Basic Electrical Technology
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Lecture - 27
DC Motors

Hello everybody. In the last class we discussed about the DC generator. We were we were doing we were discussing about the modeling of the DC generator towards the end of the class so we will continue along with that one, finish the modeling of DC generator and then we take up the new topic which is the DC motor which which has almost the same construction features of the DC generator and the operation the operational mechanisms and the basic concepts are also very similar to the DC generator. So we will look at that after we finish the modeling of the DC dynamic modeling of the DC generator.

So, in the last class we had drawn this figure we have here the DC generator shown as a circle here to which is connected the brushes and from the brushes wires are drawn to be connected to the external circuit load R L. There are two non-idealities which are brought out: R a which represents the armature winding resistance, L a which is also inbuilt into the coils of the armature which is the inductive reactance or the inductance of the inductance of the armature coils.

Now on the mechanical side we have the shaft which is connected to the DC generator and the shaft is coupled to a prime mover which is giving the energy. So the energy flows from the prime mover in the mechanical domain rotational mechanical domain gets into the magnetic domain and then from the magnetic domain to the electrical domain we get the DC generator equipment. The induced emf E g here is given by this equation which we have seen and discussed in the last class E g is number of pole pairs into N rpm into phi flux per pole into Z the total number of conductors in the armature by 60 a constant. As P, phi, Z and 60 are all constants which for a given motor E g is proportional to N or E g is proportional to omega that is radians per second; the conversion between radians per second and and omega would be if I have N rpm now N rpm into 2 pi by 60 will give radians per second. Or if you have omega in radians per second if you have to represent it in terms of rpm into 60 by 2 pi will give you rpm this is how ones goes about converting.

Now E g is proportional to omega the shaft speed and it is equal to K omega where K would be a constant which is fixed for a machine and that sometimes is given as a name plate value. Likewise, just as the E g we said that the DC generator is a gyrator wherein the E g on one side is linked to the E g which is a potential barrier on one side that is the electrical side is linked to the flow variable on the mechanical side. Likewise, the potential variable torque on the mechanical side is linked to the current the load current on the electrical side. So T is proportional to I T is equal to KI the same k into the load current I. So this is basically the principle of the the whole principle of the DC generator in terms of the equivalent circuit.

So we have here basically the DC gen which is a gyrator. Now this gyrator has two ports and the one port is the electrical port as shown here and this is the generated emf and through electrical port we have the load connected and this load load let could let us say R L or R 0.

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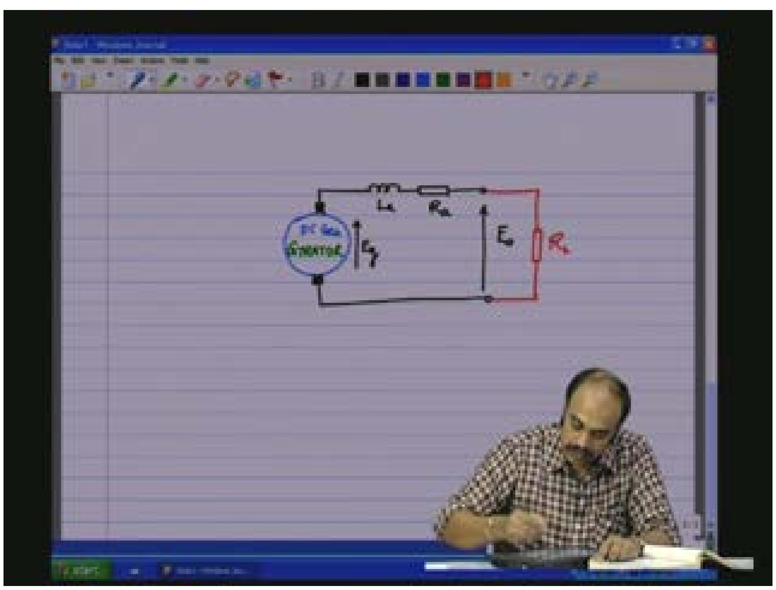

Now on the mechanical side on the mechanical side there is a coupling of course and then that is connected to a source a prime mover which is going to generate the torque. Now the load on the the load on the electrical side is the load current I and that gets reflected as Lorentz force as a torque T L. Now this is the load torque for the prime mover which will try to move against the load torque to overcome the load torque and that would be the generated torque T g the prime mover torque or let us say T p or we will call this one as T p the torque of the prime mover.

Now the shaft has an inertia reflected inertia including all the things that are coming from the prime mover side which is J and there could be a bearing friction B on one side the mechanical side and on the electrical side we have L a which is the armature reactance which is represented as an inductance L a here called the armature inductance which we use for the dynamic model; steady-state model that is not coming into the picture because the DC currents are flowing and it is zero there any voltage induced is zero and R a is the armature winding resistance.

So now we construct a state space model. So what are the methods for constructing........ The method or approach to state space model?
First we identify the dynamic elements. So dynamic elements are j and L a and associated with the dynamic elements are the state variable elements, omega is a state variable element associated with J, I is the state variable element let us represent the load current instead of instead of a capital I let me represent it as lower case i and then we also have the speed of rotation omega and the i which is the load current on the electrical side.
So omega and i are the state variables.

Now once we take one dynamic element by after the other and try to construct the dynamical relationship; now J is an inductive type dynamic element. So therefore we start with J and then let us say L a, L a is also an inductive type dynamic element, let us take the term the inertial element.

So if it is inductive type we know that J d flow by dt or d kinetic variable by dt will be equal to the potential variable. So J d omega by dt omega is the flow variable on the mechanical side should be equal to the act the actual the torque that is produced by prime mover T p minus the load torque for the prime mover which is actually the Lorentz torque which is coming from reflected from the electrical side. Now this is equal to T p and there is also a friction element which is coming B friction is a dissipative element into the flow which is minus B omega. So T p......... because the T p which is the torque produced by the prime mover has to overcome the J d omega by dt the inertial the inertial torque it should overcome the Lorentz force torque which is reflected from the load, it should overcome the friction torque which B into omega due to the bearing friction, so T p minus............... now T L should be represented in terms of only state variables so we we we stated right in the beginning at the time of forming state equations that all the variables with which that you (fo.......11:44) the state equation should be input variables and state variables only. So T p is the input variable. Now T L is the load torque. Now by gyrator action the link is K into i this would be K into i minus B omega now all are in form of state variable so this is one equation.

