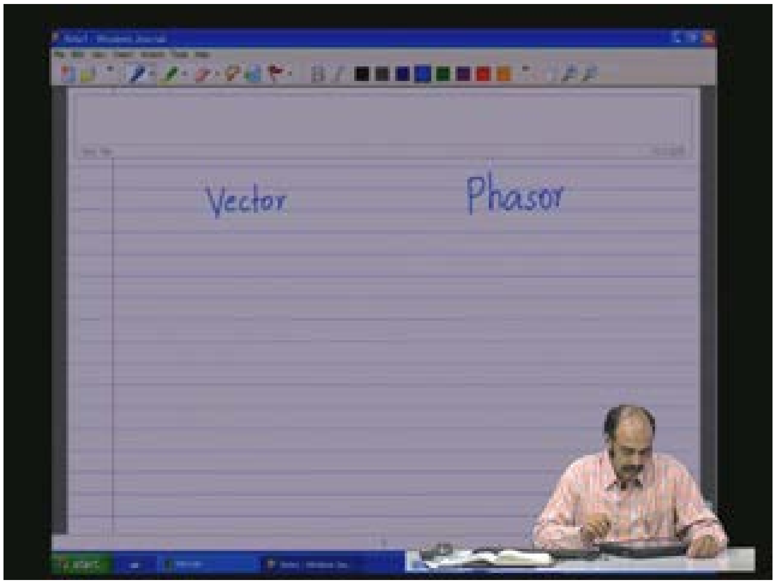
Basic Electrical Technology
Prof. Dr. L. Umanand
Department of Electrical Engineering
Indian Institute of Science, Bangalore

Lecture - 14
Phasor Analysis Part – II

Hello everybody, in the last session we started discussion on the phasor analysis analysing the circuits by means of phasors. In this session we shall consolidate on those concepts and try to use it on real circuits like the R, RC, RLC circuits and see what information we can gain from the phasor diagrams. Till now in the last session I have been interchangeably using phasor and vectors. So before going further I would like to put forth to you the question what is the difference between a vector and a phasor. So we have two terms here: one is the vector and the other is a phasor.

What is the difference between the two; because when we saw in the other session that is the last session we had been drawing vectors and we have been using the term vectors, vectors in the temporal coordinate system, vectors in the spatial coordinate system so on and so forth, the space vectors, space phasors we have been using these terms interchangeably. But before going further it is nice to understand what is the difference between them even though it may be a small difference.

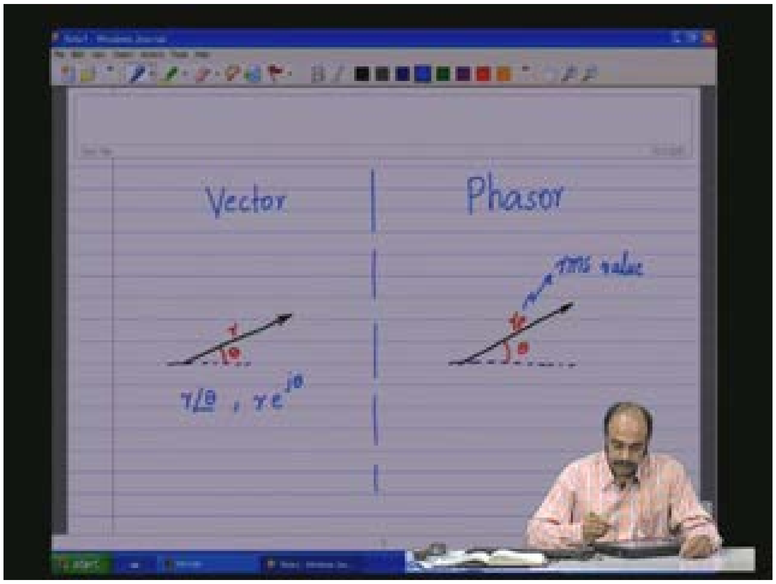
(Refer Slide Time: 2:50)



Now in the case of a vector as we know it has it is represented by an arrow which has an amplitude r and a direction as given by the arrow with respect to a reference with respect to a reference the direction is specified. So if I have a reference as shown dotted, with respect to the dotted reference the vector or the vector arrow is at an angle theta so this is usually represented as r angle theta or r e to the power of j theta where there is the polar coordinate representation. So this is a vector where r here is an amplitude.

In the case of a phasor, phasor also is represented by an arrow. This also has a direction with respect to the reference let us say I have a reference here, you can have a direction theta with respect to the reference and it has also an amplitude and let me call that as rp to distinguish it distinguish it distinguish it from the vector amplitude r. The difference lies here in rp. Instead of using the exact vector amplitude there, in most of our sinusoidal analysis we would like to use the effective value of the effective value of the waveshape rather than the peak value. So the rp here represents actually the rms quantity, this is a rms value that is the root mean square value. So effectively the phasor and vector looks the same but in the amplitude whereas in the case of the vector diagram the vector amplitude is the exact vector amplitude, in the case of the rms sorry in the case of the phasor the vector amplitude or the amplitude of that arrow is the effective value which is the rms value of the signal that is pretty useful in sinusoidal circuit analysis because we use the rms value in almost all cases for power and the heating content, i rms square r all those things and therefore the phasor diagrams are extensively used in sinusoidal analysis.

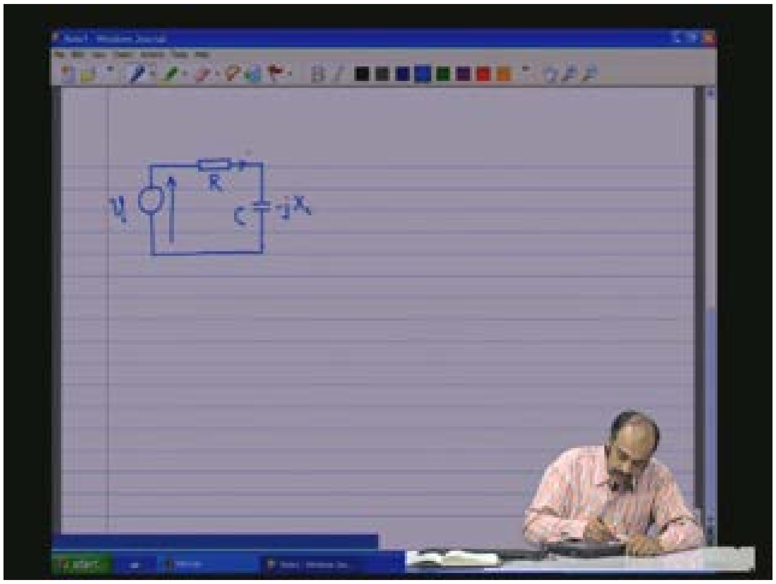
(Refer Slide Time: 6:10)



So when we say phasor remember that the amplitude is in the rms value and when we say vector the amplitude is in the exact peak value of the…………….. (00:06:20….). So with that small note let us go back to what we were doing in the last session.

