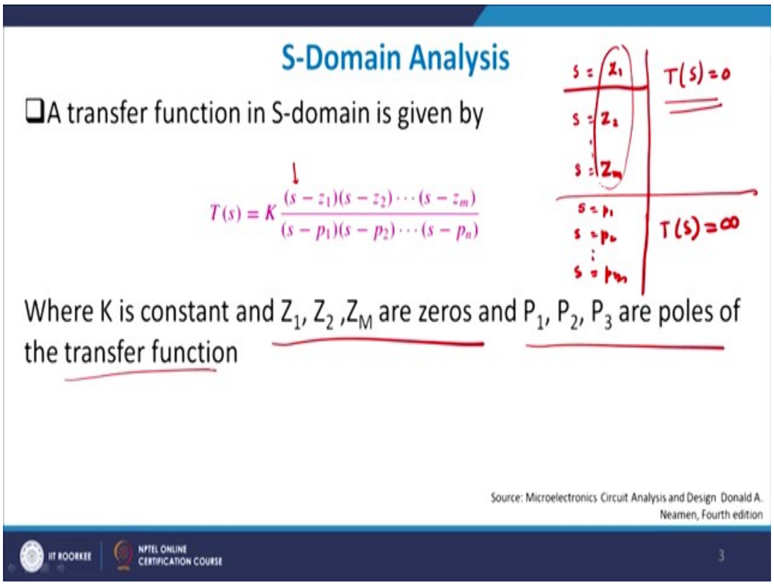

Therefore, it gives me an idea that, I can explain to you from this basis equation itself that if, let us suppose my s happens to be equal to Z_1 let us suppose. Then what happens is that, (this z minus) s is equal to Z_1, so z minus Z_1 is 0 and all of T(s) is equals to 0. Similarly, when s equals to Z_2 or till s equals to zm for all the cases I get T(s) equals to 0, which means that in the numerator if the solution of one of the points of the numerator happens to be either Z_1 to zm then the whole function actually goes to 0.

Similarly, in the denominator if each of the function s goes to p_1, s goes to p_2 and so on s goes to pm then the output T(s) goes to infinity, right. So, these are the points, Z_1,Z_2 till zm which are referred to as 0 are the points where if you are biasing your device in frequency domain at those particular points, the gain will be exactly equals to 0 whereas, if you are biasing your points at p_1, p_2, pm, then your gains will be infinitely large.

So, if you biased your amplifier whose transfer function is given by this equation which is shown in front of you, then, and if you biased it at p_1, p_2, p_3 to pm, I would expect to see a maximum gain and if you are able to biased it at Z_1, Z_2, Z_3, Z_4 then we define that the gain will be equals to 0. So, therefore we define this to be as poles and zeros, right.

Poles are the points in frequency domain where your gain becomes infinitely large and zeros are the points in the frequency response where the gain is approximately equals to 0 or exactly equals to 0. Therefore, Z_1 and Z_2 to zm are defined as zeros and p_1,p_2 and p_3 are referred to as poles of the transfer function. K is the constant which is independent of frequency. So, this is the basic understanding of this thing.

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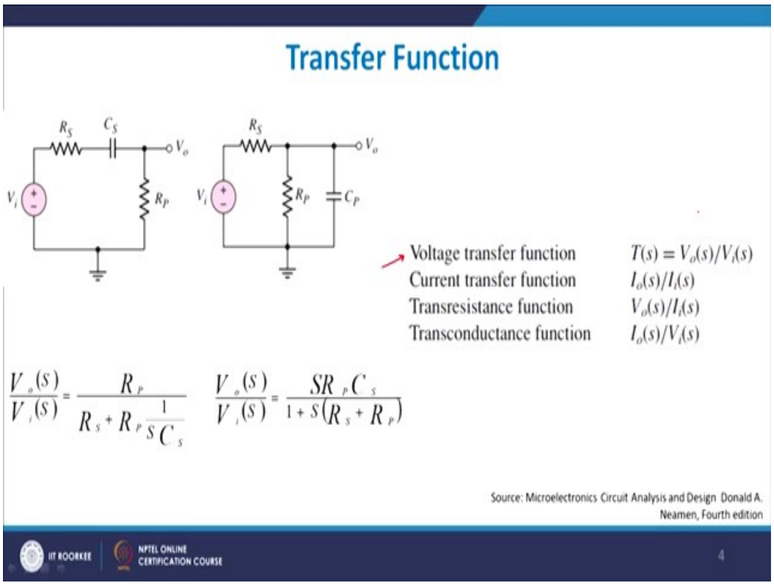

Therefore, as I discussed with you just now, since there are 2 quantities (current and voltages) and their 2 ports (input and output) I can have therefore about 4 types of Transfer Function. The first Transfer Function is very simple, Voltage transfer which refers to as output voltage by input voltage. So, $V_0(s)$ or Vo(s) divided by Vi(s) is basically giving meaning that output voltage by input voltage.