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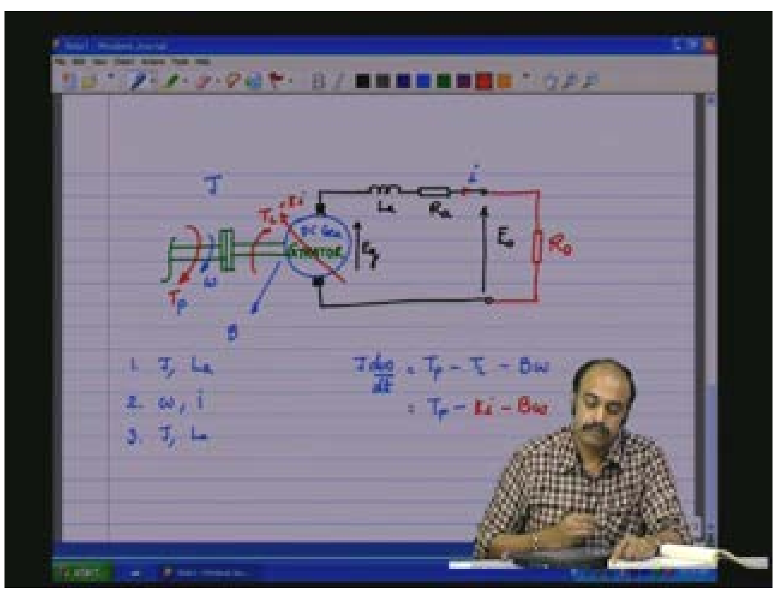

Now the second equation is on the electrical domain side. Again that is also inductive. L a into d by dt of the flow variable or the kinetic variable which is i is equal to......... now generated emf the generated voltage E g minus the drop across R a minus the load voltage but we have to express everything either in terms of input or state variables. Now E g is given by the cross link variable here which is K omega so you have K omega minus i R a minus i R 0 all expressed in terms of the state variables.

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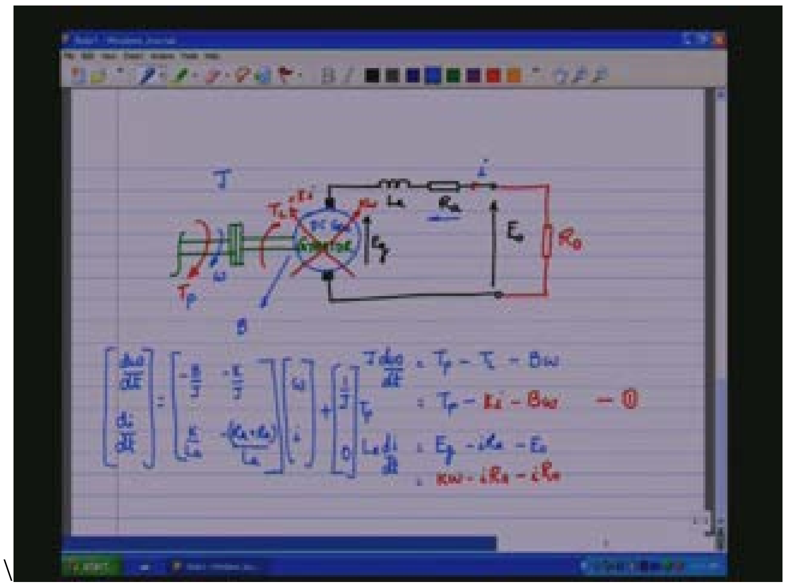

Now this would give the state equation which we shall write it as d omega by dt and di by dt which is equal to now omega so minus b by j minus K by J K by L a minus R a plus R 0 by L a so we have omega and i plus 1 by J 0 T p. So this is a state equation and dynamic equation which is of the form x dot is equal to Ax plus Bu T p is the input and of course we could have an output equation let us say E 0 now E 0 is equal to i into R 0 so therefore which is equal to 0 R 0 omega i plus state vector plus 0 into T p that is y equals Cx plus du. So this forms the state equation representation of the DC generator or the mathematical representation of the DC generator in the state equation form and this is the dynamic model.

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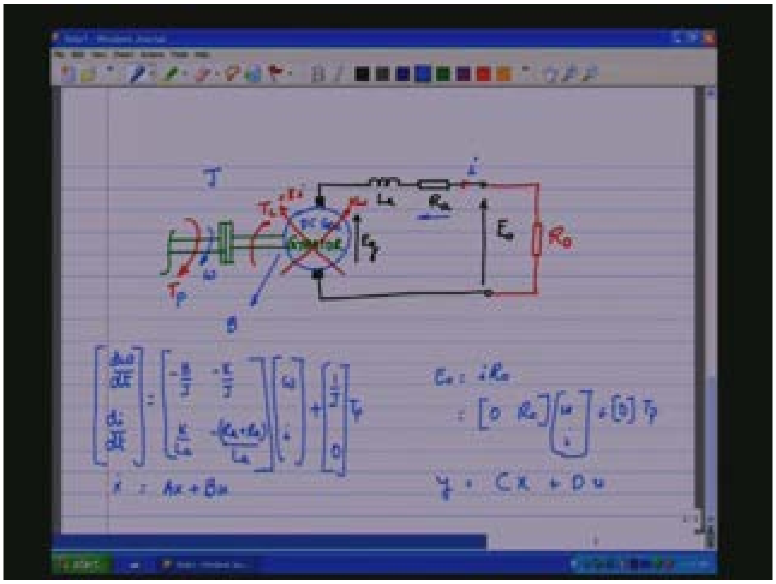

So as we said, in the steady-state model L a will vanish and we will just have the resistive drop which comes in to the picture. So with this we complete our discussion on the DC generator.

We now go on to the next machine which is the DC motor. Actually the DC motor is the reverse of the DC generator. Structurally the DC motor and the DC generator will will look very similar. So all our discussions on the DC generator can be directly carried forward to the discussions on the DC motor. In fact, all our relationships that we used in the DC generator can also be used carried forward to the DC motor concepts also. So our next topic is the DC motor.

So, in the DC motor the black box is still a gyrator in the DC motor also the black box is a gyrator like in the DC generator only difference is that you have the input port port 1 through which the energy is flowing energy input is an electrical port that is an electrical port and then the other port through which the energy flows out port 2 is the mechanical port, so this is the port in the mechanical domain. So therefore the variables here are emf e, voltage e or v and the current i for the kinetic variable, flow variable and on this side you have torque and the speed omega and then there is a parameter K which links the two with the equation T is equal to k i and the e will be equal to K omega, this would be the gyrator link operation between the two. so this is where the DC motor will be used the same machine the gener the machine that was designed for the DC generator will act as a DC motor when it is when the energy is applied reversed in the case of DC generator the shaft is rotated through a prime mover and you get an electrical voltage across the brushes. In the case of a DC motor we apply a voltage across the electrical brushes and the shaft rotates giving us the mechanical motion which can be used for driving a prime mover driving another mechanical load.