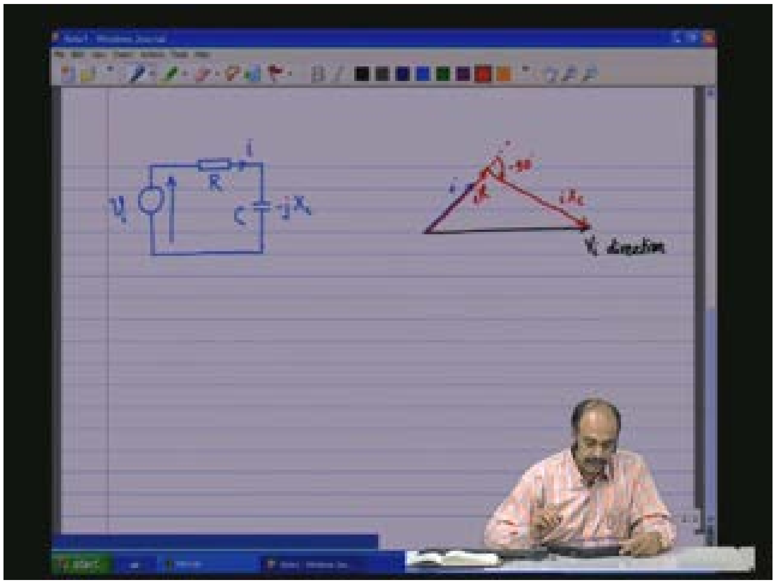
We had a circuit the familiar RC circuit which I am writing down here for clarity, there is a v i, there is R, there is a C; C is going to be a capacitive reactance which is minus 1 by so I will rub this I can just put it as here minus j X c.

(Refer Slide Time: 7:20)



Now there is a current through the circuit and that current is i. Now for this circuit we drew the phasor diagram in the last class, but remember that we chose a reference. The reference axis in this spatial coordinate system that we chose was along the v i the v i direction and here we see that the current is leading the voltage and therefore we have the current i. Now the voltage which is in-phase that is which is i into R is going to be along the current vector so this is going to be iR. Now the voltage across this is going to be i into X c but there is a minus j which means minus 90 degrees rotation and that is going to this is going to be a minus 90 degree rotation and this is i X c, this is the phasor diagram that we obtained in the last class.

(Refer Slide Time: 9:03)

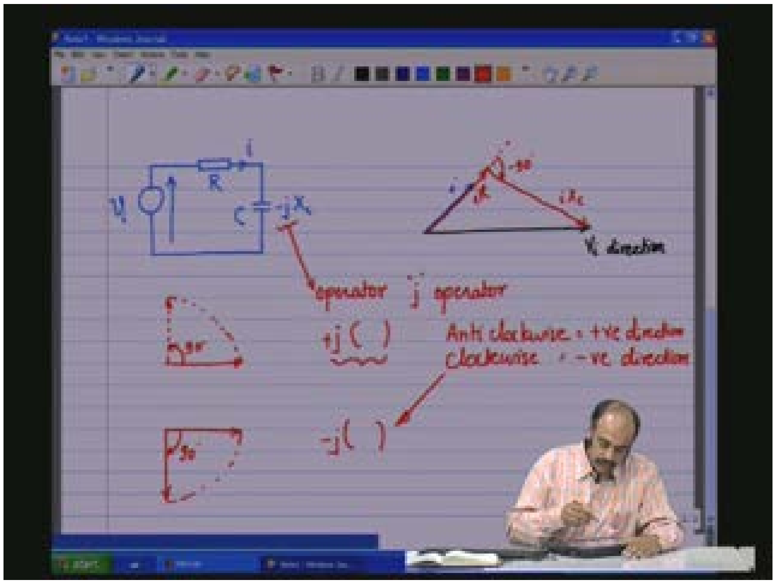


Just to recap, note one point. This j here (Refer Slide Time: 9:16) can be treated as an operator called j operator. Whenever you have multiplication by j into something then this quantity is going to be phase shifted by 90 degrees anti-clockwise. Internationally there is a convention for signs of rotation; anti-clockwise is positive direction for rotation and clockwise is negative direction for rotation.

Now a plus j is going to give you anti-clockwise plus 90 degrees rotation. So, if a vector is in this rotation in this form, let us say now you apply plus j that is it is multiplied by a factor of plus j then the vector will be rotated anti-clockwise, other positive direction, by 90 degrees to get this to get this vector.

Now if you had minus j into something then that is going to result in negative direction which is clockwise. So if I had the same vector in this form and on this you operate on minus j that is going to rotate minus 90 degrees clockwise to obtain this new position. So remember that plus j into something is going to cause a rotation of the vector by plus 90 degrees which means anti-clockwise direction plus 90, a minus j in the factor is going to cause a rotation in the clockwise direction by 90 degrees because minus j is negative direction for rotation.

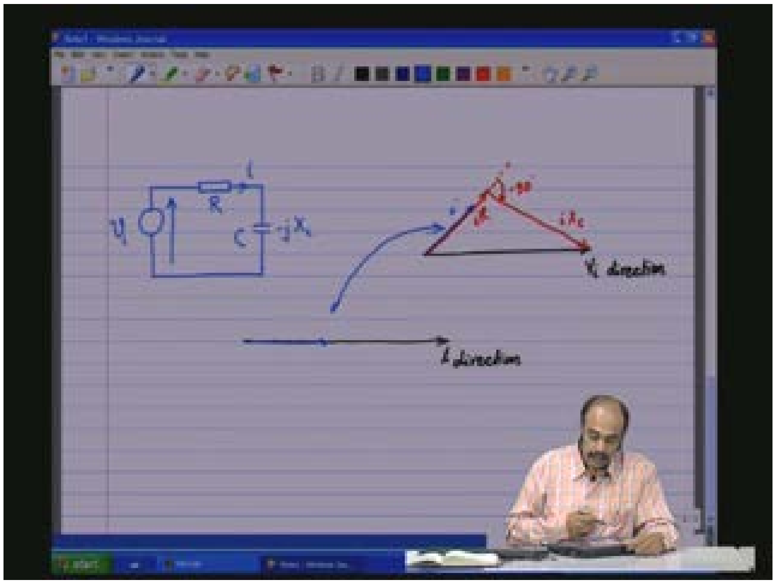
(Refer Slide Time: 12:00)



Now, going back to our circuit here, let me just copy this circuit and all these things, go to the next page.

Now you see here that we had used v i direction as the reference that is the reference in the spatial coordinate system. But we could as well use the i direction also as the reference. It is up to you to choose a particular vector as a reference but the relative positions still would remain the same. So let us say this direction here represents the i direction, the i vector direction which means now the i is the current i is like this, notice the correspondence here (Refer Slide Time: 13:17)

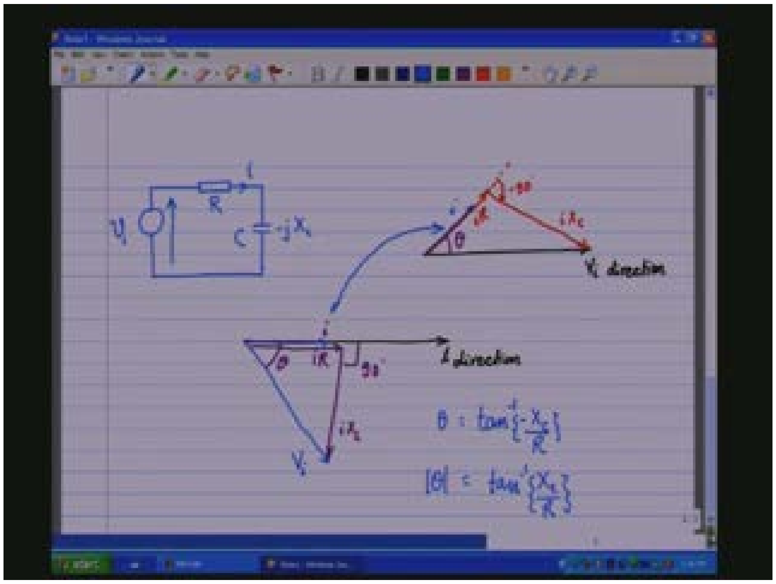
(Refer Slide Time: 13:21)



