Basic Electrical Technology
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Lecture - 12
The Sinusoid

Hello everybody, till the last session we have been focusing on modelling of electric circuits. We went through the process of obtaining the model in the form of state equation and then we obtained the sinusoidal steady-state equation and then we looked at the model in the form of transfer function which gives you a much better picture of the pole-zero domain and we saw how it is applied to the RC, RL, RLC circuits. We will be using the sinusoidal voltage source quite frequently in the electrical systems because sinusoidal voltage sources watt is also obtainable from the wall socket; it is also the fundamental source in terms of the motors like the induction motor, the generator, the alternators all those give the sinusoidal waveforms. So also when we are talking of the transformers and the coils we talk in terms of fundamentals, in terms of the sinusoidal waveforms. So sinusoidal is a pretty important waveshape.

This session will focus on trying to understand this sinusoidal waveshape and the various nomenclatures which are associated with this sine wave. So today we start with a discussion on the sinusoids so that we try to gain a much better understanding of this type of waveform which will help us in further analysis and analysing the future components that we will be discussing later on which is the transformers, the magnetic domain, the induction motors, the DC motors and their controls. So sine waves are something that you should become familiar with and very adopted with.

Therefore, if we take a typical sinusoidal waveform, let me draw the X and Y axis. This is the Y axis and then we have the x axis (Refer Slide Time: 3:41), the x axis is of course with respect to time. Let us draw the picture of the sine wave. You should also try to practice drawing the sine wave as good as possible because this is something that you will be using more and more frequently in your future analysis applications and design with respect to the electrical engineering field.

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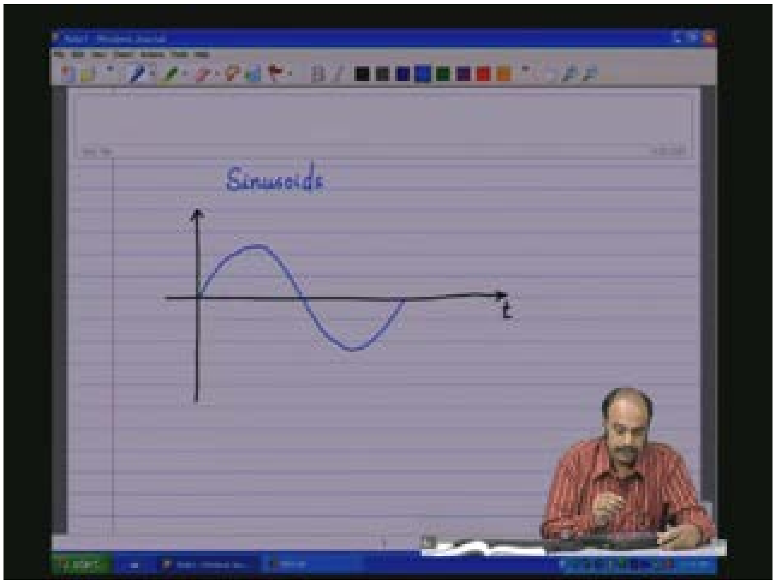

So, if we take the sine wave here it is with respect to the amplitude. It could be voltage waveform or a current waveform and based on that this can be confused a can be confused with amps so I will just write it as voltage or a current waveform. So let us say we have this sinusoidal picture here; one critical point of the sine wave is the peak value. So this is the peak value and normally if it is a voltage waveform we are denoting it my V m V max or V p V peak depending upon different literatures, they follow different conventions buy we will follow V m to represent the maximum value or the peak value of the sine wave and that is exactly this point (Refer Slide Time: 5:14).

Then when you say peak to peak value, peak to peak amplitude it is sometimes denoted by V p hyphen p, V peak to peak and that is from the positive top to the negative bottom the amplitude as shown here. So this is V peak to peak. Now this sine wave also has some definitions on the x axis. One of the most important definition is the time that it takes for completing one period. Meaning from this that is it starts from zero, reaches its peak, goes back to zero, goes to the negative direction and then comes up and goes to zero. Then from there on the same waveshape is repeated on and on so on infinitely which means this is one segment which is repeated in a time period T and in the next time period T the same waveshape gets repeated and it keeps on happening after every time period T. So this is a repetitive waveform, repeats every period. So this time period T is called the period of the waveforms; it is just called the period of the sine wave, this is called the period of the sine wave.

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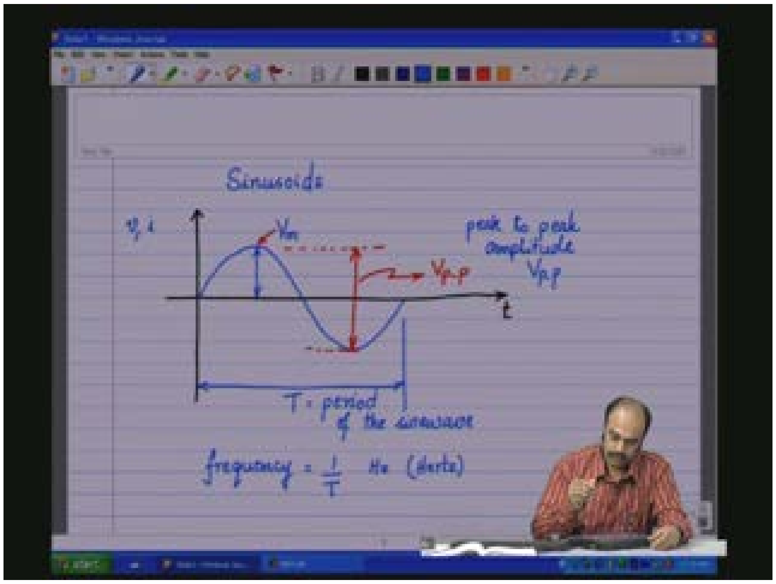

Now let us say the frequency. The frequency at which this sine wave repeats is nothing but 1 by T and this in hertz Hz after the famous scientist Hertz, time is in seconds, frequency is in so many cycles pulse second which is Hertz. So we have these definitions of the sine wave as you are seeing it on the paper or on the picture.

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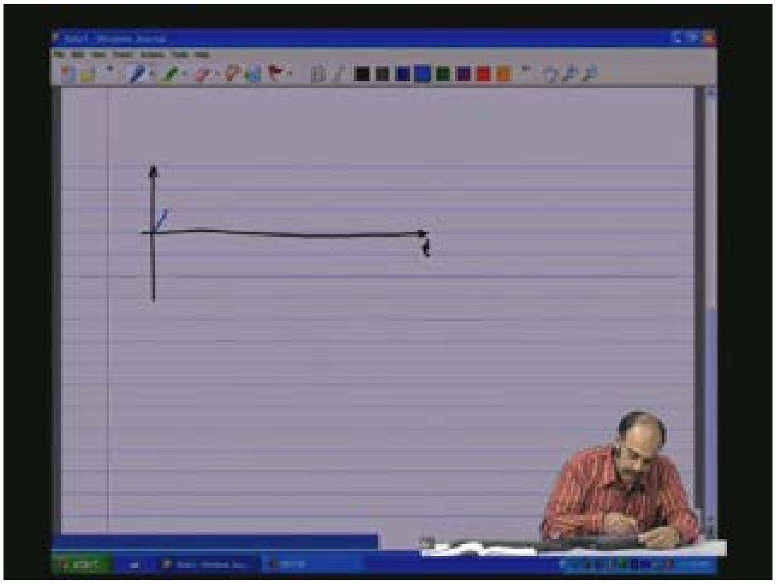

Now let us try to get some information which is not directly seen from the picture. One is, now this we stated was the peak which is V m. let us say this is voltage waveform with respect to time. This is the zero point and this is the T by 2 point and this is the T point (Refer Slide Time: 8:54) and the frequency f equals 1 by T Hertz.

Now this we would like to draw some conclusions about this waveform in terms of its average value and few other characteristic values of this particular waveform. Now let us say what is the average value of this waveform. What is what is the average or the mean AVERAGE or MEAN of this waveform; what do you understand by the AVERAGE or the MEAN?

It means the area under this curve divided by a base. so the area under this curve is nothing but this what is being hashed, this is the area under the sine curve the portion that is hashed divided by the base and the base in terms of the x axis is T. So the average value and let us say V average is we have the area under the curve by time base which is in this case T.