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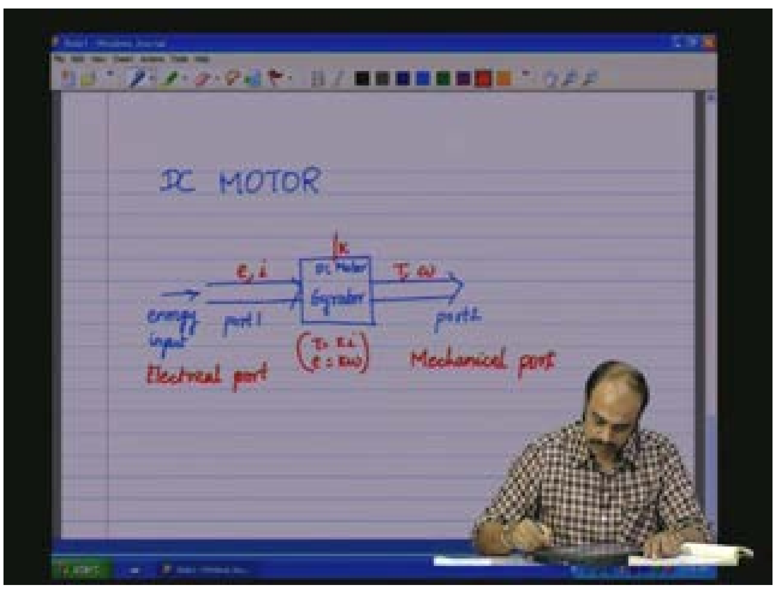

So the DC motor also is represented like in the case of DC generator by a circle. So this is the DC motor and to the DC motor we have the brushes and to the brushes we connect we connect the external terminals and on the mechanical side we have the shaft coming out of the DC motor, the shaft is connected to the armature and that armature shaft is coupled to whatever the load.

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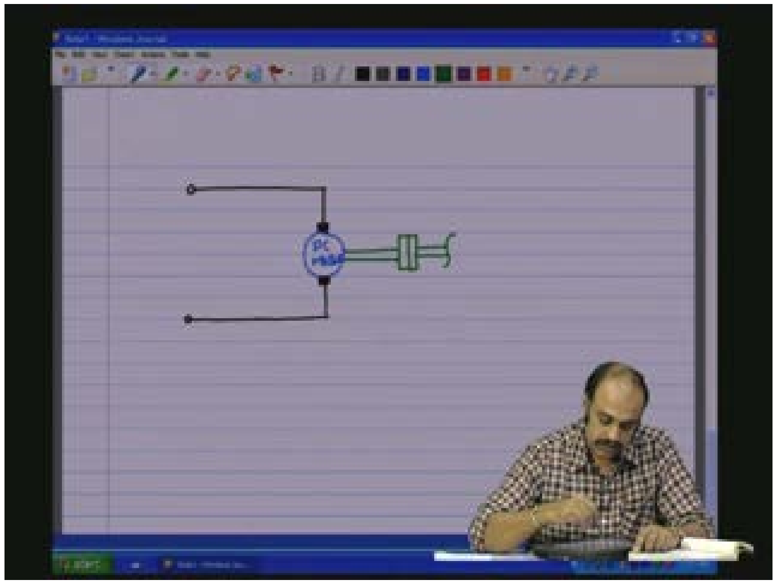

Like in a DC generator this wire is basically connected to the brush and the brush is connected to the commutator segments and the commutator segments are connected to the coils of the armature and therefore the coils of the armature will have a finite resistance winding resistance which we shall represent it as a resistance in series R a or armature R a the armature winding resistance. there is also going to be another element in series L a the armature winding inductance which we shall introduce later on while we are discussing the dynamic model of the motor which will be similar to the dynamic model of the generator. But for most of the discussions that we will be carrying now we need the steady-state model for which the inductive reactance is not required because the flow of current here will be DC and so the applied voltage which will be DC.

We now have an input which is the applied voltage and let me call this as E a. So the voltage that is applied E a or V a some from the literature is applied across the terminals electrical terminals of the motor. Now that is going to cause a rotation T g generated torque from the motor from the motor in this case this is the generated torque at the armature shaft. Now we saw that in the case of the DC generator due to an external load connected there was a low current i which was flowing and the i got reflected back through the magnetic domain and then finally into the mechanical domain as a Lorentz force or the Lorentz torque against which the prime mover had to had to force the torque thereby giving the mechanical energy. Here also if there is a load connected now that load gets reflected as a back emf in this case, it will get reflected as a back emf E b meaning E a has caused the rotation of the shaft and because the shaft is rotating because any shaft in the DC machine if theif the if the if any shaft in the DC machine if it rotates it is going to generate a voltage across the brushes. Though the applied primary voltage has caused the rotation this rotation of the shaft is going to cause currents to flow through the windings of the armature and produce an emf across........ is going to produce an emf across the brushes along the same principles that we discussed for the DC generator.

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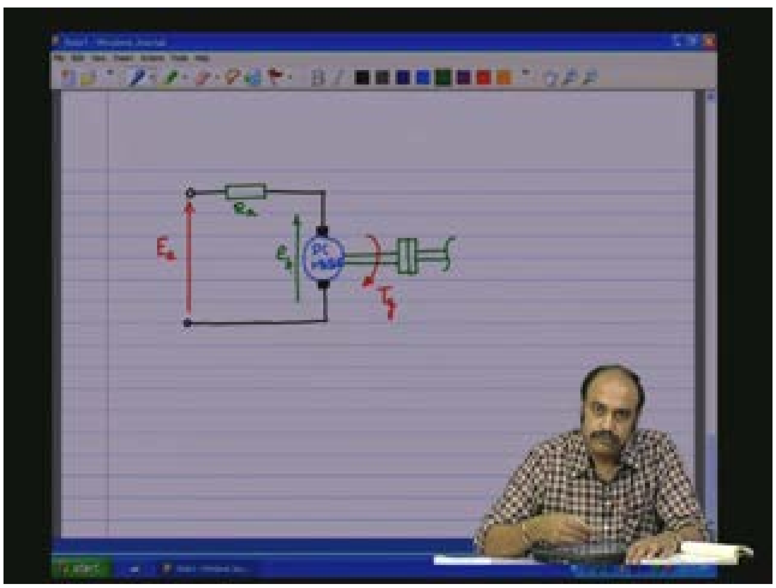

Now this emf will always be produced irrespective of what causes the rotation of the shaft and this emf is called the back emf because this is applied back........ the the voltage is applied to make the torque rotated and this rotating shaft produces a back emf and therefore an emf which is applied back on to the electrical side and it is the difference in this voltage which is going to cause which is going to get applied to the winding resistance and which is going to cause the current flow through this so that will cause a current i a the armature current i a to flow through the armature.