Now this being a capacitive circuit the i leads the voltage the input voltage. Therefore, the voltage input voltage is lagging the i. So I have the input voltage which is lagging the i, this is going to be v i and that is formed by, I will be having i into R, this is the vector in-phase with i and then i into minus j X c minus j means minus 90 degrees rotation so this has to go, this is i X c this completes to give you the v i. See that these two are similar vector diagrams or phasor diagrams; only that here the reference was chosen as the v i direction, the v i vector direction as the reference whereas here the current vector current vector or the i vector has been chosen as the reference. However, the relative phase shifts between these two still is maintained, relative phase shifts between this and this is still theta and that is maintained.

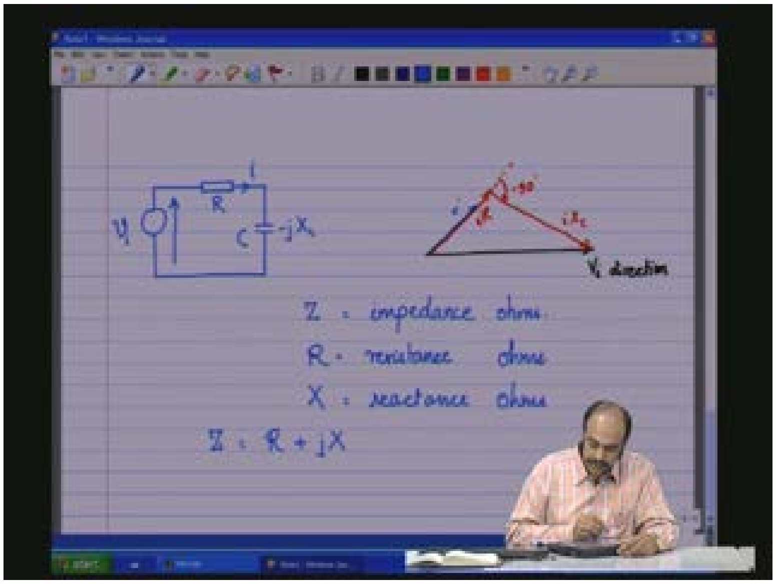
Now here what is angle theta (Refer Slide Time: 15:15). Look at this, theta is tan inverse of minus X c by R, minus here to indicate that it is minus j the minus j X c term and minus here also indicates a negative direction for the angle of rotation theta or the absolute value of theta would be tan inverse of X c by R.

(Refer Slide Time: 16:05)



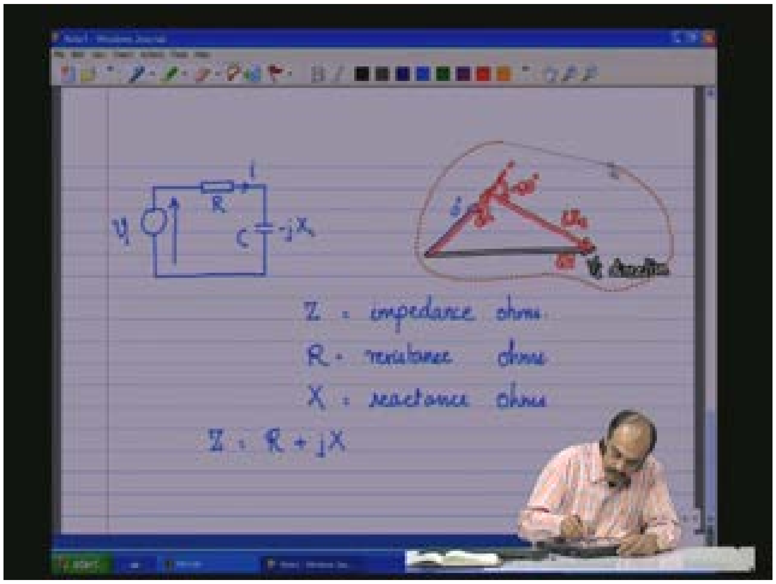
And here let us let us make a small new introduce a new component a new variable into this picture here and that is Z. Now Z is called the impedance also in ohms, R is the resistance this also is in ohms and X is the reactance also in ohms. Now, impedance is actually the vectorial sum of R plus j X, how does this come about?

(Refer Slide Time: 17:30)



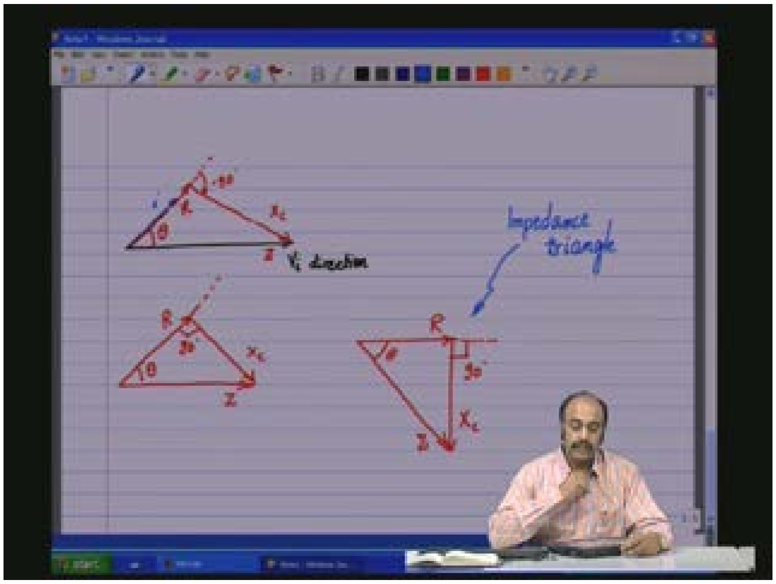
When you look at this phasor diagram here it could be either the v i direction phasor diagram or the i reference phasor diagram, this is i into R, this phasor is i X c, (Refer Slide Time: 17:57) this phasor which completes the Kirchhoff's voltage law is i into Z. therefore, let me choose that.