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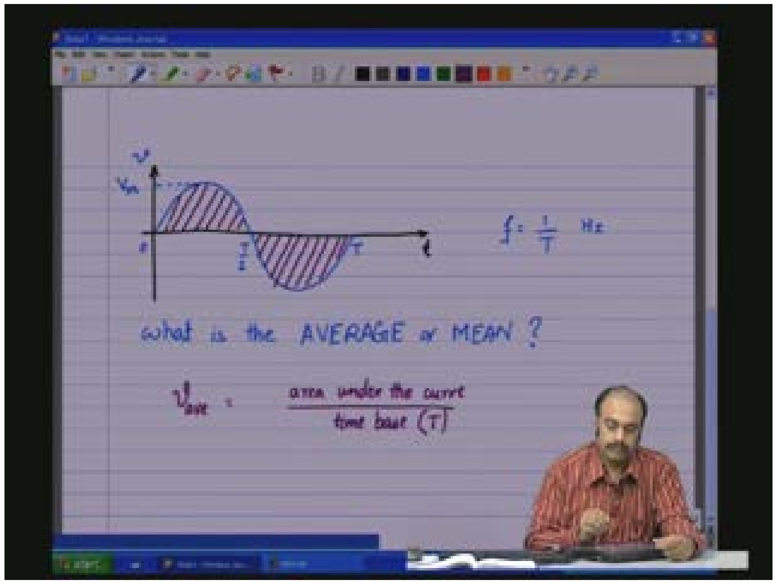

Now if you see the area under the curve; of course it is ever in just by looking at it as the top half of the sine wave is having a similar curvature like the bottom half of the sine wave; the top area because this is on the plus side this is on the minus side (Refer Slide Time: 11:30) which means this is an area which is on the plus side, this is an area which is on the minus side and they both cancel and ultimately you are going to get zero at the end of 0 by T which is equal to 0. So the mean value or the average value of the sine wave is zero because there is no DC offset, the top area and the bottom area exactly cancel and this is a pure AC signal. However, there are few modifications of the sine wave that you would see depending upon different circuits.

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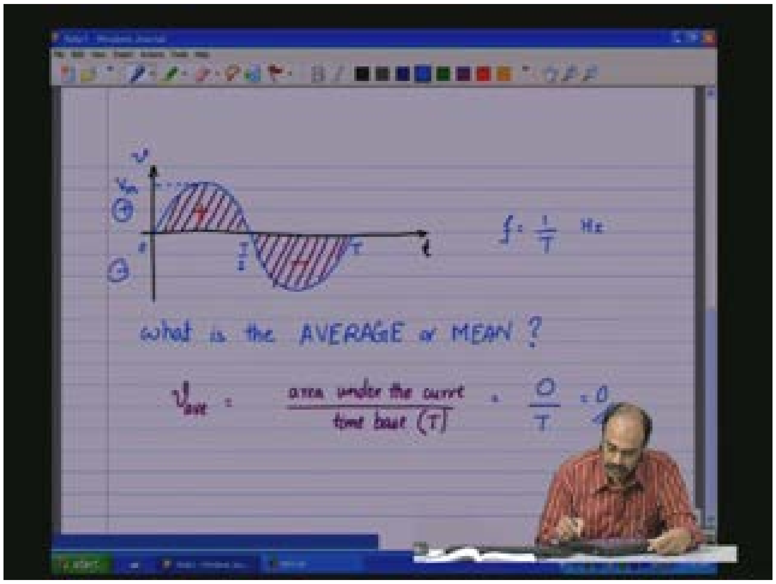

For example; if you pass it through a rectifier it could be a half-wave rectifier or a full-wave rectifier where you see portions of the sine wave which is coming out. let me indicate to you, let us say this is t, let us say the original sine wave was something like that for a period always, I am going to draw for a period starting from zero so voltage waveform; I could have let us say this waveform passed through a full-wave rectifier which means you get an output V out which is equal to V absolute value or it is nothing but…… a full-wave rectifier is nothing but having a characteristic something like that; you feed in V you get V 0 which is equal to absolute value of V which means……… look at this particular effect, what it basically does is this is 0, (Refer Slide Time: 13:48) this is a positive axis, this is the positive X axis that is in terms of time sorry this is in terms of the input, this is V and this is V 0. So whatever V be V 0 is positive which means if you have V positive V 0 is positive, if V is negative V 0 is still positive so that is what this graph represents, this is an absolute value circuit. A full-wave rectifier is an absolute value circuit. This is nothing but a full-wave rectifier.

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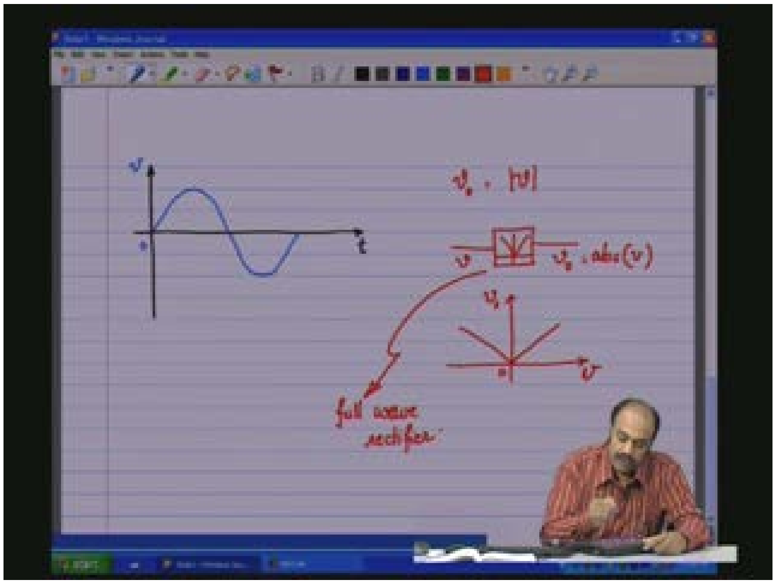

In terms of waveshape how does it look like?

V is like this. What does V 0 look like? V 0 will follow the positive of as is. Once it becomes negative of negative also is made positive and this type is coming equivalently positive and then comes like that. So if this is V the blue one, the red one is V 0 which is a full-wave rectified waveform. This is the full-wave rectified waveform rectified sine waveform because we are taking of the sine.

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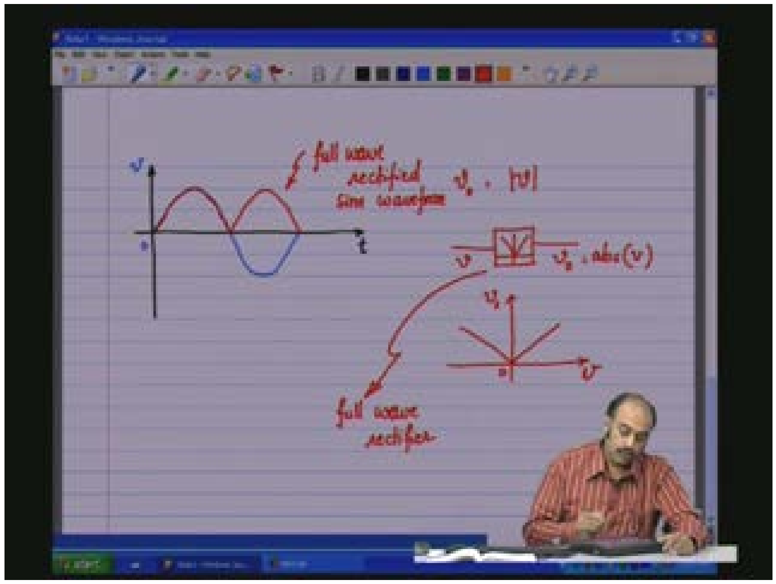

Now what is the average value of this?

Let me make some space here; so what is the average value of this full-wave rectified waveform? Quite evidently we see that as the waveform is always positive there is no negative portion, there is no area that can try to cancel out the positive area, there is no negative area which will try to cancel out the positive portion and therefore you will have a non-zero average value. Thus, all are positive. Therefore, this has an average. We average this case, it will be the area under area under the red curve divided by the time base which is T and that is what is here. Or we could also say area under one half of this red curve up to here (Refer Slide Time: 16:53) delayed by the time base which is T by 2 because that is repeating every 3 by 2 times so we get you could now say that the new period is T by 2. So area under one half of the red curve divided by T by 2.