If the speed omega here reduces due to some due to some reasons like loading then this will induce......... because reduction in the speed will induce lesser voltage across the coils because the induced voltage is given by P N phi Z by 60 N is.......... the induced emf is proportional to N the speed so reduction in the speed will cause reduced induced emf or reduced back emf or reduced counter emf; it is also called counter emf in some of the literature and reduction in the emf here means there is greater drop across R a which will force a higher current which will try to which will try to increase the power which is spread to the machine and thereby make it rotate at the higher speed bring it back to a higher speed.

This emf is called the back emf or also called the counter emf. Now i a is given by E a minus E b divided by R a. So E b E b is the instantaneous value let us put the effective value the DC values here because in this case the effective value will be the same E b and we put also E b here. So when we talk about dynamics we need to put in the instantaneous values in that and this can also be replaced with the DC value because we are talking about the steady-state so that becomes i a and this becomes also i a. So i a the armature current is given by E a minus E b by R a; it is an interesting relation, what happens at the start. So let us say you are powering on, the shaft is not rotating, omega is equal to zero so you have powered on. At the moment it has powered on, the shaft has not picked up speed, it is starting from zero omega is equal to 0 and therefore E b or the back emf is 0. So the whole applied voltage is going to come across R a and therefore I a will be E a by R a. R a is a very small quantity because it is only the armature winding resistance so I a will be orders of magnitude higher than the rated armature value of the DC machine of the DC motor.

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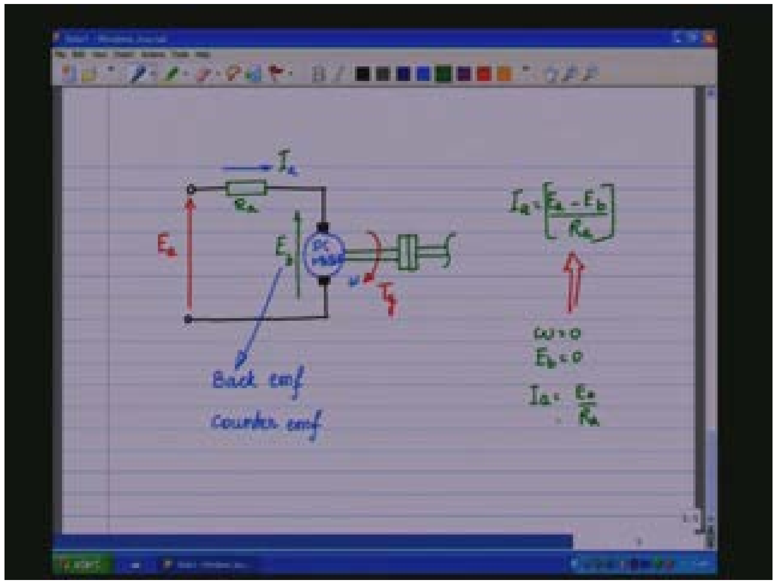

So initially you will see a very huge current search due to the factor there is no back emf and then as the current flow in it produces the torque it starts rotating the shaft starts rotating and the speed finite speed comes in which will induce E b the back emf and this back emf gradually picks up rises as the speed picks up and at no-load when there is no load E b will be a value which is just slightly less than E a and the differential voltage and the differential voltage across R a will be just a few volts and that divided by R a will be the I a. So I a the armature current will have a very high value at start up and gradually reduced to a value which is E a minus E b by R a.

So, to understand this let us let us work out a small example. So let us say the armature has a resistance of 1 ohm, this is R a and it generates that machine generates generates 50 volts when the speed is 500 rpm. What it means is if the machine is used as a generator it would at its brushes generate 50 volts when the speed is 500 rpm.

Now if the armature is connected to a voltage source E a of 150 volts then what is the starting current; the starting current, the back emf E b, the back emf when he motor rotates at............ back emf when the motor rotates at 1000 rpm like this. So, using the relationship that we just saw that is we saw that i a is equal to E a minus E b by R a, at starting omega is equal to 0, E b is equal to 0, i a is equal to E a by R a so the starting current the starting current is given by E a by R a which is equal to 150 volts by 1 ohms which is equal to 150 amps this is 150 amps.

Now the back emf at 1000 rpm we have been given that it generates 50 volts when the rpm is 500 and we know that the back emf is proportional to N. Note that E b is equal to P N phi Z by 60, back emf is proportional to N so E b 1 so therefore we can say that E b 1 by E b 2 is equal to N 1 by N 2 and therefore if you have 50 volts by E b 2 which is equal to 500 rpm by 1000 rpm and therefore E b 2 is equal to 100 volts.

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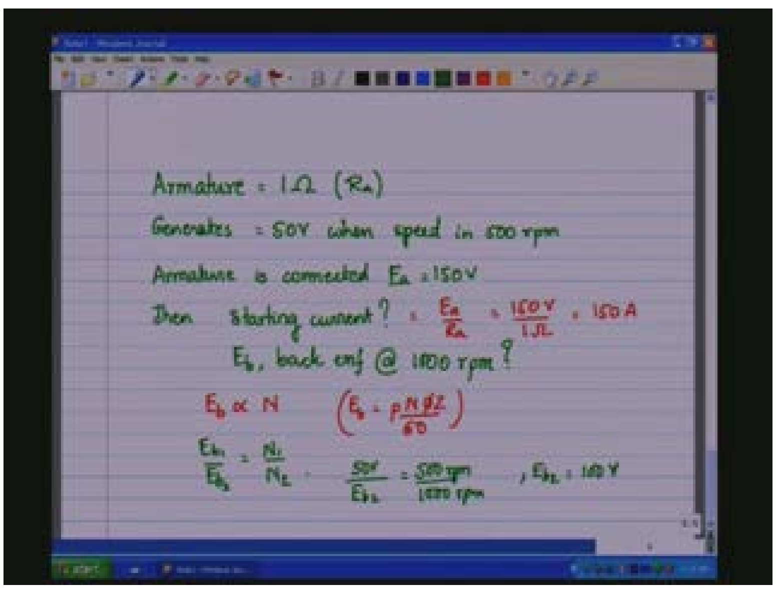

So now the back emf at 1000 rpm will be 100 volts. Now what will be the current at this situation when it is 1000 rpm?

The current now would be, i a would be E a minus E b by R a which is 150 volts minus 100 volts divided by 1 ohm which is equal to 50 amps at 1000 rpm is it not?

So this is how one goes about calculating the armature currents with special importance to be given to the starting currents because the ....starting current gives the devices which are connected along with it should also handle such high starting currents.