(Refer Slide Time: 18:12)



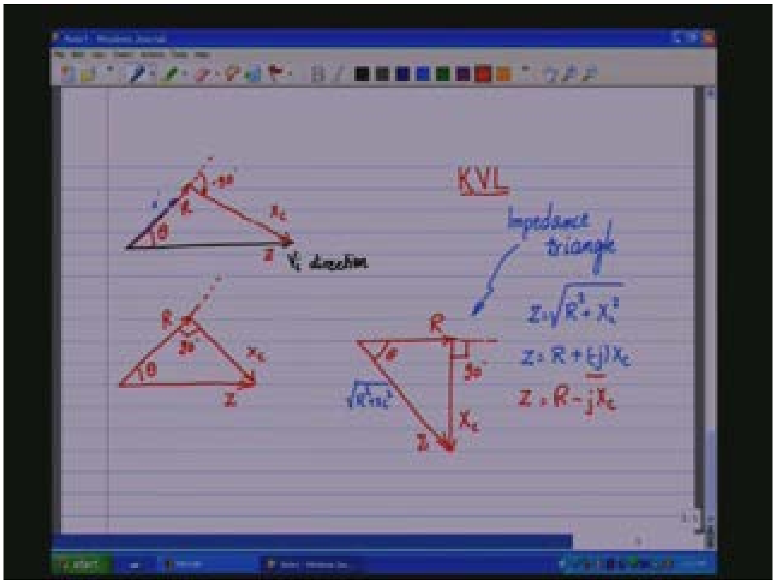
Therefore, let me remove the i's, let me erase the i from this, so you have an R, an X c and Z which means i could I could write it as an R and exactly at 90 degrees you have X c and then you have a Z you have equivalently three vectors and there is a theta here of course same theta as it would be could have been here, or you could write it as in the current axis direction that is I have an R just vertical down X c which is 90 degrees with respect to this horizontal axis and then from here we have the Z and this angle is theta (Refer Slide Time: 20:03). It is just this repositioned here with this as the reference now taken horizontal. Now this triangle this triangle is called the impedance triangle. This is called the impedance triangle.

(Refer Slide Time: 20:38)



So if this is R and this is X c, this is Z and Z will be equal to square root of R square plus X c square by applying the Pythagoras theorem, so this is nothing but root of R square plus X c square. As X c is orthogonal to R we could also write in complex form: Z equals R minus, let me put it as plus minus j into X c. Look at this minus j operator here, minus j means that it is going to rotate that vector by minus 90 degrees and that is what is done here. So this results in R minus j X c, this is the impedance which is the impedance of the circuit as seen from the input terminals. The impedance triangles are obtained when we when we are applying the Kirchhoff's voltage law to the loop.

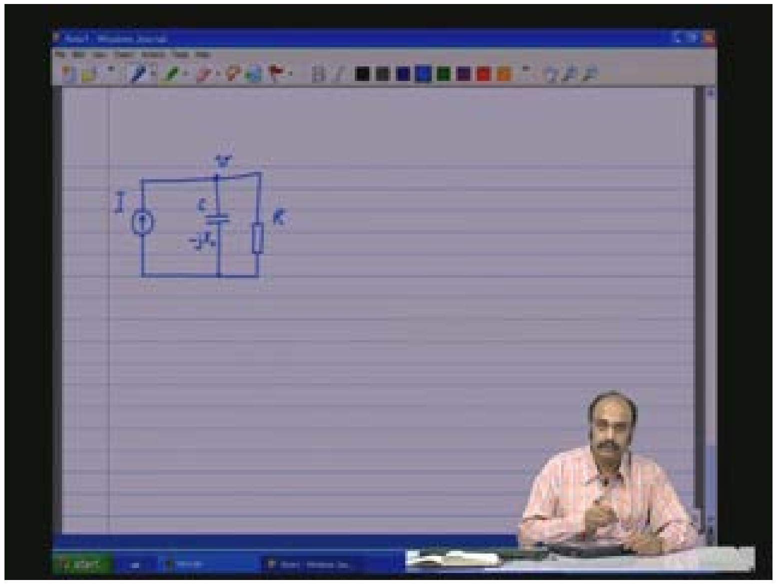
(Refer Slide Time: 22:19)



What happens when we try to apply Kirchhoff's current law?

Let us take a circuit which looks like this. So I have a circuit like this, so you have a C and a R and you have a…….. instead of having a voltage source now I am going to have a current source, let me take a dual of that circuit, so I am going to have an I, this node is going to have potential v (Refer Slide Time: 23:17), this is of course C and R and this is going to give you an impedance or a capacitive reactance minus j X c.

(Refer Slide Time: 23:30)



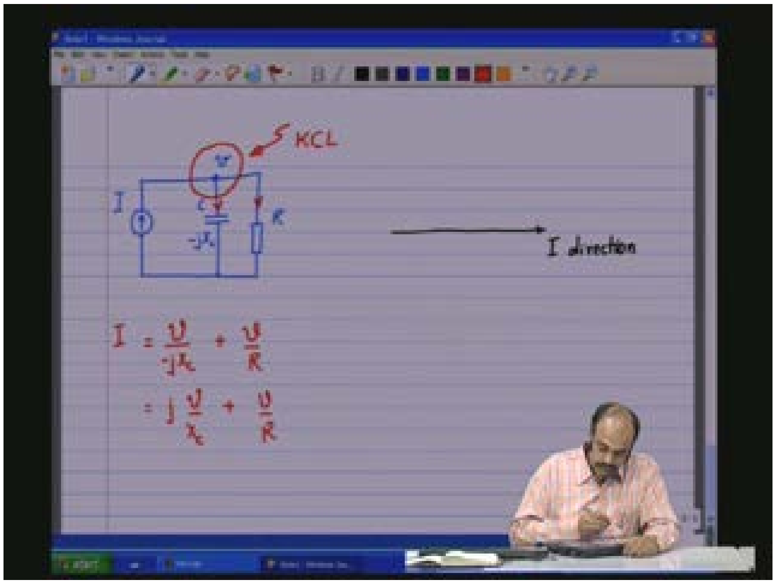
Now let us draw the phasor diagram for this. Let us first have an axis. so all our phasor diagrams are in the spatial coordinates and note that the phasor diagrams in this spatial coordinates can be mapped on to the temporal coordinates which will result in the actual waveshapes and the phasor diagrams will give you a picture at an instant of various waveshape relative it to one another taken with respect to some reference.

So let us have a reference. Now, this is the horizontal axis. We can of course decide what reference that we would like to have. Now let us say we take the reference here as the current; let us take the reference here as the current direction that is I direction. Now what is the relationship of v with respect to I?

Now we know that the current leads the voltage in a capacitive circuit. So in this case the voltage will lag, the voltage will lag the current. Now let us come back and just see how it how it reflects here. Let us have a look at KCL that we want to apply here. We want to apply KCL; to the loop we apply KVL.

Now if you apply KCL there v by minus j X c that is the current through the capacitance that is the current through the capacitance plus v by R that is the current through the resistance should equal I is it not? Now this is equal to j v by X c plus v by R. This is nothing but multiplying the numerator and denominator by j so in the denominator it becomes minus j square which is 1 so j v by X c plus v by R.

(Refer Slide Time: 27:17)



So we have………. so let me put it like this; we have a voltage v direction. So in that v direction there is one vector which is along it and I will call that one as v by R. Now v by R plus j v by X c…….. now v by X c is multiplied by j which is along this it is plus 90 degrees so anti-clock anti-clockwise direction 90 degrees which will be like that this is v by X c (Refer Slide Time: 28:17) that is going to give you I is it not.

