VLSI Physical Design with Timing Analysis

Dr. Bishnu Prasad Das

Department of Electronics and Communication Engineering

Indian Institute of Technology, Roorkee

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Lecture 23

Fidduccia-Mattheyeses (FM) Algorithm

Welcome to the course on VLSI Physical Design with Timing Analysis. In this lecture, we discussed about another partitioning algorithm that is called FM algorithm. The content of this lecture includes basically the Fidduccia Mattheyeses or FM algorithm. First of all, we discussed about the terminologies involved in this algorithm. Then we will discuss one example how this FM algorithm is useful for partitioning two blocks. So, we already discussed about the KL algorithm. But the FM has some new features compared to KL algorithm. So, here we have a graph G (V, E) with nodes and the weighted edges actually. So, our goal is to basically partition or by partition the nodes to create two disjoint partition and the final goal is to minimize the total cost and it has one extra constraint which is not there in the KL algorithm that is the area constraint of the each partition. In here we are basically whenever we are moving a cell from one partition to the other, we need to check that whether it is satisfying my area constraint or not.

Here in case of KL algorithm, we are allowed to basically swap two nodes and we are not allowed to take one single node from one partition to the other in case of KL algorithm. But in case of FM algorithm, the single cell can be moved independently instead of basically swapping a pair of nodes. So, basically it will not look into the number of nodes or the cells in a partition, but it will look into the area of the partition. So number of nodes does not matter, but the area of the partition is important. So here if you can see the area of each individual cell should be mentioned in case of FM algorithm because that is used as a criteria for moving a cell from one partition to the other.

So this should be given as input to the algorithm. Area of individual cell need to be given. So it is useful for partitioning of unequal size partition. So here what is happening is that the two partitions comes out from the final partition, whatever the final partition will come out, the nodes or the cells in each of the partition may not be equal that is also allowed. So then there is one new concept is there which is not there in the KL algorithm, which is called the hypergraph or hyperedges.

What is this hypergraph and hyperedges is that, so if I take an example here, let us say I have a two input NAND gate, it is driving one XOR gate. Now it is going to inverter let us say or now it is going to one NOR gate. So I can convert this diagram into graph. So this there is a node is here for the NAND gate. So this is you have XOR gate another node then inverter another node then your basically NOR gate is another node. So here you have node 1, node 2, node 3, node 4. So one net, you have either minimum two or more than two pins. In this case, so here what is written nets with 2 plus pins. In this example, we have three pins which is greater than two. So here these are the these edges is called your hyperedge actually and the graph is called hypergraph. This graph is called this edge is called hyperedge and the graph is called hypergraph.

So this FM algorithm can support basically hyperedges, which is not possible in case of KL algorithm. It minimizes a cut cost based on the nets, number of cut cost based on this hyperedges, which is not possible in case of KL algorithm. So let us discuss into some of the terminology which is used in case of FM algorithm. The first one is the gain actually. If I move a cell what is the gain? So this gain function is very important which basically drives our partitioning algorithm. So if you have a very good gain function, your algorithm will be totally separated. It will have a uniqueness and adds uniqueness to your algorithm. So here that FM algorithm has delta G which is a new gain compared to your KL algorithm.

$$\Delta g(c) = FS(c) - TE(c)$$

It has two terms one is called the FS of C and TE of C. What is FS of C? FS of C is the number of nets connected to C but not connected to any other cells in the C's partition. Basically the nets connected to the node C and it is passing through the cut line actually. So that is called your moving force. Moving force if I tell in another word, it says that if I move that cell from one partition to the other, how much benefit I am getting? Moving force. If it is there in one partition, if I move to another partition, how much gain I am getting? That is why it is called the moving force. Then the second one is called TE because it is called retention force. Retention force means basically if I keep that cell in the same partition, what is the benefit? So, the retention force is basically saying that what is the non-oncut nets connected to C.

If I have a node C, how many uncut nets are there? Uncut nets means that the nets which is not passing by the cut lines. So, retention force says that you have to keep that cell in the same partition. So, higher the value of delta G, then we should give more priority to move that cell to the other partition. Delta G is more, then you move that cell from one partition to the other partition. So, let us take some example here. We have basically FS of 2 is there. So if I see here, nets which is cut by the cut line. So here if you can see that node 2, you have this net is there. And this is also called a hypergraph. This is a hypergraph and these are called the hyperedges actually. The same node is connected to 2, 1 and 3. So here if you can see it is passed by the cut line. It is also connected with 2 and 3.

So if I go back, here it is written only. So FS of 2 will not include that net. That is why FS of 2 is 0. Now TE of 2, what is the TE of 2? TE of 2 has two nets A and B. But we have to consider the nets which is not passing by the cut line, uncut nets.The uncut nets is the A here. So this will give me TE of 2 is 1. My gain, what is the gain? If I move the 2 to other side, what is my gain? The minus 1. So my number of cut line will be increased by, number of cut cost will be increased by 1, if I move the 2 to other partition. So similarly I can find the FS and TE for each nodes or each cell.