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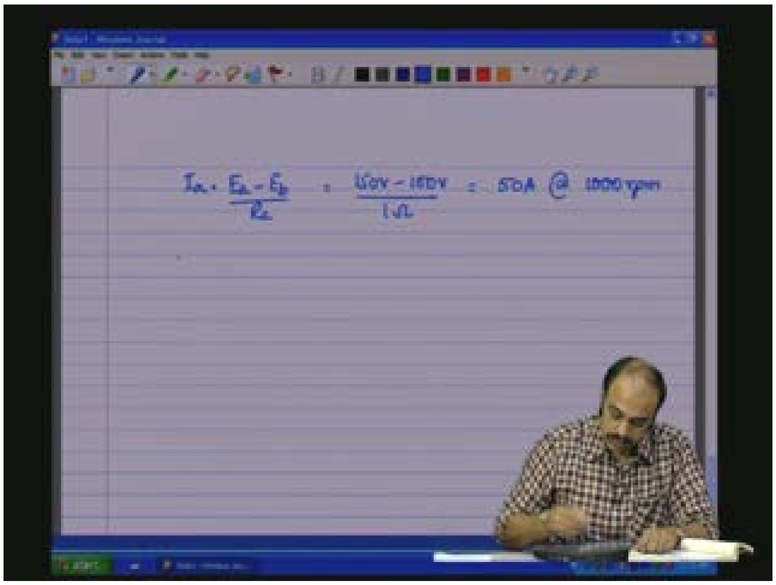

So now we need to know the power and the torque relations relationship. So we know that the back emf E b is given by P N phi Z by 60 volts, this is the back emf. The power drawn from the source........ so let us go back and then copy this picture, we will need this picture to have a better understanding (Refer Slide Time: 35:26) let us copy that, go to this place here and paste, so let us have this picture alongside to have a better idea of what we are doing.

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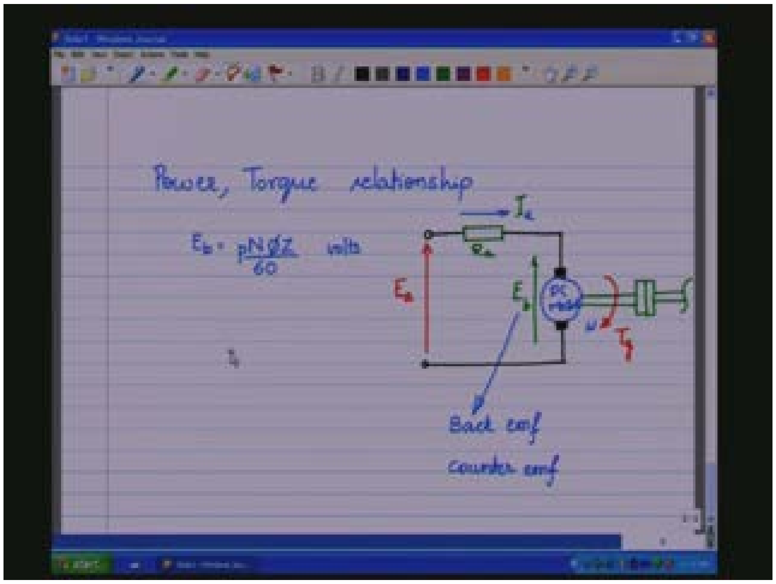

Now the power drawn from the primary P a would be E a into i a so this is the power; this voltage terminal voltage into i a will give the power drawn from the primary source which is electrical in this case across the brush terminals. And out of this E a if you apply Kirchhoff's voltage law here KVL, E a is equal to the voltage drop across this plus voltage drop across the motor which is the induced emf or the back emf. So this is equal to E b plus I a R a into I a or this is equal to E b into I a plus I a square R a. So the power that is drawn from the source the power that is drawn from the source here a portion of it I a square R a goes off as heat in the resistance armature winding resistance R a and the remaining portion E b I a is the one that goes into the machine. In fact this is the power E b I a is the power that is responsible for generating the mechanical power which goes in as the mechanical power.

So the mechanical power actually P m will be equal to E b into I a this is the mechanical power that goes into the armature. P m is nothing but the mechanical power developed by the motor, E b is the induced emf and I is the armature I a is the armature current.

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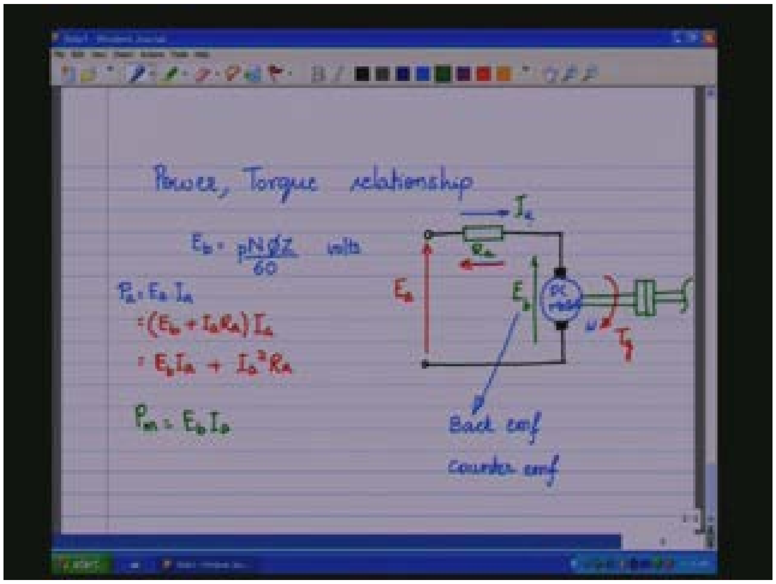

Now T, omega on the mechanical side if you look at the mechanical side T and omega are the potential and kinetic variables. And we know that the power is multiplication of the potential and kinetic variables. So therefore the power on the mechanical side is also T T g that is the generated torque T g into omega into omega.

Now this is in Newton meter and this is in radian per second. Let us express the rotation in terms of rpm. So this becomes T g into omega is in radians per second to convert it into rpm we are making into 60 by 2pi. sorry we normally have We normally have the speeds in rpm. What we have is N because that is the variable that we are using also here which is in rpm so we want to express everything in terms of N and therefore N which is in rpm to convert it into radians per second we are multiplying it by 2pi by 60. So therefore T g into n into 2pi by 60 will give you the power. So this is in Newton meter, this is in rpm and this whole together is in radian per second.