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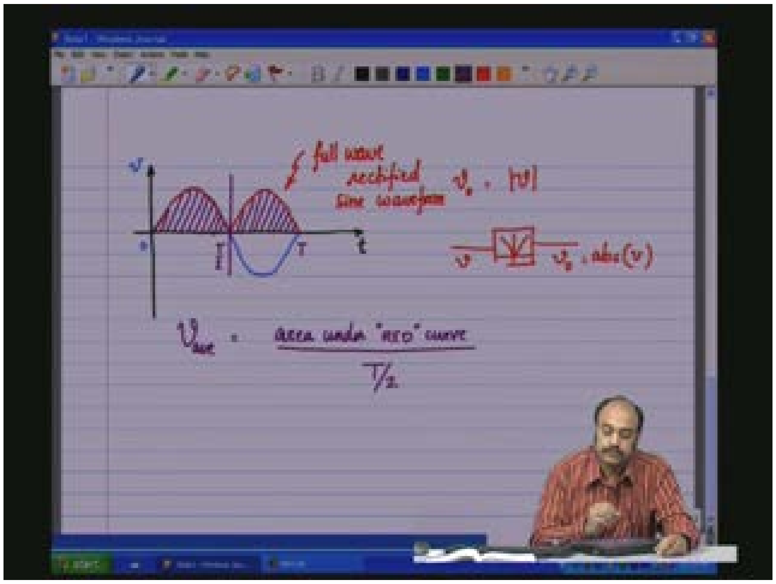

Now, what is the waveform for the sine wave?

If you take the blue that can be represented as V m sin (omegat) V m sin (omegat) is the mathematical representation of the sine wave. This we saw earlier when we were discussing the subject of sources. Omega is the frequency the radiance frequency which is 2pi into f or which is equal to 2pi by T and f is 1 by T into T. This would be the equation of the sine wave.

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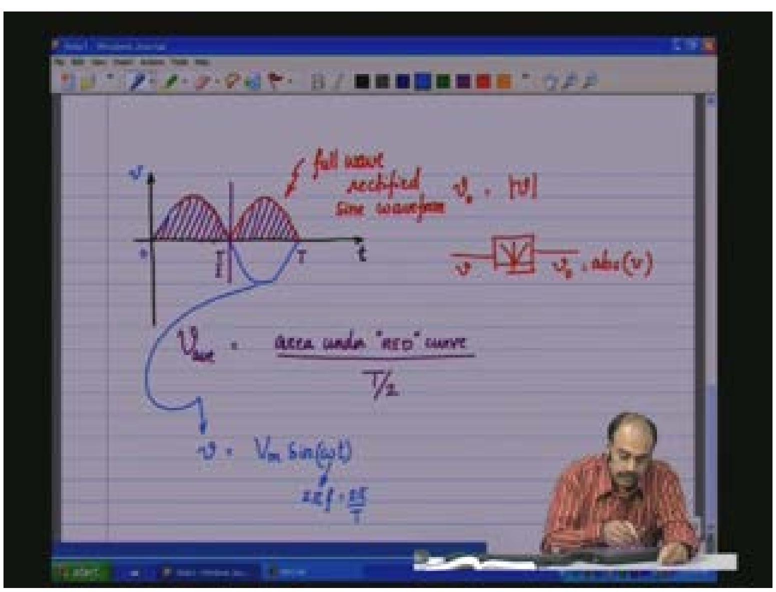

Now let us try to find out the area under the red portion of the curve for one half of the cycle using this equation. How do you find area? Area under the curve is found by integrating the curve which will give you the area under the curve.

Let us go to a fresh page. Let me write only the half portion. This is the time axis, now we have the full-wave rectified waveform and let us take after this point that is T by 2 (Refer Slide Time: 19:00) 0 T by 2 T and this is V m. So what is the area under this curve? It is the integral of this curve which will give you the area of the curve. So the integral with the curve divided by the base which T by 2 will give you the average value. So V average will be equal to integral 0 to T by 2. So we want to perform the integral from 0 to T by 2 and we integrate in the voltage waveform divided by the base which is T by 2. This is going to give you the average value.

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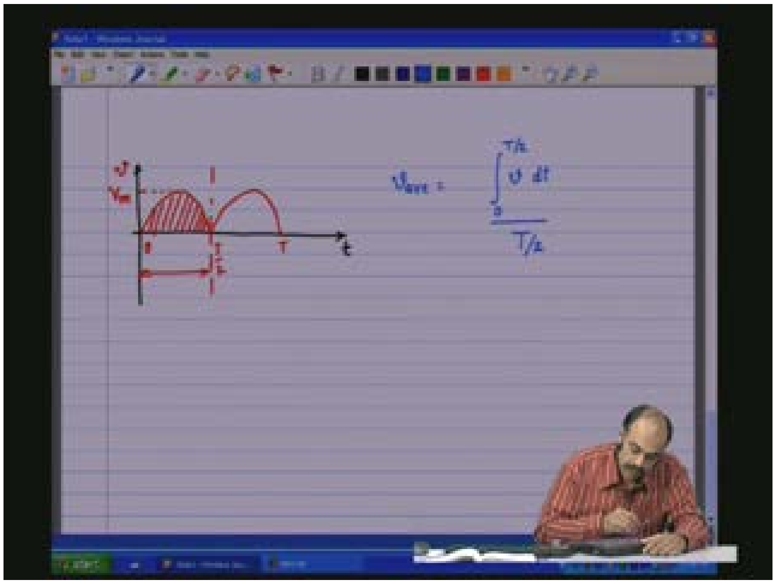

So we know what V is, therefore V average which is equal to 2 by T integral 0 to T by 2 V m sin (omegat) dt which is equal to 2V m by T; integral of sin omega T is minus cos (omegat) by omega by omega and that has to be evaluated at T by 2 and 0 which will give you 2V m by omega T and minus……… now this will be minus cos (2pi by T into T by 2) minus of minus plus so cos 2pi by T into 0 so this is going to be 2V m by omega T into cos of……… so if see here the T 2 goes off this is will be equal to minus 1 minus here so you have a plus and a plus 1. So this is equal to 4 so I am writing it here V m by 2pi by T that is omega into T. So we have a cancellation of T, cancellation of T this becomes 2 and therefore this becomes 2V m by pi. So that gives you an average value for a full-wave rectified waveform which is equal to 2V m by pi.

This is the average of a full-wave rectified waveform with V m as the parameter. So this is something that you may have to remember.

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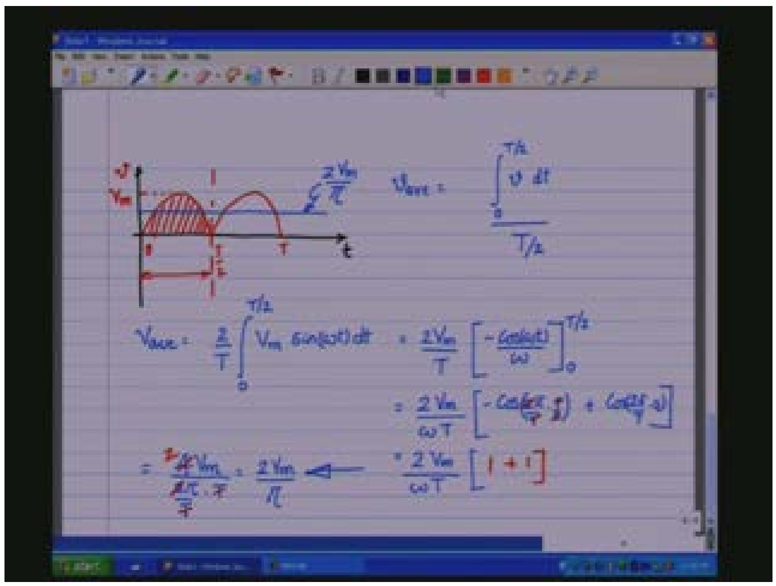

Now a slight modification to this circuit is a clamper, that is from the absolute value circuit you could use a clamper where the voltage of the sine wave is not passed through an absolute value circuit but it goes through a clamping circuit such that as long as it is positive you get the waveform, the input and the output follow; as long as the waveform goes negative the output is clamped to zero and so on.