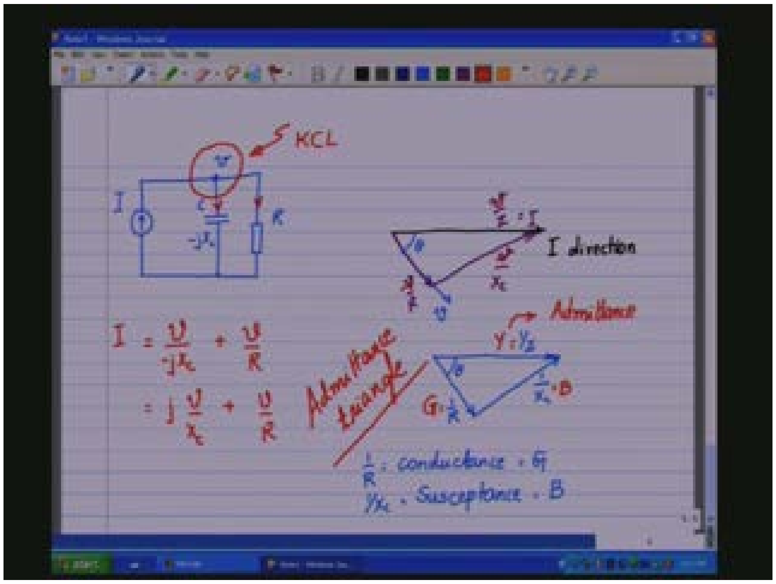
Now let us we have the one which is in-phase with v by R and then you have v by X c and this I should be equal to v by Z. Now here you can remove that, that and that v common. So we land up with a triangle which is like that. So this is the triangle (Refer Slide Time: 29:12) that you land up with. This is 1 by R, this is 1 by X c, this is by Z and you have some angle theta here same as angle theta here.

What is 1 by R?

R is resistance and therefore 1 by R is the conductance, conductance which has the unit of MOS 1 by ohms dimension is MOS or CMOS and this has a symbol G. Likewise 1 by X c is called susceptance and it has a symbol B.

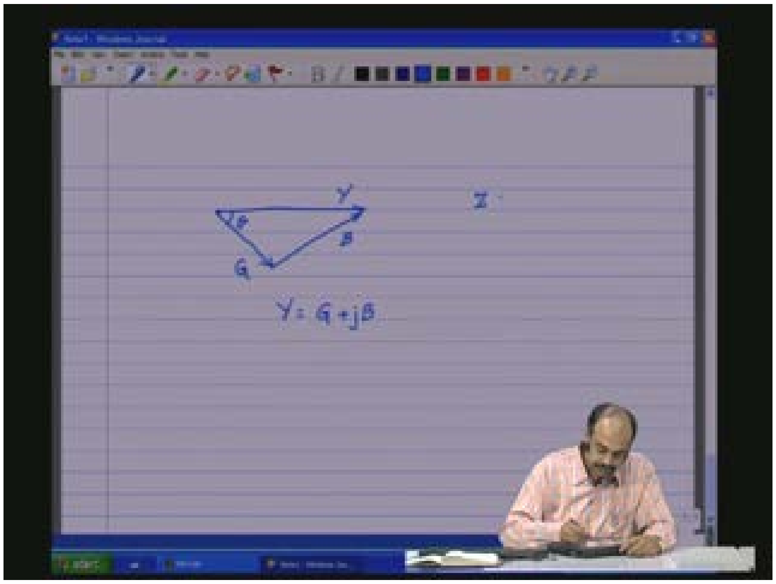
Now you see here; this is 1 by R which is G, 1 by X c which is B and 1 by Z will be let us say Y. So, Y is called Y is called admittance and this is called the admittance matrix triangle.

(Refer Slide Time: 31:18)



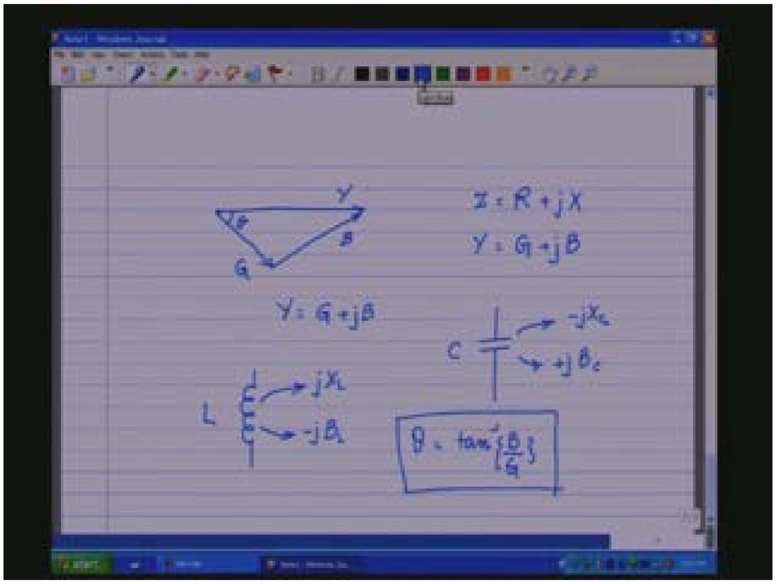
Now, in the admittance triangle we have Y, we have G and we have B, this is theta; and Y is equal to G plus j B.

(Refer Slide Time: 32:14)



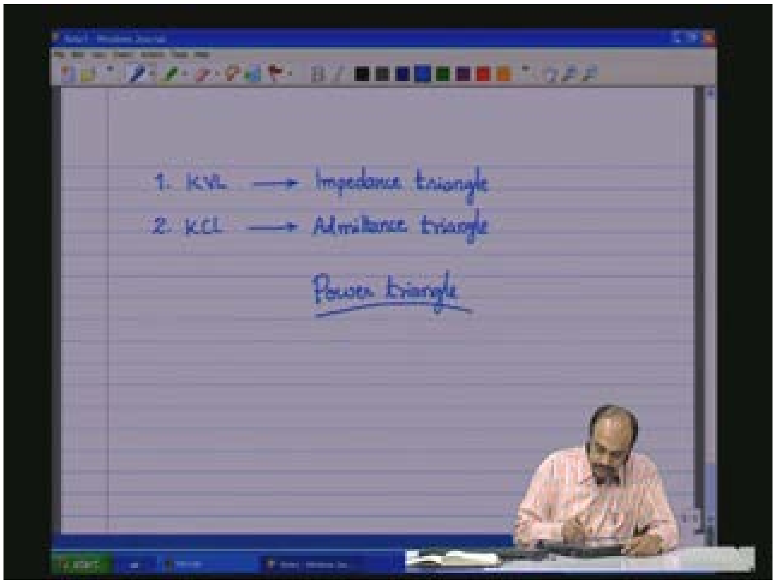
Now see here, the impedance was resistance plus j reactance, admittance is conductance plus j susceptance. If there is a capacitive reactance that has a negative sign minus j X c, a capacity susceptance, so this capacitance has a reactance which is minus j X c or a susceptance B which is 1 by this which is equal to jB and an inductance and that is positive and that is why in a capacitive circuit you see the rotation which is positive here and in the case of an inductance it has a reactance which is j X L and a susceptance which is 1 by j X L which becomes minus jB we can give a subscript L here and c here. So this is the admittance triangle and here also theta can be obtained using theta equals tan inverse of B by G. This is the angle from the impedance matrix between the I and the voltage phasors.