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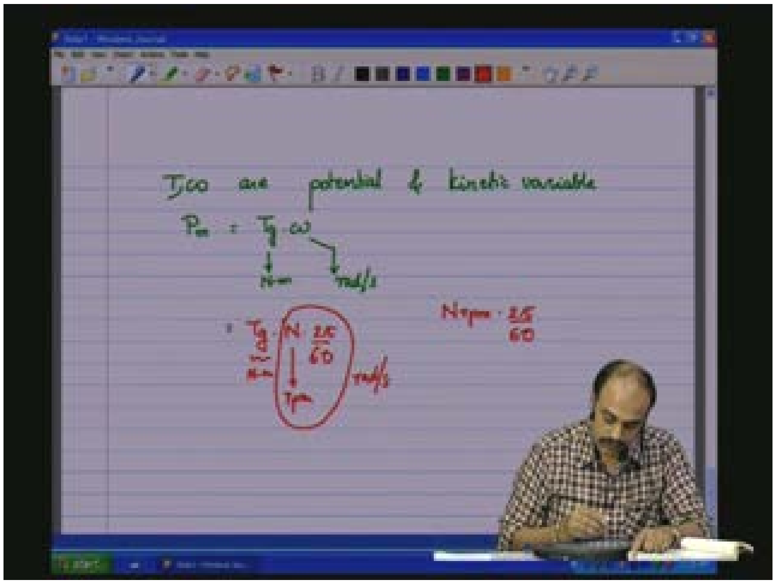

Equating this along with this (Refer Slide Time: 41:16) we have on one side P m which is equal to T g into N into 2pi by 60 which is also equal to E b into I a on the electrical side, this E b is given by P N phi Z I a by 60. So we can remove this we can remove this, remove this so this would result in the generated torque T g which is the number of pole pairs, flux per pole, number of conductors I a by 2pi this is the equation.

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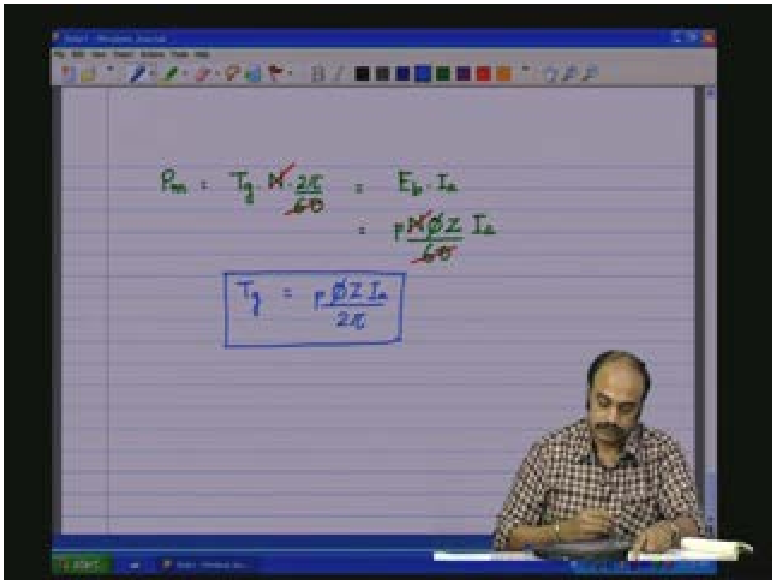

So you see that number of pole pairs pi, Z and 2pi of course which is a constant all these are dependent on the machine parameters and for a given machine T g will be proportional to I a and also of course the flux if the flux is constant otherwise we could say T g is proportional to flux into I a. So this is the field and this is the armature current or the load component load component of the torque. This is the field, flux component of the torque, load component of the torque.

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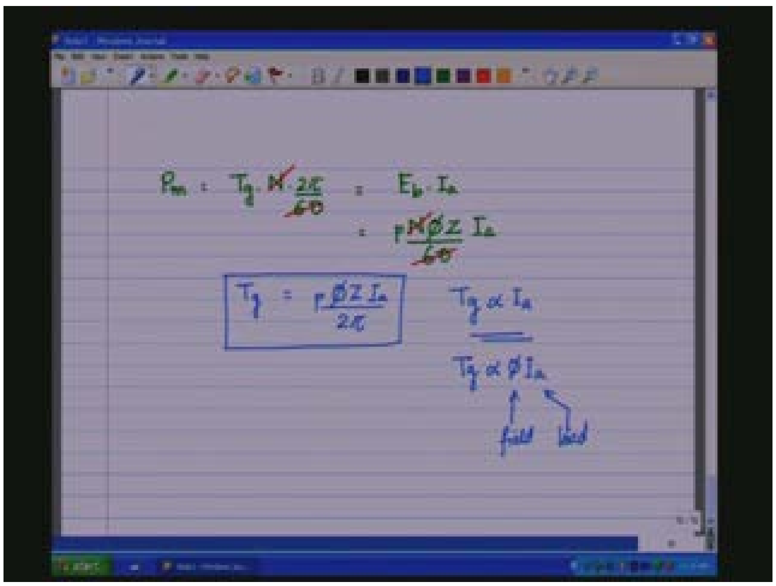

Now let us now take an example to get an idea about the discussion till now about mechanical power and the torque; a machine has the following details. So let me write down the details.

It is a 225 kilo watt machine, 250 volts, 1200 rpm DC motor and some of the details are armature coils armature coils 243 turns per coil – 1, winding type type of winding lap winding; there are many other types of windings also but lap winding is what we discussed in the DC motor generator structure and the same thing is what we are applying, this is also something called as wave winding but for the purpose of discussion we will leave it ourselves to lap winding, then you have the armature slots – 81, the commutator segments segments 243, field poles – 6, diameter of the armature armature 559 mm, axial length of armature 235 mm. So these are the specs of the machine that is given to you.

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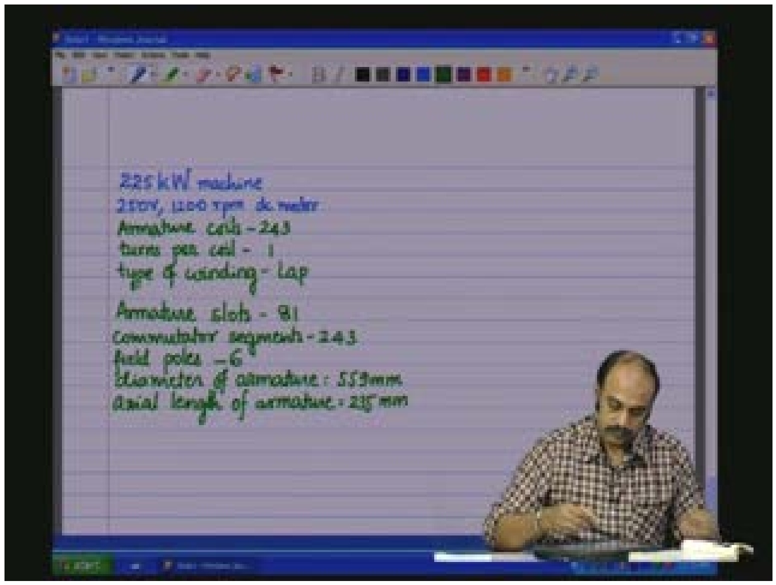

Now here try to find out calculate rated armature current then the number of conductors per slot and flux per pole these are the things that you need to find out. So now let me select this because you may have this on every page that we copy.