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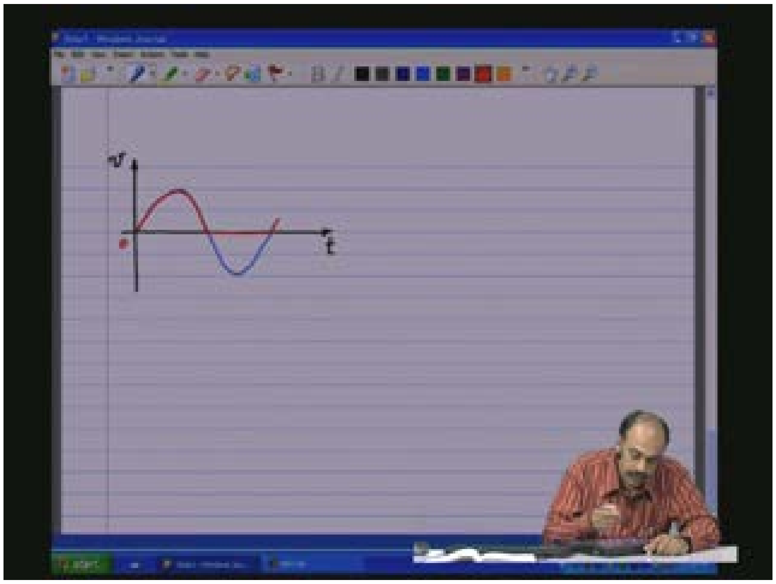

So this is a clamper's circuit, gets clamped to zero and this results in a half-wave rectification. So as the name indicates as the name indicates only half the wave gets rectified every cycle in a period.

Now what is the average of this waveform?

If we are having the peak value which is V m we have to find the average from 0 to T. You should see that it is not repeating every half cycle like it was in the case of the full-wave rectified waveform and therefore we have to consider the whole period. So in the whole period it is zero for half the time and it is having half it is having an area only from 0 to T by 2. Of course one could find v average which is equal to 1 by T which is the base into the area which is 0 to T by 2 V m sin (omegat) dt. This is the average value that you would get for this one. This is, as we see here that this is not 2 by T this is just 1 by T, this is for the whole period but you are integrating from 0 to T by 2 because from T by 2 to T it is zero and there is no output which comes into the picture. So this would give you 1 by T into V m into minus cos omega t by omega that is evaluated 0 to T by 2 and this will result in V m by pi. There is a factor of 2 missing with respect to the full-wave which is evident because it is half that area. So you will get a value which is V m by pi. So, for half wave rectified waveforms sinusoidal waveforms the average is V m by pi.

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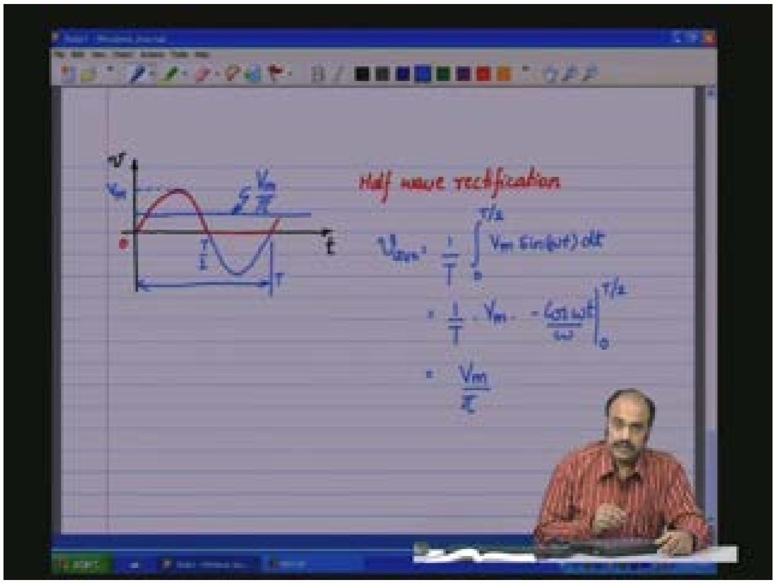

So this is how you obtain the average. There is one more important feature which is commonly used everywhere that is the Root Mean Square or the RMS value RMS value of the sinusoid. What does RMS value mean? This has a relation to the power. It is that value of the voltage or that value of the current which relates to the power or which is the power generating or the power producing part. Therefore let us first look at how the power waveform looks like.

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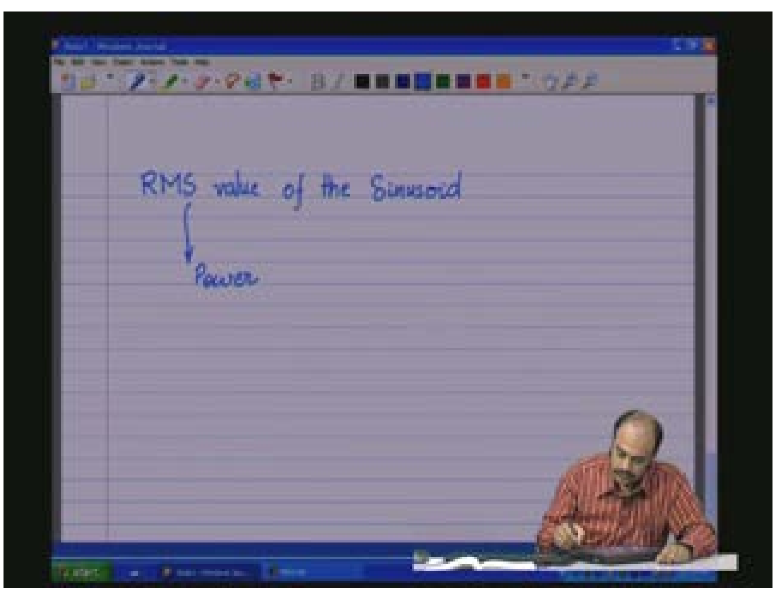

Let me consider a simple voltage source which is pure sine wave and just connected across a resistance R just connected across the resistance R so this is V and there is a i which is flowing in the circuit through the R.

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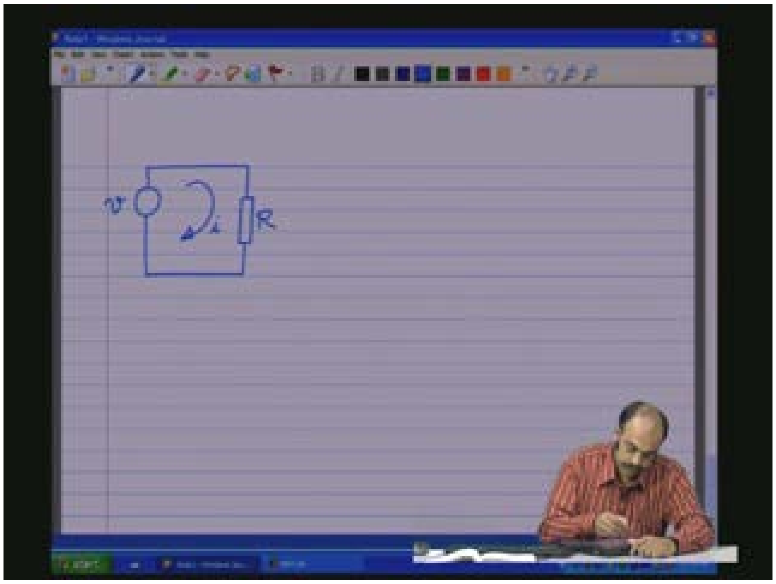

Let us say v is a pure sinusoidal waveform which is defined as V m sin omega t. So what is i? i is V by R because there is only one element which is R which is in series with the voltage source therefore i has to be V by R by Ohm's law which is V m sin (omegat) by R. So let us say Im the peak current is Vm by R then i will be equal to Im sin (omegat). So just keep a note of this equation for the voltage and this equation for the current (Refer Slide Time: 29:28) and of course the current and voltage are related through this.

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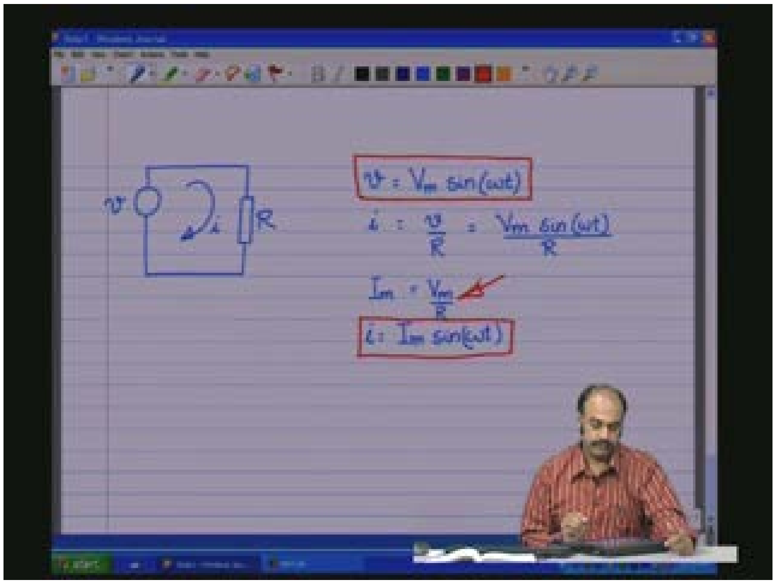

Now let us look at the waveform of the current and the voltage together. We have the time axis. Let me take the blue waveform for the voltage so we have the voltage waveform which is like that this is V this is V (Refer Slide Time: 30:12) and let me take the red waveform for the current we have let us say R is greater than unity and therefore so they are the voltages and the currents. They are pure sinusoids of course while drawing it could be a widget. Now this is the voltage and the current waveforms.