(Refer Slide Time: 34:33)



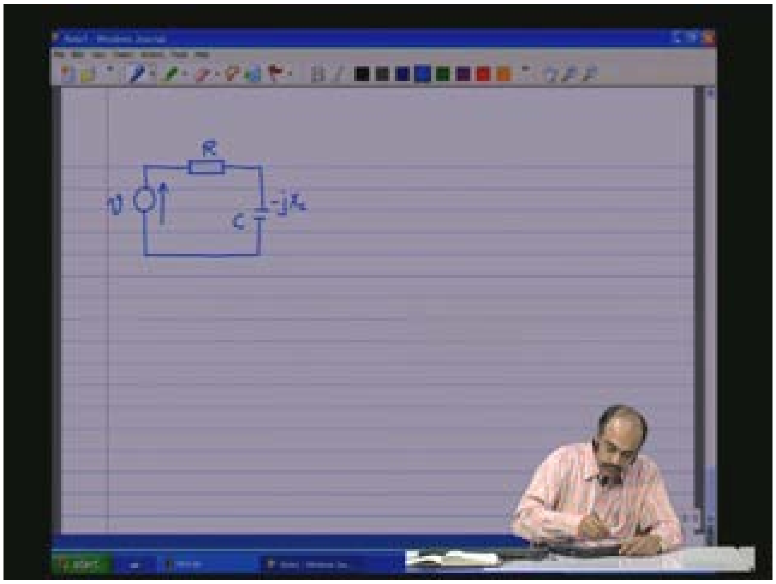
So you see that if we use KVL for solving a loop equation we would land up with an impedance triangle. If we use KCL to solve a nodal equation node voltage equation then we would land up with an admittance triangle. There is one more triangle which you will come across and that is the power triangle. Let us see what this power triangle is.

(Refer Slide Time: 35:44)



Let us go back to our………….. now more common RC circuit. So you have the R and C as usual as it looks here, you have the R, you have the C, you have the voltage input voltage and minus j X c is the impedance that this capacitance provides.

(Refer Slide Time: 36:28)



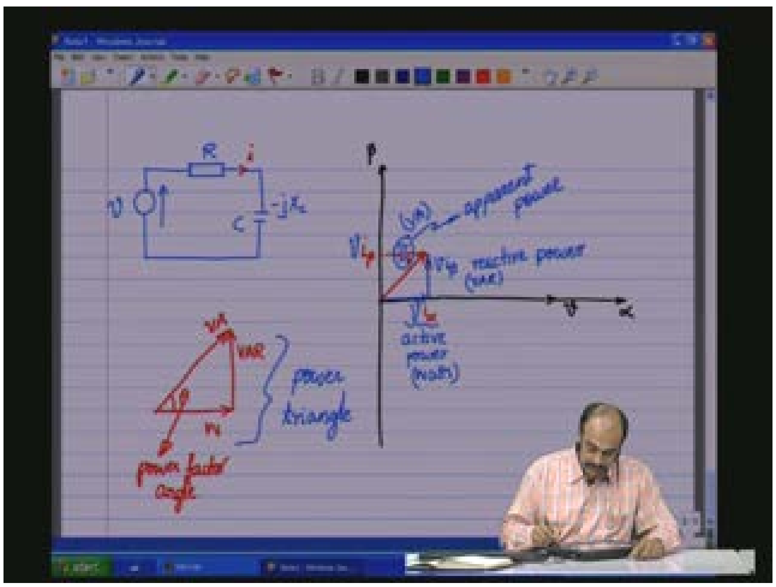
Now let us take a reference which is the v reference or the voltage reference. Now there is a current which is flowing through the circuit and that is i. Now this being a capacitive circuit this current i has to lead the voltage.

Now here there are two important things to be noted and let me also have the vertical orthogonal axis here. the projection of i on to the v axis is called i alpha, yes, because this v reference is called the alpha axis and this would be the beta axis and the projection of this i on to the beta axis is called i beta, so i alpha is in-phase with v, i beta is orthogonal to v. Now multiply all these with the voltage amplitude v.

This being a phasor, when you are multiplying with it v meaning that we are multiplying with rms quantity so i alpha rms with v rms, i rms with v rms, i beta rms with v rms with the same so relatively the vector amplitudes only will change. So what would happen is I have v into i beta, v into i, v into i alpha. That quantity which is in-phase with the voltage v into i alpha is called the active power. That quantity which is orthogonal to the v axis or the alpha axis which is having an amplitude v into i beta is called the reactive power.

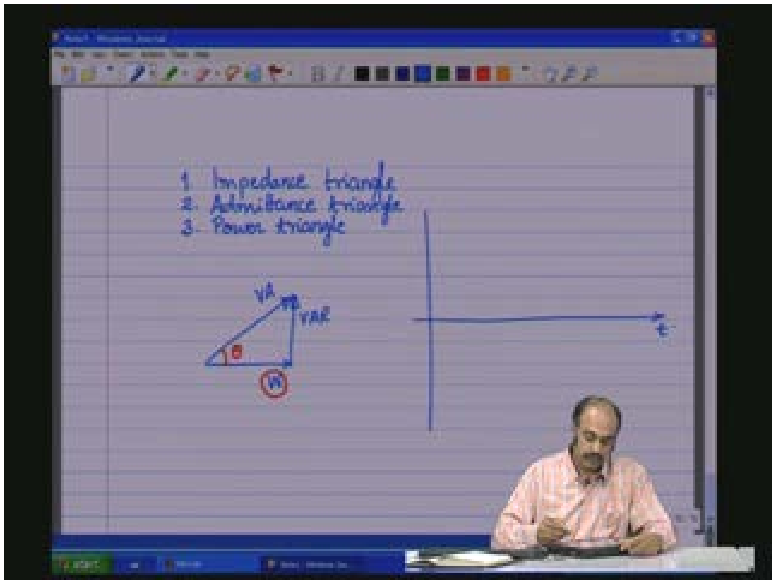
This quantity v into i the resultant of the active power and the reactive power is called apparent power. This has units of watts. This is called volt-amp reactive VAR, this is simply VA. So this triangle if we write it here we have watts, we have reactive VA and the resultant which is VA and that angle theta is called the power factor angle and this triangle is called the power triangle.

(Refer Slide Time: 41:43)



Therefore, now you see that there are three triangles that you need to know while doing the phasor diagrams: one is the impedance triangle and that is obtained while doing the KVL while solving the KVL the Kirchhoff's voltage law, the admittance triangle this is obtained while solving the Kirchoff’s current law and then the power triangle. And one important thing in the power triangle you have VA, you have VAR and you have watts. this is the portion (Refer Slide Time: 42:53) which is actually given to the node that node component active power, this reactive power is actually put back into the mains and this is the VA or the resultant power that is drawn from the mains or the mains should be rated for, or the source should be rated for and this is the power factor angle. This has very important implications, one should understand that properly.

(Refer Slide Time: 43:40)

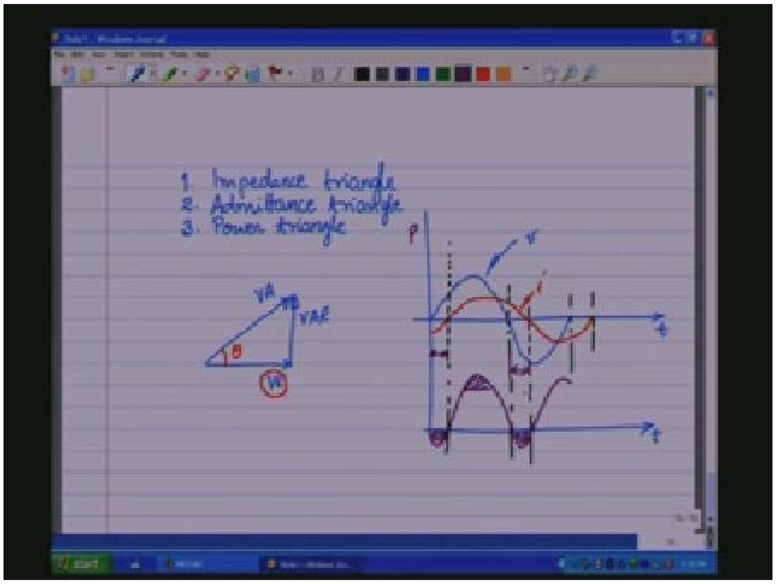


Let us say for example; I have the temporal coordinate system which is like that, let us have a sine wave, this is the voltage wave voltage wave, let us also have the current wave and let me make the current lagging by some amount so that current will lag by some amount and this one from like that.