If you look at the Current transfer function here then it is basically output current by input current, right. If you are doing a Transresistance function then output voltage by input current and transconductance if you see then output current by input voltage, right. So, the first two are dimensionless quantities and (the next two are) these two are basically your dimensionless and this is of the order of Ohms and this is 1 upon Ohms.

So, we have got semens here. So, we have got therefore 1, 2, 3, 4 types of Transfer Function available to us in a distinct sense of it. If we give you a brief idea about. Let us say for e.g. How to derive a Transfer Function? So given a circuit, given any circuit how can you derive the Transfer Function. I will just show you one example for that using basic this thing, we can actually go ahead and show it to you.

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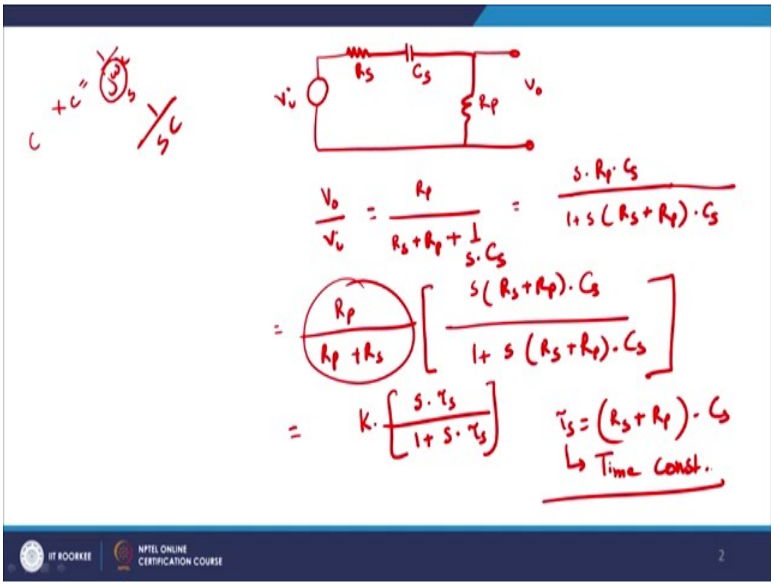

For example we will take the same circuit as this one and let us see how it works out. Let me just draw for you. I have got this and then we have got Rs here, then we have C_s here and then we have R_p here and then we have V_{out} coming like this. So, this is R_p, we have V_i here. So, if you plot V_0 by V_i, I get R_p upon, because it is a core potential divide sort of technique plus 1 by SCs. SCs basically meaning that, if you remember (you will) the capacitance, any capacitance see will have its capacitive reactance to be given as 1 upon jwc, right.

So, this jw is replaced by s, so I get 1 upon SC, so that is the reason I get 1 upon SC here in this manner, right. And therefore, you can write down this to be as equal to S into R_p into C_s divided by 1 plus s times R_s plus R_p into C_s, right. And then if you break it down, I get R_p upon R_p plus R_s, right. And then if you write it down I get S upon Rs plus R_p, right, into C_s divided by 1 plus S times R_s plus R_p times C_s, right.

I get this into consideration into all these things and therefore I can write down this. So, this basically a K which is basically a constant independent of frequency. I get S times Tow S into 1 plus S times Tow S, right. Where Tow S is given as R_s plus R_p multiplied by C_s also referred to as time constant of the circuit. So, I get K times S Tow S upon 1 plus S Tow S and circuit.