Now what is V into i?

The voltage into the current is going to be the power p and that is having the units of watts. So the volts into amps volts into amps is watts the power. So instantaneously you take the voltage at that point of time, the current at that point of time, multiply and then you plot the power waveform. you see that when this crosses zero; when the current crosses zero voltage also crosses zero and minus current into a minus voltage will result in a positive power which means that the power waveform is always going to be positive and this will let say rise like that, go towards zero, rise like that goes towards zero and so on. So this will be the power waveform P which is equal to V i.

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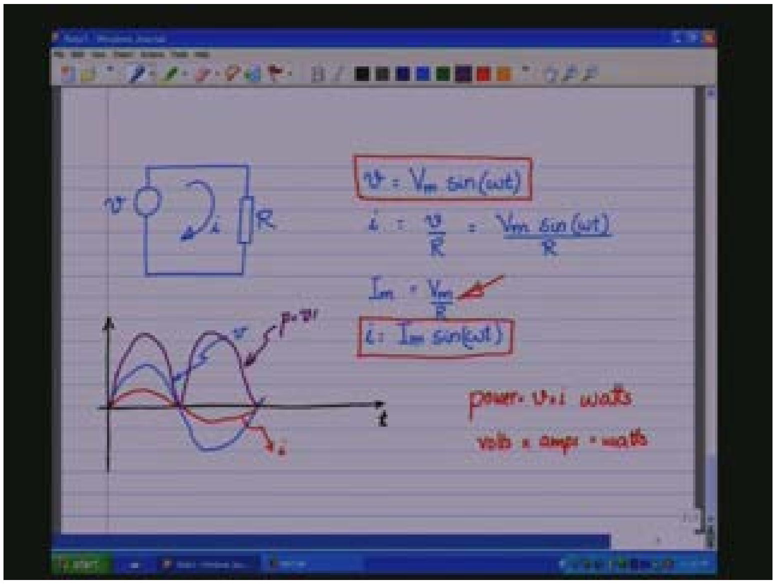

So if you look at the power waveform power p which is equal to V into i which is nothing but V m sin omega t into I m sin (omegat) this is equal to V m I m sin square (omegat). Now we have the trigonometric relation: sin square theta is equal to 1 minus cos 2 theta by 2 or cos 2 theta is equal to 1 minus 2 sin square theta. So if we apply that we have V m I m 1 minus cos 2 theta by 2 which is V m I m by 2 minus V m I m by 2 into cos 2 omega t this is omega t.

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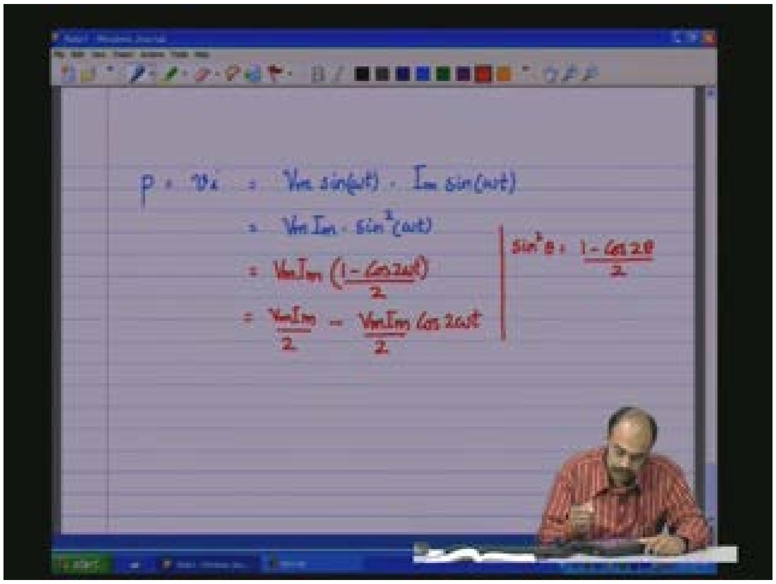

So, if you look at this equation you see that there is a DC term and then there is an AC term. Here you see that there is a DC term and here there is an AC term and the AC term is having twice the frequency 2 omega twice the frequency as the one that the voltage of the current that you started off with which is having a frequency of omega. If you go back and look at the power curve you see that there is an average quantity, this is the DC term and then there is a varying AC term quantity which is having twice the frequency as the fundamental; you see there is one complete sine wave for this period, by then it would have completed two sinusoidal waveforms in that period. So there is a 2 omega or twice the frequency term which comes into the picture which is the AC term here. So that is one of the feature of the power curve which you should take note of.

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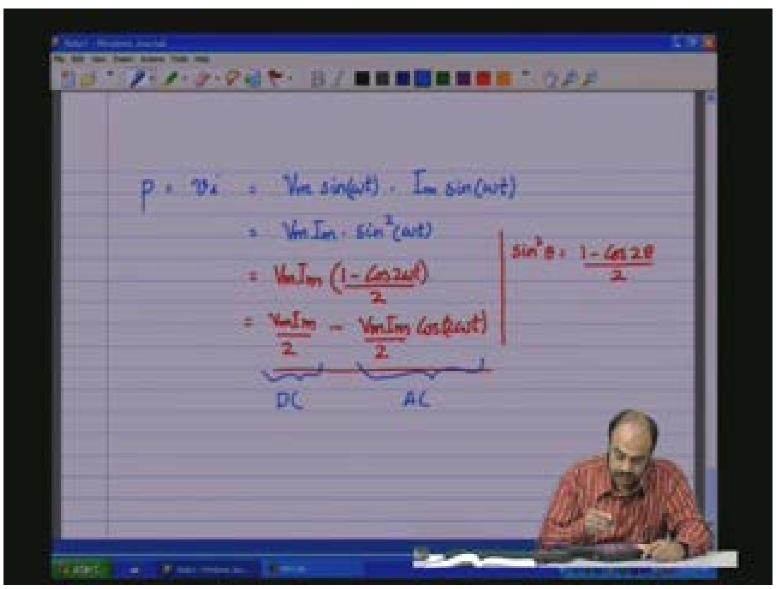

Now if you draw only the power curve versus time, power which is equal to V into i so there is a DC term. So let me write it in dotted blue dotted here, this is the DC term which is V m I m by 2 and there is a varying term or the AC term which is which starts from zero, you have a kind of a waveform which is like that sinusoidal in nature about the DC term and so on its keeps going. So if you take this point to this point that would be T the zero this is T. So this point would be T by

2 and that is point T. So you have 2 sin, this is one and this is the other (Refer Slide Time: 36:45) this is the other sine which you see in a period, this is the power curve.

So let us see this power curve and try to get the average of this one. So what is the average p average which is equal 1 by base which is the time period for the whole time period integral 0 to T v into i dt which would be 1 by T integral 0 to T V m I m by 2 (1 minus cos 2 (omegat)) dt so V m being a constant, time being a constant we can bring that out so that will be V m I m by 2 T integral 0 to T 1 minus cos (2omegat) dt.