Now let us get the power. This is of course the current wave. Now power is instantaneous multiplication of v and the i curves. Let me put some lines of demarcation. There is one point of demarcation which is this; this is another point of demarcation. There is one more important point of demarcation here (with a……45:05) because so on it goes on.

Here if you look at the power during this period what I have marked here, the current is negative, the voltage is positive, so when we multiply the power is going to show negative and during this portion (Refer Slide Time: 45:33) voltage and current is positive and therefore it is going to be positive, during this portion again the voltage is negative, current is positive and therefore the power value is going to be negative and here both are negative and therefore power is going to be positive and so on which means if we have the power curve drawn along this line, you will see that at these points it is going to go negative so it will go negative, positive, negative, positive and so on. So, this much amount of area will get subtracted from this positive area, will get subtracted from the positive area and only this much amount of area will on an average get passed on to the load. So, this much amount of extra power extra V i is drawn from the mains which gets put back in the following cycle into the mains which is indicated by the negative power which means the power is flowing back to the source; this is the power which is put back into the source and that is the implication of negative power.

(Refer Slide Time: 47:09)

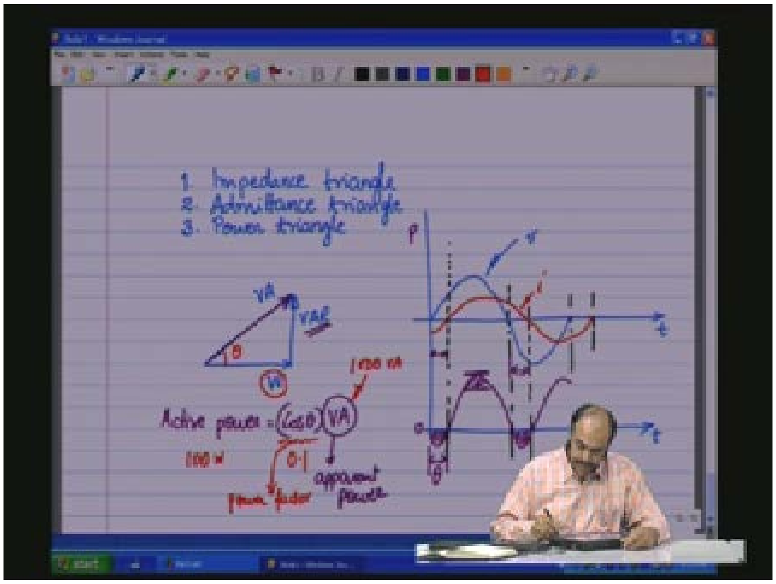


So this component which gets put back to the source is called the reactive power component and that portion of the power which gets sent to the load is called active. And on the whole a resultant of so much is this plus the active is to be drawn from the mains and that is the VA and the power factor theta here represents this angle, the power factor angle theta. What it implies is if this angle is large more negative component, in the limit when this angle becomes 90 the negative component of the power and the positive component will be equal and there is no effective active power which is so when will it be in 90, it will be when it is purely reactive whether it is a capacitive or inductive circuit and if it is lagging it is a pure inductive circuit. So no power is put to the load and all the power which is taken from the mains is put back to the mains in which case this will be zero that is when theta will be 90 degrees.

The other extreme is when theta is zero which means when the current is in-phase with the voltage, power is always positive which means everything goes to the node and that is the active component, nothing is put back to the mains so V R will be equal to 0. So the active power the active power is cos theta into V A this is the apparent power. So, active power is cos theta or cos of the power factor angle into the apparent power. What this implies is when theta is 0 cos theta is 1 so active power is same as apparent power, when theta is 90 degrees cos theta is 0 active power is 0.

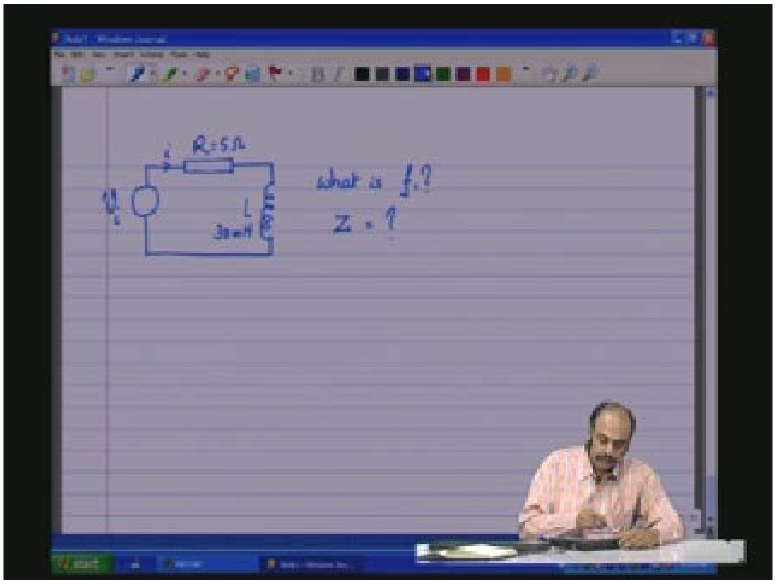
So if you want to have some active power, if cos theta is closer to 1 then VA is almost close to the t power, if cos theta is closer to 0 to get that same active power VA has to be very high which means the mains has to be rated that much higher. So, if cos theta is let us say for example 0.1 and I want a 100 watt load, what it implies is that the apparent power that you should draw from this source should be 1000 VA. So my source should be rated for 1000 VA to deliver a 100 watt load if the power factor is poor, this is called poor power factor. This cos theta is called power factor. This is an important concept that one should understand when dealing with sinusoidal sinusoidally excited circuits and you will see this power factor more and more in future circuits; both in transformer circuits and the motor drive circuits, generator circuits and so on and so forth. So it is good to have a better understanding of this. Of course in the later session, in the next session we will discuss more in detail about the power factor.