I can find it for each cell. So after I find for each cell, I need to take which is giving me the highest gain, delta G which is giving me the highest gain. So here I have basically two options are there. I have two options are there, cell 1 and cell 5. But I have to take any one of them. So here basically the cell 1 is moved to one partition, other partition. So if I go by, if I go move the cell 1 to the other partition, now I have one partition, let us say partition A which is having cell 2 and partition B which is having 1, 3, 4, 5. So now I need to find out the maximum positive gain, GM of a pass. So maximum positive gain GM is the cumulative cell gain of M moves that produce a minimum cut cost. So each step we are calculating the gain, then we are adding that delta G with the previous gain of the pass, previous gain of the iteration, then we are accumulating that.

That is called cumulative cell gain. So here GM is determined by the maximum sum of the cell gain delta G over a prefix of M move in a pass.

$$G_m = \sum_{i=1}^{m} \Delta g_i$$

So for each iteration I need to accumulate or the basically sum of all the gains of the previous gain together to find the gain of that iteration. Then we have another terminology called ratio factor. This ratio factor is used for balancing the two partitions. So here it is a relative balance between the two partitions with respect to the cell area. If you know all the cell area and the ratio factor, we can find a bound which is allowed for a partition to support. So it prevents all the cell to cluster into one of the partitions. So the ratio factor is basically will not allow clustering of all the cells into one of the partitions. So it is very useful factor. So what is the definition of ratio factor R is basically area of A divided by area of A plus area of B.

$$r = \frac{area(A)}{area(A) + area(B)}$$

So this is the ratio factor. So area of A and area B are the total respective area of partition A and B. Now we have to define a balance criteria. So how much basically the partitions can support? So this is basically allowed through the ratio factor that each of the partition what is the maximum size or what is the minimum size allowed.

So here we have this balance criteria. This is the partition A, area of a partition should be bounded by in the upper side by R times area of B plus maximum area of any of the cells in the total the cell which is offering the maximum area.

$$[r.area(V) - area_{\max}(V)] \leq area(A) \leq [r.area(V) + area_{\max}(V)]$$

This basically this factor says that the area of a cell which is having the maximum area that should be added in one of the side. In the other side R times area of B minus that cell which is having the maximum size or area. Then we have a concept called base cell. The base cell is a cell that has the highest cell gain delta G. So if you are calculating the delta G which is giving me the highest cell gain that is called your base cell. So we need to move those cells basically and also it should satisfy balance criteria. So whenever I am finding the base cell it should have a highest gain that is one thing among the all the free cells we should not calculate the gain of the fixed cell. The free means which is allowed to move and it should satisfy the balance criteria.

Two things the highest gain cell among the free cells and the second one is the it should satisfy the balance criteria. Then here if I go by this example the base cell is the cell 1 which is having delta G of 1 is 1. Then the cell 1 is allowed to move. Now we will go by the FM algorithm. First we need to compute the balance criteria because that balance criteria will be used throughout the algorithm.

Then we will compute the delta G of each cell. First of all we will calculate the FS and TE of each cell from FS and TE we will calculate the delta G. Your delta G is basically FS of a cell is basically FS of C minus TE of C. So we need to find out FS of C and TE of C to find out delta G of C. Now after you find out the delta G then we will go for finding the which is with the base cell that has having the maximum gain and it should satisfy my balance criteria. Then we will move that cell. So one cell move is allowed in case of FM algorithm. Then after we move that cell that will come under the category of fixed cell. So that cell cannot be moved in any future iterations. Then after we move my all the delta this FS and TE values will be changed after we move one cell from one partition to the other my FS and TE whatever I have calculated all will change. Then we need to find that FS and TE then of each cell after the move then we will update the gain delta G and connected to the nets via the base cell CI. Then there are two case, case one when all the cells are fixed then you are you have to go to the step 5. If not all the cells are not fixed then you have to find the next base cell which is having the maximum gain and also it will satisfy the balance criteria. Then move that cell and go to the step 3. Then this step 3 and 4 will be repeated till all the cells are fixed all the cells are fixed. Then I have to go to the step 5 where I have to find out the capital GM which is summation of i equal to 1 to m delta gi and find out the step at which my gain is gm value is maximum.

Find out the iteration where my gm is maximum. Find the step which is giving me the maximum gain. If my gm is greater than 0 then go to step 6 otherwise then you have to end the algorithm. Then in the step 6 you have to execute m different moves and reset all the fixed nodes then you have to start the new pass. You have to go to the step 1 like we did in case of a KL algorithm. So now take an example how things are happening. So these are the given factors in case of fm algorithm first thing is that your ratio factor is given this is the first thing then area of each of the cells are given. So area of the cells are given and my ratio factor is given then I can first step is the compute the balance criteria. So which is the cell having the maximum area the cell 5 the cell 5 is the basically area of the cell 5 is highest.