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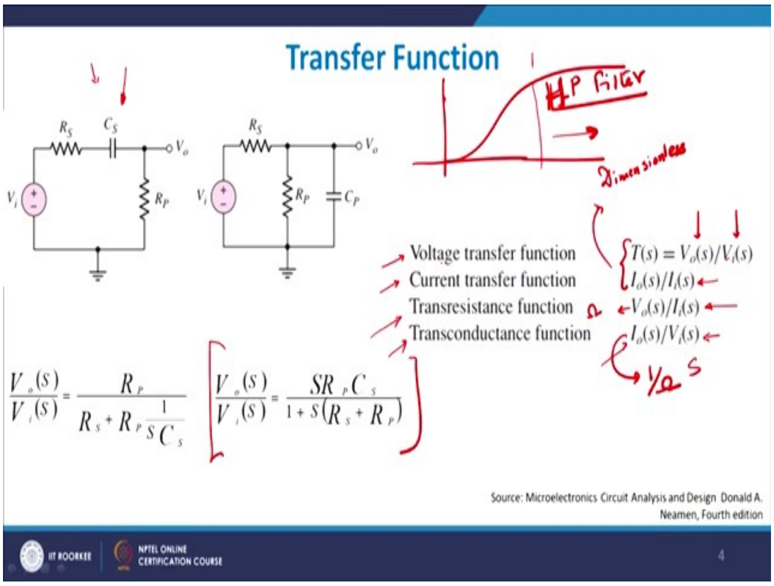

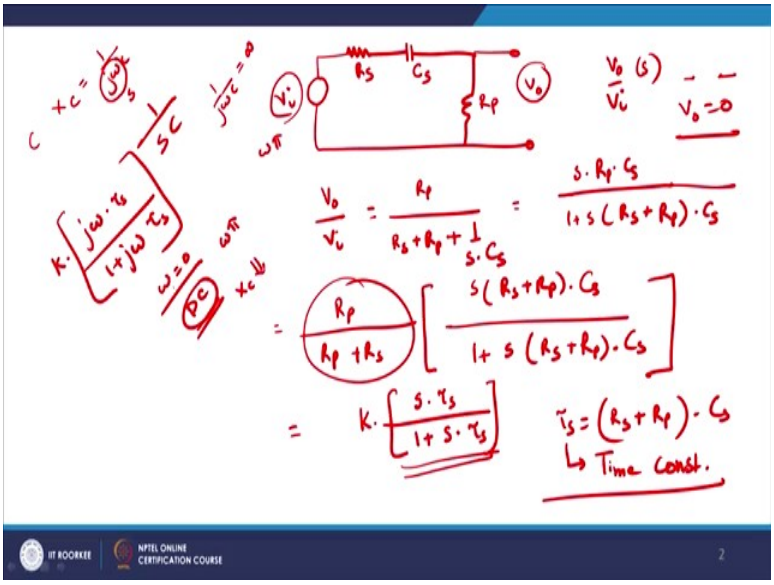

Now, if you come back to this, from this explanation as you can see, this is what I got also in the previous discussion just now. And you will see that, sorry just a minute, so I get yes, this is what I got, (I got) so if you see if I replace this S by j omega I get K times j omega Tow S upon 1 plus j omega Tow S. You might be asking the reason, why not eliminate 1 because 1 will be very small compared to j omega Tow S.

The reason we are not doing it is because for very low values of omega this condition will not hold good that j omega Tow S will always be greater than 1, right. So, for not all values of omega you will have that condition. For certain value of omega of course it will be greater than one then it becomes independent of K.

Now, let me give you a physical insight and then we will mathematically connect that with this equation which we just now derived, right. See, if you look at this 2 port network where this is my V_{in} and this is my V_{out} and then maybe fix the value of V_i, that is ok, so we find out V_0 by V_i which is function of (s). Now, if you start varying my omega frequency from a very low value to a very high value, let us see how it works out when your omega is very very small.

Let us say omega is 0, which means you are doing a DC bias, then what will happen, the output will be 0 and the reason is when omega equals to 0, 1 upon j omega c will be infinitely large and therefore the capacitance will behave like an open circuit and we already know that DC bias condition capacitance starts to behave like an open circuit and therefore, V_0 equals to 0, right. At omega equals to 0, V_0 will be equals to 0 and therefore your Transfer Function will give you a gain equals to 0.

Now, if you go on increasing the value of omega, right, then this X_c value starts to fall down from the infinity value it comes down. And therefore, it allows more and more signal to be transferred in the output side and the gain therefore starts to increase as we move ahead, right, the gain starts to increase.