So now let us try to make a calculation for this. Now E b is the back emf E b is the back emf and P m is the mechanical power; I a is equal to P m by E b. Now 225 kilo watt machine 225 kilo watt machine means the mechanical power that the armature is capable of delivering is 225 kilo watts which is 225 kilo watts that is 1 2 3 zeros divided by 250 volts, 250 volts is the applied voltage E a so here we are taking E a or E b is approximately equal to............ E b is approximately equal to E a and therefore we say it is 250 volts. So this will result in 900 amps. So that would be the rated armature current that would be the first portion of the problem.

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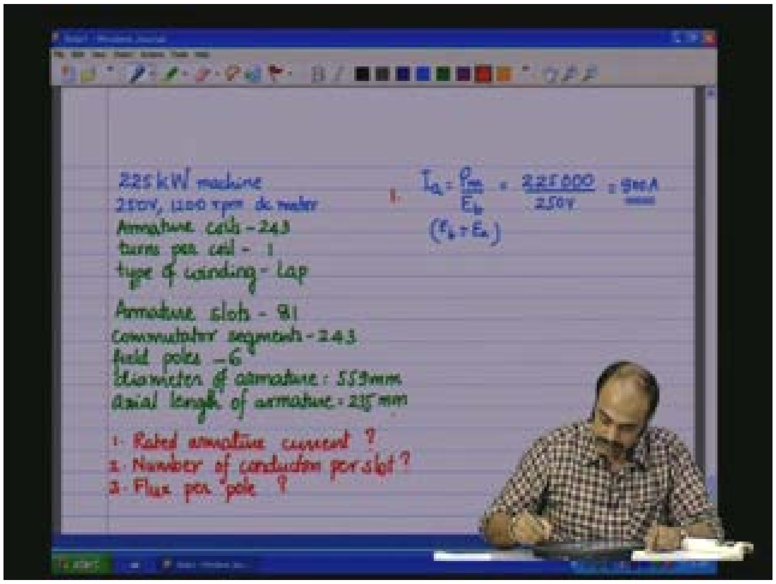

Now let us take the second portion of the problem. So we have it here so that we can refer back to these values.

Now each coil is made up of two conductors as we saw; each coil has two conductors so there are 243 coils and therefore 243 into 2 conductors so you have 243 coils into 2 which is equal to 486 conductors in the armature.

There are armature slots 81 slots so conductors per slot is 486 by 81 which will be 6 conductors per slot and the coils sides per slot is also 6, the coils sides per slot and the which is also equal to coil sides per slot is also equal to 6 because there are six coil sides in the whole coil.

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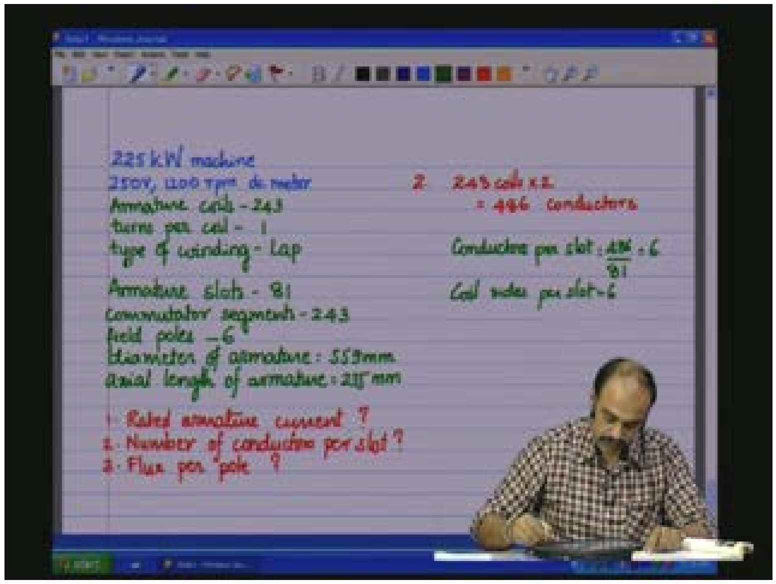

Then the third portion which is let us say the flux per pole that we need to calculate. Let us again paste this (Refer Slide Time: 51:28). So the third aspect is to find the flux per pole. The motor torque T g is given by as we saw in the equation here P by n that is P m is equal to T g N into 2pi by 60 and therefore T g will be equal to P m by N into 60 by 2pi which will give you 225 kilo watts. So 225 kilo watts or 225000 watts divided by 1200 rpm motor 1200 rpm into 60 by 2pi which is equal to 1790 Newton meter.

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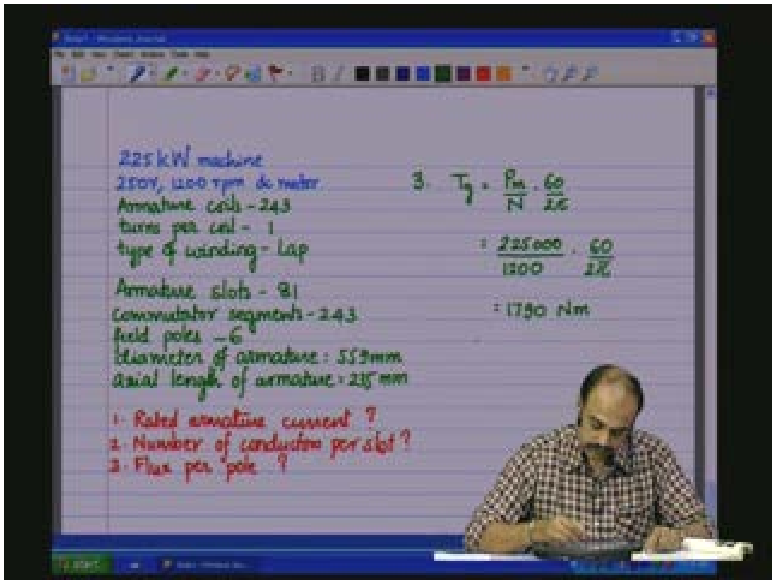

Now, from the other equation that is phi that is we also know that T g is given by P phi Z I a by 2pi. So phi is given by T g into 2pi divided by P Z I a which is equal to 2pi into 1790 Newton meter divided by total number of conductors is 486 and I a is 900 900 and there are six field poles. So, if we calculate it with 1 that is p is equal to 1 this will be 25.7 milliweber, if there are six field poles which means there are three pole pairs and therefore flux so this is flux per pole sorry this is the flux (Refer Slide Time: 55:10) and the flux per pole would be by three pole pairs for p that is this one which is approximately 8 milliweber so this is how one goes about calculating the various parameters of the DC motor must have given you an idea about the usage of the equations.