(Refer Slide Time: 51:45)



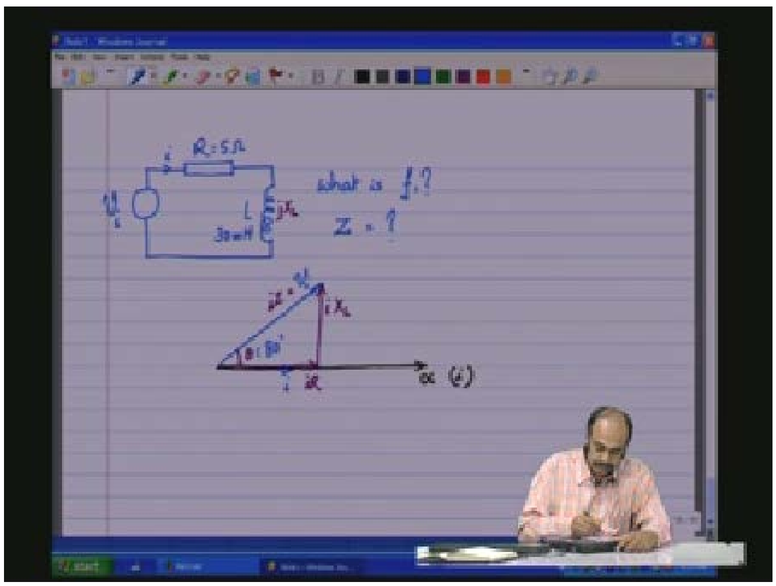
Let us take a small example now to make our understanding a bit clear. Let us take a RL circuit for a change. There is an input source R L. Now let us say there is of course a current i, I use an R of 5 ohms and let us say you use a inductance of 30 milliHenry. Now the question is what is the frequency f of the source and also what is the impedance Z? So how do we go about doing this?

(Refer Slide Time: 53:06)



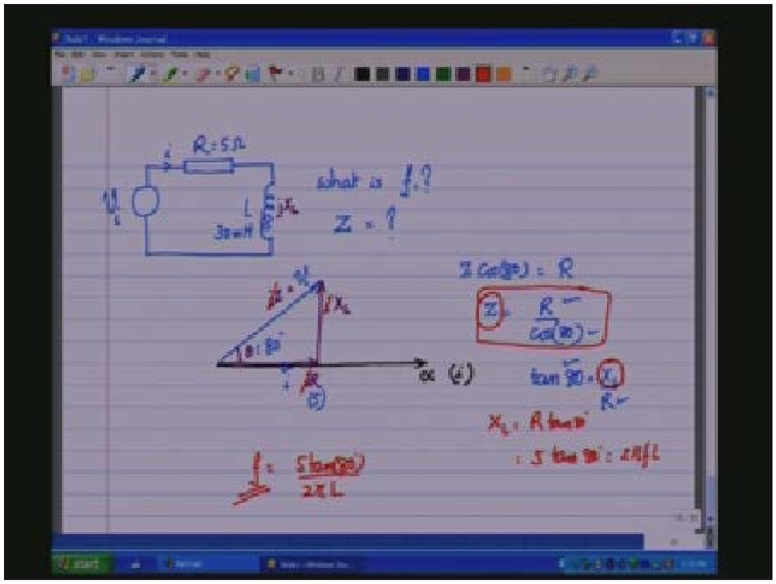
So we know that it is an inductive circuit and therefore the current latch the voltage. Let us take the current i as the reference. So let us have the alpha axis along the i axis which means you have i along this, so the current latch the voltage so the voltage has to be in this direction and let us say that is your voltage V i. There are two vectors: one in-phase with the current which is i into R and the other orthogonal which is i into X L because this contributes j X L j X L into i is going to cause anti-clockwise 90 degrees rotation and this is equal to i into Z and there is an angle theta of course here and let us have further information here and let us say for example that theta is given as 80 degrees.

(Refer Slide Time: 54:58)



Now we have the impedance triangle as this, R is 5 and therefore Z cos 80 is equal to R therefore Z is equal to R by cos 80; this is known, this is known and therefore Z is known (Refer Slide Time: 55:50), R tan 80 is equal to X L by R this is known, this is known and therefore we can find out this. So once X L is known which is X L equals R tan 80 degrees which is equal to 5 tan 80 degrees. This is equal to omega L or 2pifL therefore the frequency is 5 tan 80 degrees divided by 2piL; L is given 30 milliHenries and all Ls is known and therefore the frequency can be calculated and Z is calculated from this equation.

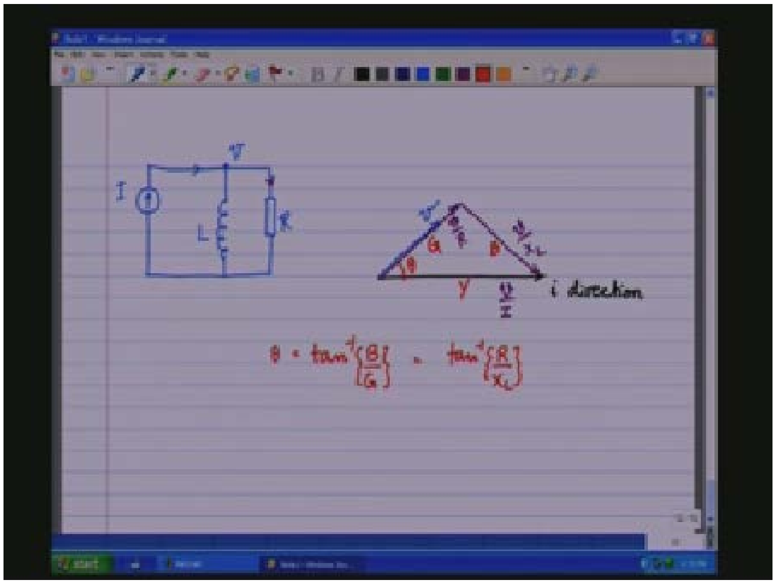
(Refer Slide Time: 57:06)



So this how we go about performing the analysis. Just one more issue. Let us take an inductive circuit and L in parallel with R, so here this is the current source. Now this is an I, it is coming in here, this is a L this is a R there is a node voltage v there is a node voltage v. So let us take the i direction i direction as the reference direction as the reference and the voltage the current has to lag the voltage or the voltage has to lead the current and therefore you have a voltage which leads I.

Now, along this voltage space phasor in-phase with that is going to be v by R which is the current here and then there is an inductive inductance coming in picture; as we are talking of solving the Kirchhoff's current law this is going to be the susceptance inductive susceptance which is minus j v and therefore it has to be minus 90 degrees so this is v by X L and this is v by Z, so this is the G, this is the B and this is the Y, this of course is theta then the angle theta is given by tan inverse of B by G, theta is given by tan inverse of B by G or which is equal to tan inverse of R by X L so this is the admittance triangle for an LR circuit.

(Refer Slide Time: 59:58)



We saw the admittance triangle for a RC circuit; it is good to know how it would look like for an LR circuit. So like this, one has to construct the phasor diagram for any given circuit whatever may be the complexity based on only these two rules. For the KVL you will under with the impedance triangle and for the KCL you will land up with an admittance triangle. you can interact You can interchangeably use them and as the circuit complexity progresses the phasor diagram can be phasor diagram will gradually grow and build-up from the simple triangles to more complex shapes. But of course later on when we start covering transformers and induction motors and so on, we will see how these phasor diagrams will grow for more complex topologies of the circuit. But this is the basic that you should know about the phasor diagrams especially for the R, RC, RL and RLC circuit.

At this point we will conclude the phasor analysis by summarizing that there are three triangles that you need to remember. One is the impedance triangle, the admittance triangle and the power triangle. the power triangle is (a…….01:01:33) important triangle in the sense that it gives good information about the amount of active power and about the amount of reactive power of a particular circuit contributes and what should be the amount of power apparent power that has to be drawn from the mains which means what should be the source ratings all these information can be gathered from the power triangle. The power factor angle theta is also a very important parameter which characterizes the circuit which is actually a measure for the goodness of the circuit in many cases. And we will see more about the power factor in the next session. Thank you for now.