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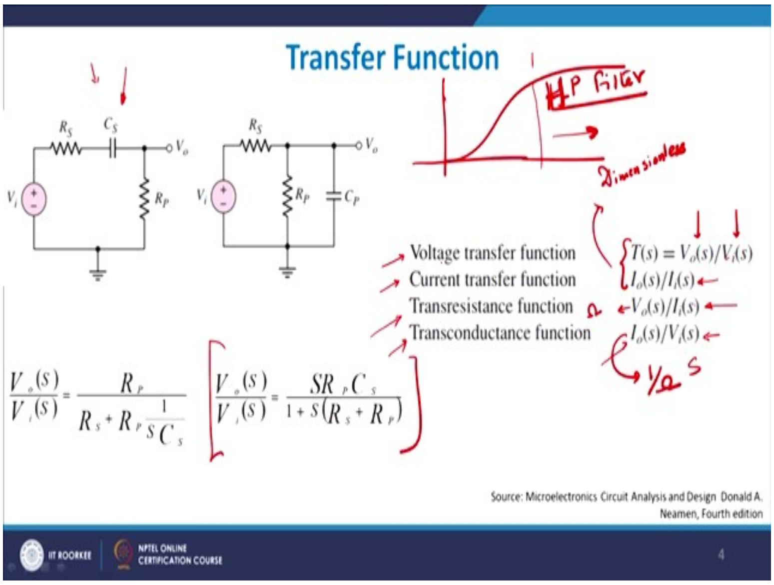

So, if you look back here, therefore for e.g. if you look at this value, then as you therefore make your omega larger and larger, the xc, the resistance offered by the C_s, right, that starts to come down and therefore (it gives you) the gain starts to increase. So, what happens is that at omega equals to 0 the gain is almost equals to 0. But, as the omega value goes on increasing the gain output voltage goes on increasing and therefore the gain goes on increasing.

So, I can safely say that the output function is something like this, right. Therefore, it starts to behave as a low-pass filter. Why low pass? Sorry, it is a high-pass filter. I am sorry, high-pass filter because at high frequencies it is allowing you to the system to pass and at low frequencies it is blocking the system in a detailed manner, right.

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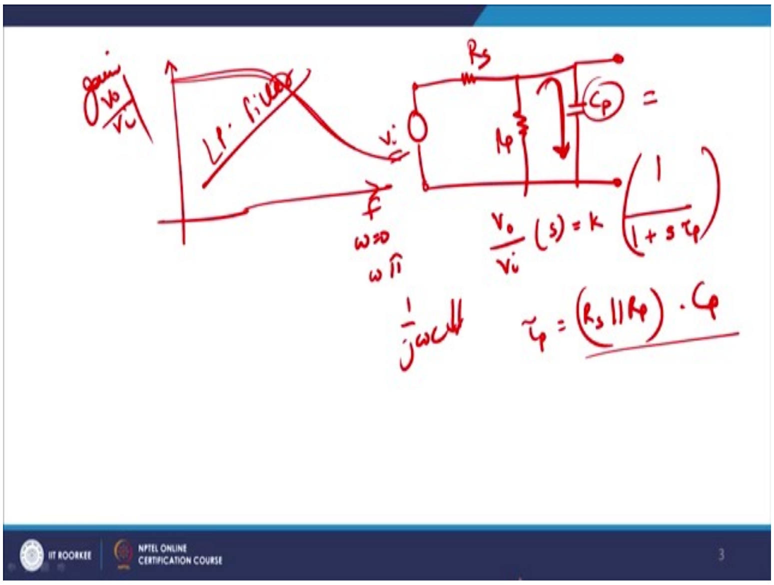

So, in reality if you look very carefully therefore, that it is acting as a filter design, basic filter and it is basically high pass filter which you are actually finding out here. Now, if you do a small change in this whole thing and then try to do that you have R_s here and then you have got R_p here and then parallel to R_p you have C_p. So, rather than a series resistance capacitance, you have now a parallel capacitance here, right, and this is R_s, this is R_p and this is C_p which you get, V_i which you get here, then I get V_0 by V_i as a function of (S) to be equals to K into 1 upon 1 plus s times Tow p.