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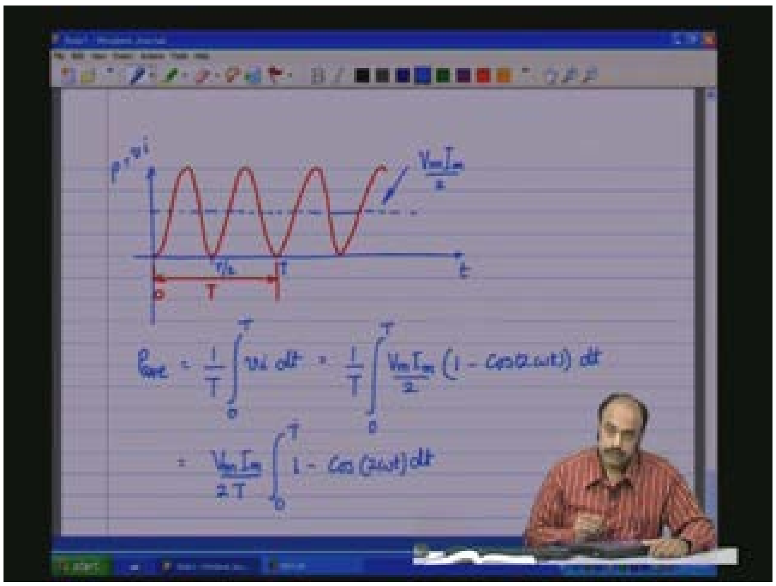

Now if you look at the integral of this it is pretty very straightforward. Let me select that (Refer Slide Time: 38:43). So if you see the integral of this this is pretty straightforward, this is nothing but t with 0 to T minus sin 2omegaT by 2omega from 0 to T. This of course multiplied by V m I m by 2 T. So here if you see this is T minus 0 that is T and sin T that is 2pi by T which will result in sine 4pi or sin 0 both are zero and therefore this will be zero at both the limits and this means that we have this means that we have V m I m by 2 this is the average power P average.

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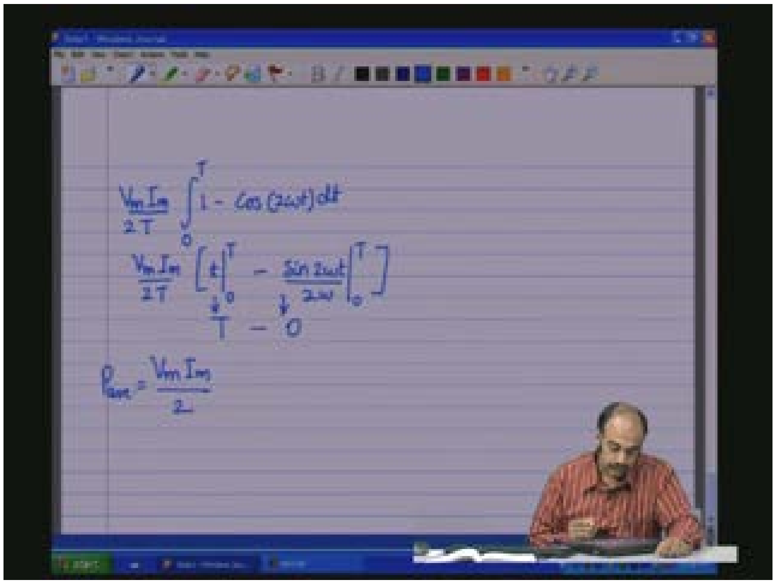

Now we know that I m is equal to V m by R which means P average is equal to V m square V m square by 2 R; R substituting the other way which is also equal to I m square by 2 into R. Look at this equation (Refer Slide Time: 41:10) power which is equal to either V m square by 2 R or I m square by 2 into R.

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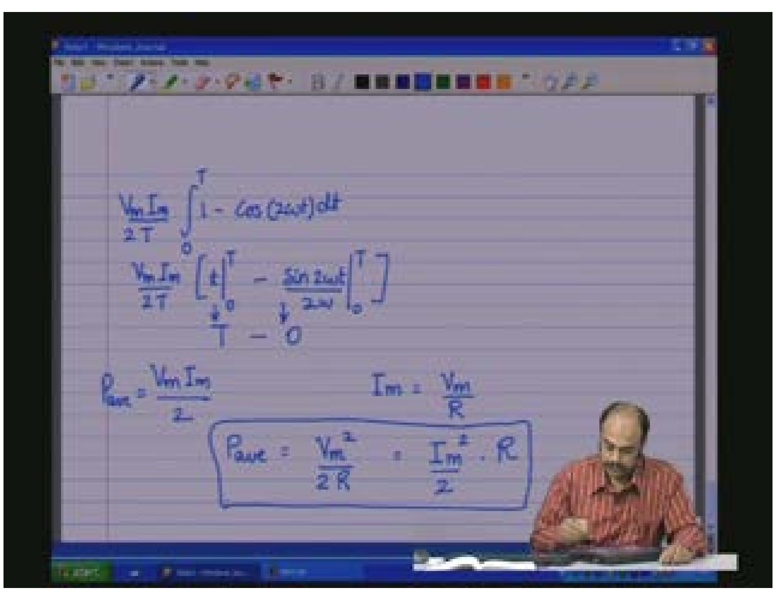

So let us take the case of P average which is equal to I m square by 2 into R, this is amp square into ohms which is giving you watts, this could also be split into amps into Amps into ohms equivalently to give you volts into Amps which is the watts.

Now here (Refer Slide Time: 42:06) there is a term I m square there is a term I m square by 2, let us split that let us take the square root of that one. So what happens; you have I m by root 2 into I m by root 2 into R. So you have Amps Amps ohms these two together is going to be volts which is going to give you Amps into volts power or Amps square into ohms which is power which is in watts.

Now this current (Refer Slide Time: 43:02) which is what is equivalently producing the Joule energy loss across the resistor or which is basically the energy that is put into the resistor is the equivalent current component that is the power giving or the power generating and this we call the RMS current. Why do we call the RMS current?
This is the next topic that we need to take up which means this can be written as I rms into I rms into R. Or I rms square into R where this is nothing but this value I m square by 2 and therefore I rms is equal to I m by 2; I rms equals I m by root 2.

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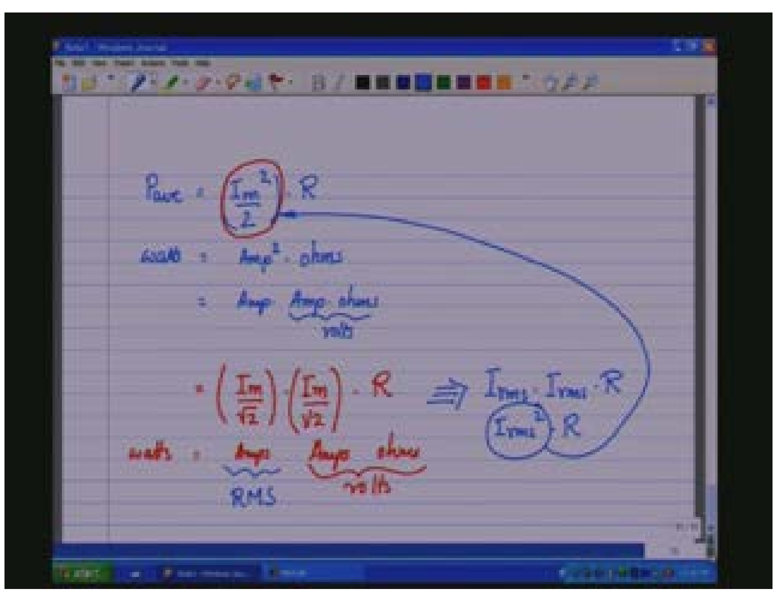

Likewise by the same argument we have V rms which is equal to V m by root 2. Now let us come from the integral point of view. Let us say power average that is which is equal to 1 by T integral of 0 to T V m sin omega t into I m sin omega t dt. Let us replace this by I m into R which is 1 by T integral of 0 to T I m into R sin omega t into I m sin omega t dt. So we see here 1 by T integral of 0 to T I m square sin square omega t dt into R will result in the power which is watts. In the case of the sinusoidal waveshapes you see from here (Refer Slide Time: 46:20) R can be taken out it is a constant and then you have 1 by T integral. Now this we saw, this portion we saw was equal to Im square by root 2 sorry I m square by 2 so this is I rms square.

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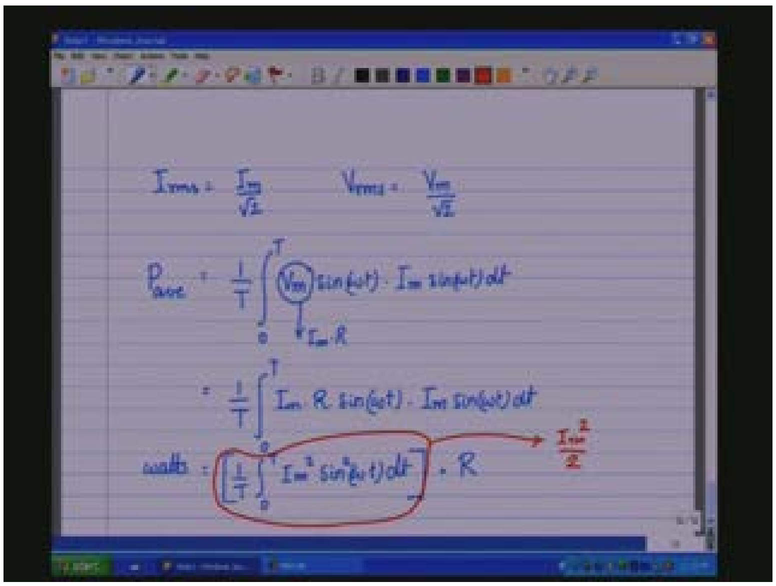

Now if you look at this equation here what do we see as you as you progress through this current equation. You have a mean 1 by T integral 0 to T I m square sin square omega t dt which is I square. So this is amp square and this is ohms. So rms has to be equivalent to the root of this circled equation.