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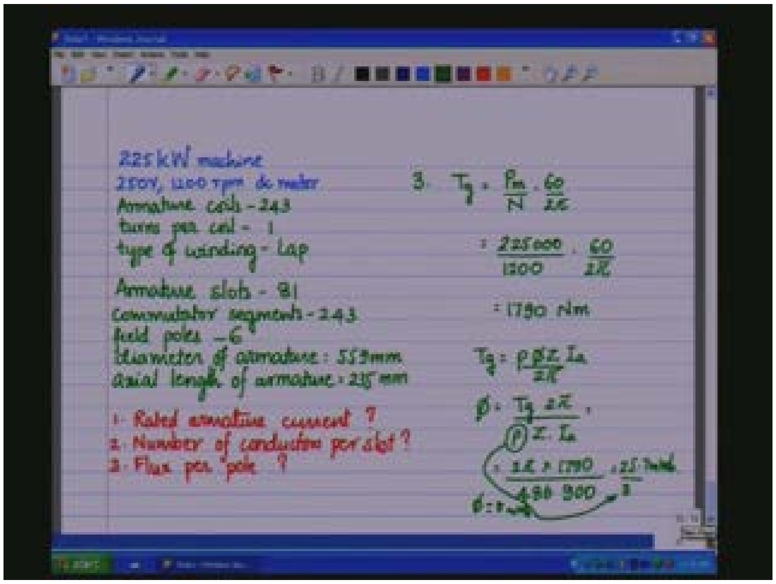

Now the DC motor itself is classified again based on the type of excitation how we give the flux as shunt motor, we have the series motor and we have the compound motor. Each type of this motor classification the name in itself indicates the type of excitation that you would give to generic flux. So, in the shunt motor the field coil is connected in shunt across the applied voltage, the series motor the field coil is connected in series where in the armature current I a flows along the field coil and the compound motor is a combination of the two which has both series and the shunt connection a part of the....... there is a series coil which connects the series with the armature armature coil and a shunt coil which connects across the terminal voltage.

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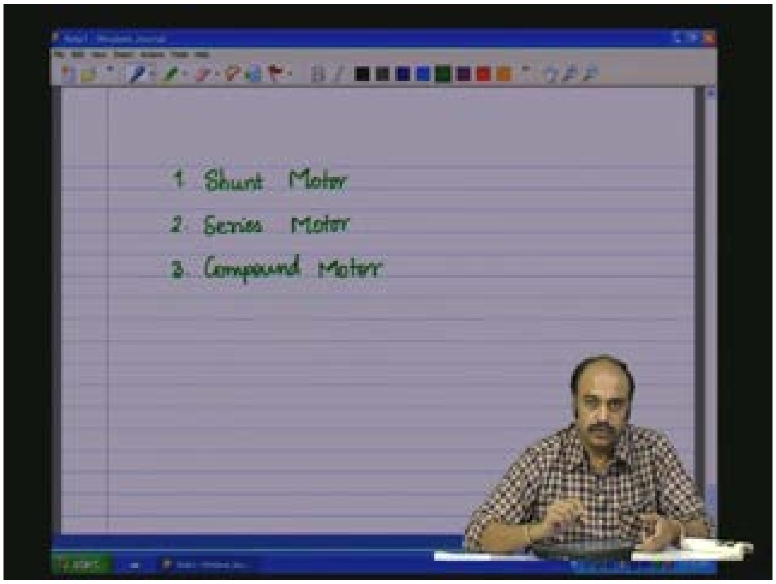

it has There are various advantages, benefits and disadvantages of each of these. We will have a look at this in the next class, in the next session. Each has their own torque speed their own unusual or unique torque speed characteristics which are useful for certain applications and thereby we try to use a particular type of motor for a particular type of application because the torque speed can give an idea whether the particular motor can handle the load.

See the generator torque speed curve and the load torque speed curve will give you an idea which type of motor can be used for a given application. So one point here before we close for the session close the session is about the speed.

So we have here E b which is equal to p N phi Z by 60. Now N of course is given by 60 into E b divided by p phi Z, this would be the speed of rotation. So we use this equation to obtain the speed of rotation on the given motor. Now then E b is the back emf and the back emf is generally not measurable because the armature resistance is distributed in the armature coil. E b is actually p phi Z is actually E a minus I a R a. So this would be the exact speed that one obtains in a DC motor for a given applied voltage and in many cases we can approximate as this drop will be low at higher speeds because at higher speeds I a is higher for a given power sorry for a given power as the speed increases the back emf is going to be higher and therefore I a will be armature current will be lower and R a is already low and therefore I a R a drop at higher speeds will be negligible compared to the applied voltage and therefore this will approximately be equal to 60 applied voltage by p phi Z where Z is the total number of armature conductors, phi is the flux per pole, p is the number of pole pairs and this is the speed equation in rpm in rpm.

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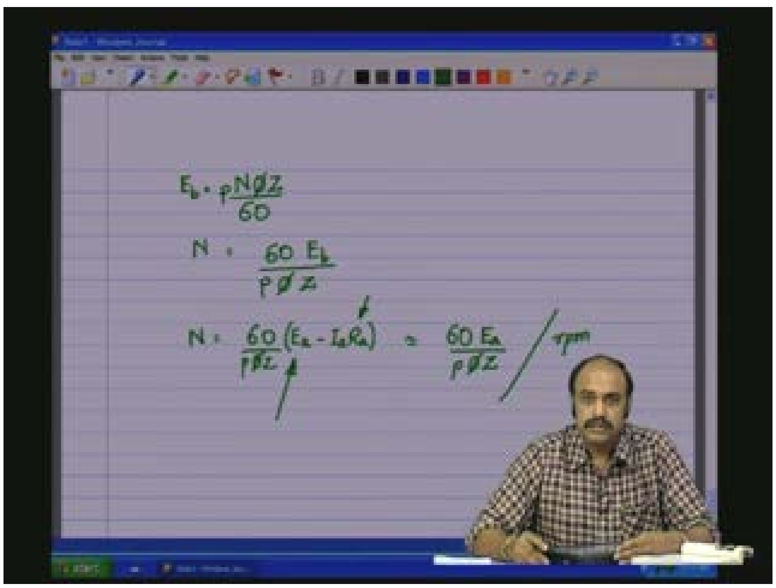

With this let us stop the discussion of DC motor at this point and continue further and consolidate our concepts with new topics on the DC motors in the next session.

Thank you.