This is you will get, where Tow p is incidentally R_s parallel to R_p multiplied by C_p, right. So, it is R_s parallel to R_p multiplied by C_p. When parallel means, you will always, the output will always be less than the least. So, out of R_s and R_p whichever is the least value, you will get for that and multiplied by C_p, where C_p is this value which you get. Now, you see incidentally here, if you look very carefully, then at omega equals to 0, this will behave as an open circuit, right, this will behave as an open circuit and as a result all V_i will appear at the output side, right, or part of it will appear at the output side.

Whereas, as you make your omega go on increasing 1 by j omega c starts to decrease and more and more frequency curves, high frequency domain circuits are inserted on to the system, right. And this does not appear in the output side, the output starts to fall. So, if you look very carefully, it will be something like this, the output profile will look something like this. If you plot frequency versus (gain) output gain, output gain is V_0 by V_i, right.

If you plot it you get something like this and therefore it starts to work as a low-pass filter, right. So, I can therefore by simply changing the placement of my capacitor from a series connection to a parallel connection, I can actually convert the high pass to low pass and the ociated with that also gives you the same principle, right, and that you can find out from here, you can find it incorporate that, let me see how it works out.

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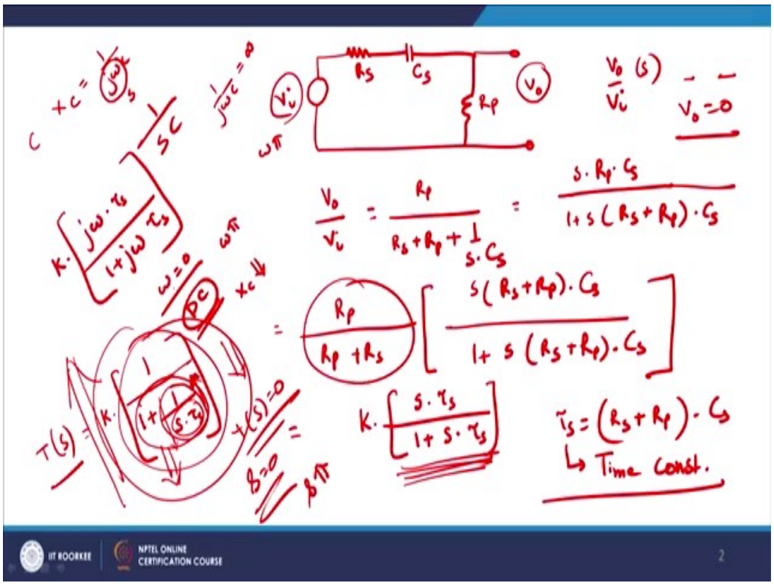

Let us suppose I found out this one, right. So, if you look at this expression of the transfer function then if I divide the numerator and the denominator by s Tow s, I get K times 1 over 1 plus 1 by s times Tow s and this will be equals to your T(s) or the transfer function of s. So, you see as (s) was equals to 0, right. If (s) equals to 0 which is a DC bias, when this is 0 this quantity actually shoots very very high, becomes very very large, right. It becomes very very large, primarily therefore it means that this quantity comes down to a very low value and therefore T(s) is almost equals to 0 at s equals to 0, fine.

So, s equals to 0 at T(s) equals to 0 and s equals to 0. Now, as S starts to increase or as S becomes more and more higher, this quantity, right, this quantity starts to reduce and therefore 1 plus this quantity starts to reduce and therefore this quantity starts to increase, fine. And as a result you will see that the gain will start to become higher. So, the plot which I made just now, this plot exactly summarizes the plot in the earlier cases, right. So, I just wanted to make it sure that you understand whatever we are doing is in consonance with the output characteristics.