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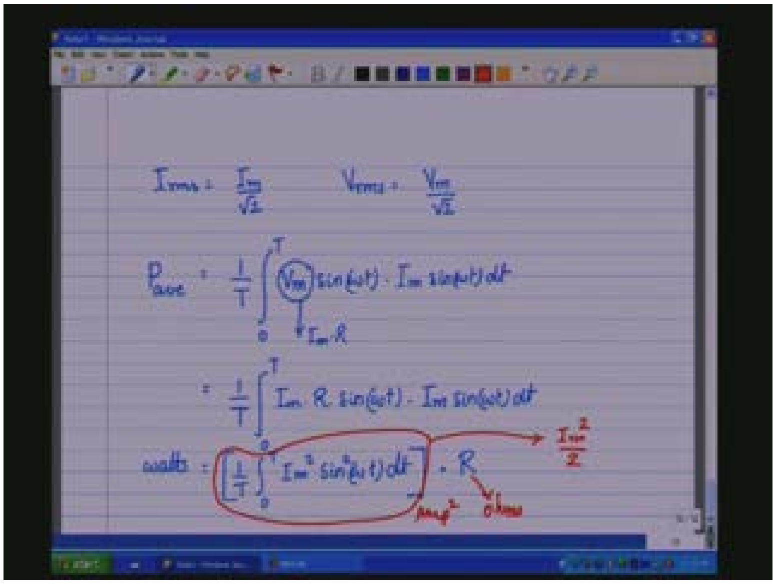

This means if we write that down: I rms is root of whatever was written in the circled portion because that is equal to I m square by 2 that is Amps square which means 1 by T integral of 0 to T I m sin omega t square dt. So here if you look at this equation there is a square root and therefore we are talking about the root and then you have the division by the base which is the mean another mean and the current being squared square. The root mean square is basically what comes out in the progression of the equation as you do as you write from left to right the root mean square or you take the square of the current in the mean and the root going the other way direction. That is what the rms means and that is the power generating component and that is the one which is equivalent to the power generating component

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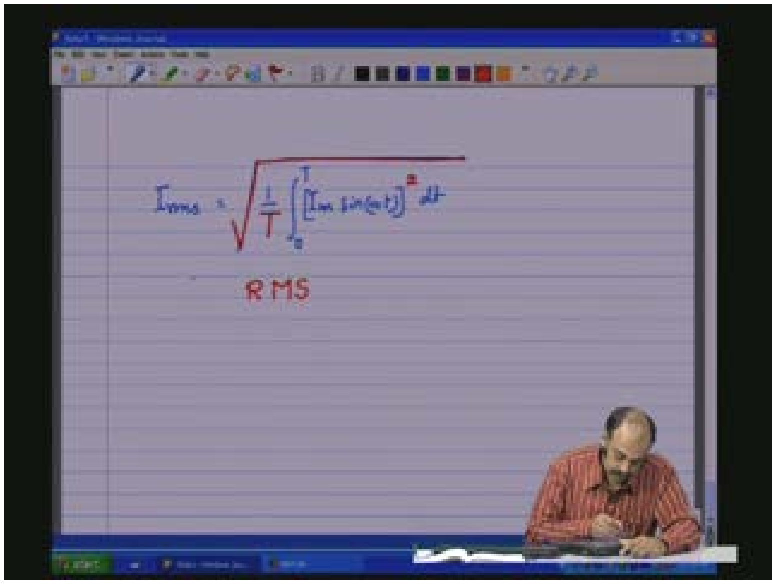

So power is equal to I rms square into R always in the case of sinusoidal quantities or it is equal to V rms square by R if you are taking the voltage across the resistor.

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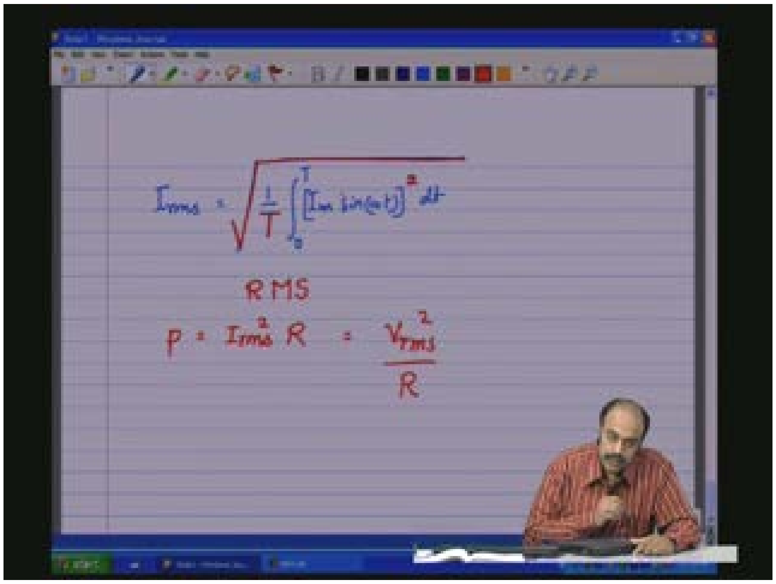

Now if you integrate this you should be getting I m by root 2. Let us just verify that. I rms equals square root of 1 by T integral of 0 to T I m square sin square omega t dt which is equal to square root 1 by T. Let us take I m square outside integral of 0 to T this is nothing but 1 minus cos 2 omega T by 2 dt square root which is equal to square root of the mean 1 by T I will put I m square, we can take out the two integrating one that is t from 0 to T minus sin 2 omega T by 2 omega 0 to T. This results in T, this results in 0 (Refer Slide Time: 51:29) then t and t cancels which gives you I m square by 2 this is the inner one is I rms square, you take the root of it which will be I m by root 2. So even if we come from the integration side we will land up with Im by root 2 and that is your I rms value.

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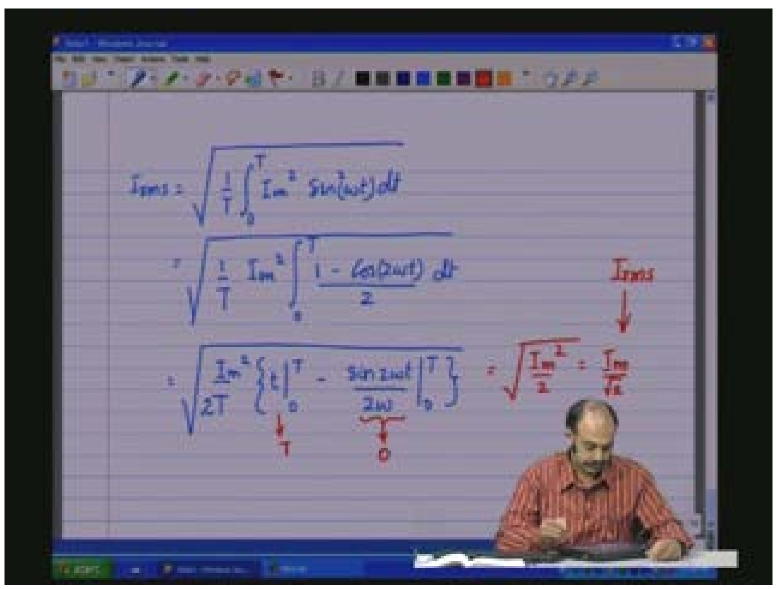

Therefore, in the sine picture if you draw a sine wave with respect to time let us say the voltage waveform, I draw this sine waveform and if this is V m the peak value we have the average value let us say half the full-wave rectified let us say that is 2V m by pi and let us have the rms value slightly higher and that value is V m by root 2 so this is the rms value and this is the average value of full-wave rectified waveform full-wave rectified waveform and that is the peak value.