So that one should come here. So this will be 5 this will be 5. Then the area of A what is the constraint for area of A? R times total area. So it comes out to be 11. So the upper bound of partition A is 11 and the lower bound is basically 1. So upper bound of partition area of partition A is 11 the lower bound of partition A is 1. So whenever I am doing this all the steps I need to check this whether my balance criteria are met or not. So the compute the gains of each cell. So we need to compute the gains of each cell again. First of all I need to find the fs then te then delta g for each of the cells. And if I see here my cell 1 and cell 5 has the same gain delta g of the cell 1 and cell 5 are 1. So then if they have the delta g is same then I have to look into my look into the balance criteria. So basically here I have two candidates for the base cell the cell 1 and the cell 5 then the area of basically the balance criteria I need to check if I move the cell 1 the area of A is basically 4. Then if I move the cell 5 the area of the partition A is 11. So both the moves are satisfying my balance criteria so I can choose any one of the cells but here we have taken the cell 1 is selected for the move. Now after I move one of the partition A have basically two other partition has all the cells 3, 4, 5 and 1.

Now I need to again find out the delta g whatever I told you after I find the delta g then I need to check that which is giving me basically the best gain. The best gain is offered by basically cell 2 which is giving me gain 2. So now I have to move to the iteration 2 here we have cell 2 has a maximum gain the cell 2 is maximum gain of 2 then area criteria 0 is violated here if I move this one the area criteria is violated then I have to choose the next node either 3 or 5 I have taken the 3 then delta g 2 is minus 1 area of A is now 5 then the balance criteria is met. Here in the first case the case 1 the balance criteria is not met the case 2 the balance criteria is met.

So we need to choose the cell 3 for a move. So also we can check it for the cell 5 which is having area of 9 it is also meeting the balance criteria. So either 3 or 5 should be used can be taken for a move but we have taken the cell 3 here. So then we have 2 and 3 in one partition 1, 4, 5 is in the other partition and this 1 and 3 is now the fixed cell, 1 and 3 will not move in the future iterations. So now if you can see here my I have 3 more cells cell 2 cell 4 and cell 5 where I am finding the delta g then I have to check the which is having giving me the best gain. So the gain was offered by the cell 2 here. So here the balance criteria is met and I can move the cell 2. So here I have A3 is basically 3 and B3 is basically 1, 2, 4, 5 with the fixed cell 1, 2, 3 this 3 1, 2, 3 will not move in the future iterations. Now I have only 2 more cells are there 4 and 5 with gain 0 and minus 1 the cell 4 has the highest gain that is 0 then the cell 4 has a gain maximum gain of 0 and the area of partition is basically 5 it is meeting the balance criteria. So I have 3, 4 in one partition 1, 2, 5 in the other partition with these I have 1, 2, 3, 4 as the fixed cell 1, 2, 3, 4 are the fixed cell. So now after this I have the last cell the cell 5 is left out the cell 5 is having the gain of minus 1 then the cell 5 is the only cell left out. So it is whatever the gain that will be the maximum. So the area of the partition A is 10 which is also meeting my balance criteria. So the area of partition A4 is 3, 4, 5 area of B3 is 1, 2 all the cells are now fixed. Now whatever we are doing here each step we are finding the gain. The first step the gain is 1 then the accumulation happens in the G2 which is 2 which is G2 is 0, G3 is 1, G4 is 1, G5 is G5 is 0. So then we need to find out which is giving me the best gain among all the iterations.

So I need to find out the maximum positive cumulative gain 1, G1, G3 and G4. 1, 3, 4 I am getting iterations I am getting maximum Gm. So the move prefix 4 is selected because it has a better balance criteria 5 the 4 cells 1, 2, 3, 4 are then moved actually. So this is the results of the past one which is having 3, 4 in one partition and 1, 2, 5 in the other partition. The cut cost is reduced from 3 to 2 in this partition. So this is the partition A, this is the partition B. I have two cut costs 1 and 2.

So in this lecture we discussed about one of the famous partitioning algorithm that is called the FM algorithm. It has many merit points such as it takes area of the cell into account while doing the partitioning that is the first point. The second point is that it takes basically it can allow a single cell movement that is the second advantage and the third advantage is that it can be applicable for hypergraph or hyper edges which is really happens in case of digital circuit design. So our circuits have multiple pins are connected to the same net. So that is why this FM algorithm is suitable for hypergraph. We also discussed about one example using the FM. Thank you for your attention.

Thank you very much.