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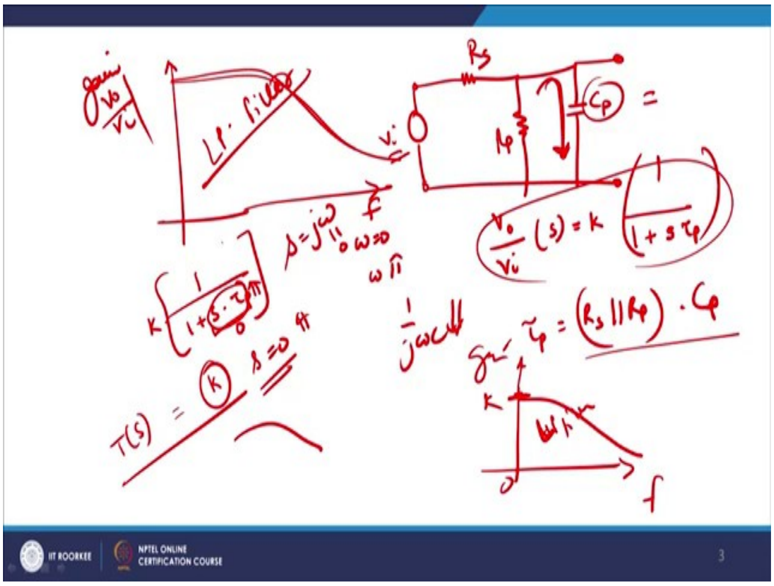

I will show you here also, say for e.g. if you take this into consideration, I get K times 1 over 1 plus s times Tow p, right. Now, if your S is 0 for example then I get output equals to K. If S equals to 0, S equals to basically equals to j omega, that omega equals to 0 I get S equals to 0. S equals to 0 implies that this quantity goes to 0 implies that the output T(s) will be always equals to K and therefore at S equals to 0, if I plot frequency versus gain here, I will get K here at T equals to 0.

As my S goes on increasing, this quantity goes on increasing and therefore 1 upon this quantity goes on decreasing and therefore my gain starts to fall down and (())(21:28) starts falling down like this at higher value of F. So, it starts to behave as a low-pass filter. So, physically also and numerically also, I can show it to you by using transfer function I can predict output characteristics of the device or the circuit in a sense that I will be able to predict its behaviour in terms of output gain with respect to change in the frequency in the input side, right.

That is what is written in this, that is what it is shown in this module or in this worksheet and it gives me an idea that simply by changing the position of the capacitor in this case, for example, from series to para combination, I am able to change the filter from a high pass to a low pass one. So, this is the basic understanding or operation of the design which we get.

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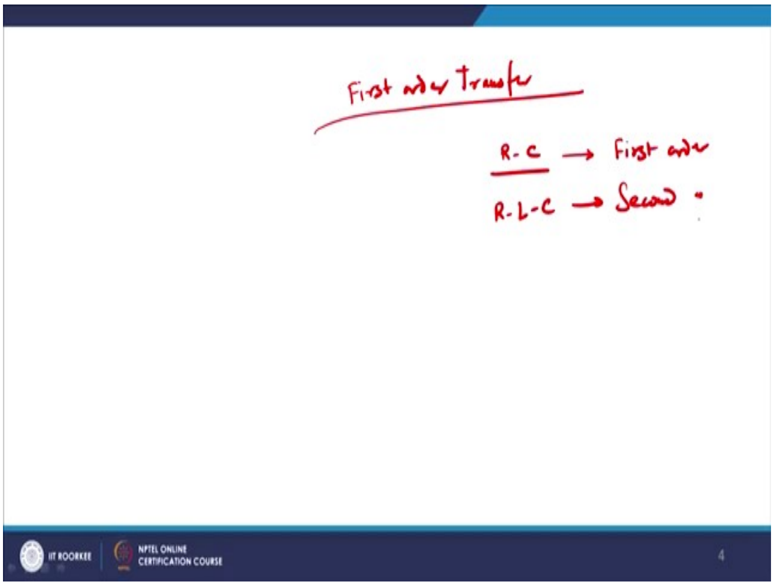

These are all defined as first order transfer functions. First order basically means that where, apart from register you have only one capacitor into consideration, right. Now, if you have a capacitor as well as the inductor RLC then we would define that as a second order transfer function, right. So, if you have a simple R_L, suppose you have got simple R and C network then the equation which defines it is basically or the transfer function defines basically as first order transfer function.

But, if it is an R, L and C, then we define this to be second order transfer function. We will stick ourselves to first order and explain to you from first order itself how you can derive various quantities here. So, we see that therefore you have got 4 types of functions. Transfer function can be given, we can evaluate and we can get the values.