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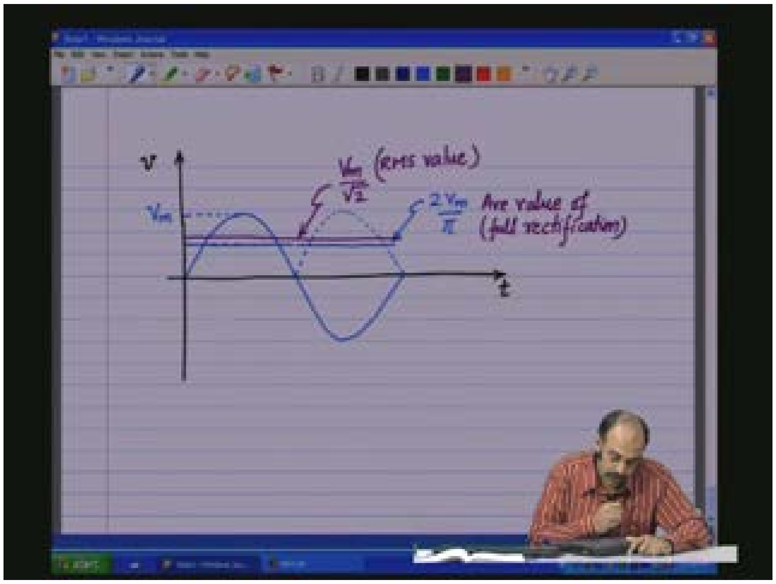

Here some definitions are in order. You will see that in the literature one definition is the peak value by the rms value root mean square value which is let us say V m by V m by root 2 which is root 2 which is 1.41 approximately; this is peak by the rms value. There is also another ratio which is peak by the average value which is V m by 2V m by pi which is equal to pi by 2 which is approximately 1.57 or so half of 3.14 this is the peak by the average value. Sometimes this is called the form factor and this is called the crest or peak factor in the literature. Of course the nomenclature is not so important but what is important is the ratios of these things.

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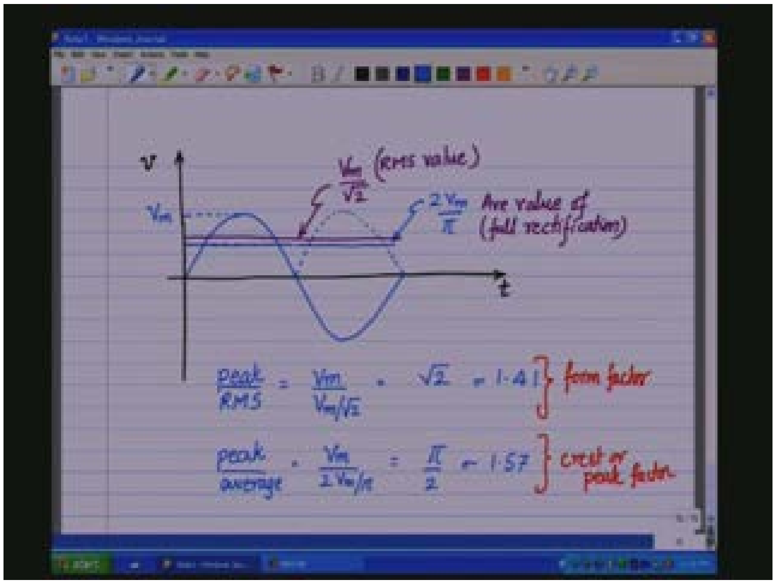

There is one more ratio probably you could remember that is the rms by average which is V m by root 2 divided by 2V m by pi. So V m goes off so this results in pi by 2 root 2 and this is approximately 1.1 so the ratio of the rms to average is 1.1. These are some of the definitions that you need to know and the full picture of the sine wave is something like that (Refer Slide Time: 57:00); you have the full sine wave defined and it has two important parameters as you see: one is on the X axis or the time axis that is at period T which is equal to 1 by frequency and the radiant frequency omega is given by 2pi into f or which is equal to 2pi by T and the other is in the Y direction which is the peak value or the crest value which is V m; you could also take the peak to peak value which is 2 times V m and then there is an average value. The average value of the actual sine is 0 because the positive area cancels with the negative area; the average value of a full-wave rectified sine wave meaning taking the absolute value of the voltage waveform will be 2V m by pi at 0.637 V m; the average value of a half-wave rectified waveform would be V m by pi which is half of that value and then there is a rms quantity which we saw, the rms quantity is something that relates to the power or equivalent power producing or power generating quantity see the power is in terms of voltage square by R or it is Amps square into R and that equivalent voltage is called the rms voltage so it is V rms square by R or it is I rms square into R.

In fact, in the wall socket ratings what you see the 230 volt rated the rated voltage or the rated Amps 5 Amps or 6 Amps or even 15 Amps socket the Amps that relates there is a rms quantity, not the peak value or the average value. So what is mentioned in all the wall sockets and all the power rating or the name plate ratings are generally the rms quantities because they relate to the power so the rms values are pretty important there.

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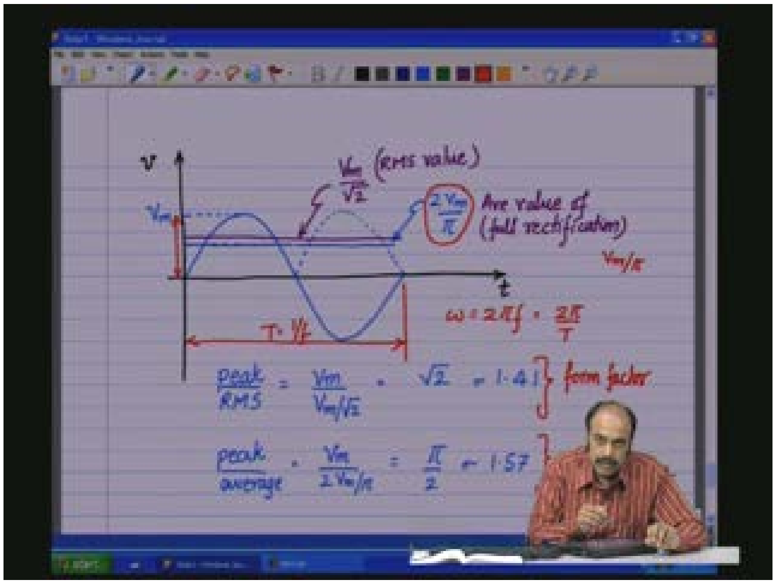

So the rms value of the voltage waveform is V m that is the peak value by root 2 that is 0.707 times the peak value. So this is 0.707 V m 70 percent about 70 percent the peak value or the current is also is also represented in the rms value because I rms square into R is the power so I rms is I m by root 2 and the power is V m I m by 2 or V rms into I rms. We saw the power is V m I m by 2 let us split it into rms which means V m by root 2 into I m by root 2 this is V rms into I rms. So, for sinusoid the power is equal to V rms into I rms. Note that this is for a resistive load that we have been defining it for other loads like the inductive or capacitive loads. We will look at it much later in the later class.

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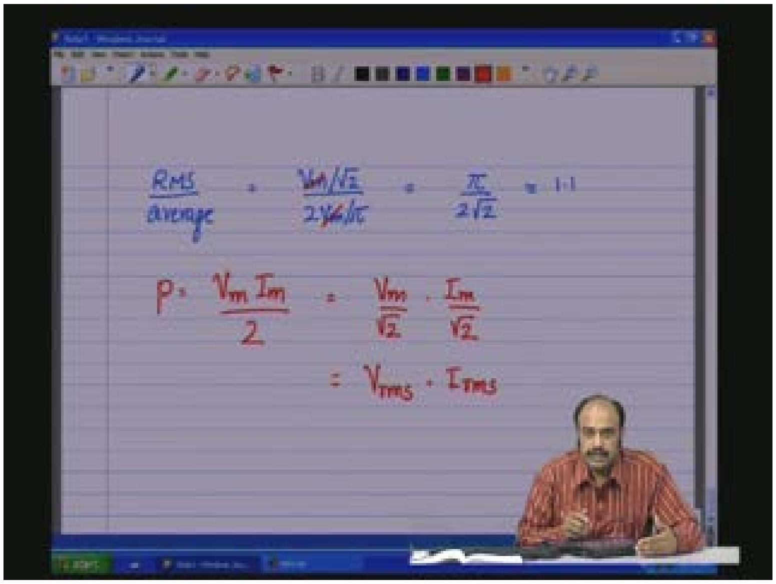

For now this is how the sinusoid looks like. Try to understand the concepts of the sine wave because you will be using that frequently in the future circuits. Thank you.