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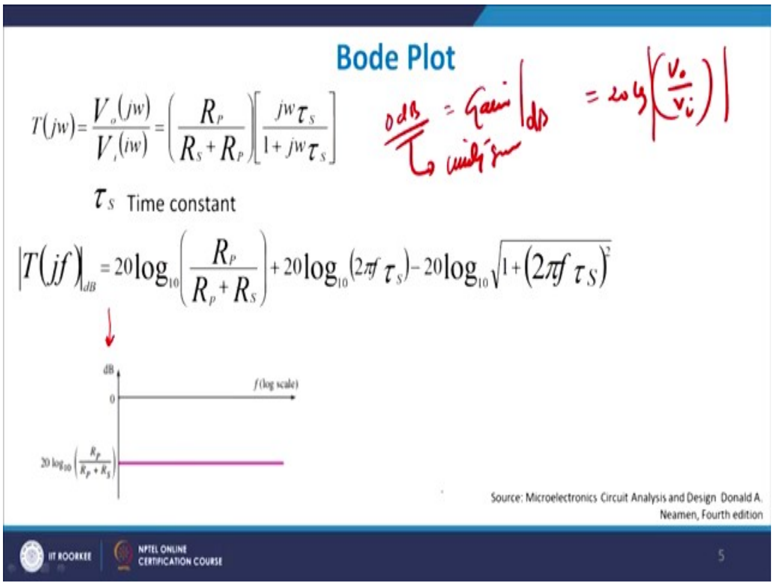

Now, let me come to a very important term or a very important aspect of designing is basically the Bode Plot. The Bode Plot is basically this one, the variations of the (gain) voltage gain in dB with respect to frequency in log scale because, if you remember frequency can vary from 0 to maybe gigahertz value or maybe megahertz or gigahertz value.

Therefore, if I keep it in a linear scale I will never be able to get the actual profile because, linear scale will give you a very large linear scale with us. Therefore, what people have proposed is that the X axis or the X scale should be basically a log scale, right. And in the log scale we have the frequency which is there and on the Y axis we have the gain and gain is in dB.

So, it is basically 20 log V_0 by V_i, this is your gain in, voltage gain in dB. It is 20 log V_0 by V_i, right. So, when V_0 equals to V_i, log 1 will be equals to 0 and therefore our gain will be equals to 0 dB. So, 0 dB primarily means that you do not have any gain. It is unity gain. So, it is basically unity gain available to me, right and so on and so forth. We should also therefore, we will see how you can generate a Bode Plot or a Bode plot, we will see each one of them individually.

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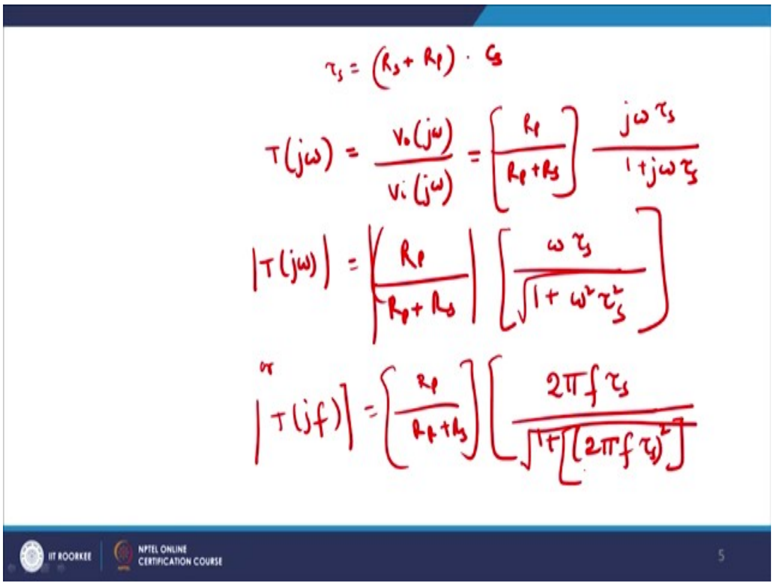

Now, we just now saw that, we just now saw that, let me just again come back to this same concept here and let me explain to you that, we just now saw that Tow S was equals to Rs plus R_p into C_s and T of j omega because T of S was equals to V_o of j omega upon V_i of j omega, right, and is given as R_p upon R_p plus R_s into j omega Tow S divided by 1 plus j omega Tow S.

So, if you take the mode of T of j omega then I get square of this whole quantity and I get R_p upon R_p plus R_s, magnitude of this one ofcourse remains the same, into omega Tow S upon 1 plus omega square Tow S square or if I do it in terms of T (j f) because it is 2 pi f equals to omega, I get R_p upon R_p plus R_s, right, into 2 pi f of Tow S upon 1 plus 2 pi f of Tow S whole square and then this will be square root, so this will be square root here. So, I will get this consideration in this particular form.

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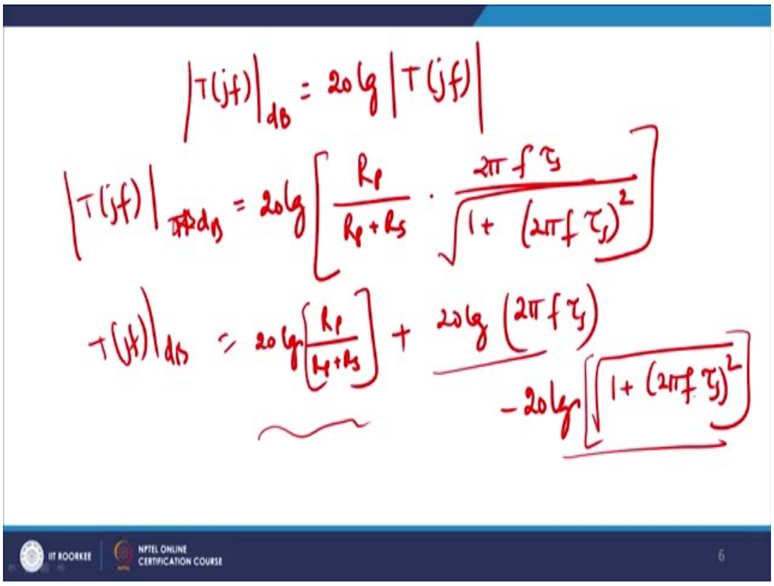

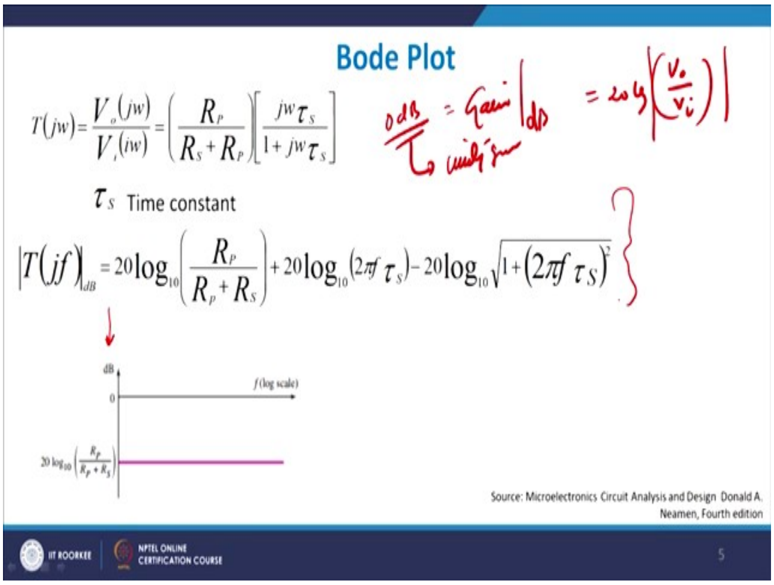

Now, if I therefore write T(jf), right, in dB, I just need to find out 20 log of T(jf), right. And therefore, T(jf) in tB, sorry in dB, will be given as 20 log of R_p upon R_p plus R_s multiplied by 2 pi f Tow S upon 1 plus 2 pi f of Tow S whole square square root, right. If you look at this, it is exactly the same thing as this quantity. Because, why this quantity? Because if you just break it down I get, I can break it down into 2 parts. I get 20 log of R_p upon R_p plus R_s, right. I can break it into this, plus I get 20 log of 2 pi f of Tow S, right.

And then I can get a minus sign 20 log of 1 plus 2 pi f Tow S whole square square root of this one. That is what we defined now as T(jf) in dB. So, the whole quantity which we found out just now, can be therefore broken up into 3 parts, 1, 2, 3 and we will see in the next turn, in the next module that when we start with this and we add these 3 together how the profile comes out in the output side, right. So, we will just do a principle of super position for all these 3 and check out the values of the output Bode Plot, fine. In the next turn when we take up, we will discuss this point. Thank you.